Real time image subtraction and “exclusive or” operation using a self-pumped phase conjugate mirror

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Real time “exclusive or” operation with an interferometer using a self-pumped phase conjugate mirror is reported. Also, results of image subtraction and intensity inversion are shown.

In this letter we report a real time “exclusive or” operation obtained with an interferometer using a phase conjugate reflector. Figure 1 illustrates the system. A wave with amplitude $E_{in}$ was divided by beam splitter $BS_1$, whose reflection and transmission coefficients are equal to $r$ and $t$, respectively. Without making any assumptions let $r'$ and $t'$ be the amplitude reflection and transmission coefficients for wave incident from the opposite side of the beam splitter. Each of the two waves then passed through a transparency with amplitude transmittance $T_1$ for beam 1 and $T_2$ for beam 2. The two beams are then reflected by a self-pumped phase conjugate mirror (SPPCM) with phase conjugate reflectivities $R_1$ and $R_2$ for beam 1 and beam 2, respectively. ($R_1$ and $R_2$ are in general not the same.) The phase conjugated beams recombine interferometrically at beam splitter $BS_2$ to form an output field intensity $I_{out}$ at detector $D_1$ given by

$$I_{out} = |r'r^* R_1 T_1|^2 + |t't^* R_2 T_2|^2 |E_{in}|^2,$$

where $E_{in} = |E_{in}|^2$.

From Stokes' principle of the time reversibility of light

$$r'r^* + t^* t = 0$$

so that

$$I_{out} = |R_1 T_1|^2 + |R_2 T_2|^2 |E_{in}|^2.$$

If the two phase conjugate mirrors are identical, i.e.,

$$R_1 = R_2 = R,$$

then

$$I_{out} = |T_1|^2 + |T_2|^2 |E_{in}|^2$$

$$\propto |T_1|^2 + |T_2|^2,$$

where $\propto$ represents the Boolean “exclusive or” operation.

Similarly, the field intensity $I'$ measured by detector $D_2$ is

$$I' \propto |T_1|^2 + |T_2|^2 |E_{in}|^2.$$

We note that the $\pi$ phase shift between the complex fields of the two images, which is the key element of the “exclusive or” operation, is introduced naturally by the time reversibility of light. This is an essential difference between this method and other methods in which the $\pi$ phase shift was artificially provided by a piezoelectric or an electro-optical modulator. This device is only sensitive to intensity differences of the two transparencies and is independent of the phase information of the transparencies or the optical path lengths of the two arms.

In the experiment a single TEM$_{00}$ mode argon laser beam (5145 Å, 50 mW) was expanded and split into two beams by 50% beam splitter $BS_1$. Each beam was then passed through separate transparencies $T_1$ and $T_2$. A lens $L$ ($f = 30$ cm) was used to focus the two expanded beams which were adjusted to overlap completely inside the poled BaTiO$_3$ crystal. The crystal was then aligned to form a self-pumped phase conjugate mirror$^2$ by setting the angles between the beams and the crystal $c$ axis to $\theta_1 = 50^\circ$ and $\theta_2 = 40^\circ$. The two image bearing beams were phase conjugated simultaneously with no cross talk.$^3$ The magnitude of the phase conjugate reflectivities of beam 1 and beam 2 was approximately the same and equal to $25\%$. The phases of the complex phase conjugate reflection coefficients of the two beams are also the same. Since the SPPCM regards the combination of the two input beams as a single complex input wave, and due to the beams’ overlap in the crystal, they are both reflected from the same set of gratings. Another method for obtaining phase locking between the two phase conjugate beams is described in Ref. 4, in which a self-induced oscillation locks the relative phase between the two phase conjugate beams. The phase conjugate reflected image bearing beams were then combined interferometrically at beam splitter $BS_2$. The two transparencies and the detectors were placed close to the beam splitters to reduce diffraction aberration.

The transparencies $T_1$ and $T_2$ used in the experiment are pictures of a semiconductor and a colon, respectively. The phase conjugate images of these two transparencies are shown in Figs. 2(a) and 2(b), respectively. Figure 2(c) is the image detected by $D_1$, which represents the “exclusive or” operation (or, in this case, image subtraction) between the two images, $|T_1|^2 \oplus |T_2|^2$. Figure 2(d) is the image recorded by $D_2$, which represents $|T_1|^2 + |T_2|^2$. Figure 2(e) is proportional to the sum of intensities, $|T_1|^2 + |T_2|^2$, when $|r|^2 = 0.5$. Slight edge enhancement effects were also observed in these figures which are probably due to large object

![FIG. 1. Experimental arrangement to demonstrate the “exclusive or” operation.](image-url)
beam intensities as compared to the weaker pump beam intensities. These results are independent of the optical path lengths of either beam between the BS$_1$ and the crystal.

The response time of the self-pumped phase conjugate mirror obeyed approximately the relation $\tau \approx 10/|I|$, where $I$ is the total intensity of the interaction beams in mW/mm$^2$.

Optical intensity inversion was also observed by simply removing transparency $T_2$ so that the intensity detected by $D_1$ is proportional to $|1 - |T_1|^2|^2$ which result follows from Eq. (3) when we put $|T_2|^2 = 1$. Figures 3(a) and 3(b) are the phase conjugate images of a uniform illumination and a resolution chart, respectively. Figure 3(c) is the intensity inversion image detected by $D_1$. Figure 3(d) is the image intensity observed by detector $D_2$, which is proportional to $|1 + |T_1|^2|^2$.

Intensity inversion by a different method which uses four wave mixing was reported recently by Ochoa et al.\cite{6} In their method the object beam intensity is required to be
FIG. 3. (a) Image of the uniform illumination after phase conjugation. (b) Image of resolution chart after phase conjugation. (c) Intensity inversion of the resolution chart. (d) Intensity addition of the resolution chart and the uniform illumination.

much larger than that of the reference beam in order to ensure that the diffraction efficiency of the index grating is inversely proportional to the object beam intensity.

In summary, we have demonstrated a real time “exclusive or” operation by an interferometer with a phase conjugate mirror. Examples of image subtraction and intensity inversion are also shown.

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