counted for by the instrumental resolution of 0.7 to 0.8 eV. Another part can be attributed to the instrumental angular resolution coupled to the rapidity of the energy dispersion; a reasonable estimate of the angular resolution would be 5° full width at half maximum, implying a further energy spread of ~0.8 eV. It is therefore quite possible that the residual width due to the inherent lifetime of the surface states is consistent with the above expectations. Clearly, experimental work at higher angular and energy resolution would be desirable to resolve this point.

In summary, we have extended the technique of KRIPE to the study of surface states. We have mapped the $E(k)$ relations for an unoccupied surface state on Pd(111)—information unobtainable in any other technique. In general, KRIPE promises to deliver the half of the information on surface states not accessible with use of ARUPS. Some particular questions concerning the energy width and dispersion of the Pd(111) surface state would be worthy of further investigation.

5D. P. Woodruff, N. V. Smith, P. D. Johnson, and W. A. Royer, to be published.

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**Sliding Conductivity of Charge-Density Waves**

Leigh Sneddon

*Physics Department, Princeton University, Princeton, New Jersey 08544, and Bell Laboratories, Murray Hill, New Jersey 07974*

and

M. C. Cross and Daniel S. Fisher

*Bell Laboratories, Murray Hill, New Jersey 07974*

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A classical model of charge-density wave motion at large velocities is used to describe the nonlinear response of the charge-density wave in fields well above the threshold. The charge-density wave is regarded as a charged, deformable medium and its interaction with pinning centers is treated in perturbation theory. The results fit the available high-field data on NbSe$_3$ well. The observed interference effects between a large dc field and an ac field are also explained.

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Peierls and Fröhlich showed how the coupling between conduction electrons and the phonons of an anisotropic metal could lead to a charge-density wave (CDW): a periodic modulation of the electron density and a corresponding distortion of the lattice. Fröhlich also suggested that a CDW could slide and carry electrical current. This phenomenon has now been observed in NbSe$_3$; the CDW is pinned at small electric fields but moves in the presence of fields larger than a threshold depinning field. This results in a nonlinear current-voltage curve, which has to date been analyzed in terms of both a single classical degree of freedom (or "particle") and a Zener-tunneling model. The latter theory involves the quantum mechanical tunneling of macroscopic portions of the CDW, which we believe implausible. Neither theory is consistent with all of the data.

The purpose of this Letter is firstly to present a model which we believe is a more realistic and complete model of a moving CDW than has been given to date. It is classical and includes the internal degrees of freedom of the CDW. Secondly
we calculate its properties, within perturbation theory, at large dc electric fields.

The first result is that the leading terms in the current density, \( j \), for high electric field, \( E \), are
\[
j = \sigma E - CV E
\]
with \( \sigma \) the conductivity in the absence of pinning and \( C \) a constant. This form fits the available data very well. We have also investigated the effects of an alternating current superposed on the dc current. The results explain the observed interference effects. The characteristic field, \( (C/\sigma)^2 \), determines the range of validity of the perturbation theory and is equal, to within numerical factors, to the weak pinning threshold field of Lee and Rice. The voltage oscillations measured with a dc current are, however, not present in the theory. This indicates that either the model, or its leading order perturbative solution, is incomplete, or that finite-size effects are being observed. This issue is the subject of continuing study.

We consider a CDW interacting with impurities, which are treated perturbatively following a method which has been successfully used to study vortex lattice motion in superconducting films. Dimensionality is crucial. The exponent in the second term of (1) is \((d-2)/2\) where \( d \) is the dimensionality of phase space for coherent excitations of the internal degrees of freedom of the CDW. These classical internal degrees of freedom are thus of central importance, the fit of the experiments to Eq. (1) indicating the three-dimensionality of the CDW, which is consistent with x-ray studies. The impurities distort the CDW as it moves over them, resulting, at second order in the pinning potential, in a net retarding force. The important normal modes of the CDW turn out to be those of long wavelength and low frequency, and so macroscopic equations of motion may be used. At large fields, the CDW is moving rapidly over the pinning centers, causing the local distortions of the CDW to be small. The leading order correction is then expected to dominate the nonlinear transport.

Distortions \( u(\vec{r}) \) of the CDW are acted upon by various forces. Firstly, there is an elastic restoring force per unit volume \(-\int D(\vec{r} - \vec{r}')u(\vec{r}')d\vec{r}'\), where \( \vec{r}' \) labels the position of the undistorted

\[
\tilde{u}(\vec{r},t) = \frac{d^2}{dt^2} \int d\vec{r}' \int d\vec{r}'' \left( \rho_\phi \left[ G^{-1}(\vec{r},\omega) - G^{-1}(\vec{r}',\omega) \right] - \rho_\psi \Phi_\phi \Phi_\psi \right) u(\vec{r},t') u(\vec{r}',t''),
\]

where the Fourier-transformed Green's function is given by
\[
G^{-1}(\vec{k},\omega) = -m \omega^2 + D(\vec{k}) + i\omega \lambda.
\]

CDW at time 0 and \( u(\vec{r}) \) is the local distortion in the incommensurate \( z \) direction. (Since commensurability energies are much larger than pinning energies due to dilute impurities, only distortions in the \( z \) direction need be considered.) Secondly, the CDW is acted on by the applied uniform electric field \( E \), which we take to be along the \( z \) axis. In the long-wavelength limit the field couples to the full mobile charge of the CDW given by an effective charge density \( \rho_\phi \). The pinning forces \(-\nabla \Phi(\vec{r})\), which have structure on short length scales, will couple to the spatially dependent CDW charge density \( \rho(\vec{r}) \).

The coupling of the CDW to the other degrees of freedom in the solid can be represented by a phenomenological damping force \(-\lambda \tilde{u}(\vec{r})\). In considering long-wavelength distortions of the CDW, it is necessary to consider the uncondensed electrons which act to screen the long-range Coulomb interactions. In the absence of this screening, the low-frequency phase modes with finite \( q \) components will be shifted to high frequencies and become plasmonlike. However, for \( NbSe_6 \), the uncondensed electrons are sufficiently mobile to screen out the Coulomb forces at the frequencies of interest, and the result of including this effect is an enhanced \( \lambda \) of the CDW. We will ignore for simplicity other scattering between normal conduction electrons and the CDW.

When we combine all the terms, the equation of motion for \( u \) becomes
\[
m \tilde{u}(\vec{r},t) + \int D(\vec{r} - \vec{r}')u(\vec{r}',t)d\vec{r}' + \lambda \tilde{u}(\vec{r},t) - \rho_\phi E(t) = -\rho(\vec{r})\Phi_\phi + \tilde{u}(\vec{r},t),
\]
where \( m \) is the effective mass density of the CDW and the Cartesian subscript on \( \phi \) denotes differentiation.

For a given dc field \( E_0 \), and with a superposed ac field of frequency \( \nu \), the average velocity \( \nu \) of the CDW can be computed in perturbation theory about the weak-pinning or high-velocity limit. We subtract off the CDW displacements in the absence of pinning:
\[
u(\nu) = \nu(t) + (a/v) \sin(\nu) + \bar{u}(\vec{r},t),
\]
where the applied field is
\[
E(t) = E_0 + \left[ (a/2\rho_\phi) (\lambda + \Im \nu) e^{ivt} + c.c. \right].
\]

The CDW distortion may then be written
\[
\tilde{u}(\vec{r},t) = \int \left[ G^{-1}(\vec{r},\omega) - G^{-1}(\vec{r}',\omega) \right] \left( \rho_\phi - \rho_\psi \Phi_\phi \Phi_\psi \right) \tilde{u}(\vec{r}',t') dt',
\]
The solution $\tilde{u}$ can be obtained in perturbation theory with contributions $u_0$, $u_1$, and $u_2$ given by replacing the quantity in square brackets in Eq. (5) by $\rho_\phi E_0 - \lambda v$, $-\rho(\tilde{r})\phi_1(\tilde{r} + \tilde{v} + (\tilde{A}/\nu) \sin t)$, and $-\rho(\tilde{r}) u_1(\tilde{r}, t')\phi_2(\tilde{r} + \tilde{v} + (a/\nu) \sin t)$, respectively. We obtain to second order an expression for the volume-averaged incremental dc velocity,

$$\langle \tilde{u} \rangle = \lambda^{-1} \langle \rho_\phi E_0 - \lambda v \rangle + \sum_{\epsilon, n} \langle \rho_\phi \rangle \langle \tilde{v} \rangle \langle \phi_2(\tilde{r} + \tilde{v} + (a/\nu) \sin t) \rangle J_n^2(q, a/\nu) \text{Im} G(q - g, q_s v - n v) \rangle,$$

where $\rho_\phi$ is the structure factor of the CDW density $\rho(\tilde{r})$, $\Lambda$ is the impurity potential correlation function

$$\Lambda(q) = \int d^3r e^{-i\tilde{r} \cdot \tilde{r}} \frac{1}{V} \int d^3s \phi(\tilde{s} + \tilde{r}) \phi(\tilde{s}),$$

and $J_n$ is the Bessel function of order $n$. The condition that $\langle \tilde{u} \rangle$ must vanish then leads to an expression for $v$, the self-consistent dc velocity, in terms of the field $E_0$.

The evaluation of the integral in Eq. (7) can be simplified by using bounds on the characteristic frequencies obtained from frequency-dependent Ohmic (i.e., small field) conductivity measurements. If we consider replacing the right-hand side of Eq. (2) by $-K_0 v$ where $K_0$ is an average stiffness due to pinning, then we obtain for the ac Ohmic conductivity the simple harmonic-oscillator result

$$\sigma(\omega) = i \omega \rho_\phi^2 (-m \omega^2 + i \omega \lambda + K_0)^{-1/2} + \sigma_n,$$

where for small $k$ we have assumed $D(k) = K_0 k^2 + K_0^2 k^2 = K_0 k^2$. The velocity in the absence of pinning, $v_0$, is given by $v_0 = \rho_\phi E_0 / \lambda$ and we thus see that Eq. (10) has the form of Eq. (1) with $C$ second order in the pinning. The nontrivial term in Eq. (10) is the first term in an expansion, valid for high velocities, in powers of $\lambda E_0^{-1/2}$.

To test the form, Eq. (1), for the non-Ohmic transport, the data of Fleming and Grimes are plotted in Fig. 1 as $j/E$ vs $1/\sqrt{E}$. The high-field data are very well fitted by a straight line, as required by Eq. (1).

It is instructive to compare our result with the predictions of two other theories which yield results for the high-field dc response. Grüner, Zawadowski, and Chalikin model the CDW by a single particle moving in a viscous medium in a periodic potential. Expanding their results about the high-field limit, one obtains

$$j = \sigma E + O(1/E),$$

with this form the tangent to the $j$ vs $E$ curve at large $E$ intersects the $j = 0$ axis at the origin.

$$\sigma_n = \text{the conductivity of the normal electrons. Comparison of the parameters with experiment yields the bounds } \lambda/m > 10^{10} \text{ Hz, } \sigma_n > 10^{15} \text{ Hz, and } (\rho_\phi^2/m)^{1/2} > 10^{12} \text{ Hz. The characteristic frequency } \omega_n \text{ which enters the argument of the Green's function is believed to be in the range } 5 \times 10^9 \text{ to } 10^{10} \text{ Hz, and we may have to take it to be small in comparison to the other characteristic frequencies.}$$

The dominant contribution to the integral in Eq. (7) comes from long-wavelength, dissipative modes, $\omega_n(k) = -i\lambda^{-1} D(k) \text{ with } k = q - g < 0$, for which we may neglect the inertial term in Eq. (6). Since the frequency $g_s v$ is small, the integral is dominated by small $k$ and is given by replacing $q$ with $g$ everywhere except where it occurs as $q - g$, or $q_s v - n v$ when this is small. To obtain the dc current in the absence of an ac field, we set $a = 0$ in Eq. (7), whence only the $n = 0$ term appears in the sum over $n$ and we obtain

$$j = (\sigma_n + \rho_\phi^2/\lambda) E_0 - (4\sqrt{2\lambda})^{-1} (K_0 K_f K_s)^{-1/2} (\rho_\phi^2/\sqrt{\lambda}) \sum_n \rho_\phi^n |g_s|^2 \Lambda (q_s) |g_s v_0|^{1/2},$$

where $\sigma_n$ is the conductivity of the normal electrons. Comparison of the parameters with experiment yields the bounds $\lambda/m > 10^{10}$ Hz, $\sigma_n > 10^{15}$ Hz, and $(\rho_\phi^2/m)^{1/2} > 10^{12}$ Hz. The characteristic frequency $\omega_n$ which enters the argument of the Green's function is believed to be in the range $5 \times 10^9$ to $10^{10}$ Hz, and we may have to take it to be small in comparison to the other characteristic frequencies.

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$$j = \sigma E + O(1/E),$$

where $j_0$ is independent of $E$. However, it also disagrees with Eq. (1). If Eq. (12) were plotted in Fig. 1, the resulting curve would be a concave downward curve. While data at higher fields are required to determine which theory, if any, is correct, the result of the present theory fits the available dc data very well, and rather better than the tunneling theory.\(^6\)

In addition to the high-field dc response, we can compute from Eq. (7) the dc response to an ac field added to a large constant dc field by retaining
as the fundamental frequency of the voltage fluctuations in the dc experiments, with $g_{z0}$ the $z$ component of the smallest reciprocal-lattice vector. The steps corresponding to $g_{x0}$ and various $n$ have been observed experimentally by Monceau, Richard, and Bernard although at this stage a detailed analysis of the amplitudes and step shapes has not been carried out.

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7For a review, see G. Grüner, A. Zettl, W. G. Clark, and J. Bardeen, to be published.


9Schmid and Hauger, Ref. 5.

10A. I. Larkin and Yu. N. Ovchinnikov Zh. Eksp. Teor. Fiz. 55, 1704 (1973), and 65, 1915 (1973) [Sov. Phys. JETP 32, 854 (1974), and 31, 960 (1975)] consider a moving lattice in a three-dimensional type-II superconductor and also obtain a velocity correction proportional to $(v/\mu)^{1/2}$.


12The vectors $\hat{u}$, $\hat{v}$ point in the $z$ direction and their amplitudes are denoted by $u$, $v$, respectively.


14We have assumed that one of the principal axes of $D(E)$ for small $b$ lies along the $z$ direction. This may not be the case for monoclinic NbSe$_3$, but the errors introduced by this assumption will not change the form of the results, but only the detailed step shape in Eq. (13).