quiet places. This is true, but I hardly think that the relative intensity of the voice sounds and the noise is such as to enable the person afflicted with middle ear deafness to hear better because of the increased voice energy. Perhaps there are two factors involved which now can be better understood. In the first place when people talk louder than usual, their voices become high pitched and the overtones become much more prominent. In the second place the acuity of hearing for these higher pitch sounds is greater than for those of lower pitch. Through long experience with deaf people I always try to make them understand first by raising the pitch of my voice and at the same time talking louder. I am thus able to excite more of the flexural type of vibration of the drum and less of the piston type for which we are considering that there is a considerable restraining action. Probably the louder talking has less effect than the increased pitch, because usually women's voices are more easily understood than men's by people afflicted with middle ear deafness. The reason, I think, is because of differences pointed out in this discussion. In any case, the effect we are discussing is probably not large.

Another explanation of paracusis advanced by some writers is that loud noises cause the drum and ossicles to vibrate with more intensity and under this condition they become freer and can then become more sensitive for speech sounds. I think probably this is hardly the case for even more or less normal ears would observe the effect. On the whole, I should feel inclined to give greater weight to the explanation advanced in the preceding paragraph, particularly in view of the fact that it has as its basis experimental observations which seem to fit in satisfactorily with the theory which I have advanced.

[1] (*a*) Seashore, Dr. Dean and Bunch, Iowa City, Iowa; (*b*) Sabine and Kranz, Geneva, Ill.; (*c*) Dr. Fowler, Fletcher and Wegel, New York City; (*d*) Dr. Wilson and Minton, Chicago, Ill.; (*e*) Dr. Rae and Minton, New York City.

A NEW CRYSTAL FOR WAVE-LENGTH MEASUREMENTS OF SOFT X-RAYS

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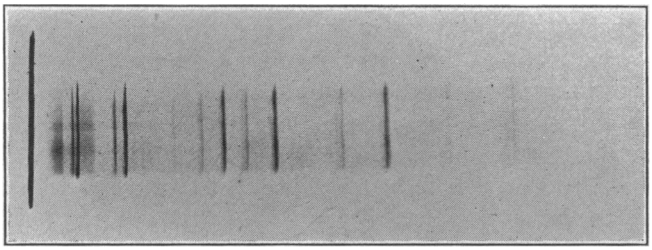
During an investigation of the crystal structures of hematite and corundum[1] spectral photographs of the K-radiation of molybdenum reflected from the plane (00.1) of the hexagonal crystal β-alumina,[2] Al_2O_3, were made. One such photograph is shown in the accompanying reproduction.

Upon substituting in the equation $n\lambda = 2d \sin \theta$ the wave-lengths λ of the X-rays used and the corresponding angles of reflection θ, the grating-constant d was found to be about 11.2 Å, assuming that n is unity for the reflection at the smallest angle. This result shows that this substance has an unusually large grating-constant; and, since this is accompanied by strong reflection, the crystal should prove of great value in the measurement of wave-lengths of soft X-rays.



In order to determine accurately the value of the grating-constant, photographs were taken of the lines $L\alpha_1$ of silver (first order) and $K\alpha_1$ of copper (second order), using a vacuum spectrograph similar to that of Siegbahn,[3] and using his double exposure method.[4] As source for the X-radiation hot sparks[5] were used. The following data were obtained:

LINE	WAVE-LENGTH	ORDER OF REFLECTION	ANGLE OF REFLECTION	GRATING-CONSTANT
Cu $K\alpha_1$	1.53730 Å	2	7° 51′ 41″	11.240 Å
Ag $L\alpha_1$	4.14564 Å	1	10° 38′ 29″	11.225 Å

Several photographs were taken, and the values of d calculated from them varied by less than 0.002 Å from the averaged values given above. The dependence of the grating-constant on the angle of reflection seems to be a real effect. This involves deviation from the Bragg equation, but is in agreement with previous investigations.[6]

The following relative intensities of reflection in successive orders were observed: first and second, very strong; third, weak; fourth and fifth, strong; sixth, very weak; seventh, medium strong; eighth, very weak; ninth, medium weak; tenth, medium.

The crystals previously used with soft X-rays have smaller grating-constants than this crystal; that for (100) of sucrose being 10.57 Å and for the cleavage face of mica 10.1 Å. Moreover, these crystals are soft, and the reflections they give in small orders are not strong. These disadvantages do not exist in the case of β-alumina. The crystals of β-alumina are in the form of basal plates, some of which are of perfect internal structure and give very sharp reflections. They can be obtained with faces large enough for use in spectrographic work; the specimen[7] used in obtaining the values reported in this note possessed a good face about 1 cm.

square. The hardness of the crystal is equal to that of corundum (9 on Mohs' scale), so that specimens are not easily scratched or deformed. The theoretical maximum wave-length of X-rays with which the crystal can be used is $2d$, or about 22.5 Å, corresponding to an angle of reflection of 90°; the wave-length corresponding to an angle of reflection of 60° is about 19.5 Å.

Previous attempts to find crystals with large grating-constants have been restricted to substances with complicated chemical formulas; such a procedure appears unnecessary in light of the surprising discovery of the unusually large constant possessed by crystals of the simple inorganic substance β-alumina.

[1] Linus Pauling and S. B. Hendricks, *J. Amer. Chem. Soc.*, **47,** 781 (1925).

[2] G. A. Rankine and H. E. Merwin, *J. Amer. Chem. Soc.*, **38,** 568 (1916).

[3] M. Siegbahn, "Spektroskopie der Röntgenstrahlen," Julius Springer, Berlin, 1924, p. 70.

[4] Ref. 3, p. 60.

[5] Albert Björkeson, *Proc. Nat. Acad.* This number, p. 413.

[6] Ref. 3, p. 24.

[7] We wish to thank Dr. A. A. Klein of the Norton Company, Worcester, Mass., for this speciman; also Dr. R. G. Dickinson of this Institute, and Dr. R. W. G. Wyckoff of the Geophysical Laboratory, for supplying us with some crystals, from the same source, used in our preliminary work.

ON QUASI-ANALYTIC FUNCTIONS

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A recent note of G. Julia in the *Comptes Rendes de l'Academie des Sciences in Paris* (**180,** p. 720, March, 1925) may have most remarkable consequences in the coming years, as appears if we pay attention to the general properties of quasi-analytic functions and their mutual relations.

As yet, calculations and special studies in Calculus have, above all, concerned analytic functions. Where does this quite special importance come from? It is clear that it is due to two main circumstances, which are:

(1) That the simplest known functions—beginning with $y=f(x)=x$ itself—those which we want it the most elementary and usual calculations, are analytic:

(2) That they generate each other by the main operations of Calculus: algebraic operation, substitution of functions in functions, differentiation, integration, and so on—i.e., carrying out such operations on analytic functions always leads to functions which are again analytic.

Quasi-analytic functions, or, more exactly, definite classes of quasi-analytic functions, also admit of the latter property. Therefore, if amongst