Validity of Chiral Perturbation Theory for K^0-$\bar{K}^0$ Mixing

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Chiral perturbation theory relates the $|\Delta S|=2$ matrix element $\langle \bar{K}^0|(\bar{s}d)_{V-A}\times(\bar{s}d)_{V-A}|K^0\rangle$ to the $|\Delta S|=1$ matrix element $\langle \pi^+\pi^0|(\bar{s}d)_{V-A}(\bar{u}u)_{V-A}+(\bar{s}u)_{V-A}\times(\bar{u}d)_{V-A}-(\bar{s}d)_{V-A}(\bar{d}d)_{V-A}|K^+\rangle$. The latter matrix element is measured in $K^+\to\pi^+\pi^0$ decay and the former matrix element is relevant for the predictions that the standard model makes for CP nonconservation in K^0-$\bar{K}^0$ mixing. In this paper the corrections of order $m_K^4 \ln m_K^2$ to lowest-order chiral perturbation theory (i.e., order m_K^2) for these matrix elements are computed. The correction is large for the $|\Delta S|=2$ matrix element indicating a breakdown of chiral perturbation theory.

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In the standard six-quark model for strong, weak, and electromagnetic interactions the W bosons couple to the weak current

$$J_\mu^{(+)}=\frac{g_2}{2\sqrt{2}}(\bar{u},\bar{c},\bar{t})\gamma_\mu(1-\gamma_5)U\begin{pmatrix}d\\s\\b\end{pmatrix}. \tag{1}$$

Here U is a 3×3 unitary matrix that arises because the charge-$(+\frac{2}{3})$- and charge-$(-\frac{1}{3})$-quark mass matrices are not diagonalized by the same unitary transformation. If we adjust the phases of the quark fields U can be put in the form

$$U=\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3\\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta}\\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta}\end{bmatrix}, \tag{2}$$

where $c_i\equiv\cos\theta_i$, $s_i\equiv\sin\theta_i$, $i\in\{1,2,3\}$. The angles θ_1, θ_2, and θ_3 are chosen to lie in the first quadrant. In general the phase δ cannot be removed from the matrix U by a redefinition of quark fields and it is a source of CP nonconservation.[1] In the standard model the only other source of CP nonconservation is the QCD vacuum angle $\bar{\theta}$. However, the stringent upper limit on the electric dipole moment of the neutron implies that $\bar{\theta}$ is too small to be responsible for the CP nonconservation observed in the neutral kaon system.[2]

Experimental information on neutron β decay and semileptonic hyperon decays gives[3] (for small s_3)

$$s_1\simeq 0.22. \tag{3}$$

Experimental information on semileptonic B-meson decays gives[4] (for small s_2 and s_3)

$$s_2^2+s_3^2+2s_2s_3c_\delta\simeq 3\times10^{-3}(10^{-12}\text{ s})/\tau_B, \tag{4a}$$

$$s_3^2\lesssim 1\times10^{-3}(10^{-12}\text{ s})/\tau_B. \tag{4b}$$

Recent measurements of the B-meson lifetime[5] imply that τ_B is about 10^{-12} s, and so the weak mixing angles θ_2 and θ_3 are very small.

The CP nonconservation observed in kaon decays is consistent with it arising from second-order weak (i.e., order G_F^2) K^0-$\bar{K}^0$ mixing. CP nonconservation in K^0-$\bar{K}^0$ mixing is characterized by the parameter

$$\epsilon\simeq(1/\sqrt{2})e^{i\pi/4}(\mathrm{Im}M_{12})/(m_{K_S}-m_{K_L}), \tag{5}$$

which measures the deviation of the K_S and K_L states from the CP eigenstates $(1/\sqrt{2})(K^0-\bar{K}^0)$ and $(1/\sqrt{2})(K^0+\bar{K}^0)$, respectively. In Eq. (5)

$$M_{12}\simeq\langle\bar{K}^0|H_{\text{eff}}^{|\Delta S|=2}|K^0\rangle/\langle K^0|K^0\rangle. \tag{6}$$

Experimentally $|\epsilon|\simeq 2.3\times10^{-3}$.

In order to compare the experimental value of ϵ with the prediction of the standard model one needs to know the effective Hamiltonian for K^0-$\bar{K}^0$ mixing and the matrix element of this Hamiltonian between K^0 and $\bar{K}^0$ states. The effective Hamiltonian density for K^0-$\bar{K}^0$ mixing has been computed by successively treating the W boson, t quark, b quark, and c quark as heavy and integrating them out of the theory. The result, to leading nontrivial order in these large masses, is[6]

$$\begin{aligned}H_{\text{eff}}^{|\Delta S|=2}=\frac{G_F^2}{16\pi^2}s_1^2m_c^2[\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha][\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta]\\ \times\{\eta_1c_2^2(c_1c_2c_3-s_2s_3e^{-i\delta})^2+\eta_2s_2^2(c_1s_2c_3+c_2s_3e^{-i\delta})^2(m_t/m_c)^2\\ +2\eta_3s_2c_2(c_1c_2c_3-s_2s_3e^{-i\delta})(c_1s_2c_3+c_2s_3e^{-i\delta})\ln(m_t^2/m_c^2)\}+\text{H.c.}\end{aligned} \tag{7}$$

Here η_1, η_2, and η_3 are QCD correction factors. They are roughly independent of the top-quark mass and have the values $\eta_1 \simeq 0.7$, $\eta_2 \simeq 0.6$, and $\eta_3 \simeq 0.4$ [with use of $m_c = 1.5$ GeV, $m_b = 4.8$ GeV, $M_W \simeq 80$ GeV, $\Lambda_{\rm QCD} \simeq 0.1$ GeV, and $\alpha_s(\mu^2) = 1$].

In Eq. (7) long-distance contributions to the effective Hamiltonian have been neglected since they do not contain a factor of the square of a heavy-quark mass. It is likely that long-distance contributions dominate $\mathrm{Re}M_{12}$. Long-distance contributions to ϵ involve CP nonconservation in $|\Delta S| = 1$ amplitudes and are expected to be of order $20\epsilon'$. In anticipation of an improvement in the experimental limit[7] on ϵ'/ϵ, long-distance contributions to ϵ will be ignored. Then, from Eqs. (5)–(7) it follows that

$$\epsilon \simeq -\frac{s_1^2 B G_F^2 f_\pi m_K^2 m_c^2 s_2 s_3 s_\delta}{16\sqrt{2}\pi^2 (m_{K_S} - m_{K_L})}\left[-\eta_1 + \eta_3 \ln\left(\frac{m_t^2}{m_c^2}\right) + \eta_2\left(\frac{m_2^2}{m_c^2}\right)(s_2^2 + s_2 s_3 c_\delta)\right] e^{i\pi/4}, \tag{8}$$

to leading nontrivial order in small angles. Here f_π is the pion decay constant (≈ 130 MeV) and the dimensionless parameter B is defined by

$$\langle \bar{K}^0 | O' | K^0 \rangle = B f_\pi m_K^3, \tag{9}$$

where

$$O' = [\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha][\bar{s}_\beta \gamma^\mu (1-\gamma_5) d_\beta]. \tag{10a}$$

Under the chiral-symmetry group $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ the operator O' transforms as $(\mathbf{27}_L, \mathbf{1}_R)$. Chiral perturbation theory relates the matrix element of O' to the $K^+ \to \pi^+\pi^0$ decay amplitude.[8] The operator

$$O = [\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha][\bar{u}_\beta \gamma^\mu (1-\gamma_5) u_\beta] + [\bar{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha][\bar{u}_\beta \gamma^\mu (1-\gamma_5) d_\beta] - [\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha][\bar{d}_\beta \gamma^\mu (1-\gamma_5) d_\beta] \tag{10b}$$

is in the same representation of $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ as O'. The effective Hamiltonian density for $|\Delta S| = 1$, $|\Delta I| = \frac{3}{2}$ weak nonleptonic kaon decays is

$$H_{\rm eff} = -(G_F/2\sqrt{2}) s_1 c_1 c_3 C O. \tag{11}$$

Here C is a factor that takes into account strong-interaction corrections. With the strong-interaction parameters used previously[9] $C \simeq 0.4$. To leading order in derivatives and quark masses there is a unique operator involving the pseudo-Goldstone-boson fields that transforms as $(\mathbf{27}_L, \mathbf{1}_R)$.

The pseudo-Goldstone-boson fields are incorporated in a 3×3 special unitary matrix

$$\Sigma = \exp(2iM/f), \tag{12}$$

where

$$M = \begin{bmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -(\frac{2}{3})^{1/2}\eta \end{bmatrix}. \tag{13}$$

Under chiral $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ Σ transforms as

$$\Sigma \to L \Sigma R^\dagger. \tag{14}$$

Thus the leading operator that transforms as $(\mathbf{27}_L, \mathbf{1}_R)$ is

$$\alpha T^{ij}_{kl} (\Sigma \partial_\mu \Sigma^\dagger)^k_i (\Sigma \partial^\mu \Sigma^\dagger)^l_j, \tag{15}$$

where T^{ij}_{kl} is a traceless symmetric tensor and α is a constant. To leading order in small quark masses, matrix elements of O' and O between pseudo Goldstone bosons may be replaced by matrix elements of the above operator, where T^{ij}_{kl} has nonzero components

$$T^{33}_{22} = 1 \tag{16}$$

for O' and nonzero components

$$T^{13}_{12} = T^{31}_{21} = T^{31}_{12} = T^{13}_{21} = \tfrac{1}{2},$$
$$T^{23}_{22} = T^{32}_{22} = -\tfrac{1}{2}, \tag{17}$$

for O.

Expanding the operator (15) gives

$$\langle \bar{K}^0 | O' | K^0 \rangle = -8\alpha m_K^2/f^2, \tag{18}$$

$$\langle \pi^+\pi^0 | O | K^+ \rangle = -(12ia/\sqrt{2}f^3) m_K^2. \tag{19}$$

Here the matrix elements conserve four-momentum and the up- and down-quark masses have been neglected.

To leading order in chiral perturbation theory the Lagrangian density for the strong interactions of the Goldstone bosons is[10] (with neglect of up- and down-quark masses)

$$L = \frac{f^2}{8}\mathrm{Tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \frac{m_K^2}{4} f^2 (\Sigma_3^3 + \Sigma_3^{3\dagger}). \tag{20}$$

If we apply the Noether procedure to the above Lagrangian, it follows from the transformation law in Eq. (14) that the left-handed current is given by

$$L_\mu^\alpha = -(if^2/4)\mathrm{Tr}(\Sigma^\dagger T^\alpha \partial_\mu \Sigma), \tag{21}$$

where T^α is an SU(3) generator. Evaluation of the matrix element of L_μ^α between a pion and the vacuum gives $f = f_\pi$ to leading order in chiral perturbation theory. Thus

$$B = -8\alpha/f^3 m_K, \tag{22}$$

and the invariant matrix element for $K^+ \to \pi^+\pi^0$ decay is

$$M(K^+ \to \pi^+\pi^0) \simeq -(3iG_F/8)Bs_1c_1c_3Cm_K^3. \tag{23}$$

The measured $K^+ \to \pi^+\pi^0$ width implies from Eq. (23) that

$$|B| \simeq 0.4. \tag{24}$$

In this Letter the leading corrections to Eqs. (18) and (19) are evaluated. These corrections are of order $m_K^4 \ln m_K^2$ and arise from a one-loop evaluation[11] of the matrix elements of the operator in Eq. (15). The "large" logarithm causes these terms to dominate over those of order m_K^4. The m_K^4 corrections are not computable since they can arise from operators with four derivatives.

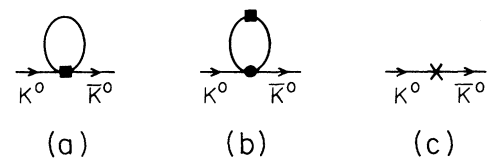

FIG. 1. Feynman diagrams for the one-loop contributions to the K^0-$\bar{K}^0$ matrix element of O'.

For the K^0-$\bar{K}^0$ matrix element of O' the graphs which contribute are shown in Fig. 1. Figure 1(c) is a counterterm that arises from one-loop field renormalization. In Figs. 1(a) and 1(b) the solid square is a vertex from Eq. (15) while the solid circle is a strong-interaction vertex from Eq. (20). With inclusion of pieces of order $m_K^4 \ln m_K^2$ from Fig. 1,

$$\langle \bar{K}^0 | O' | K^0 \rangle = -\frac{8\alpha m_K^2}{f^2}\left\{1 + \frac{m_K^2}{(4\pi f)^2}\left[-\frac{26}{3} - \frac{14}{3} + \frac{5}{3}\right]\ln\left(\frac{m_K^2}{\mu^2}\right)\right\}. \tag{25}$$

In Eq. (25) the three terms in the square brackets are the contributions of Figs. 1(a)–1(c), respectively. For Fig. 1(a) there are nonzero contributions from eta and charged-kaon loops. These are in the ratio 11:2.

For the $K^+ \to \pi^+\pi^0$ matrix element of O the graphs which contribute are shown in Fig. 2. Again, the solid square is a vertex from Eq. (15) while the solid circle is a vertex from Eq. (20). Figure 2(e) is a counterterm that arises from one-loop field renormalization of the pseudo Goldstone bosons. With inclusion of pieces of order $m_K^4 \ln m_K^2$ from Fig. 2,

$$\langle \pi^0\pi^+ | O | K^+ \rangle = -\frac{12i\alpha m_K^2}{\sqrt{2}f^3}\left\{1 + \frac{m_K^2}{(4\pi f)^2}\left[-\frac{17}{3} + 1 - \frac{7}{9} + \frac{22}{9} + \frac{3}{2}\right]\ln\left(\frac{m_K^2}{\mu^2}\right)\right\}. \tag{26}$$

Here μ is the subtraction point. The subtraction-point dependence in Eqs. (26) and (25) cancels against the subtraction-point dependence of the contributions of operators with four derivatives. In Eq. (26) the five terms in the square brackets are the contributions from Figs. 2(a)–(e), respectively. Figure 2(a) gets contributions from eta, charged-kaon, and neutral-kaon loops. These are in the ratio 4:7:6. Isospin invariance of the strong interaction implies that Fig. 2(b) is nonzero only for the $\pi^+\pi^0$ loop. Figure 2(c) can have a $K^+\pi^0$, $K^0\pi^+$, or $K^+\eta$ loop. These possibilities contribute in the ratio $(-31):(-18):21$. Figure 2(d) can have a $K^+\pi^-$, $K^0\pi^0$, or $K^0\eta$ loop. These possibilities contribute in the ratio 11:12:21.

There is a one-loop correction to the matrix element of the axial current which gives[12]

$$f = f_\pi[1 + (4\pi f_\pi)^{-2} m_K^2 \ln(m_K^2/\mu^2)]. \tag{27}$$

With use of this, Eqs. (25) and (26) become

$$\langle \bar{K}^0 | O' | K^0 \rangle = -\frac{8\alpha m_K^2}{f_\pi^2}\left\{1 - \frac{41}{3}\frac{m_K^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_K^2}{\mu^2}\right)\right\}, \tag{28}$$

$$\langle \pi^+\pi^0 | O | K^+ \rangle = -\frac{12 i a m_K^2}{\sqrt{2} f_\pi^3}\left\{1 - \frac{9}{2}\frac{m_K^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_K^2}{\mu^2}\right)\right\}. \tag{29}$$

The correction to the K^0-$\bar{K}^0$ matrix element of O' is large, for a subtraction point μ of about 1 GeV, which indicates a breakdown of chiral perturbation theory for this matrix element. The correction to the $K^+ \to \pi^+\pi^0$ matrix element of O is not as large.

The corrected value of B is obtained from

$$M(K^+ \to \pi^+\pi^0) = -\frac{3iG_F}{8} B s_1 c_1 c_3 C m_K^3 \left[1 + \frac{55}{6}\frac{m_K^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_K^2}{\mu^2}\right)\right]. \tag{30}$$

The large difference between Eqs. (30) and (23) indicates that chiral perturbation theory for B has broken down. This has implications for analyses of the parameters of the standard model[4] which use Eq. (8).

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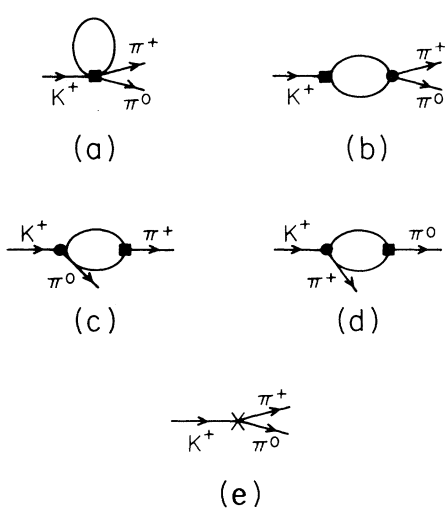

FIG. 2. Feynman diagrams for the one-loop contributions to the $K^+ \to \pi^+\pi^0$ matrix element of O.

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