The Latitude and Longitude Effects in Cosmic Rays Over the United States and Canada at 30,000 Feet*

A. T. BIEHL AND H. V. NEHER
California Institute of Technology, Pasadena, California
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The change with geomagnetic latitude of the ionizing particles at 310 g/cm² atmospheric pressure (30,000 ft.) that could penetrate various thicknesses of absorber was measured by Biehl, Neher, and Roesch1 over a range of latitude of 64° geomagnetic north to the geomagnetic equator along longitude 80°W. These flights show that most of the latitude effect is over at 50° as one proceeds north and this was independent of the absorber used or of whether the total radiation at that altitude was measured or only that near the vertical. On the other hand Swann, Morris and Seymour² find a 10 percent increase for a 10° increase in geomagnetic latitude at 30,000 feet even beyond the “knee” of the curve. Although no mention of an absorber was made, presumably their measurements were made with a lead absorber.

A number of flights having a further bearing on this point have recently been made, covering the latitude range particularly from 40° to 64° geomagnetic north with counter telescopes and ionization chambers, the latter both unshielded and shielded with 10 cm of lead.

In Fig. 1 the measurements made with two of our ionization chambers, which were sealed off 12 years ago, are shown, the one with no lead shield, the other with a 10 cm lead shield. Curve A is that taken in June 1948 along longitude 80°W while B was taken in October, 1949 along 115°W using the same instrument as for A. Curve C was taken with a companion instrument along 115°W but surrounded with 10 cm of lead. B is the result of 3 flights made within a period of five days. Where these flights overlapped they differed by ±2.0 percent, which was well outside the experimental errors and are hence assumed to represent real changes in the primary radiation during this period. Curve B is therefore drawn to represent the mean of these flights.

In Fig. 2 are shown the corresponding results with counter telescopes over the same range of latitudes at the same two longitudes, namely 80° and 115°W. Curve A' and C' have been published previously1 but are here given in greater detail. Curve B' was taken in October, 1949. In addition curve D' is given which represents the mean of six flights made during the summer of 1948 along longitude 117°W at 33,000 ft. pressure altitude.

Comparing corresponding curves in Figs. 1 and 2 the following conclusions are evident: (a) There is a small increase both with counter telescopes and ionization chambers of 2.5 to 3 percent from latitude 50° to 64°N and is approximately the same with or without a 10 cm lead absorber. (b) In going south with latitude 50°N the curves taken along longitude 80°W fall off more slowly than along longitude 115°W.

Table I gives the summary of the experimental situation. (Numbers given in the table are in percent change of radiation per degree change in geomagnetic latitude.)

In seeking an explanation for why the curves along 80°W longitude fall off at a different rate from along 115°W, we measured the film taken with the unshielded ionization chamber along a constant geomagnetic latitude line on a flight at 30,000 ft. from California to Florida on August 1, 1949. The route varied less than 0.5° from geomagnetic latitude 40.8°N and a slight correction taken from Figs. 1 and 2 of approximately 1 percent per degree

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3. The Auger electrons were assumed for this experiment to represent a ratio of intensities for the K-capture processes in the two isotopes under consideration. In reality a small amount of the intensity of the Auger line for Cu⁶⁺ would be caused by the internal conversion electrons. However, their intensity is much less than the K-capture intensity and could not cause a detectable change in the experimentally observed value. [See Blom, Blauer, Marmier, and Preiswerk, Phys. Rev. 77, 295 (1950) and Owen, Cook, and Owen (article to be submitted to Phys. Rev.)]

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FIG. 1. These curves show the ionization as a function of latitude at two different longitudes at an altitude corresponding to 310 g/cm² air pressure. The probable errors in the points, as determined by the scatter of the individual discharges on the film, are given approximately by the diameter of the points.

FIG. 2. Similar curves to those in Fig. 1 but taken with counter telescopes pointing vertically.
Energy Band Structures in Semiconductors

W. Shockley
Bell Telephone Laboratories, Murray Hill, New Jersey
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A method of attack is proposed which for semiconductors has the potentiality of actually determining the shapes of the energy surfaces in the Brillouin zone in detail by proper comparison with experiment. The essential feature is that the most complicated family of energy surfaces arising from a degenerate energy band in a cubic crystal can be described by three parameters. Similar results for the restricted case of tight binding have been discussed by Sommerfeld and Bethe.1 We shall illustrate the more general case in terms of the wave functions near the bottom of an energy band which has its lowest energy $H_0$ at the center of the Brillouin zone and is triply degenerate with $p$-type wave functions,2 type $\delta$, which have the periodicity of the lattice. These are denoted by

$$\psi = x_u(x^2, y^2, z^2) = x_u(x^2, z^2, y^2)$$

A wave function near the lowest energy of the band may be written as

$$\Phi = \sum \psi_\alpha \psi_\beta + a_\psi \psi_\beta + \sum \delta_\psi \psi_\alpha \exp(i\mathbf{P} \cdot \mathbf{r}/\hbar),$$

where the $\psi_{\alpha \beta}$ are also periodic and satisfy $H_{\psi_{\alpha \beta}} = H_{\psi_{\alpha \beta}}$. In order to solve the eigenvalue problem $\langle H - \overline{W} \rangle \Phi = 0$, a set of equations for the coefficients is found by multiplying by $\exp(-i\mathbf{P} \cdot \mathbf{r}/\hbar)\psi_{\alpha \beta}^*$, etc., and the resulting system treated by a method similar to that of Van Vleck2 so as to eliminate the $\delta$'s. This gives

$$[A \mathbf{P}^2 + B(\mathbf{P}^2 + \mathbf{P} \cdot \mathbf{L}) - K] x_{\alpha \beta} + C \sum \mathbf{P} \cdot \mathbf{r} x_{\alpha \beta} = 0,$$

and similar equations with permuted indices $(x, y, z)$, where $K = W - H_0$ is essentially the kinetic energy of motion and

$$A = \frac{1}{3} m = \sum \sigma_\alpha (x | \mathbf{p}_{\alpha} | \mathbf{a}_\alpha (x) | \mathbf{p}_{\alpha} | x),$$

$$B = \frac{1}{3} m = \sum \sigma_\alpha (x | \mathbf{p}_{\alpha} | \mathbf{a}_\alpha (x) | \mathbf{p}_{\alpha} | y),$$

$$C = \sum \sigma_\alpha (x | \mathbf{p}_{\alpha} | \mathbf{a}_\alpha (x) | \mathbf{p}_{\alpha} | y) / \hbar.$$

These equations are formally identical with equations for the frequency and polarization of an acoustical wave of displacement

$$\partial \mathbf{r} / G \exp(i\mathbf{k} \cdot \mathbf{r} + i\omega t)$$

which gives

$$[\mathbf{c}_{11} k_x^2 + \mathbf{c}_{12} (k_y^2 + k_z^2) - \hbar^2 / \mathbf{m}] G_x + (\epsilon_{12} - \epsilon_{12}) (k_x k_y G_x + k_x k_z G_z) = 0.$$

The secular equations for both the quantum and the mechanical cases give surfaces of three sheets in $k$ space for a given eigenvalue, the “polarization vector” $(\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z)$ of the wave function being analogous to $G$. The present treatment was suggested by the analogy of the surfaces in Fig. 1 computed for the $3p$ band in NaCl to acoustical surfaces. The symmetry of (1) follows directly, of course, from the fact that it is quadratic in $P$ and that the crystal is cubic.

A similar procedure for wave functions of the form $x^2 - y^2$, $y^2 - z^2$ (type $\gamma$) leads to secular equation of two constants:

$$K^2 + 2FK + G^2 + 3(L^2 + P^2 + P^2) = 0,$$

where

$$A = \pm \frac{1}{3} m = \sum \sigma_\alpha (x | \mathbf{p}_{\alpha} | \mathbf{a}_\alpha (x) | \mathbf{p}_{\alpha} | x),$$

$$B = \frac{1}{3} m = \sum \sigma_\alpha (x | \mathbf{p}_{\alpha} | \mathbf{a}_\alpha (x) | \mathbf{p}_{\alpha} | y),$$

$$C = \sum \sigma_\alpha (x | \mathbf{p}_{\alpha} | \mathbf{a}_\alpha (x) | \mathbf{p}_{\alpha} | y) / \hbar.$$