Magnitude of High-Energy Meson-Meson Total Cross Sections*

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(Received 4 November 1970)

The total cross section for meson-meson scattering at high energy is calculated using a simple, but plausible, SU(n)-symmetric multiperipheral model. The resulting σ_T is of the order $16\pi^3/NM_V^2$, where M_V is the central mass of the dominant low-energy resonance multiplet in elastic meson-meson scattering and N is the dimensionality of the multiplet of the incident mesons. The nongeometric character of this result is discussed.

The magnitude of high-energy hadron-hadron total cross sections is generally considered an elementary geometrical phenomenon. A typical expectation is πR^2 with R on the order of a pion Compton wavelength. The purpose of this note is to suggest that such reasoning may be misleading and that the magnitude of high-energy hadron total cross sections constitutes an important test for theoretical models. To illustrate this point we show that a simple, but plausible, SU(n)-symmetric multiperipheral model for meson-meson scattering leads to a total cross section of the order

$$(16\pi^3/N)(1/M_V^2), \tag{1}$$

where M_V is the central mass of the dominant low-energy resonance multiplet in elastic meson-meson scattering and N is the dimensionality of the pseudoscalar meson multiplet. With $N=8$ and $M_V=900$ MeV, this number is an acceptable 30 mb, but there appears no reasonable manner in which to interpret the total cross section given by formula (1) as a geometrical πR^2, since the pion (or pseudoscalar) mass does not enter. The important resonance mass is so large that without the surprising $16\pi^3$ to compensate, total cross sections interpreted on a geometrical basis with a radius M_V^{-1} would be only about 1.5 mb. Finally, there is the important and unequivocally nongeometric factor of $1/N$.

Our reasoning starts with the multiperipheral model of Amati, Fubini, and Stanghellini and Bertocchi, Fubini, and Tanin[1] and the assumption that the input of elastic meson-meson scattering is primarily given by the on-shell transition via a single sharp resonance. With SU(n) symmetry the crossing matrix guarantees that the leading output Regge pole will be a singlet in the t channel when the resonance multiplet structure is not pure singlet. Physically one would have some appropriate admixture of SU(3) $\underline{8}$'s and $\underline{1}$'s. The diagonalized t-channel singlet absorptive part equation for zero momentum transfer takes the form[2]

$$A_\lambda(t,t') = K_\lambda(t,t') + \frac{1}{16\pi^3(\lambda+1)} \times \int_{-\infty}^{0} dt'' \frac{K_\lambda(t,t'')A_\lambda(t'',t')}{(M_P^2-t'')^2}, \tag{2}$$

with

$$K_\lambda(t,t') = C_{VP}\exp[-(\lambda+1)\eta(M_V^2,t,t')] \tag{3}$$

and

$$\cosh\eta(s,t,t') = (s-t-t')/2[(-t)(-t')]^{1/2}; \tag{4}$$

λ is the usual continuous label for the SO(1,3) representation, M_P the pseudoscalar meson mass, and M_V the resonance mass. The coefficient C_{VP} measures the resonance elastic width. Since we are dealing with the diagonalized forward equation, any effects of spin carried by the resonance in approximating the input elastic cross section are absent. The amplitude $A_\lambda(t,t')$ arises from the usual s-dependent forward absorptive part, $A(s,t,t')$, by the Laplace transform

$$A_\lambda(t,t') = \int_{4M_P^2}^{\infty} ds \exp[-(\lambda+1)\eta(s,t,t')] \times A(s,t,t'). \tag{5}$$

When the leading t-channel singlet pole dominates the amplitude, the total P-P cross section is recovered by

$$\sigma_{PP}^{\text{tot}}(s) = \frac{A(s,M_P^2,M_P^2)}{[s(s-4M_P^2)]^{1/2}}\frac{1}{N}. \tag{6}$$

The factor N^{-1} is the crossing matrix element connecting the t-channel singlet to any individual s-channel state. This result, of course, is model independent.

In the neighborhood of the leading singularity of $A_\lambda(t,t')$ in the λ plane, the partial-wave amplitude is well represented by the approximate[3]

$$A_\lambda(t,t') = K_\lambda(t,t')/1-\text{Tr}K_\lambda \tag{7}$$

with

$$\text{Tr}K_\lambda = \frac{1}{16\pi^3(\lambda+1)}\int_{-\infty}^{0} dt \frac{K_\lambda(t,t)}{(M_P^2-t)^2}. \tag{8}$$

Since $M_P^2 \ll M_V^2$ it is adequate for $\lambda \gtrsim \frac{1}{2}$ to evaluate the trace indicated in Eq. (8) by setting $M_P = 0$. This leads to

$$\mathrm{Tr}K_\lambda = \frac{C_{VP}}{16\pi^3 M_V^2} \frac{2}{\lambda(\lambda+1)(\lambda+2)}. \tag{9}$$

These approximations are further discussed below.

From the Laplace inversion formula we recover the full on-shell absorptive part, which for large s behaves as

$$A(s, M_P^2, M_P^2)_{s \tilde{\to} \infty} 16\pi^3 \left(\frac{s}{M_V^2}\right)^\alpha J(\alpha), \tag{10}$$

where α is the largest value of λ for which $\mathrm{Tr}K_\lambda = 1$, and

$$\frac{-1}{J(\alpha)} = \frac{\partial}{\partial\lambda}\left[\frac{2}{\lambda(\lambda+1)(\lambda+2)}\right]_{\lambda=\alpha}. \tag{11}$$

The total cross section for any meson-meson collision in the s channel is, from Eq. (6),

$$\sigma_{PP}{}^{\mathrm{tot}}(s)_{s \tilde{\to} \infty} \frac{16\pi^3}{N} \frac{J(\alpha)}{M_V^2} \left(\frac{s}{M_V^2}\right)^{\alpha-1}. \tag{12}$$

The value of α depends on the value of the coupling constant C_{VP}, but if we choose C_{VP} so that $\alpha = 1$, we have finally

$$\sigma_{PP}{}^{\mathrm{tot}}{}_{s \tilde{\to} \infty} \frac{16\pi^3}{N M_V^2} J(1), \tag{13}$$

where $J(1) = 18/11$. As noted by Tow,[3] C_{VP} must be quite large to make $\alpha = 1$. Our C_{VP} represents a weighted sum over the contributions from several resonances in each channel and from the many channels involved in the SU(n) symmetric problem. Thus the contribution from any individual VP coupling need not be unacceptably large.

Our confidence in the accuracy of the approximate partial amplitude given in Eq. (7) is based on its relationship to rigorous separable upper and lower bounds on the exact kernel, Eq. (3).[4] These separable bounding kernels are trivially soluble and lead to total cross sections like Eq. (13) with $J(1)$ lying somewhere between 18/11 and 2. Further, the finiteness of M_P may be readily taken into account to lowest order, the largest correction term being of the form $(M_P/M_V)^2 \times \ln^2(M_P/M_V)$. The net effect is to increase $J(1)$ by 30%.

Giving support to the physical plausibility of the model is its prediction for the rate of increase with energy of the average multiplicity of produced resonances. Using the formula of Amati, Fubini, and Stanghellini and Bertocchi, Fubini, and Tonin,[1]

$$\frac{d}{d\ln s} \bar{n}_{V s \tilde{\to} \infty} C_{VP} \frac{d\alpha}{dC_{VP}} \tag{14}$$

$$= -\{d/d\lambda(\ln \mathrm{Tr}K_\lambda)_{\lambda=\alpha}\}^{-1}$$

$$= (1/\alpha + 1/\alpha + 1 + 1/\alpha + 2)^{-1}$$

$$= 6/11, \text{ if } \alpha = 1. \tag{15}$$

Since each resonance decays into two pseudoscalar mesons, the model predicts

$$d\bar{n}_P/d\ln s \sim 12/11, \tag{16}$$

an entirely acceptable rate of multiplicity increase from the point of view of experiment.[5]

It is evidently possible to refine this type of model to take into account several resonance contributions to the kernel and to consider breaking the SU(n) symmetry. Furthermore the questions of self-consistency raised by incorporating the effects of the relatively small high-energy portions of the kernel must be addressed.

At the moment, however, we wish only to emphasize the striking qualitative features of the resulting total cross-section formula, Eq. (13). First, the very existence of such a simple formula is a pleasant surprise, much of the multiperipheralist community supposing the observed magnitudes of hadron total cross sections to be incalculable, even qualitatively, by simple models. Next, the nongeometric nature of the result is most unexpected. Geometrically inclined physicists would have been faced with $\sigma_{PP}{}^{\mathrm{tot}} \to \infty$ as $M_P \to 0$. Our result, at the least, presents a challenge to geometers building optical models. Finally, perhaps the most intriguing implication of our result is that hadron total cross sections are "small" because there exists not just one but many different low-mass mesons.

We wish to acknowledge private communications from D. Tow, D. R. Snider, and S. S. Shei that helped to direct our attention to the model discussed here.

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*Research sponsored by the U. S. Air Force Office of Scientific Research under Contract No. AF49(638)-1545.

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[1]D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento 26, 896 (1962); L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962).

[2]S. Nussinov and J. Rosner, J. Math. Phys. 7, 1670

(1966).

[3]G. F. Chew, T. W. Rogers, and D. R. Snider, Phys. Rev. D 2, 765 (1970); D. M. Tow, Phys. Rev. D 2, 154 (1970).

[4]This point will be discussed in a forthcoming paper by the present authors.

[5]L. W. Jones et al., Phys. Rev. Lett. 25, 1679 (1970). These authors give $d\bar{n}_c/d\ln s = 0.7 \pm 0.1$, where $\bar{n}_c$ is the averaged multiplicity of charged particles produced. Assuming most particles to be pions, one should multiply $\bar{n}_c$ by a factor $\frac{3}{2}$ to achieve $\bar{n}_p$. With exact SU(3) symmetry the factor would be 2.

ERRATA

ION-WAVE INSTABILITIES AND ANOMALOUS RESISTIVITY. J. R. Kan [Phys. Rev. Lett. 25, 348 (1970)].

We have found a mistake in our numerical computations. Therefore Fig. 1 of the paper is in error and should be replaced by the accompanying figure. The expressions given in the paper are correct as they stand. The corrected figure shows that the results are β dependent and that the critical streaming velocity for ion-wave instabilities is significantly reduced only for $\beta \gtrsim 0.01$. In particular, for $\beta = 0.1$ and $T_e \sim T_i$, the critical streaming velocity is reduced from the electron thermal speed (Fried and Gould's ion-acoustic mode) to approximately twice the ion thermal speed. The theory is likely to find applications in high-β, rather than in low-β, plasmas. Our previous discussion concerning anomalous resistivity is thus also erroneous.

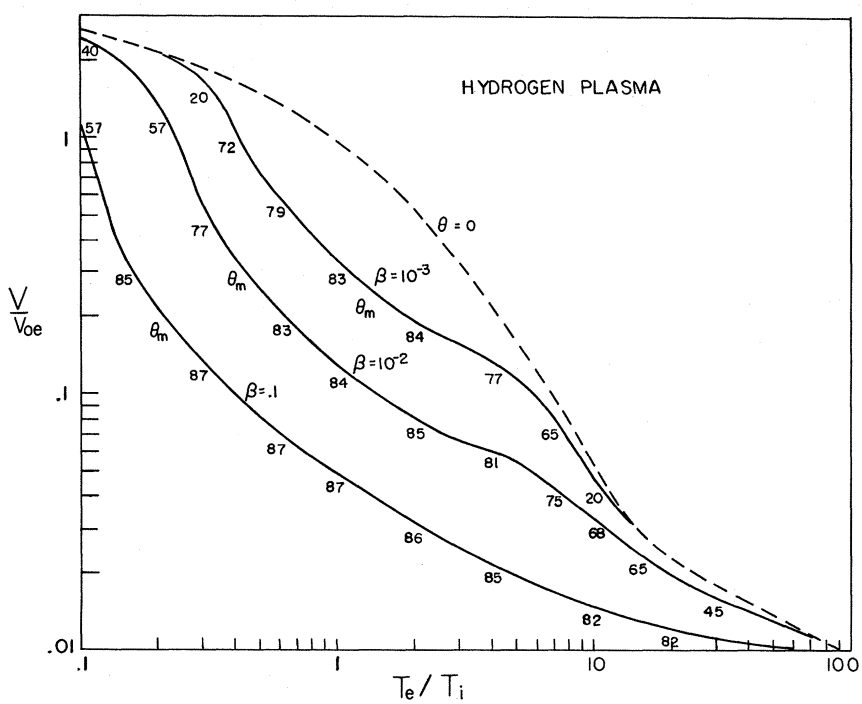

FIG. 1 (revised). Critical streaming velocity in units of the electron thermal speed for ion-wave instabilities vs the electron-ion temperature ratio. θ (in degrees) is the angle between $\vec{k} = (0, k_y, k_z)$ and the dc magnetic field $\vec{B}_0 = (0, 0, B_0)$; θ_m is a value of θ at which the critical streaming velocity is reached; β is the ratio of kinetic pressure to magnetic pressure. In this figure we used $\omega/\Omega_i = 10^{-1}$.