TRAVEL TIMES OF LONGITUDINAL WAVES FROM SURFACE FOCI

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Tables and graphs for travel times of seismic waves propagated through the earth are based on earthquake records. This requires accurate knowledge of the longitude and latitude of the corresponding epicenters, focal depths, and origin times. It is highly desirable to compare the resulting travel time curves and tables with results derived from experiments in which all four fundamental quantities are accurately known from independent data. Seismologists are indebted to scientists and organizations for measurements of records and other corresponding information from which accurate travel times of longitudinal waves originating at known surface sources in the Pacific area could be found.

Residuals for 33 observed direct longitudinal waves, $P$, were calculated relative to travel times of Jeffreys and Bullen for average continental surface foci, as well as relative to similar tables by Gutenberg and Richter. The following average residuals (with standard errors) from the Jeffreys-Bullen tables are found in four ranges of distances containing most of the observations:

\[
\text{Distances, deg.} \quad 35-45 \quad 50-65 \quad 68-79 \quad 80-91 \\
\text{Residuals, sec.} \quad -1.7 \pm 0.6 \quad -3.9 \pm 0.4 \quad -1.3 \pm 0.6 \quad -1.1 \pm 0.7
\]

It has been suspected previously that travel times from surface sources in the Pacific basin are about 2 seconds shorter than the corresponding times from continental surface foci due to differences in velocity in the surface layers. Consequently, 2 seconds were added to all residuals from Pacific sources. Averages based on graphs of the individual residuals similar to figure 1 are given in the lines marked $S$ in table 1.

\[
\text{TABLE 1} \\
\text{CORRECTIONS TO THE TRAVEL TIMES OF JEFFREYS-BULLEN (JB) AND GUTENBERG-RICHTER (GR) FOUND FROM PACIFIC SURFACE SOURCES ($S$, REDUCED TO CONTINENTAL STRUCTURE) AND THE EARTHQUAKE OF JULY 21, 1952 ($E$); FOR DETAILS, SEE TEXT.} \\
\Delta = \text{Angular Distance} \\
\begin{array}{cccccccc}
\text{\(\Delta\)} & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \text{ deg.} \\
\text{JB} & E & +1 & +1 & 0 & -1 & 0 & +1 & +1 & +1 \text{ sec.} \\
& S & +1 & 0 & -1 & -2 & 0 & +1 & +1/2 & 0 \text{ sec.} \\
\text{GR} & E & -1 & -2 & -3 & -3 & -3 & -1 & -11/2 & -1 \text{ sec.} \\
& S & 0 & -21/2 & -3 & -4 & -2 & -1 & -2 & -2 \text{ sec.}
\end{array}
\]

FIGURE 1

Residuals from Gutenberg-Bullen (1945) h = 33 km
Residuals from Gutenberg-Richter (1935) h = 25 ± 4 km

APPROXIMATE EFFECT OF DEPTH
25 ± 10 km

HALF TIME DIFFERENCE BETWEEN h = 33 km and h = 24 km

July 21, 1952
0:11:52:14.3 GCT

July 21, 1952
0:11:52:14.3 GCT

OBSERVED VS. PREDICTED TIMES

DISTANCE, DEGREES

0 10 20 30 40 50 60 70 80 90

0 1 2 3 4 5 6 7 8

0 2 4 6 8 10 12 14

0 0.5 1 1.5 2 2.5 3 3.5

0 0.5 1 1.5 2 2.5

RESIDUALS FROM GUTENBERG-RICHTER
RESIDUALS FROM GUTENBERG-BULLEN

0 10 20 30 40 50 60 70 80 90

0 1 2 3 4 5 6 7 8

0 2 4 6 8 10 12 14

0 0.5 1 1.5 2 2.5 3 3.5

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0 0.5 1 1.5 2 2.5

0 0.5 1 1.5 2 2.5
Records of the Kern County, California, earthquake of July 21, 1952, gave an opportunity to calculate travel times from a well-located continental source. Over 150 stations all over the world responded to a request to mail originals or copies of their records to Pasadena. Readings of arrival times from 40 additional stations are available at the time of writing. Careful determination of the elements of this shock (to be described elsewhere) resulted in an origin time of 11$^h$52$^m$14.3 with a standard error of about 0.2 second and an epicenter at 35$^\circ$00' North Longitude, 119$^\circ$02' West Latitude, probably correct within 3 km.

For all measured and reported arrival times of the direct longitudinal wave $P$ travel times were calculated as well as the corresponding geocentric distances. Residuals from the travel times of Jeffreys-Bullen$^2$ for their focal depth "$h = 0.00"$ (supposed to correspond to 33 km.) and from the travel times of Gutenberg-Richter$^4$ for a focal depth $h$ of 25 $\pm$ km. were calculated for each reading and plotted as a function of distance in the respective two sections. Half the travel time differences between the curves of Jeffreys-Bullen for "$h = 0.00"$ and "surface focus" are also indicated in the lower part of figure 1. Since the earthquake of July 21, 1952, probably had a focal depth of about 16 km., the deviation of the observed times from this curve corresponds approximately to the corrections to be applied to the Jeffreys-Bullen tables. Similarly, the differences in travel time between $h = 25$ and $h = 16$ km. is indicated in the upper part of the figure. It has been found previously$^6$ that the zero point of the Gutenberg-Richter travel times is about 1 second too early. To adjust for this difference 1 second would have to be added to all residuals in the lines $GR$ of table 1.

Table 1 shows that the assumptions made, especially those for the difference between Pacific and continental velocities in the earth's crust, are good approximations.

From table 1 and the corresponding graphs (including figure 1) corrections to the $JB$ and the $GR$ curves were estimated for every degree and applied to the corresponding tables for surface focus. Table 2 contains a smoothed revised travel time curve for continental surface focus on the basis of these results.

The data for the Pacific surface sources contained five instances of $P''$ (longitudinal waves through the outer and inner core) observed at distances between 110 and 140 degrees. Three of them showed two phases between 5 and 15 seconds apart. The earlier phases probably correspond to early $P''$ waves observed in earthquake records; "these early readings are probably not due to longitudinal waves approximately following the laws of geometrical optics; a phenomenon of diffraction is probably involved."$^6$

For the five later impulses the residuals based on the tables of Gutenberg and Richter$^7$ for surface focus are very small if the hypothetical effect of Pacific structure is considered.
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**Table 2**

**Travel Times of Longitudinal Waves from Continental Surface Focus; Δ = Distance in Degrees**

*Geophysics: B. Gutenberg*
SKP (along one path through the mantle longitudinal, along the other transverse; along the path through the outer core longitudinal) was reported by one station a few seconds earlier than calculated from the tables. One $P'P'$ (longitudinal wave which has passed twice through the whole earth, travel time about 40 minutes) had a residual of $-8$ seconds from the Gutenberg-Richter tables (corresponding to $-4$ seconds of $P'$) and one $PKKP$ (reflected inside the core from the core boundary) observed near a distance of $110^\circ$ had a travel time exactly in agreement with the tables of Gutenberg-Richter.

The errors of the average travel time curves for direct longitudinal waves are now probably smaller than the effects of the local structure near the source and the station. The corresponding differences in travel time depend mainly on the depth of the Mohorovičić discontinuity at the source (in shallow earthquakes) and the station and may amount to 3 seconds or even more at distances of a few degrees and at least 2 seconds at greater distances.

* Division of the Geological Sciences, Contribution No. 637.

**APPROXIMATE METHODS IN MATHEMATICS**

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The following simple ideas have proved advantageous in solving systematically certain problems in geometrical optics and appear to be of general mathematical interest. The method is not restricted to power series approximations, but is applicable to any system of approximating functions, $P_r$, which have the quality that $P_rP_j$ can be approximately represented as a linear combination of polynomials $P_r$ with $r$ smaller than a constant $n$. However, in this note we shall restrict ourselves to polynomials.