a reflection cavity containing the spins; or more specifically from the change in the reflection coefficient between the “on resonance” and “off resonance” conditions.

The measurements can be performed with the aid of the conventional equipment for the measurement of reflection coefficients. Great simplification is realized when a variable coupling cavity is used.

The reflection coefficient of a reflection cavity containing paramagnetic centers is given by

$$\Gamma = \Gamma_0 + \frac{8\pi Q \sigma \beta}{(1 + \beta)^2} \chi'' - \frac{8\pi Q \sigma \beta}{(1 + \beta)^2} \chi', \quad (1)$$

In Eq. (1), the radio frequency is assumed equal to the resonance frequency of the cavity and the magnetic losses are a small fraction of the cavity losses, i.e., $|4\pi Q \sigma| << 1$.

$$\Gamma_0 = (\beta - 1)/(\beta + 1)$$

is the reflection coefficient far away from resonance,

$$\beta = \text{cavity } Q/\text{external } Q = Q_e/Q_m,$$

and $\chi''$ and $-\chi'$ are the real and imaginary parts of the rf susceptibility.

For operation near the critical coupling point $\beta = 1$, for a spectrometer tuned to $\chi''$ only, Eq. (1) simplifies to

$$\Gamma = \Gamma_0 + 2\pi Q \sigma \chi''$$

which at resonance becomes

$$\Gamma_{\text{res}} = \Gamma_0 + 2\pi Q \sigma \chi''_{\text{max}}. \quad (2)$$

Consider an ion with effective spin $S$ in a crystalline field of trigonal or higher symmetry. Let $z$ be the coordinate along the symmetry axis. The interaction of such an ion with an external magnetic field $H = (H_x, H_y, H_z)$ is given by the Hamiltonian

$$H_{\text{interaction}} = g_L \beta H \sigma z + g_L \beta (H_x \sigma_z + H_y \sigma_y).$$

$S_z, S_y$ and $S_z$ are three components of the spin vector operator, $\beta$ is the Bohr magneton, while $g_L$ and $g_L$ are, respectively, the parallel and transverse components of the $g$ tensor. In the presence of a steady magnetic field applied in the $z$ direction and a linearly polarized rf field of frequency $\nu_0$ applied in the $x$-$y$ plane, the $\chi''$ due to the $M \leftrightarrow M - 1$ transition is given by

$$\chi''_{\text{max}} = \frac{N \delta g_L^2 \nu_0 (S + M)(S - M + 1)}{2kT \Delta H g_L (2S + 1)}, \quad (3)$$

where $N_0$ is the number of spins/cm$^3$ and cgs units have been used.
Equation (3) is a modification of an expression given by Bloembergen et al., applied to the case of an ion in a crystal. The following conditions are implicit in Eq. (3):

1. The radio frequency is resonant, i.e., \( \nu_0 = \frac{E_M - E_{M-1}}{\hbar} \);
2. The temperature \( T \) is high enough so that \( \hbar \nu << kT \);
3. The dc magnetic field is oriented along the crystal symmetry axis.
4. The \( \chi' \) vs \( H \) curve is a Lorentzian whose width at half maximum is \( \Delta H \). This width is then related to the peak of the normalized Lorentzian \( f(\nu_0) \) by

\[
f(\nu_0) = \frac{2\hbar}{\pi g_i \beta H}.
\]

The filling factor \( \eta \) is related to the sample volume \( V_s \) by a proportionality constant whose value depends on the mode of resonance and the cavity dimensions

\[
\eta = aV_s.
\]  \hspace{1cm} (4)

Substitution of Eqs. (3) and (4) in Eq. (2) leads to

\[
N_0 = \left( \frac{k}{\pi \hbar} \right) \frac{(g_i^2)(T\Delta H)}{Q_{av} \nu_0 V_0} \frac{(2S+1)}{(S+M)(S-M+1)} \times [\Gamma_{\text{res}} - \Gamma_0].
\]  \hspace{1cm} (5)

Replacing the first factor on the right-hand side of Eq. (5) by its numerical value and multiplying by \( V_s \), results in

\[
N = 7.08 \times 10^9 \left( \frac{T\Delta H g_i^2}{Q_{av} \nu_0 V_s} \right) \frac{(2S+1)}{(S+M)(S-M+1)} [\Gamma_{\text{res}} - \Gamma_0],
\]

where \( N = N_0 V_s \) is the total number of “spins.”

The usefulness of Eq. (6) is twofold. It can be used to estimate the optimum size of sample to be used in a paramagnetic resonance experiment, if the spin density is known. This will also involve a consideration of the system’s sensitivity and the saturation power level. The second application, one with which this paper is chiefly concerned, is the determination of \( N \).

After assuming a knowledge of \( Q_{av}, \nu_0, a, T, \Delta H \), and the appropriate quantum numbers \( S \) and \( M \) for the transition considered, we have to measure \( \Gamma_{\text{max}} \) and \( \Gamma_0 \), the voltage reflection coefficients “on” and “off” resonance. This can be achieved by measuring the voltage standing wave ratios for the two conditions and using the relation

\[
\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}
\]

to derive the reflection coefficients. An alternative method, and one actually used in our experiment, is to measure the reflection coefficients by measuring the ratio, power reflected/power incident, with a precision attenuator. An indication of the incident power is obtained by causing a total reflection to occur by extreme mismatching of the cavity (which was made possible by the variable coupling feature). Series attenuation is then introduced until the receiver indication is equal to that obtained “on” and “off” resonance. The necessary attenuation is equal to the reflection coefficient, expressed in db.

To test the procedure described above, we used it to determine the number of Gd²⁺ ions in a host CaWO₄ crystal. The pertinent data were: \( Q_{av} = 25,000, a = 3.04 \text{ cm}^{-3} \), (a T2e₁₁₁ circular cavity having a volume of 3.12 cm³ and a centrally placed sample) \( \Delta H = 1.8 \text{ gauss}, T = 20.94^\circ \text{K}, g_i = g_i \approx 2, \nu_0 = 23 \text{ kHz}, S = 7/2, M = -5/2 \). The quantity \( (\Gamma_{\text{res}} - \Gamma_0) \) was found to be 0.143. Substitution in Eq. (6) gives

\[
N = 1.15 \times 10^{18}.
\]

This result was in good agreement with the result \( N = 1.00 \times 10^{18} \) obtained by an independent method based on measurement of the characteristic Gd²⁺ fluorescence and which is believed to be accurate to within ±10%.

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1 J. P. Gordon (to be published).
2 Equation (1) is valid when the reflection coefficient is measured at the position of the “detuned short.”
4 N. Bloembergen et al., Phys. Rev. 73, 679 (1948).

Letters to the Editor

Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Comments on “Erroneous Readings of Large Magnitude in a Bayard-Alpert Ionization Gauge and their Probable Cause”

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THE intent of the paper recently published by Barnes is to show, by indirect measurements of the ion current in a field ion vacuum gauge, that Bayard-Alpert gauges are unreliable. This writer believes that the purpose has not been accomplished. The objections are as follows: