Some Unsteady Fluid Forces on Pump Impellers

R. S. Miskovish
C. E. Brennen
California Institute of Technology, Pasadena, Calif. 91125

1 Introduction

In previous publications (Chamieh et al., 1985, Jery et al., 1985, Adkins and Brennen 1986, Franz et al., 1990) considerable information has been presented on the flow induced radial and rotordynamic forces in centrifugal and axial (Arndt and Franz, 1986) flow pumps. The effect of cavitation on these forces has also been explored (Franz, 1989). The forces in question can be visualized by reference to Fig. 1, which is in a plane perpendicular to the axis of rotation. Steady radial forces which result from asymmetries in the volute, the discharge flow or the inflow are denoted by $F_{ox}$ and $F_{oy}$. If, in addition, the axis of rotation undergoes a small displacement given by $x(t)$, $y(t)$ from some mean position then the instantaneous fluid forces on the impeller, $F^*_{x}(t)$ and $F^*_{y}(t)$, can be expressed by

$$
\begin{bmatrix}
F^*_{x}(t) \\
F^*_{y}(t)
\end{bmatrix} =
\begin{bmatrix}
F_{ox} \\
F_{oy}
\end{bmatrix} + [A^*]
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix}
$$

(1)

where $[A^*]$ is the rotordynamic matrix. In addition to the radial forces, the corresponding bending moments should be divided into steady radial moments and a rotordynamic moment matrix as follows:

$$
\begin{bmatrix}
M^*_{x}(t) \\
M^*_{y}(t)
\end{bmatrix} =
\begin{bmatrix}
M_{ox} \\
M_{oy}
\end{bmatrix} + [B^*]
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix}
$$

(2)

The forces and moments are presented in dimensionless form denoted by the same symbols without the asterisk. The steady radial forces and moments are non-dimensionalized by $\rho \pi \Omega^2 r_d b_2$ and $\rho \pi \Omega^2 r_d b_3$, respectively, the matrices $[A^*]$ and $[B^*]$ by $\rho \pi \Omega^2 r_d b_2$ and $\rho \pi \Omega^2 r_d b_3$ and the displacements are non-dimensionalized by $r_d$. If for simplicity we focus on a circular whirl orbit (see Fig. 1) of eccentricity $\epsilon$ and frequency $\omega$, then $x = \epsilon \cos \Omega t$ and $y = \epsilon \sin \Omega t$, and

$$
F^*_x = F_{ox} + \epsilon[A]
\begin{bmatrix}
\cos \Omega t \\
\sin \Omega t
\end{bmatrix}
$$

(3)

$$
M^*_x = M_{ox} + \epsilon[B]
\begin{bmatrix}
\cos \Omega t \\
\sin \Omega t
\end{bmatrix}
$$

(4)

Virtually all of the previous experimental data confirms the reasonable supposition that the matrices $[A]$ and $[B]$ should be independent of the particular choice of axes, $x$ and $y$. Such rotational invariance requires that

$$
A_{xx} = A_{yy} = F_{x}; A_{xy} = -A_{yx} = F_{y}
$$

$$
B_{xx} = B_{yy} = M_{x}; B_{xy} = -B_{yx} = M_{y}
$$

(5)

where $F_x$, $F_y$, $M_x$, $M_y$ are the rotordynamic forces and moments normal and tangential to the whirl orbit non-dimensionalized by $\rho \pi \Omega^2 r_d b_2 \epsilon$ in the case of the forces and by $\rho \pi \Omega^2 r_d b_3 \epsilon$ in the case of the moments.

Contributed by the Fluids Engineering Division for publication in the JOURNAL OF FLUIDS ENGINEERING. Manuscript received by the Fluids Engineering Division April 16, 1991. Associate Technical Editor: U. S. Rohatgi.
2 Spectra of Forces on a Centrifugal Impeller

During the present investigation a comprehensive examination was undertaken of all the unsteady fluid-induced forces and moments acting upon a typical centrifugal pump impeller. The specific impeller and volute was that combination designated Impeller X/Volute A (see Jery et al., 1985). Impeller X is a five-bladed centrifugal pump with a discharge radius of 8.1 cm. and a design specific speed of 0.57 made by Byron Jackson Pump Company. Volute A is a vaneless, spiral volute made to match Impeller X at a flow coefficient \( \phi = 0.092 \).

A sketch of the volute and of the impeller mounting is included in Fig. 2. Note that since the impeller moves on an eccentric orbit the clearances between the impeller and the volute vary with time.

The forces described in the preceding section were evaluated from measurements of the radial forces, \( F_r \) and \( F_z \), and moments, \( M_r \) and \( M_z \), in the coordinate frame of the shaft-mounted rotating balance to which the impellers were directly mounted. This balance measured the unsteady components of all six forces and moments, including the torque \( M_z \), and the axial thrust, \( F_z \).

As part of the present investigation, the dynamic signals associated with all six forces and moments during operation of the pump at a particular flow coefficient, \( \phi \), shaft frequency, \( \Omega \), and whirl frequency, \( \omega \), were closely examined and revealed at least one surprise which we shall come to shortly. To discuss the results (of which Fig. 3 represents a typical example), it is necessary to describe briefly how the motions are controlled in the experiment. A generator produces a low frequency signal denoted by \( \Omega \) where \( J \) is an integer typically 10 or 20. This is then multiplied by \( J \) to produce the signal which drives the whirl motion so that \( \omega = \Omega J / J \). Both motions are closely controlled and monitored as part of the data acquisition system. It follows that in the frame of reference of the rotating balance the radial forces and moments which are steady in the laboratory frame will show up at the frequency, \( \Omega \). The rotordynamic forces and moments will be represented by signals at frequencies of \( (\Omega \pm \omega) \) or \( \Omega(J \pm I) / J \). Blade passage frequencies will also be manifest; in the case studied here the Impeller X had five blades and, in combination with the vaneless volute, would generate a blade passage signal at \( \Omega J \).

Fourier components were obtained by cross-correlation for all integer multiples of \( \Omega J \). Tests revealed no significant amplitudes above this frequency. Spectra like this were obtained at a number of shaft speeds, whirl ratios, \( I / J \), and operating conditions represented by the flow coefficient, \( \phi \). They all had similar characteristics exemplified by Fig. 3, in which the results for each of the six forces and moments are normalized with respect to the maximum component occurring in that particular spectrum. Those normalizing values are attached to each spectrum and should be compared with the later data of Figs. 6, 7, and 8 in order to assess whether the amplitudes are significant or not. The following conclusions can be drawn from Fig. 3 and other similar spectra:

(a) The strong peak at \( \Omega \) in all of the radial forces and bending moments \( (F_r, F_z, M_r, \text{ and } M_z) \) is generated by steady radial forces and bending moments due to the asymmetry of the volute.

(b) The peaks at \( (J \pm I) \Omega / J \) in \( F_r, F_z, M_r, \text{ and } M_z \) are generated by nonzero \( F_{r0}, F_{z0}, M_{r0}, \text{ and } M_{z0} \). It is particularly noticeable that the \( (J - I) \Omega / J \) component is usually much larger than the \( (J + I) \Omega / J \) component and hence

\[
A_2 = \text{impeller discharge area} = 2 \pi r_2 b_2
\]

\[
b_2 = \text{impeller discharge width}
\]

\[
F_r, F_z = \text{radial forces on the impeller observed in the rotating frame of the balance}
\]

\[
F_3 = \text{thrust on the impeller}
\]

\[
M_{r0}, M_{z0} = \text{normalized mean radial forces acting on the impeller where } x \text{ is the direction of the volute cutwater and } y \text{ is a perpendicular direction rotated in the direction of shaft rotation. Unlike otherwise stated, all forces are normalized by } 0.5 \rho u_2^2 A_2 \text{ and all moments by } 0.5 \rho u_2^2 A_2 r_2.
\]

\[
F_{n}, F_{z} = \text{mean forces normal to and tangential to the whirl orbit; these are normalized using } 0.5 \rho u_2^2 A_2 / r_2.
\]

\[
M_{n}, M_{z} = \text{bending moments observed in the rotating frame of the balance}
\]

\[
\psi = \text{head coefficient} = g \Delta h / u_2^2, \text{ where } \Delta h \text{ is the total head rise across the pump}
\]

\[
\Omega = \text{pump rotational speed (rad/s)}
\]

\[
\omega = \Omega J / J = \text{whirl rotational speed (rad/s) where } I \text{ and } J \text{ are integers}
\]

\[
r_2 = \text{impeller discharge radius}
\]

\[
u_2 = \text{impeller tip speed at discharge} = \Omega r_2
\]

\[
\epsilon = \text{radius of the eccentric whirl orbit}
\]

\[
\phi = \text{flow coefficient} = Q / \mu_2 A_2, \text{ where } Q \text{ is the volume flow rate through the pump}
\]

\[
\psi = \text{head coefficient} = g \Delta h / u_2^2, \text{ where } \Delta h \text{ is the total head rise across the pump}
\]

\[
\Omega = \text{pump rotational speed (rad/s)}
\]

\[
\omega = \Omega J / J = \text{whirl rotational speed (rad/s) where } I \text{ and } J \text{ are integers}
\]

\[
r_2 = \text{impeller discharge radius}
\]

\[
u_2 = \text{impeller tip speed at discharge} = \Omega r_2
\]

\[
\epsilon = \text{radius of the eccentric whirl orbit}
\]

\[
\phi = \text{flow coefficient} = Q / \mu_2 A_2, \text{ where } Q \text{ is the volume flow rate through the pump}
\]

\[
\psi = \text{head coefficient} = g \Delta h / u_2^2, \text{ where } \Delta h \text{ is the total head rise across the pump}
\]

\[
\Omega = \text{pump rotational speed (rad/s)}
\]

\[
\omega = \Omega J / J = \text{whirl rotational speed (rad/s) where } I \text{ and } J \text{ are integers}
\]
In this section we shall examine the relative magnitude of this unsteady force. Since the balance does not record the steady thrust and since we do not have independent means of measuring the thrust, we shall compare the magnitude of the measured unsteady thrust with an estimate of the steady thrust based on measurement of the pressure rise across the pump and on the pressure distribution around the impeller.

From the geometry of the five-bladed centrifugal pump impeller, Impeller X, and assuming (i) that the discharge pressure acts on the back-face of the impeller and (ii) that the pressure on the exterior of the shroud varies linearly with radial position between the inlet tip and the discharge tip, we find by integration of the pressure distribution on the impeller that

\[
\frac{\text{Mean Thrust}}{0.5p_{\text{ud}}A_2} = F_{\text{sd}} = 3.09\alpha - a\psi
\]

in which \( a = 3.0 \). This provides values which are acceptably close to those of Beitz and Ktittner (1981) who recommend a value of \( a \) in the range 3.6-4.6. With this estimate of steady thrust, ratios of the magnitude of the unsteady thrust at the blade passing frequency to the steady thrust were evaluated for a range of flow coefficients, \( \phi \), and whirl ratios. With respect to the latter, Fig. 4 demonstrates that, as expected, the ratio is independent of the whirl ratio since the interaction mechanism occurs irrespective of whirl. Indeed Fig. 4 merely indicates the typical scatter in the magnitude of the unsteady thrust.

The variation with operating point is exemplified by the values in the following table:

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Steady thrust</th>
<th>Unsteady thrust magnitude/steady thrust</th>
<th>( \theta_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>( \psi )</td>
<td>( F_{\text{sd}} )</td>
<td>( \text{deg} )</td>
</tr>
<tr>
<td>0.060</td>
<td>0.510</td>
<td>-1.52</td>
<td>0.0069</td>
</tr>
<tr>
<td>0.092</td>
<td>0.425</td>
<td>-1.25</td>
<td>0.0028</td>
</tr>
<tr>
<td>0.104</td>
<td>0.375</td>
<td>-1.09</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

The nominal design flow coefficient of this impeller/volute combination is 0.092. Note that, as might be expected, the unsteady thrust at the blade passing frequency is smallest at the design flow coefficient. Though the magnitude of unsteady thrust is less than one percent of the steady thrust, it is possible to envisage circumstances in which it could cause serious axial resonance problems.

Finally, the relative location of the impeller blades to the volute cutwater at the time of the peak thrust is of interest in attempting to understand the origins of this unsteady thrust. Since Impeller X is five-bladed the angle between the tips of the five blades is 72 deg. If we define an angle between the line from the impeller center to the volute cutwater and the line from the impeller center to the discharge blade tip at the time when the instantaneous thrust is a maximum then this angle would take values of \( \theta_m = \theta_0 \pm 72n \text{ deg} \) (\( n = 0, 1, 2, \text{ etc.} \)) where the observed values of \( \theta_m \) are as listed in Table 1. The positive direction of \( \theta_m \) is defined as being in the direction of rotation of the impeller. Note that in all three cases listed in Table 1 the maximum instantaneous thrust occurs when the discharge tip of a blade is close to the volute cutwater.

4 Origins of the Radial and Rotodynamic Forces

Previous publications, (Chamieh et al., 1985; Jery et al., 1985; Adkins and Brennen, 1988; Franz et al., 1990) have presented data on the steady radial forces and on the rotodynamic forces for a number of impellers, diffusers and volutes. Visualizing the centrifugal pump impeller as a control volume, one can recognize that there exists the possibility of contributions to the radial force from three different sources. First, circumferential variation in the impeller discharge pres-

3 Unsteady Thrust at Blade Passage Frequency

In the previous section we have noted the significant component in the unsteady thrust at the blade passing frequency, the former frequency provides the major contribution to \( F_n, F_p, M_n, M_p \), and the rotdynamic matrices.

(c) The moments \( M_t \) and \( M_r \) seem substantially noisier than the forces \( F_n \) and \( F_p \). This is rather misleading; it is due to the fact that the moments are small since the origin of the force/moment coordinate system is in the center of the discharge of the impeller and the line of action of the radial forces is close to this axial location.

(d) The unsteady axial thrust, \( F_n \), contains a surprisingly large component at the blade passing frequency, 5\( \Omega \), though other multiples of the impeller rotation frequency are also present. Note that since the balance only records unsteady forces, neither the steady thrust nor torque can be measured by this device.

(e) The unsteady torque, \( M_r \), was usually quite small compared with the steady torque which was estimated using separate strain gauges on the drive shaft. As demonstrated by Fig. 3, no consistent pattern was observed in the frequency content of this unsteady torque. In all cases, the largest harmonic is 60 Hz, which corresponds to frequency of the electrical input to the balance. In light of the blade passing frequency content in the thrust, one might have expected a similar contribution to the torque. However this was not observed.
and found that these pressure distributions were in accord with experimental measurements. Also, integration of the experimental pressure distributions yielded radial forces in good agreement with both the overall radial forces measured using the force balance and the theoretical predictions of the theory. These results demonstrate that it is primarily the circumferential non-uniformity in the pressure at the impeller discharge which generates the radial forces, but that the non-uniformity in the pressures acting on the exterior of the shroud may also contribute.

The origins of the rotordynamic forces were also explored by Adkins and Brennen (1988) who used the same model to evaluate the rotordynamic forces acting on the impeller discharge. It was found that the theoretical values for the impeller discharge contributions were significantly smaller than the total measured forces. Thus it was concluded that the leakage flow around the shroud exterior can be an important contributor to the rotordynamic forces. To confirm this, Adkins and Brennen (1988) made experimental measurements of the pressure distributions in both the impeller discharge flow and in the leakage flow.

None of these previous experimental or theoretical studies addressed the issue of the location of the lines of action of either the radial or the rotordynamic forces. Clearly, a discussion of the lines of action or of the moments requires a definition of the axial location of the origin of the reference frame of the forces and moment. In this paper we arbitrarily choose the origin to be in the center of the impeller discharge. If the radial forces $F_{rx}$, $F_{ry}$ acted at this location, then the moments $M_{rx}$ and $M_{ry}$ would be zero. It is one of the purposes of this paper to present data on both the steady radial moments and the rotordynamic moments and in doing so to provide information of the location of the lines of action of the radial and rotordynamic forces.

## 5 Forces and Moments

Typical data on the radial forces and moments are presented in Figs. 5 and 6, respectively, for the Impeller X/Volute A combination at a shaft speed of 1000 rpm and four different flow coefficients. Note that the forces are independent of the whirl ratio as they should be. The data of Fig. 5 correspond to data presented previously by Jery (1987). The moments in Fig. 6 are new. It should be noted that they follow a righthand rule in which the $z$-axis points toward the inlet. Thus a radial force in the $x$ or $y$ direction whose line of action is closer to the inlet than the origin in the center of the impeller discharge, leads to a positive $M_{ry}$ or a negative $M_{rx}$. Though the data in Fig. 6 contains a few anomalous points, it does suggest that there exists a steady moment, primarily in the $x$-direction and that this changes with the flow coefficient. The following example illustrates the typical magnitude of these steady moments. At a flow coefficient of $\phi = 0.06$, the steady force vector $F_{s} = F_{sx} + F_{sy}$ has components $F_{sx} = 0.03$ and $F_{sy} = 0.06$ (see Fig. 5); it has a magnitude of about 0.067 and an angle from the $x$-axis, $\theta_{0}$, of 63 deg. The steady moment vector $M_{s} = M_{sx} + M_{sy}$, where $M_{sx} = -0.02$ and $M_{sy} = 0$, has a magnitude of 0.02 and an angle from the $x$-axis, $\theta_{m}$, of 180 deg. The position of the line of action on the $z$-axis can be computed from $r * F = M_{s}$ (or, in scalar terms $F * r = M_{s}$, where $M_{s} = M_{sx} - M_{sy}$), and the resulting value of $r$ when $\phi = 0.06$ is 2.2 cm forward from the center of the discharge. For $\phi = 0.13$, a similar estimate gives a location 1.3 cm forward of the center of the discharge.

In conclusion, the steady moments indicate that the line of action of the steady radial force is some distance ahead of the center of the discharge, though it moves backward with
of the lines of action of \( F_r \) and \( F_t \) relative to the center of the discharge can be computed in a manner similar to that used for computation of the lines of action of the steady forces. Typical values from Figs. 7 and 8 indicate that these displacements are much smaller than for \( F_{nn} \) and \( F_{nn} \) and are less than 1 cm. Thus we conclude that the line of action of the rotordynamic force matrix is close to the center of the discharge. This, too, is consistent with previous analysis which suggest that for the impeller/volute combination used in these experiments the shroud force contributions to the rotordynamic matrices are quite small.

6 Conclusions

The conclusions drawn from this study of the fluid forces on a typical centrifugal impeller are:

(i) The spectral analysis of the forces shows the largest peaks at the frequencies \( \Omega \) and \( \Omega \pm \omega \) for \( F_{nn}, F_{nt}, M_{nr}, \) and \( M_{nt}. \)

(ii) On closer examination, the magnitude of the unsteady thrust is found to contain a strong harmonic at the blade passing frequency.

(iii) The steady radial moments are equivalent to a line of action for the steady radial forces which is as much as 0.25 radii from the center of the impeller discharge (radius 8.1 cm) in the direction of the inlet. This is consistent with the conclusion of Adkins and Brennen (1988) that the forces on the shroud provide important contributions to the radial forces. The line of action moves closer to the center of impeller discharge as the flow coefficient increases.

(iv) The rotordynamic moment matrices are close to the rotationally invariant form that has been used to describe the rotordynamic force matrix. Thus, it suffices to present the moment matrices in terms of \( M_r \) and \( M_t \) without loss of information. In comparing the steady and unsteady moments with the steady and unsteady forces, the line of action in the \( z \)-direction is shown to be close to the center of the impeller discharge (within about 0.1 radii).

Acknowledgments

This work was performed under Contract NAS 8-33108 from NASA George Marshall Space Flight Center, Huntsville, Alabama; the authors are very grateful for their support. We should also like to express our thanks to R. Franz, N. Arndt, A. J. Acosta, and T. K. Caughey for their help during the conduct of this research program.

References


Fig. 7 Normal and tangential rotordynamic forces, \( F_n \) and \( F_t \), for Impeller/Volute A at 1000 rpm and various flow coefficients as in Figs. 5 and 6. Uncertainty expressed as a standard deviation: \( F_n, F_t \pm 0.05.\)

Fig. 8 Normal and tangential rotordynamic moments, \( M_r \) and \( M_t \), for Impeller/Volute A at 1000 rpm and various flow coefficients as in Figs. 5 and 6. Uncertainty expressed as a standard deviation: \( M_r, M_t \pm 0.1.\)


For Your ASME Bookshelf

Measurement and Modeling of Environmental Flows 1992
Editors: S.A. Sherif, D.E. Stock, F.E. Michaelides, L.R. Davis, I. Celik, B. Khalighi, R. Kumar

Topics include: mass flow measurement of gaseous breathing air through the liquid air pack respirator; quantitative flow measurement and numerical simulation of lid-driven rotating cavity flow; interfacial transport in river-reservoir systems; numerical prediction of a turbulent plume in a stably stratified environment; effects of coastal topography on lake flows; effects of viscosity on gravity currents in the inertial regime; lee waves, benign and malignant; more.

Order No. G00772 $62.50 List/$50 ASME Members

To order write ASME Information Central, 22 Law Drive, Box 2300, Fairfield, NJ 07007-2300 or call 800-THE-ASME (843-2763) or fax 201-882-1717.