A model for fluvial bedrock incision by impacting suspended and bed load sediment

Michael P. Lamb, William E. Dietrich, and Leonard S. Sklar

Received 24 September 2007; revised 5 May 2008; accepted 23 July 2008; published 17 September 2008.

[1] A mechanistic model is derived for the rate of fluvial erosion into bedrock by abrasion from uniform size particles that impact the bed during transport in both bed and suspended load. The erosion rate is equated to the product of the impact rate, the mass loss per particle impact, and a bed coverage term. Unlike previous models that consider only bed load, the impact rate is not assumed to tend to zero as the shear velocity approaches the threshold for suspension. Instead, a given sediment supply is distributed between the bed and suspended load by using formulas for the bed load layer height, bed load velocity, logarithmic fluid velocity profile, and Rouse sediment concentration profile. It is proposed that the impact rate scales linearly with the product of the near-bed sediment concentration and the impact velocity and that particles impact the bed because of gravitational settling and advection by turbulent eddies. Results suggest, unlike models that consider only bed load, that the erosion rate increases with increasing transport stage (for a given relative sediment supply), even for transport stages that exceed the onset of suspension. In addition, erosion can occur if the supply of sediment exceeds the bed load transport capacity because a portion of the sediment load is transported in suspension. These results have implications for predicting erosion rates and channel morphology, especially in rivers with fine sediment, steep channel-bed slopes, and large flood events.


1. Introduction

[2] River incision into bedrock is one of the fundamental drivers of landscape evolution and propagates climatic and tectonic signals throughout drainage networks. Incision into rock occurs relatively slowly and during large infrequent events making it difficult to investigate mechanistically. To characterize river incision geomorphologists typically have relied on reach-scale rules, for example, by setting the rate of erosion to be a function of boundary shear stress [Howard and Kerby, 1983] or stream power [Seidl and Dietrich, 1992; Howard et al., 1994; Seidl et al., 1994; Whipple and Tucker, 1999]. These models are limited in application, however, because they mask the physical mechanisms by which bedrock erosion occurs. More realistic model predictions require advances in our quantitative understanding of erosion processes [e.g., Dietrich et al., 2003; Whipple, 2004].

[3] One such model proposed by Sklar and Dietrich [2004] explicitly models the wear of bedrock by bed load particles (referred to as the saltation-abrasion model herein). Application of the saltation-abrasion model and related efforts have led to significant insights into the controls of bedrock river morphology including, channel slope [Sklar and Dietrich, 2006; Gasparini et al., 2007], knickpoints [e.g., Chatanantavet and Parker, 2005; Wobus et al., 2006; Crosby et al., 2007], slot canyons [Carter and Anderson, 2006; Johnson and Whipple, 2007], and channel width [Finnegan et al., 2007; Nelson and Seminara, 2007; Turowski et al., 2008]. Nonetheless, the saltation-abrasion model is incomplete because it neglect other important mechanisms for riverbed erosion such as cavitation, plucking of jointed rock and abrasion by suspended sediment [Whipple et al., 2000]. Abrasion by suspended sediment in particular has been argued to be an important or dominant erosion mechanism in some streams [Hancock et al., 1998; Whipple et al., 2000; Hartshorn et al., 2002] owing in part to the frequent occurrence of polished bedrock surfaces, flutes, potholes, and undulating canyon walls.

[4] In this paper, we investigate erosion by suspended particles by deriving a total load erosion model, which expands on the saltation-abrasion model of Sklar and Dietrich [2004] to include suspended particles. Cavitation and plucking of jointed rock are not investigated here. In section 2, the saltation-abrasion model is reviewed briefly and the assumption that the impact rate is zero at the onset of suspension is discussed. In section 3, we propose that suspended particles do interact with the bed and that the impact rate scales with the product of the near-bed sediment concentration and the particle impact velocity. The near-bed
sediment concentration is found by partitioning a given sediment supply between the bed and suspended load. In section 4, commonly used formulas are adopted to solve the model, including the Rouse concentration profile to describe the vertical distribution of suspended sediment. In section 5, predictions of the total load erosion model are shown and compared to the saltation-abrasion model for different values of transport stage, sediment supply, particle size, and channel slope. Finally, the entrainment capacity, viscous damping of impacts, and implications for natural streams are discussed in section 6.

2. Saltation-Abrasion Model

[Sklar and Dietrich [2004], following the work of Foley [1980], Beaumont et al. [1992], Tucker and Slingerland [1994], and others, present a model for fluvial incision of bedrock by saltating sediment, which is briefly reviewed here. The saltation-abrasion model was formulated by neglecting abrasion by all modes of sediment transport except saltation. A planar bed, rectangular channel cross section, and uniform size sediment are assumed. The model assumes that the net effects of spatial heterogeneity in hydraulics, rock strength, and sediment supply can be adequately represented in terms of a unit bed area.

[6] The rate of vertical erosion $E$ is defined as the product of the average volume of rock detached per particle-bedrock impact $V_p$ the rate of particle impacts per unit bed area per unit time $I_r$ and the fraction of exposed bedrock on the river bed $F_e$

$$ E = V_p I_r F_e. $$

The volume of eroded bedrock per particle impact $V_p$ is scaled by the kinetic energy of the particle impact

$$ V_p = \frac{1}{2} \frac{V_p \rho_p w_i^2}{\varepsilon_v}. $$

where $V_p$, $\rho_p$, and $w_i$ are the particle volume, density, and impact velocity normal to the bed. A threshold kinetic energy needed to cause erosion is not based on the results from abrasion mill experiments [Sklar and Dietrich, 2001]. The kinetic energy required to cause erosion of a unit volume of bedrock $\varepsilon_v$ (units of energy per volume) depends on the capacity of the rock to store energy elastically

$$ \varepsilon_v = k_v \frac{\sigma_Y^2}{2Y}, $$

where $\sigma_Y$ is the tensile yield strength and $Y$ is Young’s modulus of elasticity of the bedrock. The dimensionless coefficient $k_v$ was found to be of the order $10^6$ [Sklar and Dietrich, 2006].

[7] The rate of particle-bedrock impacts per unit bed area $I_r$ is given by

$$ I_r = \frac{q_b}{V_p L_b}, $$

where $q_b$ is the volumetric bed load sediment transport capacity per unit channel width traveling as bed load and $L_b$ is the saltation hop length. Note that $q_b$ in this paper is the same as $q_b/\rho_s$ defined by Sklar and Dietrich [2004], since they defined $q_b$ to be a mass flux rather than a volumetric flux.

[8] Following the hypothesis of Gilbert [1877], the fraction of the river bed that is exposed bedrock and not covered with alluvium $F_e$ is assumed to vary as

$$ F_e = \left(1 - \frac{q_b}{q_{bc}}\right), $$

where $q_{bc}$ is the volumetric bed load sediment transport capacity per unit channel width [Sklar et al., 1996; Slingerland et al., 1997; Sklar and Dietrich, 2004]. This linear relationship has yet to be tested in nature, and others have argued that an exponential relationship is more appropriate [Turowski et al., 2007]. Herein we use equation (5) to simplify later comparison of the saltation-abrasion model with the total load erosion model. Equation (5) must be true in end-member cases at steady state. Where the supply of sediment exceeds the transport capacity, sediment is deposited on the bed and the bedrock is protected from erosion. This is typically the case in alluvial, transport-limited rivers and many formulas exist to predict the sediment transport (and hence the transport capacity) under such conditions [e.g., Fernandez Luque and van Beek, 1976]. On the other hand, if the sediment supply is zero, the river bed will be free of cover. In this case, however, no erosion will occur because there are no particles to impact the bed.

[9] Combining equations (1)–(5) yields the composite expression of the saltation-abrasion model

$$ E = \frac{\rho_s q_{bc}^2 e_f Y}{L_b k_v \sigma_Y^2} \left(1 - \frac{q_b}{q_{bc}}\right). $$

[10] Most important for the present study is evaluation of the saltation hop length $L_b$. Sklar and Dietrich [2004] compiled data from numerous experimental and theoretical studies on particle saltation [Francis, 1973; Abbott and Francis, 1977; Wiberg and Smith, 1985; Sekine and Kikkawa, 1992; Lee and Hsu, 1994; Nino et al., 1994; Hu and Hui, 1996] and found the best fit relationship to be

$$ \frac{L_b}{D} = 8.0 \left(\frac{\tau_s}{\tau_{c*}} - 1\right)^{0.88}, $$

where $D$ is the particle diameter and $\tau_s/\tau_{c*}$ is the transport stage. The nondimensional bed stress or Shields stress is given by

$$ \tau_s = \frac{u_s^2}{R g D}, $$

where $R = (\rho_s - \rho_f)/\rho_f$ is the submerged specific density of the sediment, $\rho_f$ is the density of the fluid, $g$ is the acceleration due to gravity, and $u_s$ is the bed shear velocity. The critical value of the Shields stress ($\tau_{c*}$) is the value of $\tau_s$ at the threshold of particle motion [Shields, 1936].

[11] In the saltation-abrasion model, particle-hop length is assumed to be infinite for particles transported in suspension. A flow is typically considered competent to suspend sediment if

$$ u_s/w_{st} \geq 1, $$
where \( w_{st} \) is the terminal settling velocity of the sediment [Bagnold, 1966]. Therefore, Sklar and Dietrich [2004] modified equation (7) to be

\[
\frac{L_b}{D} = 8.0 \left( \frac{\tau_*}{\tau_*^*} - 1 \right)^{0.88} \sqrt{1 - \left( \frac{u_s}{w_{st}} \right)^2}
\]

(10)

and the erosion rate (equation (6)) is zero if \( u_s/w_{st} \geq 1 \).

[12] The experimental particle trajectory data used to calibrate equation (10) does not extend into the regime \( u_s/w_{st} \geq 1 \), and thus the validity of equation (10) over equation (7) cannot be verified. We hypothesize that suspended sediment does contribute to bedrock erosion due to particle-bedrock impacts. In the next section, we develop this hypothesis and present a model for bedrock erosion from suspended and bed load sediment.

3. Total Load Erosion Model

[13] Our model development follows the assumptions and limitations of previous work on erosion by bed load discussed above. In particular, our model considers incision into a flat bed of unit area by impacts of single sized particles. The model is based on the concept that suspended sediment actually is not held in a fluid indefinitely. Instead, particles are continuously falling through the fluid due to gravitational settling and are advected toward the bed due to turbulence. Where \( u_s/w_{st} \geq 1 \), sediment travels both in suspension and bed load [Bagnold, 1966; van Rijn, 1984; Nino et al., 2003]. Therefore, the incision model is developed to include impacts by both bed load and suspended particles (i.e., the total load) under a wide range of conditions including \( u_s/w_{st} \geq 1 \).

3.1. Settling Flux

[14] During conditions of suspended sediment transport (i.e., \( u_s/w_{st} \geq 1 \)), particles do impact and interchange with the bed. Particles are entrained from the bed by coherent flow structures, which produce bursts of upward moving fluid [Grass, 1970; Jackson, 1976; Sumer and Deigaard, 1981; Nelson et al., 1995; Bennett et al., 1998]. As these structures dissipate, particles tend to settle toward the bed at a rate near their settling velocity in still water [e.g., Sumer and Deigaard, 1981; Nino and Garcia, 1996]. This gravitational settling results in a volumetric flux per unit area of sediment toward the bed given by

\[
f_s = c_b w_s,
\]

(11)

where \( c_b \) is the near-bed volumetric sediment concentration and \( w_s \) is the gravitational settling velocity of the sediment (which can be less than \( w_{st} \)). Despite this downward sediment flux, an equilibrium concentration of particles can be attained because there is a dynamic balance between the upward and downward fluxes of particles [Rouse, 1937; Smith and McLean, 1977; Parker, 1978; García and Parker, 1991; Bennett et al., 1998].

[15] This concept is well illustrated in the experiments of Einstein [1968] in which a recirculating flume was used to create a steady, uniform flow over an open framework and immobile gravel bed. The flow was highly turbulent and capable of suspending the silt that was introduced into the flume (\( u_s/w_{st} \) ranged from 74 to \( 7.2 \times 10^3 \)). Despite the fact that \( u_s/w_{st} \gg 1 \), the suspended particles did indeed impact the bed, as the turbid flows eventually clarified, and a steady state concentration profile was not attained. This was because the suspended silt settled through the gravel on the flume bed and the downward flux of sediment was not balanced by a commensurate entrainment flux from the bed.

3.2. Particle-Bed Impacts

[16] Few experimental studies have traced the flow paths of individual suspended particles, which, along with the stochastic nature of such trajectories, makes it difficult to directly formulate an effective particle hop length for suspension. Since classic suspension theory is based in terms of sediment concentration [Rouse, 1937], it is useful to formulate the impact rate as a function of sediment concentration instead of hop length. Following the above arguments and equation (11), the rate of particle impacts per unit bed area can be expected on average to be proportional to the product of the near-bed sediment concentration and the particle velocity normal to the bed,

\[
I_r = \frac{A_1 c_b w_s}{V_p}.
\]

(12)

The impact velocity normal to the bed (\( w_i \)) is used here as a measure of the particle velocity instead of the gravitational settling velocity (\( w_s \), as in equation (11)) because \( w_s \) might not be normal to the bed and impacts also can occur because of turbulent fluctuations (discussed in section 4.4). The coefficient \( A_1 < 1 \) accounts for the fact that some of the particles near the bed are advected upward because of lift forces.

[17] Equation (12) is not specific to suspension and also holds for bed load. For example, the downstream flux of bed load sediment can be written as

\[
q_b = c_b U_b H_b,
\]

(13)

where \( U_b \) is the vertically averaged stream-wise particle velocity and \( c_b \) is the vertically averaged sediment concentration within the bed load layer of height \( H_b \). The average bed load velocity can be scaled as

\[
U_b = \frac{L_b}{t_i} \approx \frac{A_2 w_s L_b}{H_b},
\]

(14)

where \( t_i \) is the timescale between bed impacts for an individual particle. \( A_2 < 1 \) accounts for the fact that the average fall velocity within the bed load layer might be less than the near-bed settling velocity, and that the total time between impacts should also include the particle ejection or risetime as well as the fall time. For example, Sklar and Dietrich [2004] suggest \( A_2 \approx 1/3 \). Combination of equations (4), (13), and (14) results in

\[
I_r = \frac{A_2 c_b w_s}{V_p},
\]

(15)

which is the same as equation (12) provided that \( A_2 w_s = A_1 w_i \).
3.3. Sediment Supply

[18] In alluvial rivers with an unlimited supply of sediment on the bed and a steady state concentration profile, the settling flux of sediment near the bed $f_s$ is equal to the entrainment capacity of the flow (per unit bed area) $F_e$, which can be written as

$$f_s = \alpha w_s,$$  \hspace{1cm} (16)

where $\alpha$ is a nondimensional sediment entrainment parameter (which is a function of $u*/w_s$ [e.g., Garcia and Parker, 1991]). Thus, where $f_s = f_e$, the near bed sediment concentration $c_b$ can be determined directly from the hydraulics and sediment size because combination of equations (11) and (16) results in $\alpha = c_b$. This is not the case in bedrock rivers.

[19] For supply limited conditions typical of bedrock rivers, the concentration of particles in suspension (and therefore $c_b$) is not dependent on the entrainment capacity (i.e., $\alpha > c_b$) and instead is determined by the sediment supply from the bed, banks, and upstream. By continuity

$$q_s = \int_{h_b}^H cudz = c_b U H \chi,$$  \hspace{1cm} (17)

where $q_s$ is the volumetric flux of sediment per unit channel width traveling in suspension, $c$ and $u$ are the depth-dependent concentration and downstream flow velocity per unit channel width averaged over turbulent fluctuations, $U$ is the depth-averaged flow velocity in the downstream direction, $H$ is the flow depth, $z$ is the coordinate perpendicular to the river bed, and $0 \leq \chi \leq 1$ is the integral that describes the vertical structure of velocity and concentration. In equation (17), it is assumed that the average stream-wise particle velocities are equal to the fluid velocities, as is typical for suspended sediment [e.g., McLean, 1992].

[20] To evaluate the impact rate given by equation (12), the near-bed sediment concentration must be known. Here, we seek an expression for the near-bed concentration by partitioning the supplied sediment flux into bed and suspended load. To simplify matching the concentration profile between the bed load and the suspended sediment above, we assume that within the bed load layer ($z \leq H_b$) sediment is well mixed [e.g., McLean, 1992] with a concentration of $c_b$ (Figure 1). Equations (13) and (17) can be summed and solved for $c_b$ as

$$c_b = \frac{q}{U H \chi + U_b H_b},$$  \hspace{1cm} (18)

where $q$ is the total volumetric flux of sediment traveling as both bed and suspended load per unit width, which is equivalent to the total sediment supply (per unit width) in the supply limited conditions considered here. Thus, inclusion of suspended sediment (rather than considering only bed load) reduces the near-bed sediment concentration and therefore the rate of impacts for a given sediment supply. Equation (18), however, predicts a finite near-bed sediment concentration for all flow conditions.

Figure 1. Schematic showing vertical profiles of sediment concentration $c$ (equation (26)) and velocity $u$ (equation (21)) for the conditions of the Eel River (Table 1) and for (a) 60-mm gravel and (b) 1-mm sand. Also shown are the calculated heights of the bed load layer $H_b$ (equation (25)), weighted average particle fall heights $H_f$ (equation (32)), flow depth $H$ (Table 1), and the near-bed sediment concentration $c_b$ (equation (18)).

3.4. Composite Expression for the Total Load Erosion Model

[21] Substituting equations (2), (3), (5), (12), and (18) into equation (1) yields the combined model for erosion by bed and suspended sediment

$$E = \frac{A_1 \rho_s Y}{k_c \sigma_f} \left( \frac{qw^3}{(UH \chi + U_b H_b)} \right) \left( 1 - \frac{q_b}{q_w} \right),$$  \hspace{1cm} (19)

where $q_b$ is found from equations (13) and (18) to be

$$q_b = \frac{q}{U_b H_b \left( UH \chi + U_b H_b \right)}.$$  \hspace{1cm} (20)

4. Empirical Expressions and Calculation Procedure

[22] Following Sklar and Dietrich [2004], the total load erosion model is explored here by holding some variables to constant values typical of a reference field site, the South
Table 1. Model Input and Output Values for Representative Field Case: South Fork Eel River, California

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel slope, S</td>
<td>0.0055</td>
</tr>
<tr>
<td>Channel width, W</td>
<td>18 m</td>
</tr>
<tr>
<td>Sediment discharge, q_s</td>
<td>8.9 \times 10^{-4} m^3/s</td>
</tr>
<tr>
<td>Flow velocity, U</td>
<td>2.2 m/s</td>
</tr>
<tr>
<td>Flow depth, H</td>
<td>0.95 m</td>
</tr>
<tr>
<td>Shear velocity, u_s</td>
<td>0.22 m/s</td>
</tr>
<tr>
<td>Rock tensile strength, \sigma_r</td>
<td>7 MPa</td>
</tr>
<tr>
<td>Young’s elastic modulus, Y</td>
<td>5.0 \times 10^6 MPa</td>
</tr>
<tr>
<td>Rock resistance parameter, k_c</td>
<td>1.0 \times 10^6</td>
</tr>
<tr>
<td>Critical Shields stress, \tau_{sc}</td>
<td>0.03</td>
</tr>
<tr>
<td>Sediment density, \rho_s</td>
<td>2650 kg/m^3</td>
</tr>
<tr>
<td>Water density, \rho_w</td>
<td>1000 kg/m^3</td>
</tr>
<tr>
<td>Kinematic viscosity of water, \nu</td>
<td>10^-6 m^2/s</td>
</tr>
<tr>
<td>Sediment size, D</td>
<td>60 mm, 1 mm</td>
</tr>
<tr>
<td>Transport stage, \tau_t/\tau_{sc}</td>
<td>1.7, 102</td>
</tr>
<tr>
<td>Particle fall height, H_f</td>
<td>79 mm, 38 mm</td>
</tr>
<tr>
<td>Terminal settling velocity, \nu_t</td>
<td>0.98 m/s, 0.13 m/s</td>
</tr>
<tr>
<td>Bed load velocity, U_b</td>
<td>1.26 m/s, 2.2 m/s</td>
</tr>
<tr>
<td>Bed load concentration, c_b</td>
<td>0.0089, 0.0151</td>
</tr>
<tr>
<td>Bed load layer height, H_b</td>
<td>72.3 mm, 14.5 mm</td>
</tr>
<tr>
<td>Bed load transport capacity, q_{bc}</td>
<td>1.0 \times 10^{-3} m^3/s, 3.8 \times 10^{-3} m^3/s</td>
</tr>
<tr>
<td>Erosion rate, E</td>
<td>31 mm/a, 10 mm/a</td>
</tr>
</tbody>
</table>

4.1. Flow Velocity

For turbulent boundary layer flow in a channel, the downstream velocity can be calculated as

\[ u = \frac{u_\ast}{\kappa} \ln \left( \frac{z}{z_0} \right), \]

where \( z_0 \) is a function of the boundary roughness and \( \kappa \) is von Karman’s constant (~0.41) (Figure 1). The shear velocity is calculated from \( u_\ast = (gH \sin \theta)^{1/2} \), where \( \theta \) is the channel-bed slope angle. Strictly speaking, equation (21) is only applicable to the lower ~20% of the water column, and an adjustment to the eddy viscosity could be made for the upper portion of the flow [e.g., Coles, 1956; Gefenbaum and Smith, 1986]. Modifications to the eddy viscosity could also be made because of stratification and form roughness [Vanoni, 1946; McLean, 1992; Wright and Parker, 2004]. For simplicity we assume that equation (21) is applicable throughout the water column and integrate to find the depth-averaged velocity

\[ U = \frac{1}{H} \int_{z_s}^{H} \frac{u_\ast}{\kappa} \ln \left( \frac{z}{z_0} \right) dz. \]  

For the following calculations we set \( z_0 = nD/30 \) with the empirical coefficient \( n = 3 \) [e.g., Kamphuis, 1974]. To hold the hydraulic conditions constant for \( D = 60 \text{ mm} \) and \( D = 1 \text{ mm} \), we evaluate the roughness using \( D = 60 \text{ mm} \) for both cases. This is done to simplify comparison. We suspect, however, that this might be an inaccurate parameterization of the hydraulic roughness in natural bedrock streams where the bed is only partially covered with sediment. Furthermore, roughness might be dominated by the banks, immobile boulders, or sculpted forms on the bed [Finnegan et al., 2007; Johnson and Whipple, 2007; Yager et al., 2007].

4.2. Bed Load Transport Capacity, Layer Height, Concentration, and Velocity

Many equations exist for the bed load transport capacity. Here, we use the relation of Fernandez Luque and van Beek [1976]:

\[ q_{bc} = 5.7(RgD^3)^{1/2}(\tau_{\ast} - \tau_{e})^{3/2}. \]

The sediment transport capacity for the two representative cases is found to be \( 1.0 \times 10^{-3} \text{ m}^2/\text{s} \) and \( 3.8 \times 10^{-3} \text{ m}^2/\text{s} \) for the 60-mm gravel and the 1-mm sand, respectively (Table 1).

The depth-averaged bed load velocity and layer height are given as empirical expressions by Sklar and Dietrich [2004] derived from several different bed load studies. The best fit relationships are

\[ U_b = 1.56(RgD^3)^{1/2}(\frac{\tau_{\ast}}{\tau_{e}} - 1)^{0.56}. \]
and

\[ H_b = 1.44D \left( \frac{\tau_*}{\tau_{*c}} - 1 \right)^{0.50}. \]  

The bed load velocities and layer heights for the two representative cases are found to be \( U_b = 1.26 \text{ m/s} \) and \( H_b = 72.3 \text{ mm} \) for the 60-mm gravel, and \( U_b = 2.6 \text{ m/s} \) and \( H_b = 14.5 \text{ mm} \) for the 1-mm sand (Table 1). For the sand, equation (24) predicts a bed load velocity that is greater than the depth averaged fluid velocity. The high transport stage for the sand (\( \tau_{*c} / \tau_* = 102 \)) is beyond the range of empirical data used to formulate equation (24). At large transport stages, particle velocities instead approach the fluid velocity [e.g., Bennett et al., 1998]. To account for this effect, we set \( U_b = U \) where equation (24) predicts \( U_b > U \). Likewise, in rare cases with large transport stages, large channel slopes, and small flow depths, the empirical equation (25) predicts a bed load layer height (i.e., a saltation hop height) that is greater than the flow depth. In reality, under these conditions the bed load layer likely occupies the entire depth of flow. Therefore, where this occurs we set \( H_b = H \). Using these expressions, the near-bed concentration of particles (equation (18)) is found to be 0.0089 and 0.0151 for the 60-mm gravel and the 1-mm sand, respectively (Table 1).

### 4.3. Vertical Structure of Suspended Load

[29] To evaluate the erosion rate, the vertical structure of the suspended sediments must be known. Here we use the most widely accepted expression for the vertical profile of suspended sediment, Rouse’s [1937] equation

\[ c = c_b \left( \frac{1 - \zeta_s}{\zeta_s} \right)^{P} \left( \frac{1 - \zeta_s}{\zeta_s} \right)^{\mu}, \]  

where \( \zeta_s = z/H \), \( \zeta_b = H_b/H \), and \( P = w_{st}/\beta k u_* \) is the Rouse parameter (Figure 1). To arrive at equation (26), Rouse balanced the entrainment and settling flux of suspended sediment, and scaled the entrainment flux as a diffusive process using a parabolic eddy viscosity profile for steady, uniform flow

\[ v_T = \beta u_* \kappa z (1 - z/H). \]  

The coefficient \( \beta \) is typically thought to be a constant of order unity and accounts for any differences between the diffusivity of momentum and sediment.

[30] As discussed above for the logarithmic velocity profile, several authors have argued that the Rouse profile should not apply because equation (27) is only applicable to the lower 10–20% of the water column. Nonetheless, experimental data support use of the Rouse equation throughout the water column, with \( \beta \) ranging from approximately 0.5 to 3 [Bennett et al., 1998; Graf and Cellino, 2002; Nezu and Azuma, 2004; Wren et al., 2004; Muste et al., 2005]. Because of the present uncertainty in the value of \( \beta \), we assume that \( \beta = 1 \) in the following calculations.

[31] To apply equation (26), the near-bed concentration \( c_b \) is calculated from equation (18), where the integral relating suspended sediment flux to the bulk parameters of the flow (\( \chi \)) can be found from equations (17), (21), and (26) as

\[ \chi = \frac{1}{UH} \int_{H_b}^{H} \left[ \frac{1 - \zeta_s}{\zeta_s} \right] \frac{u_{st}}{\kappa} \ln \left( \frac{z}{z_0} \right) dz. \]  

The resulting concentration profiles for the representative cases are shown in Figure 1. Because of the low transport stage, most of the 60-mm gravel is contained within the bed load layer. In contrast, a significant portion of the sediment extends above \( H_b \) for the 1-mm sand.

### 4.4. Particle Impact Velocity

[32] For saltating sediment, Sklar and Dietrich [2004] used a scaling analysis combined with their empirical fits for \( L_b, U_b, \) and \( H_b \) to obtain an expression for the impact velocity,

\[ w_i = 0.8 (RgD)^{1/2} \left( \frac{\tau_*}{\tau_{*c}} - 1 \right)^{0.18} \left( 1 - \frac{w_{st}}{w_{mt}} \right)^{1/2}. \]  

Equation (29) cannot be used in our model because the empirical data used to calibrate the equation does not extend into the suspension regime.

[33] As an alternative approach, we consider impacts at the bed due to gravitational settling of particles and advection by turbulent eddies. First, we calculate the impact velocity due to gravitational settling directly from a momentum balance for a falling particle. It is important to calculate the settling velocity as a function of fall distance because large particles might not have sufficient settling distance to reach terminal velocity upon impact. The component of the particle settling velocity normal to the bed can be calculated from a balance between the forces of gravity and drag as

\[ w_i = w_{st} \cos \theta \sqrt{1 - \exp \left( - \frac{3C_d \rho g h_f}{2 \rho_p D \cos \theta} \right)}, \]

where

\[ w_{st} = \left( \frac{4 R_g D}{3 C_d} \right)^{1/2} \]

is the terminal settling velocity (see Appendix A). The drag coefficient \( C_d \) depends on the particle Reynolds number and grain shape, and we calculate \( C_d \) from the empirical formula of Dietrich [1982] for natural sediment (Corey shape factor = 0.8, Powers roundness scale = 3.5).

[34] The particle velocity given by equation (30) depends on the distance over which a particle falls \( (H_f) \). In a combined bed load and suspension flow, particles are falling from all distances above the bed \( (z) \), from the top of the bed load layer to the depth of the flow \( (H_b \leq z \leq H) \). For uniform-size sediment, the average height from which particles fall should depend on the fraction of particles that are suspended to that elevation. Therefore, the shape of the steady state concentration profile should reflect the relative
heights that particles are suspended (and therefore their fall distances). To incorporate these effects, we propose an average fall distance that is weighted by the proportion of the total near-bed sediment \( c_b \) that is suspended to that height,

\[
H_f = \frac{1}{c_b} \int_{H_b}^{H_f} z \frac{dc}{dz} dz. \tag{32}
\]

If all sediment is bed load, equation (32) predicts, as expected, that all particles fall from the top of the bed load layer (i.e., \( H_f = H_b \)) because we assume that sediment is uniformly mixed within the bed load layer (i.e., \( \frac{dc}{dz} = 0 \) for \( z < H_b \)). For 60-mm gravel, \( H_f = 79.2 \text{ mm} \), which is only slightly greater than the bed load layer height (\( H_b = 72.3 \text{ mm} \)) (Figure 1). For 1-mm sand, \( H_f = 38.4 \text{ mm} \) and is greater than \( H_b = 14.5 \text{ mm} \), because the high transport stage for the sand results in more of the load carried above \( H_b \).

[35] In addition to gravitational setting of particles, turbulent fluctuations can affect the average particle-bed impact rate by advecting particles both away from the bed (reducing the impact rate) and toward the bed (increasing the impact rate). Rigorously characterizing the temporal and spatial variability in turbulent fluctuations is beyond the scope of this paper. As a first-order approach, we assume that turbulent fluctuations follow a Gaussian distribution [e.g., Bridge and Bennett, 1992; Nezu and Nakagawa, 1993; Cheng and Chiew, 1999]. The probability density function \( P(w) \) of velocity fluctuations \( (w') \) is given by

\[
P(w') = \frac{1}{\sqrt{2\pi}\sigma_w} \exp\left(-\frac{(w')^2}{2\sigma_w^2}\right), \quad \tag{33}
\]

where \( \sigma_w = \sqrt{\overline{w^2}} \) is the standard deviation of velocity fluctuations perpendicular to the bed and the overbar denotes a time average. The standard deviation of these velocity fluctuations has been shown to be approximately equal to \( \overline{u_*} \) in open channel flow [Nezu and Nakagawa, 1993], which we employ here (i.e., \( \sigma_w = \overline{u_*} \)).

[36] To calculate the particle impact velocity, we assume that particles follow the fluid, so that equation (33) can be used to calculate the probability of fluctuations in particle velocity, as well as fluid velocity. Furthermore, we assume that inertial forces dominate near the bed so that particles impact the bed and are not swept laterally with the flow (see section 6 for discussion). With these assumptions, the average impact velocity can be found by summing the component of the gravitational settling velocity perpendicular to the bed with the turbulent velocity fluctuations (which by definition are perpendicular to the bed), and integrating over all possible values of fluctuations as

\[
w_i = \int_{-\omega_i}^{\omega_i} P(w)Pdw'. \tag{34}
\]

The upper limit of integration was chosen because it incorporates very near 100% of the positive fluctuations (Figure 2). The lower limit defines the condition \( w' + w_s = 0 \); where \( w' + w_s < 0 \), particles are moving upward and the impact velocity and impact rate are zero. Thus, despite the fact that the Gaussian distribution is symmetrical, the mean impact velocity can deviate from the gravitational settling velocity because the impact velocity must be nonnegative (Figure 2).

[37] The deviation of the impact velocity from the gravitational settling velocity is more important when considering that the erosion rate scales with the impact velocity cubed (equation (19)). The erosion rate depends on the cube of individual particle velocities (i.e., \( w_i^3 + w_s^3 \)), however, and not the average impact velocity \( w_i \). Thus to formulate an average impact velocity that scales with the erosion rate, we define the effective impact velocity by nonlinear averaging, as

\[
w_{i,\text{eff}} = \left[ \int_{-w_s}^{6\sigma_w} (w' + w_s)^3 Pdw' \right]^{1/3}. \tag{35}
\]
Similar to the turbulent fluctuations, the gravitational settling velocity also could be weighted to account for the cubic dependence of erosion rate on impact velocity, rather than using the velocity for the linearly averaged fall distance calculated in equation (32). We found, however, that accounting for this has a negligible effect on the results.

[38] For the gravel at \( \tau_s/\tau_e = 1.7 \), the gravitational fall velocity is sufficiently large compared to the turbulent fluctuations, so that only the very tail of the distribution is within the regime \( w^* + w_i < 0 \) (shown as a thick dashed line in Figure 2a). The result is that turbulent fluctuations tend to cancel, and therefore \( w_i \approx w_s \). This notwithstanding, the minor asymmetry in the probability density function results in an average impact velocity that is slightly greater than that predicted from gravitational settling alone. This effect is enhanced for the effective impact velocity \( w_{i,eff} \) due to the cube of the velocity fluctuations (Figure 2a). Both \( w_i \) and \( w_s \) are smaller than \( w_{st} \) for the gravel because the fall distance is not sufficient for particles to reach terminal settling velocity.

[39] Turbulence has a much stronger effect on the predicted impact velocities for the sand owing to the large transport stage (Figure 2b). A substantial portion of the distribution of turbulent fluctuations is within the regime \( w^* + w_i < 0 \). Because impact velocities must be positive, the distribution is truncated at \( w^* + w_i = 0 \) before integrating. This results in an asymmetric distribution, and an average impact velocity and effective impact velocity that are much greater than the gravitational settling velocity (i.e., \( w_{i,eff} > w_i > w_s \)) (Figure 2b). The fall distance is sufficient for the sand that the gravitational fall velocity is equal to the terminal settling velocity (i.e., \( w_i = w_{st} \)).

[40] The velocities calculated above are a function of transport stage for the case of particles falling from the top of the bed load layer (i.e., \( H_f = H_b \)) (Figure 3). For gravitational settling \( (w_s) \), the velocity increases as the bed load layer height increases (equation (25)) until a transport stage of about 10, beyond which particles are calculated to fall at the terminal velocity. The average impact velocity \( w_i \) and the effective impact velocity \( w_{i,eff} \) are nearly equal to the gravitational settling velocity for low transport stages \( (\tau_s/\tau_e < 10) \) because \( u_s \) is small. However, these velocities deviate significantly from the gravitational settling velocity where \( w_i \approx w_{st} < 0 \) because the distribution in particle velocities becomes increasingly asymmetric. The result is that \( w_i \) and \( w_{i,eff} \) are significantly greater than the terminal settling velocity for large transport stages. Note that all velocity measures calculated herein (i.e., \( w_s \), \( w_i \), and \( w_{i,eff} \)) converge with the predictions of the empirical equation (29) at low transport stages, which is expected since this is the regime in which it was calibrated. Equation (29) predicts an impact velocity of zero at large transport stages (i.e., \( u_s > w_{st} \)), which contrasts with the velocity model proposed herein.

4.5. Bedrock Erosion by Total Load

[41] Finally, to calculate the erosion rate, \( w_{i,eff} \) replaces \( w_i \) in equation (19) resulting in

\[
E = \frac{A_1 \rho_Y Y}{k_r \sigma_Y} \left[ q_{bc}/(UH_X + U_bH_b) \right] \left[ \frac{w_{i,eff}}{(gD)} \right]^3 \left[ 1 - \frac{q_b}{q_{bc}} \right].
\]

Equation (36) can by nondimensionalized as

\[
E^* = \frac{E \sigma_Y^2}{\rho_Y (gD)^{3/2}} = \frac{A_1}{k_r q_h} \left[ \frac{q_{bc}}{(UH_X + U_bH_b)} \right] \left[ \frac{w_{i,eff}}{(gD)} \right]^3 \left[ 1 - \frac{q_b}{q_{bc}} \right].
\]

This reveals that \( E^* \) is a function of the three dimensionless quantities shown in brackets: (1) the normalized sediment supply or equivalently the near-bed sediment concentration (see equation (18)), (2) the normalized effective impact velocity cubed, and (3) the relative supply of bed load. By introducing the empirical expressions proposed in section 4, \( E^* \) also can be shown to be a function of particle size, transport stage, relative sediment supply \( q/q_{bc} \), and channel-bed slope (or equivalently flow depth for a given transport stage). The dependency on flow depth and channel slope was not revealed in the salitation-abrasion model (equation (6)). In the total load model, it arises because both the near-bed sediment concentration and the gravitational fall velocity are sensitive to the vertical distribution of sediment in the water column, which in turn is a function of flow depth.

5. Model Results

[42] Model results are shown for the two cases, where the total load is composed of either 60-mm gravel or 1-mm...
The predicted erosion rates are given in millimeters per year; however, these rates are instantaneous and have not been multiplied by an appropriate intermittency factor for events that cause erosion. For the representative event of the South Fork Eel River, the instantaneous erosion rates for the gravel and sand are predicted to be 31 and 10 mm/a (Table 1), respectively. This yields an annual average erosion rate of 1.9 and 0.6 mm/a using an appropriate intermittency factor for the Eel River of 0.06 (see Sklar [2003] and Sklar and Dietrich [2004] for details). These predicted erosion rates seem reasonable given the average landscape lowering rate of 0.9 mm/a [Merritt and Bull, 1989].

To explore model predictions over a wide range of parameter space, we vary sediment supply, flow depth, or channel slope for a given grain size and hold the other variables to constant values specified for the Eel River (Table 1). In addition to our total load erosion model, the predictions of the saltation-abrasion model are shown for comparison, and we set $A_1 = A_2 = 0.36$. The integrals in equations (22), (28), (32), (34), and (35) are solved numerically.

### 5.1. Effect of Transport Stage

For a given grain size and absolute sediment supply (Table 1), the erosion rate is a function of transport stage, which in turn is a function of channel slope and flow depth. The dependence of erosion rate on transport stage is explored here for a constant slope example (solid lines in Figure 4; $S = \tan \theta = 0.0053$) and a constant flow depth example (dashed lines in Figure 4; $H = 0.95$ m).

![Figure 4](image-url) Log-log plot of erosion rate as a function of transport stage for 60-mm gravel and 1-mm sand. Two cases are shown for each particle size. For the first, shown by solid lines, the channel slope is $S = 0.0053$ and the flow depth varies with transport stage. For the second case, shown by dashed lines, the flow depth is $H = 0.95$ m and the channel slope varies with transport stage. For all cases, the sediment supply is 8.9 $m^2/s$. The saltation-abrasion model is shown only for 60-mm gravel because it predicts near zero erosion for the 1-mm sand at all transport stages. The black circles are the conditions for the representative field case of the Eel River (Table 1).

For 60-mm gravel, the total load model predicts zero erosion at transport stages $\tau_0/\tau_{e_0} \leq 1.5$ (Figure 4) because the transport capacity is less than the supply of sediment (Table 1), and the bed is therefore predicted to be covered with sediment. As transport stage increases, the rate of erosion increases as the bedrock becomes rapidly exposed. The rate of erosion initially peaks at $\tau_0/\tau_{e_0} \approx 2.5$ with an erosion rate of $\sim 70$ mm/a. For larger transport stages (but smaller than $\tau_0/\tau_{e_0} \approx 50$) the models predict a decreasing erosion rate with transport stage (Figure 4). This is because, for a constant sediment load, more sediment is held in the upper water column (i.e., $H_b$ and $U_b$ increase in equation (18)), sediment is advected over the bed at a faster rate (i.e., $U$ and $H$ increase in equation (18)), and therefore the near-bed sediment concentration and the impact rate per unit bed area decrease with increasing transport stage.

The decrease in sediment concentration with increasing transport stage is more significant for the constant slope case as compared to the constant depth case (Figure 5). An increased flow depth, in addition to transport stage, results in a reduction in near-bed sediment because a greater suspended load can be transported (i.e., $H$ increases equation 18). In calculating the erosion rate, however, the reduction in $c_b$ is offset by the increasing impact velocity with transport stage (Figure 3). For the constant depth case, the increased impact velocity more than compensates for the decrease in $c_b$ at large transport stages ($\tau_0/\tau_{e_0} > 50$), resulting in an ever increasing erosion rate with transport stage for steep slopes ($S > \sim 0.15$) (Figure 4). Where slope is held constant, the erosion rate decreases (but remains nonzero) with increasing transport stage.

![Figure 5](image-url) Log-log plot of near-bed sediment concentration as a function of transport stage for 60-mm gravel and the 1-mm sand. Two cases are shown for each particle size. For the first, shown by solid lines, the channel slope is $S = 0.0053$ and the flow depth varies with transport stage. For the second case, shown by dashed lines, the flow depth is $H = 0.95$ m and the channel slope varies with transport stage. For all cases, the sediment supply is 8.9 $m^2/s$. The black circles are the conditions for the representative field case of the Eel River (Table 1).
5.2. Effect of Sediment Supply

With constant values of transport stage, flow depth, and channel slope (Table 1), the saltation-abrasion model predicts a peak in erosion rate where the supply of sediment is one half the bed load transport capacity (i.e., $q = q_{bc}$) (Figure 6). The erosion rate goes to zero where the sediment supply is zero because there are no particle impacts. At high relative supply, the erosion rate also goes to zero because of bed coverage. This upper limit is $q = q_{bc} - 1$ for the saltation-abrasion model because all of the supplied sediment is assumed to travel as bed load (i.e., $q = q_b$). The total load model, however, indicates that erosion is possible where the supply exceeds the bed load capacity because some of the load is transported in suspension (Figure 6). Thus, the bed load flux $q_b$ can be less than the bed load capacity, even though the total load $q$ is not. This effect is more pronounced for the sand than for the gravel because a greater proportion of the sediment load is traveling in suspension (because of the higher transport stage). Erosion persists for the sand until the supply is nearly double the bed load transport capacity (Figure 6).

5.3. Effect of Grain Size

Where sediment supply, flow depth and channel slope are set to constant values for the reference field site (Table 1), the models predict a peak in erosion rate for particle sizes of about $D = 45$ mm (Figure 7). The erosion rate goes to zero for larger grain sizes because the flow is not competent to transport these sizes, such that the bed is predicted to be covered with alluvium. Because of the dependence of erosion rate on gravitational settling velocity, the erosion rate also decreases for finer grain sizes. The saltation-abrasion model predicts zero erosion for sizes smaller than about 2 mm because $u_w/w_{st} > 1$. In contrast, the total load model predicts a finite erosion rate for all particle sizes.

5.4. Effect of Flow Depth and Channel Slope

In contrast to the saltation-abrasion model, the total load model is a function of flow depth, or channel slope for a given transport stage (Figure 8). Flow depth affects the erosion rate in two competing ways. On one hand, the impact rate depends on the near-bed sediment concentration, which, among other things, is a function of flow depth. For the same bed shear stress, particle size and sediment supply, a deeper flow on a smaller slope will have less sediment near the bed and a lower impact rate than a shallower flow on a steeper slope. On the other hand, for particles that do not attain...
terminal velocity, the particle impact velocity is larger in deeper flows because of the greater fall distance.

For 60-mm gravel with a constant transport stage and sediment supply, the erosion rate is nearly constant at low channel slopes, but decreases as slope increases (Figure 8). For this sediment size, the increased impact rate in shallower and steeper flows is more than compensated for by the drop in impact velocity (because of the reduced fall distance), resulting in a decrease in erosion rate with increasing slope. In contrast, finer sediment rapidly reaches terminal velocity so that changes in flow depth have little effect on impact velocity. Thus, the erosion rate for 1-mm sand is predicted to increase with increasing slope because of the greater impact rate that results from the increased near-bed sediment concentration in steeper flows with smaller flow depths (Figure 8).

The abrupt increase in erosion rate for the gravel at $S \approx 0.04$ and $H \approx 0.2$ m (Figure 8) occurs where the bed load velocity given by equation (24) is predicted to be larger than the fluid velocity (equation (22)), and therefore we set $U_b = U$ (see section 4.2). The jump in erosion rate is because the bed load velocity is predicted to increase with transport stage (regardless of flow depth), whereas $U$ systematically decreases with increasing slope (and decreasing flow depth). This results in a heightened near-bed sediment concentration and erosion rate. The second jump in erosion rate at $S \approx 0.07$ and $H \approx 0.07$ m (Figure 8) is where $H_b = H$, which again results in a heightened near-bed sediment concentration with increasing slope (and decreasing flow depth).

### 5.5. Contour Plots of Erosion Rate

To evaluate the total load model over a wide range of parameter space, Figures 9–11 show contours of erosion rate versus transport stage and relative sediment supply. The saltation-abrasion model shows a peak erosion rate at a relative sediment supply of 0.5 and a transport stage of $\tau_s/\tau_{sc} \approx 15$ for both the 1-mm sand and the 60-mm gravel (Figure 9). The peak erosion rate occurs at a slightly different transport stage for the two different sediment sizes because the relationship between transport stage and the onset of suspension is a function of the drag coefficient, which is grain-size dependent [Dietrich, 1982]. The erosion rate goes to zero at high and low transport stages because of the onset of suspension and the threshold of motion, respectively. The erosion rate goes to zero at high and low relative sediment supply because of the effects of bedrock coverage and particle impact rate, respectively (see Sklar and Dietrich [2004] for a detailed discussion).

The contour plots of the total load erosion model are strikingly different than the model that considers only bed load (Figures 10 and 11). Like the bed load model, the

![Figure 8. Erosion rate as a function of channel slope and flow depth for the 60-mm gravel (with a constant transport stage of 1.7) and the 1-mm sand (with a constant transport stage of 102) using a constant sediment supply (8.9 × 10^{-4} m^2/s). The saltation-abrasion model would plot as a horizontal line because it is not sensitive to the relative contributions of slope and flow depth in setting the transport stage. The black circles are the conditions for the representative field case of the Eel River (Table 1).](image)

![Figure 9. Contour plots of erosion rate in millimeters per year for the saltation-abrasion model versus transport stage and relative sediment supply for (a) 60-mm gravel and (b) 1-mm sand. The dashed lines are slices through parameter space that are shown on Figures 4 and 6. The black circles are the conditions for the representative field case of the Eel River (Table 1).](image)
erosion rate increases with increasing transport stage (with a constant channel slope) because the impact velocity increases with increasing flow depth (Figure 10). The erosion rate, however, does not decline at large transport stages for a given relative sediment supply. Instead, it increases because of the heightened impact velocity due to turbulence. The dashed lines on Figure 10 show the 2-D parameter space represented in Figures 4 and 6. These illustrate that an increase in transport stage results in a decrease in relative supply \( \frac{q}{q_{bc}} \), if the absolute sediment supply \( q \) is constant. This is the reason for the decrease in erosion rate at high transport stages in Figure 4. The contour plots, however, reveal that erosion rate can increase indefinitely with increasing transport stage, as long as the absolute sediment supply also increases with transport stage. In such a case, the erosion rate does not have a maximum value (Figure 10). Furthermore, at large transport stages \( \frac{\tau_s}{\tau_c} > 100 \), the erosion rate can be nonzero for sediment loads that are much larger than the bed load transport capacity.

6. Discussion

6.1. Entrainment Capacity for Total Load

Equation (36) contains a transport capacity for bed load \( q_{bc} \), in which erosion is zero if \( q_b > q_{bc} \) because of depositional cover. For flows with significant suspended sediment, the transport capacity of the total load is typically formulated in terms of a maximum near-bed sediment concentration instead of a maximum bed load flux [Smith and McLean, 1977; Parker, 1978; García and Parker, 1991]. This maximum sediment concentration can be found by equating equations (11) and (16), i.e., \( c_b = \alpha \), as discussed in section 3.3. For most of the model results shown, the near-bed sediment concentration does not exceed \( \alpha \), where \( \alpha \) is calculated using the empirical model of García and Parker [1991]. This, however, is not true for the 1-mm sand at small transport stages. For \( \frac{\tau_s}{\tau_c} < \sim 10 \), the bed is predicted to be covered with sediment (i.e., \( c_b > \alpha \)) and thus the erosion rate is zero (Figure 12). This indicates a need for an accurate model of the maximum near-bed sediment concentration for both bed load and suspension conditions, and particularly the transition in between.

6.2. Viscous Damping of Impacts

Sklar and Dietrich [2004] assumed that there was not a threshold kinetic energy required to cause erosion in their
and is a measure of the particle inertia relative to the viscous force exerted on the particle from the fluid, where $\nu$ is the kinematic viscosity of the fluid ($10^{-6}$ m$^2$/s) and $w_p$ is the particle velocity. Both Schmeeckle et al. [2001] and Joseph and Hunt [2004] found that impacts from glass spheres were partially damped for $St < 100$, and completely damped for $St < 30$. Schmeeckle et al. [2001] also show that data are more scattered for natural sediment because of their nonspherical nature.

[59] If the erosion rate is set to zero for particle impacts with $St < 30$ (where $w_p = w' + w_s$ in equation (25)), the 1-mm sand is predicted to cause no erosion for transport stages less than about 3 (Figure 13a). For larger transport stages the sand does erode the bed because the enhanced impact velocity due to turbulence increases the Stokes number to $St > 30$. Viscous damping apparently has no effect on the 60-mm gravel because the gravitational settling velocity is great enough that $St > 30$ for all transport stages.

[60] To assess a potential threshold energy needed to cause erosion, it is useful to compare the model predictions to the abrasion mill experiments of Sklar and Dietrich [2001] (Figure 13b). The experiments were performed by mechanically stirring sediment and water in a cylindrical basin with a bedrock floor. Particle size was varied whereas the total volume of sediment, which is equivalent to $q$ in a closed system, was held constant. The saltation-abrasion model matches the data well for large particle sizes, but predicts zero erosion for the medium sand ($D = 0.4$ mm) because it was in suspension. The total load erosion model, on the other hand, captures the measured finite erosion for the medium sand ($D = 0.4$ mm) because it was in suspension. The total load erosion model, on the other hand, captures the measured finite erosion for the medium sand (Figure 13b), but over predicts the erosion rate. Although the fit seems better by including a Stokes number cutoff (Figure 13b), it is nonetheless difficult to evaluate whether the data support this threshold. For example, Sklar and Dietrich [2004] reported that fine sand ($D = 0.2$ mm) did not produce wear above their detection limit ($\sim 10^{-3}$ g/h), and model on the basis of abrasion mill experiments [Sklar and Dietrich, 2001], an assumption that we adopted in the total load erosion model. Nonetheless, considering the fine particles addressed here, it is possible that some impacts might be viscously damped. Theoretical and experimental results suggest that particle-wall impacts can be viscously damped, and the degree to which is a function of the particle Stokes number [Davis et al., 1986; Lian et al., 1996; Schmeeckle et al., 2001; Joseph and Hunt, 2004].

For spheres impacting a wall, the Stokes number can be written as

$$St = \frac{\rho w_p D}{9 \rho \nu}$$

(38)

Figure 11. Contour plots of erosion rate in millimeters per year for the total load erosion model for (a) 60-mm gravel and (b) 1-mm sand. The dashed lines are slices through parameter space that are shown on Figures 4 and 6. The black circles are conditions for the field case of the Eel River (Table 1). The flow depth is held constant at $H = 0.95$ m, so that the transport stage is a function of channel slope. The vertical axes differ for the 60-mm gravel and the 1-mm sand.

Figure 12. Contour plot of erosion rate in millimeters per year for the same model parameters as Figure 10b, except that erosion rate is set to zero where the near-bed sediment concentration exceeds the entrainment capacity of the flow (i.e., $c_b > \alpha$). The black circle represents the conditions for the field case of the Eel River (Table 1).
6.3. Implications for Natural Streams

The total load erosion model differs significantly from the saltation-abrasion model for high transport stages and high relative sediment supply rates. The large transport stages explored for the 60-mm gravel (e.g., $\tau_s/\tau_c \gg 1$) most likely occur during relatively large floods or in steep mountain terrain. For example, the bed shear stress for the Bonneville flood of the western United States has been estimated to be 2500 Pa [O’Connor, 1993]. We calculate that this flood was competent to suspend 150-mm cobbles (i.e., $u_s/w_{st} = 1$, using the $w_{st}$ relation of Dietrich [1982] for natural sediment), which is consistent with Bonneville flood deposits [O’Connor, 1993]. During this event, 60-mm gravel was at a transport stage of $\tau_s/\tau_c = 85$, and 1-mm sand was at $\tau_s/\tau_c = 5.2 \times 10^3$. In mountain terrains, such large bed stresses can be achieved more readily. For example, during Typhoon Bilis in 2000, which has a recurrence interval of about 20 years, the reach averaged bed stress of the LiWu River in Taiwan was about 2300 Pa [Hartshorn et al., 2002], making this more frequent event nearly as competent as the Bonneville flood in suspending gravel. In fact, the maximum across channel erosion rates during Typhoon Bilis occurred several meters above the channel thalweg, suggesting that erosion by suspended particles outpaced bed load [Hartshorn et al., 2002].

The total load erosion model is also important to consider for fine sediment, which can be at large transport stages during more regular flow events. For the characteristic event on the Eel River, the 1-mm sand is calculated to have a transport stage of $\tau_s/\tau_c = 102$. For these conditions the saltation-abrasion model predicts no erosion, whereas the total load model predicts an instantaneous erosion rate of approximately 10 mm/a. The erosion rate due to sand is smaller than that predicted for gravel (for the same sediment supply), but it is nonetheless significant (Table 1). The total load model might be particularly important for rivers where the load is dominated by sand, for example, because of granite or sandstone lithologies.

Deciphering between the relative roles of sand and gravel in fluvial erosion is beyond the scope of this paper. A significant limitation of the model is that it only considers sediment of a single size. It is clear from evaluation of the contour plots (Figures 10 and 11), that there are regimes in parameter space where erosion from sand can be greater than that from gravel, but this depends on the relative supply of each. Since finer particles often dominate the load of a river, it seems possible that erosion from sand might be as or more important than erosion from gravel. Incorporating multiple particle sizes and particularly bimodal distributions of sediment into the model, however, is not trivial. For example, it has been shown that the addition of sand into a gravel bed can lead to nonlinear increases in the transport capacity of both sizes [Wilcock et al., 2001; Wilcock and Crowe, 2003]. Extending the erosion model to multiple particle sizes would require reassessment of several formulas used herein to account for mixture and bimodal effects (over a bedrock bed) including the bed load transport capacity, the hydraulic roughness of the bed, the bed load velocity and the bed load layer height. Experimental and field measurements are needed to guide future theoretical work.

The total load erosion model is most sensitive to the prediction of impact velocity, and this is also a topic that deserves future study. For example, our characterization of particle fluctuations as a Gaussian distribution is undoubtedly oversimplified. The degree to which particles detach...
from the fluid near the boundary likely depends on the relative particle response time compared to the fluid turbulence timescale (i.e., a particle Stokes number) [e.g., Crowe et al., 1996]. In addition, local turbulent fluctuations can be intense, especially above a nonuniform bed. The model does not incorporate changes in hydraulic roughness or turbulence due to sediment cover or bed forms. Erosion of protruding pieces of bedrock is likely to be much more efficient than erosion into a flat bed (as assumed herein), because the impact velocity should scale with the mean flow rather than turbulence intensity or the settling velocity [e.g., Anderson, 1986]. Furthermore, erosion by suspended sediment could be substantial over bed forms such as flutes [Johnson and Whipple, 2007].

[65] Where it differs from the saltation-abrasion model, the total load erosion model should have significant implications for predicting river channel morphology. For example, variations of the saltation-abrasion model have been used to model knickpoint migration in bedrock rivers [e.g., Chatanantavet and Parker, 2005; Gasparini et al., 2007; Crosby et al., 2007], and the total load model is likely to make different predictions owing to the large transport stages that typify these steepened reaches. It has been suggested, for example, that hanging valleys might form because, on the basis of the saltation-abrasion model, steepened reaches have lower erosion rates because of increased particle hop lengths and decreased impact rates [Wobus et al., 2006; Crosby et al., 2007]. The total load erosion model, however, suggests the opposite: erosion rates increase with increasing channel slope and transport stage (at least for large transport stages, e.g., Figure 4) because of the advection of suspended particles toward the bed by turbulent eddies. Some support for this finding comes from the experiments of Chatanantavet and Parker [2006], where the erosion rate was found to increase with increasing slope, even for the case of a constant transport stage.

[66] Although the total load erosion model offers insight into channel dynamics, we caution against using it (or other fluvial abrasion models) for quantitative estimates in steep reaches with large roughness to depth ratios (i.e., k/fu). In these cases, descriptions of flow resistance [e.g., Bathurst, 1985], sediment transport capacity [Yager et al., 2007], and incipient sediment motion [Lamb et al., 2008] are likely to be different that the formulas used herein. Moreover, at near vertical slopes, other processes such as plunge pool erosion [e.g., Lamb et al., 2007] are probably more important than fluvial abrasion.

7. Conclusions

[67] We have developed a mechanistic model for fluvial bedrock incision by suspended and bed load sediment. Particles are considered to impact the bed because of gravitational settling and advection by turbulent eddies, the latter of which dominates at high transport stages. The model predicts that the erosion rate is a function of three dimensionless quantities for a given grain size: transport stage (rconstant), relative sediment supply (q/QSW), and channel slope. Inclusion of suspension is important for high transport stages (i.e., large floods, steep slopes, or small particle sizes) and high relative sediment supply rates. For a given ratio of sediment supply to transport capacity, the erosion rate is predicted to increase with transport stage because of the heightened impact velocity due to turbulent fluctuations and does not taper to zero as predicted in the saltation-abrasion model. For most cases, erosion rates increase more rapidly with transport stage by increasing slope and fixing depth, rather than the opposite. This depth (or slope) dependency on erosion rate arises because both the near-bed sediment concentration and the particle fall velocity are sensitive to the vertical distribution of sediment in the water column. The total load erosion model predicts that erosion can be substantial where the sediment supply exceeds the bed load transport capacity because a portion of the load is carried in suspension.

Appendix A: Fall Velocity

[68] The acceleration of a falling particle can be calculated from the difference between the gravitational acceleration of the particle and deceleration due to drag

\[
\frac{dw}{dt} = C_1 - C_2w^2, \tag{A1}
\]

where \(w\) is velocity in the vertical dimension, \(g\) is the acceleration due to gravity and \(C_1\) and \(C_2\) are given by

\[
C_1 = \frac{(\rho_s - \rho_f)}{\rho_s}g \tag{A2}
\]

\[
C_2 = \frac{1}{2} C_d \rho_f A_s \frac{V_p}{w} \tag{A3}
\]

where \(C_d\) is a drag coefficient, \(\rho_f\) is the density of the fluid that the particle is falling through, \(\rho_s\) is the particle density, \(A_s\) is the cross sectional area of the particle perpendicular to fall velocity, and \(V_p\) is the volume of the particle. We are interested in the acceleration over a certain fall distance rather than over a certain fall time. Equation (A1) can be written in terms of vertical distance \(z\) (positive downward) by substituting \(dt = dz/w\), which yields

\[
w \frac{dw}{dz} + C_2w^2 = C_1. \tag{A4}
\]

To solve equation (A4) analytically, we assume that \(C_2\), and therefore \(C_d\), is not a function of \(z\). In reality \(C_d\) should vary as particles accelerate and the particle Reynolds number increases. Using a simple numerical integration, we found that accounting for a variable drag coefficient typically has less than a 10% effect on settling velocity. We therefore assume that \(C_d\) is a constant for a given particle size and solve the nonlinear ordinary differential equation as

\[
w = \sqrt{\frac{C_1}{C_2} \left(1 - \exp(-2C_2z)\right)}, \tag{A5}
\]

where the boundary condition \(w(z = 0) = 0\) has been applied. Substituting equations (A2) and (A3) into equation (A5), assuming spherical particles (i.e., \(V_p/A_s = 2D/3\)), defining the
fall distance as \(z = H_f\cos \theta\), and taking the component normal to the bed results in equation (30).

**Notation**

- \(A_x\): cross sectional area of a sediment particle (L^2);
- \(c\): volumetric sediment concentration (dimensionless);
- \(c_b\): near-bed volumetric sediment concentration (dimensionless);
- \(C_d\): drag coefficient (dimensionless);
- \(D\): sediment diameter (L);
- \(E\): rate of vertical erosion (LT^{-1});
- \(I_F\): impact rate per unit bed area (L^{-2}T^{-1});
- \(F_e\): fraction of exposed bedrock (dimensionless);
- \(g\): acceleration due to gravity (LT^{-2});
- \(H\): depth of flow (L);
- \(H_b\): thickness of the bed load layer (L);
- \(H_f\): particle fall distance (L);
- \(L_h\): particle saltation hop length (L);
- \(n\): roughness coefficient (dimensionless);
- \(P\): rouse parameter (dimensionless);
- \(q\): volumetric sediment supply per unit channel width (L^2T^{-1});
- \(q_b\): volumetric bed load flux per unit channel width (L^2T^{-1});
- \(q_{bc}\): volumetric bed load transport capacity per unit channel width (L^2T^{-1});
- \(q_s\): volumetric suspended load flux per unit channel width (L^2T^{-1});
- \(q_w\): volumetric water discharge per unit channel width (L^2T^{-1});
- \(R\): submerged specific density of sediment (dimensionless);
- \(S\): channel-bed slope (dimensionless);
- \(S_t\): particle Stokes number (dimensionless);
- \(t_i\): time between particle impacts (T);
- \(u\): stream-wise flow velocity (LT^{-1});
- \(U\): depth-averaged stream-wise velocity (LT^{-1});
- \(U_h\): depth-averaged stream-wise bed load velocity (LT^{-1});
- \(u_s\): shear velocity (LT^{-1});
- \(V_i\): volume of eroded rock per impact (L^3);
- \(V_p\): volume of a particle (L^3);
- \(W\): channel width (L);
- \(w\): vertical velocity (LT^{-1});
- \(w_{st}\): terminal settling velocity of a particle (LT^{-1});
- \(w_i\): impact velocity of a particle at the bedrock interface (LT^{-1});
- \(w_i,\text{eff}\): effective impact velocity (LT^{-1});
- \(w_p\): particle velocity (LT^{-1});
- \(w_s\): velocity of a falling particle normal to the bed (LT^{-1});
- \(\nu\): velocity fluctuations perpendicular to the bed (LT^{-1});
- \(Y\): Young’s modulus of elasticity (ML^{-1}T^{-2});
- \(z\): height above the bed (L);
- \(z_0\): flow roughness parameter (L);
- \(\varepsilon_v\): energy to erode a unit volume of bedrock (ML^{-1}T^{-2});
- \(\sigma_T\): rock tensile strength (ML^{-1}T^{-2});
- \(\sigma_n\): standard deviation in vertical velocity fluctuations (LT^{-1});
- \(\alpha\): sediment entrainment parameter (dimensionless);
- \(\beta\): proportionality constant relating the diffusivity of momentum and sediment (dimensionless);
- \(k_v\): empirical rock erodibility coefficient (dimensionless);
- \(\kappa\): von Karman’s constant (dimensionless);
- \(\zeta_s\): relative height above the bed (dimensionless);
- \(\zeta_b\): relative height of the bed load layer (dimensionless);
- \(\nu\): kinematic viscosity of the fluid (L^2T^{-1});
- \(\nu_T\): turbulent eddy viscosity (L^2T^{-1});
- \(\rho_s\): density of sediment (ML^{-3});
- \(\rho_f\): density of fluid (ML^{-3});
- \(\tau_s\): shields stress (dimensionless);
- \(\tau_{sc}\): critical Shields stress for incipient sediment motion (dimensionless);
- \(\chi\): integral relating the flux of suspended sediment to \(c_{50}, H, \text{ and } U\) (dimensionless).

[69] Acknowledgments. This study was funded by NASA BioMars. We thank Michael Manga and Mark Stacey for insightful comments and Ben Crosby and Jens Turowski for helpful reviews.

**References**


Tinkler, K. J. (1997), Rockbed wear at a flow convergence zone in Fifteen Mile Creek, Niagara Peninsula, Ontario, *J. Geol.*, 105, 263–274.


W. E. Dietrich and M. P. Lamb, Department of Earth and Planetary Science, University of California, 307 McConic Hall, Berkeley, CA 94720, USA. (mpl@gps.caltech.edu)

L. S. Sklar, Department of Geosciences, San Francisco State University, 509 Thornton Hall, 1600 Holloway Avenue, San Francisco, CA 94132, USA.