Supporting Information

Wu et al. 10.1073/pnas.0900102106

SI Materials and Methods

Lottery Tasks in Experiment 2. The purpose of the decision-making session in experiment 2 was to reliably estimate both the subjective transformation of outcome [the value function, \(v(O)\)] and probability [the probability weighting function, \(w(p)\)]. To obtain reliable estimate of the parameters for \(v(O)\) and \(w(p)\), we design lottery pairs that span the range of outcomes from $30 to $150 and probability from 0.2 to 0.96.

There were 120 lottery pairs in our design, which are categorized into 4 types. We now describe them in detail.

**Type 1: The Common Ratio Task.** The lottery pairs were constructed based on the base pair \((p, s_x)\), a lottery with probability \(p\) of winning \(s_x\) or nothing, and \((q, s_y)\), a lottery with probability \(q\) of winning \(s_y\) or nothing. Let \((y/x) = (p/q) = 1.2; p = 0.24; k = 1, 2, 3, 4; and x be drawn from a uniform distribution within $[30,50]\) to construct the common ratio task. In Fig. S1, we show an example of the common ratio design. On rung 1, the subjects are asked to choose between 2 lotteries: \((0.24,30)\) and \((0.2,36)\). The probabilities are then multiplied by \(k (k = 2, 3, 4)\) in both lotteries to construct the subsequent rungs.

The common ratio task is often referred to as the multiplicative version of the Allais task, whereas the common consequence task of experiment 1 is the additive Allais. Each pair was presented 16 times, making the total number of presented pairs of this type of pairs 64.

From pilot results and Monte Carlo simulations, we found that data from type 1 lotteries alone do not provide sufficient constraint to estimate \(v(O)\) and \(w(p)\) across a wide range of parameter values. For this reason, we added 3 other types of trials.

**Type 2.** Create \((p, s_x)\) and \((q, s_y)\) such that \((p/q) = 1.2, p = 0.24, k = 1, 2, 3, 4\) and \((y/x) = m, m = 1, 1.1, 1.2, 1.3, 1.4\). For example, if \(k = 1, m = 3, and x = 50\), then the lottery pair is \((0.24,50)\) and \((0.2,150)\). In other words, we manipulated the ratio \((y/x)\) in addition to the common ratio task. Type 2 had a total of 32 pairs.

**Type 3.** Create \((p, s_x)\) and \((q, s_y)\) such that \((p/q) = 1.5, q = 0.2, k = 1, 2, 3\) and \((y/x) = 1.2\). For example, if \(k = 3\) and \(x = 40\), then the lottery pair is \((0.9,40)\) and \((0.6,40)\). Type 3 had 12 trials.

**Type 4.** Create \((p, s_x)\) and \((q, s_y)\) such that \((y/x) = 1.2\) and \(q = 0.2, (p/q) = k, k = 2, 3, 4\). For example, if \(k = 4\) and \(x = 40\), then the lottery pair is \((0.8,40)\) and \((0.2,40)\). Type 4 had 12 trials.

Parameter estimation. In every trial in both the motor and classical tasks, the subject had to chose one of 2 lotteries, \(A\) \((p, s_x)\) and \(B\) \((q, s_y)\). In cumulative prospect theory, the subjective transformation of outcome is modeled by a value function of the form

\[v(O) = \begin{cases} O^a, & O \geq 0 \\ -(O)^a, & O < 0 \end{cases} \]  

The distortion of probability is modeled by the probability weighting function \(w(p)\). Since its proposal, there have been several functional forms proposed. In this paper, we chose the form proposed by Prelec (1)

\[w(p) = \exp[(-\ln p)^\gamma], \quad 0 < p < 1. \]  

The cumulative prospect value of a lottery is the sum of the value of each outcome weighted by its decision weight. In the context of a 1 non-zero outcome lottery, the decision weight of the non-zero outcome is the probability weight associated with it. In the model, we assume that cumulative prospect value is a random variable with Gaussian noise \(\epsilon\) with mean 0 and standard deviation proportional to the \(v(O)w(p)\). To write out explicitly the cumulative prospect of both lotteries,

\[\psi(A) = v(x)w(p) + e_A\]  

\[\psi(B) = v(y)w(q) + e_B,\]  

where \(e_A = N(0, k v(x)w(p))\) and \(e_B = N(0, k v(y)w(q))\). Given that \(v(O)\) and \(w(p)\) each have 1 parameter, and 1 parameter \(k\) determines the noise standard deviation, we wish to estimate 3 parameters \((\alpha, \gamma, k)\) from each subject’s choice performance in the classical task and 3 parameters \((\alpha, \gamma, k)\) from the subject’s choice performance in the motor task.

We assumed that the subject’s decision rule in either task was determined by the difference between the 2 lotteries \(\Delta = \psi(A) - \psi(B)\). Specifically, we assumed that, if \(\Delta > 0\), the subject chooses \(A\), otherwise \(B\); \(\Delta\) is itself a Gaussian random variable with variance \(\sigma_\Delta^2 = \sigma_A^2 + \sigma_B^2\), the sum of variances of the 2 independent variables \(\psi(A)\) and \(\psi(B)\). The probability of choosing \(A\) can then be computed as

\[p_A = 1 - \frac{1}{\sqrt{2\pi\sigma_\Delta}} e^{-(\Delta/k)^2/2\sigma_\Delta^2}d\delta.\]  

Given that on the \(i\)th trial, the subjects have a choice response \(r_i\), let choosing \(A\) be denoted by \(r_i = 1\) and choosing \(B\) by \(r_i = 0\). Then we could write down the likelihood function

\[L(\alpha, \gamma, k) = \prod_{i=1}^{n} p_{\delta_i}^r(1 - p_{\delta_i})^{1-r_i}.\]  

The maximum likelihood estimate of \((\alpha, \gamma, k)\) is the choice of \((\alpha, \gamma)\) that maximizes the above likelihood function. We verified by Monte Carlo simulation that the selection of trials of 4 types described above allowed stable parameter estimates.

Fig. S1. An example of the common ratio design. In this example, the bottom rung is constructed of the lottery pair (0.24, $30) and (0.2, $36). The upper rungs in the ladder are constructed by multiplying the probability of the non-zero outcome by a constant to both lotteries. For example, the second rung is constructed by multiplying the probabilities by 2 to arrive at (0.48, $30) and (0.4, $36).