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An asymmetric information model of the bid-ask spread is developed for foreign exchange market subject to occasional government interventions. Traditional tests of the unbiasedness of the forward rate as a predictor of the future spot rate are shown to be inconsistent when the rates are measured as the average of their respective bid and ask quotes. Larger bid-ask spreads on Fridays are documented. Reliable evidence of asymmetric bid-ask spreads for all days of the week, albeit more pronounced on Fridays, are presented. The null hypothesis that the forward rate is an unbiased predictor of the future spot rate continues to be rejected. The regression slope coefficients increase toward unity, however, indicating a less variable risk premium.
The unbiasedness of forward foreign exchange rates as predictors of future spot rates, henceforth referred to as the unbiasedness hypothesis, has received considerable attention in the finance and economics literature; Assuming market efficiency and rational expectations, Fama (1984) decomposes the forward rate into an expected future spot rate and a risk premium. This decomposition suggests testing the unbiasedness hypothesis by regressing the percentage change in the spot rate onto the forward premium, defined as the percentage difference between the forward and spot rates. Under the null hypothesis, the intercept and slope coefficients of this regression will be equal to zero and unity, respectively.

The empirical evidence does not support the unbiasedness hypothesis. Using one month forward rates for different currencies relative to the U.S. dollar, the slope coefficients systematically differ from unity and, even more puzzling, are significantly negative. A time-varying risk premium might explain these results. Fama (1984) shows that the negative slope coefficients arise from the higher volatility of the risk premium relative to the volatility of the expected change in the spot rate. The volatility of the risk premium is hard to explain, however. Attempts to fit specific asset pricing models to the data have met little success. ¹

In this paper, we investigate whether the bid-ask spread explains, the strong rejection of the unbiasedness hypothesis. The foreign exchange markets are modeled as competitive dealer markets with asymmetrically Informed agents. Governments occasionally intervene in these markets, thus skewing the distribution of spot rate changes. Consequently, in the presence of traders who are better informed about pending government intervention; the expected future spot rate will not be located halfway between the bid and the ask forward quotes, i.e., the forward bid-ask spread will be asymmetric. The asymmetry will be accompanied by an increase in the bid-ask spread. Since governments usually reveal their intention to intervene during weekends when the foreign exchange markets are closed, the asymmetry and increase in the ‘bid-ask spread will be most pronounced on Fridays. ²

We investigate the implications of our model for empirical tests of


² Foster and Viswanathan (1990) Investigate market microstructure effects of insiders’ accumulating information over the weekend. In our framework, inside information is revealed during weekends. Consequently, our theoretical results differ markedly. In our model, for instance, the bid-ask spread is largest on Friday. In theirs, the price impact of order Bows is largest on Mondays. Our empirical results differ accordingly [see Foster and Viswanathan (1988)].
the unbiasedness hypothesis. Such tests usually employ average bid and ask quotes, thus implicitly assuming that the expected future spot rate is halfway within the forward bid-ask spread. A different location is shown to yield inconsistent estimates, invalidating inference on the unbiasedness hypothesis. A regression equation that explicitly models the bid-ask spread is proposed as an alternative.

The empirical evidence on four continental European currencies with respect to the French franc is consistent with the implications of the model; First, bid-ask spreads are found to be larger on Fridays (this is also true for other currencies). Second, evidence of asymmetric bid-ask spreads is presented. The expected future spot rate is closer to the forward ask or bid depending on the type of government intervention, in casu, devaluation or revaluation. Third, the null hypothesis that the forward rate is an unbiased predictor of the ‘future spot rate is still rejected. The results, however, indicate a less variable risk premium.

We should point out that the increase and corresponding asymmetry of the bid-ask spread could also be explained with a model based on transactions demand, as in Brock and Kleidon (1989), or inventory effects, as in Garman (1976) Amihud and Mendelson (1980) and Ho and Stroll (1981), but the evidence of this article conforms more readily to the asymmetric information paradigm. In addition, we want to emphasize we do not aim at questioning the existence of a risk premium in the foreign exchange market. Rather, we want to understand the high volatility displayed by the risk premium in the forward market especially relative to its low volatility in the futures market. Characteristics pertaining to the organization of the two markets, particularly the open outcry system as opposed to the dealer system, might explain the difference.

The paper is organized as follows. A model of the bid-ask spread in the foreign exchange market is developed in Section 1. The empirical implications of the model are derived in Section 2. The sample and the empirical evidence are presented in Section 3. Section 4 concludes the paper.

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3 The results obtained from the futures market contrast singularly with their forward market counterparts. See Hodrick and Srivastava (1985). McCurdy and Morgan (1989) model the time variation of the risk premium in the foreign exchange futures market. The findings on the unbiasedness hypothesis carry over to futures prices, i.e., the hypothesis is rejected. However, the slope coefficients of the regressions are positive and close to unity. The risk premium is, therefore, less volatile than expected change in the spot rate.

4 In a similar vein, Kamara (1988) examines the Treasury bill markets and finds that differences in market trading structures explain price disparities between futures and forward or spot markets.
1. A Model of the Bid-Ask Spread in the Foreign Exchange Market

1.1 Notation and assumptions

1.1.1 The spot currency. Let \( s_t \) be the spot value in domestic currency of one unit of foreign currency at time \( t \). The spot contract is assumed to have a value equal to \( s_T \) at time \( T \). Write

\[
s_T = s_t + \sum_{\tau=t+1}^{T} \eta_{\tau}^{*},
\]

where \( \eta_{\tau}^{*} = s_{\tau-1} \). Without additional assumptions, no restrictions can be imposed on the shocks, the \( \eta_{\tau}^{*} \)'s. In particular, they might have a nonzero drift, i.e., \( E[\eta_{\tau}^{*} | \phi_t] \neq 0 \). The expected shock conditional on the public information at time \( t \), denoted by \( \phi_t \), may be different from zero. Certain restrictions do follow from the assumption that participants in the foreign exchange market are risk neutral.

Let \( r_{t,\tau} \) be the yield at time \( t \) on a domestic and foreign pure discount bond maturing at time \( T \), respectively, and let \( \rho_{t,\tau} \) denote the ratio of the foreign and domestic interest rates at time \( t \), equal to \( (1 + r_{t,\tau}^{*})/(1 + r_{t,t}^{*}) \). After multiplying both sides of Equation (1) by \( \rho_{t,\tau}^{*} \),

\[
\rho_{t,\tau}^{*} s_T = \rho_{t,\tau}^{*} s_t + \sum_{\tau=t+1}^{T} \rho_{t,\tau}^{*} \eta_{\tau}^{*},
\]

a new shock \( \eta_{\tau} \) can be defined as

\[
\eta_{\tau} = \begin{cases} 
\rho_{t,\tau}^{*} \eta_{\tau}^{*} + s_{\tau}(\rho_{t,\tau}^{*} - 1), & \text{for } \tau = t + 1, \\
\rho_{t,\tau}^{*} \eta_{\tau}^{*}, & \text{for } \tau > t + 1,
\end{cases}
\]

which yields

\[
\rho_{t,\tau}^{*} s_T = s_t + \sum_{\tau=t+1}^{T} \eta_{\tau}.
\]

Consider the costless investment strategy of shorting \( s_t \) units of the domestic bond, converting the proceeds into foreign currency at the going rate, and investing them in foreign bonds. Under risk neutrality, this costless strategy should earn zero profits on average. This implies that

\[
s_t = E[s_T | \phi_t] \left( \frac{1 + r_{t,\tau}^{*}}{1 + r_{t,t}^{*}} \right).
\]

\[
= E[s_T | \phi_t] \rho_{t,\tau}^{*}.
\]
Taking the expectation of (4) and comparing the result to the expression in (5) yield

\[ \sum_{t^*}^{T} E[\eta_t | \phi_t] = 0. \]  

In other words, the sum of the future \( \eta_t \)'s has zero conditional expectation.

1.1.2 The forward currency. Let \( f_t^T \) be the forward price at time \( t \) for delivery at time \( T \) of one unit of foreign currency. To avoid arbitrage, the forward contract must pay an amount equal to \( S_T \) on the maturity date of the contract \( T \).

\[ f_t^T = S_T. \]  

The relationship between the forward and spot rates is given by the interest rate parity theorem,

\[ f_t^T = s_t \frac{1 + r_t^T}{1 + r_t^S}, \]

\[ = \frac{s_t}{\rho_t}, \quad \tau = t, \ldots, T. \]  

Equation (8) indicates that under stochastic interest rates, changes in the forward price are not perfectly correlated with changes in the spot price. The relationship between the value of the forward contract today and on its maturity date, i.e \( f_t^T \) and \( f_T \), respectively, is assumed to be given by

\[ f_t^T = f_T^T + \sum_{t=t+1}^{T} \delta_t, \]

where \( \delta_t = f_T^T - f_{t-1}^T \). Since the forward contract requires no investment, the condition

\[ E[\delta_t | \phi_{t-1}] = 0 \]

can be imposed in a risk-neutral world. Consequently, the price of the forward contract at time \( t, f_t^T \), is the expected final forward price, conditional on \( \phi_t \):

\[ f_t^T = E[f_T^T | \phi_t] = E[S_T | \phi_t], \]

where the latter equality holds from Equation (7).

Equation (8) dictates a certain relationship between the shocks in the spot and forward markets. It can be shown that \( \eta_{t+1} \) relates to \( \delta_{t+1} \) through the functional form
Equation (12) shows that (i) the shocks in the forward and spot markets are not independent, (ii) they are not perfectly correlated under stochastic interest rates, and (iii) while \( \eta_{t+1} \) is not necessarily equal to zero. On the other hand, if interest rates are equal across countries, \( \eta_{t+1} = \delta_{t+1} \).

1.2 The structure of the spot and forward foreign exchange markets

As in Admati and Pfleiderer’s (1989) model, the spot and forward foreign exchange markets are assumed to be competitive dealer systems where a large number of risk-neutral market makers commit to take either side of the market. Each one sets bid and ask quotes on the spot and/or the forward foreign exchange markets. For sake of simplicity, the bid-ask spread on the bond market is ignored. Market makers are prepared to satisfy all the orders that arrive. Unlike Hughson (1990), orders are aggregated and the market makers observe only the net order flow, and, unlike Kyle (1985) and Hughson (1990), market makers do not supply quotations conditional on the numbers of orders. Each trader is allowed to trade exactly one unit of foreign exchange. Consequently, the information sets of the market makers (at the moment they set prices) and the public (before its members observe their signals) coincide and include, for example, the history of the shocks \( \delta_{t}, \eta_{t} \) for \( \tau \leq t \), but exclude the order flow at time \( t \). The bid and ask quotes in the forward and spot foreign exchange markets are defined as follows. The lowest ask commission in the forward market at time \( t \), denoted by \( a_{t}^{f} \), is defined as the difference between the lowest ask quote, \( A_{t}^{f} \), and the forward rate, \( f_{t} \),

\[
a_{t}^{f} = A_{t}^{f} - f_{t}.
\]

Similarly, the lowest bid commission in the forward market at time \( t \), denoted by \( b_{t}^{f} \), is defined as the difference between the forward rate, \( f_{t} \), and the highest bid quote; \( B_{t}^{f} \):

\[
b_{t}^{f} = f_{t} - B_{t}^{f}.
\]

The lowest ask and bid commissions in the spot market at time \( t \) are similarly defined with the superscript \( s \) replacing the superscript \( f \) in Equations (13) and (14), respectively. Other commissions are ignored, since no market participant will trade at those commissions.

The market makers face two different types of traders in both the

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5 This might seem unrealistic, but is not inconsistent with the data. See Table 2, which will be discussed in Section 3.2.
The informed and liquidity traders trade exactly one unit of the spot or forward foreign currency. The informed traders observe at time $t$ the shock $\delta_{t+1}$ in the forward market or $\eta_{t+1}$ in the spot market. Their superior information has a limited life of one period, since the shock $\delta_{t+1}$, or $\eta_{t+1}$, becomes public information at time $t+1$. Each liquidity trader receives at time $t$ an independent and identically distributed signal $z_{s,t}^i$ (for the liquidity traders in the spot market, or $z_{f,t}^i$ (for the liquidity traders in the forward market). The signal $z_{s,t}^i$, or $z_{f,t}^i$, can take on three possible values, namely, $-1$, $0$, or $+1$. The liquidity trader submits a buy (sell) order when the signal is equal to $+1$ ($-1$). The liquidity trader does not submit an order when the signal is equal to $0$. The probabilities of the signal being $+1$ or $-1$ are equal. Further, the signal is independent of the bid-ask spread.

1.3 The location of the expected spot rate within the forward and spot bid and ask quotes

1.3.1 The buy and sell conditions in the forward and spot markets. Informed traders buy at time $t$ one unit of foreign currency forward if the increase in the forward rate to occur at time $t+1$ is larger than the lowest ask commission:

$$\delta_{t+1} > a_f.$$  \hspace{1cm} (15)

Conversely, informed traders sell forward if

$$\delta_{t+1} < -b_f.$$  \hspace{1cm} (16)

Similarly, informed traders buy at time $t$ one unit of the spot currency if they expect a profit from borrowing the domestic currency, converting it into foreign currency, and investing the proceeds in the foreign bond:

$$-(1 + r_s^f) \phi_i + (1 + r_s^f) \times \left( E[s_t \phi_i] - \frac{E[\eta_{t+1} | \phi_i]}{\rho_s^i} + \frac{\eta_{t+1}}{\rho_s^i} \right) > 0.$$  \hspace{1cm} (17)

Using Equation (5) and the definition of the lowest ask commission, Equation (17) is rewritten

$$\eta_{t+1} - E[\eta_{t+1} | \phi_i] > a_i.$$  \hspace{1cm} (18)
Conversely, they sell at time $t$ one unit of the spot currency if

$$
\eta_{t+1} - E[\eta_{t+1} | \phi_i] < -b_i^f. \quad (19)
$$

To simplify the notation further, the unexpected shock, i.e., $\eta_{t+1} - E[\eta_{t+1} | \phi_i]$, is denoted by $\eta_{t+1}^u$, where the superscript “$u$” stands for “unexpected.” Equations (18) and (19) can now be rewritten as $\eta_{t+1}^u > a_i^f$ and $\eta_{t+1}^u < -b_i^f$, respectively.

1.3.2 The market makers’ zero expected profit conditions in the forward and spot markets. The aggregate demand (offer) on the spot or on the forward market is the sum of the informed demand (offer) and the liquidity demand (offer). Let $l_{D_i}$ denote the indicator function of the event $D$ where the function takes a value equal to unity if the event occurs, and zero otherwise. Let $\omega_{f,i}$ denote the number of units of forward foreign currency ordered at the ask at time $t$. Then, $\omega_{f,i}$ is equal to

$$
\omega_{f,i} = I_f \cdot 1_{(\eta_{t+1} > a_i^f)} + \sum_{r=1}^{n^f} 1_{(s_r^f > -1)}. \quad (20)
$$

Similarly, let $\omega_{b,i}$ be the number of units of forward foreign exchange offered by the informed and liquidity traders at the bid at time $t$. Then, $\omega_{b,i}$ is equal to

$$
\omega_{b,i} = I_f \cdot 1_{(\eta_{t+1} < -b_i^f)} + \sum_{r=1}^{n^f} 1_{(s_r^f < -1)}. \quad (21)
$$

The total order flow in the spot market at the ask and the bid, $\omega_{f,i}$ and $\omega_{b,i}$, can be defined in a similar way. The superscript $s$ would replace the superscript $f$, and $\eta_{t+1}^u$ would replace $\delta_{t+1}$ in Equations (20) and (21).

Since the trading system is assumed to be competitive, the market makers set the bid and ask quotes such that they make zero expected profits. Therefore, at the ask in the forward market,

$$
E[\omega_{f,i} (A_i^f - f_i^f) | \phi_i] = 0, \quad (22)
$$

and at the bid,

$$
E[\omega_{b,i} (f_i^b - B_i^f) | \phi_i] = 0. \quad (23)
$$

Substituting Equations (13), (9), and (20) into (22) and using the mean independence condition (10) yield

$$
E[\omega_{f,i} (A_i^f - f_i^f) | \phi] = E \left[ (I_f \cdot 1_{(\eta_{t+1} > a_i^f)} + \sum_{r=1}^{n^f} 1_{(s_r^f > -1)}) \right].
$$
If the distribution of $\delta_{t+1}$ is symmetric, over $\phi_i$ by assumption. Consequently, a comparison of Equations (24) and (25) leads to the following proposition.

**Proposition 1.** If the distribution of $\delta_{t+1}$ is symmetric, the lowest ask and bid commissions in the forward market are equal, namely, $a'_f = b'_f$.

Proposition 1 states that, under the assumption of a symmetric distribution for the change in the forward price, $f^*_f$ halfway between the forward bid and ask quotes. Further, since $f^*_f = E[f^*_f | \phi_i] = E[s_f | \phi_i]$, $f^*_f = E[s_f | \phi_i] = \frac{1}{2}(A'_f + B'_f)$.

If, however, the distribution of a change in the forward rate ($\delta_{t+1}$), is asymmetric, Equation (26) will not hold, and the predictor of the future spot rate halfway between the forward bid and ask quotes is biased.

Equations (24) and (25) show that the forward bid ‘and ask commissions are strictly positive, i.e., the forward rate is inside the forward
bid and ask quotes. Indeed, by setting the ask commission, \( a'_n \), equal to zero, the first two terms of Equation (24) vanish. Since the third term is strictly negative, the market maker expects to make a loss. To prevent this, he, accordingly, increases the ask commission. An analogous reasoning works for the bid commission. -This leads to the following proposition.

**Proposition 2.** The lowest ask and bid commissions in the forward market are positive, i.e., the forward rate, equal to the expected future spot rate, is inside the forward ask and bid quotes.

In the spot foreign exchange market, on the other hand, the market makers make zero average profits when adjusted for a fair return in the domestic and foreign bond markets. Consequently, at the spot ask quote, the zero expected profit condition is

\[
E[\omega^{t \Delta} (A^t - s_T \rho^t) | \phi] = 0. \tag{27}
\]

At the spot bid quote, the zero expected profit condition is

\[
E[\omega^{t \Delta} (s_T \rho^t - B^t) | \phi] = 0. \tag{28}
\]

Using the definitions of the lowest ask commission and the total order flow at the ask in the spot market, substituting (4) into (27), and using the restriction in (6) yield

\[
E[\omega^{t \Delta} (A^t - s_T \rho^t) | \phi] = E \left( (F \cdot 1_{\{\eta_{t+1} > \eta_T\}} + \sum_{r=1}^{\ell_T} 1_{\{\xi_{t,r} = \eta_T\}}) \right) \times \left( s_T + a_T - s_T - \sum_{r=t+1}^{\tau_T} \eta_T \right) \phi_T
\]

\[
= a_T F E[1_{\{\eta_{t+1} > \eta_T\} | \phi_T] + a_T L \cdot E[1_{\{\xi_{t+1} = \eta_T\} | \phi_T] - F \cdot E[\eta_{t+1} | \phi_T]
\]

\[
= 0. \tag{29}
\]

Similarly, for the bid quote,

\[
E[\omega^{t \Delta} (s_T \rho^t - B^t) | \phi] = E \left( (F \cdot 1_{\{\eta_{t+1} < \rho_T\}} + \sum_{r=1}^{\ell_T} 1_{\{\xi_{t,r} = \rho_T\}}) \right)
\]
Comparing (29) and (30), Propositions 1 and 2 will not necessarily follow for the spot market, since does not necessarily equal Moreover, setting the bid and the ask commissions equal to zero does not unambiguously lead to sure losses for the market maker on both sides of the market, a crucial point in the proof of Proposition 2. Consequently, while the expected future spot rate is within the forward bid and ask quotes, it need not be within the spot bid and ask quotes.

2. Implications of the Model

2.1 The market microstructure effects of government intervention

Proposition 1 gives conditions under which the expected spot rate, equal to the forward rate, is halfway between the forward bid and ask quotes. The distribution of the unexpected change in the forward rate, \( \delta_{t+1} \), must be symmetric. The assumption of a symmetric distribution is questionable, however. Governments occasionally intervene in the foreign exchange market. Government intervention can take several forms ranging from direct market intervention, such as open market operations, to outright devaluation and revaluation. Whatever its form, government intervention will introduce skewness in the distribution of changes in the spot rate as perceived by investors and market makers, since it implies a low probability of a large loss but a zero probability of a large gain.

Suppose that the market makers expect the devaluation of a currency. From Equation (25), the expected loss due to sell orders from informed traders in the forward foreign exchange market is equal to

\[
\sum_{i=1}^{T} \eta_i \left( s_i + b_i - s_i + \sum_{t=1}^{T} \eta_t \right) | \phi_i |
\]

\[= b_i^p E[1_{\text{\eta}_t < -\eta_i}] | \phi_i | + b_i^a L^t E[1_{\text{\eta}_t < 1}] | \phi_i | + P E[\eta_{t+1} | \phi_i ]
\]

\[= 0. \quad \text{(30)}
\]

Comparing (29) and (30), Propositions 1 and 2 will not necessarily follow for the spot market, since \( E[\sum_{t=1}^{T} \eta_t 1_{\eta_t < -\eta_i}] | \phi_i | \) does not necessarily equal \( -E[\sum_{t=1}^{T} \eta_t 1_{\eta_t > -\eta_i}] | \phi_i | \). Moreover, setting the bid and the ask commissions equal to zero does not unambiguously lead to sure losses for the market maker on both sides of the market, a crucial point in the proof of Proposition 2. Consequently, while the expected future spot rate is within the forward bid and ask quotes, it need not be within the spot bid and ask quotes.
makers increase the bid commission until the zero profit condition is again satisfied. In contrast, the expected losses and gains at the ask side of the market are unlikely to change. The ask commission should, therefore, neither increase nor decrease. An asymmetric bid-ask spread results. The expected future spot rate is closer to the forward ask quote, than to the forward bid quote. The asymmetry works in the other direction when a revaluation is expected, i.e., the expected future spot rate is closer to the forward bid than to the forward ask quote.

Though expectations about government intervention change over time, the market knows when an intervention is most likely to occur. Decisions about currencies are mostly taken during weekends when the foreign exchange markets are closed; All realignments within the European Monetary System, henceforth E.M.S., for example, have been decided during weekends. The distribution of the price change of the forward currency will, therefore; be more skewed on Fridays than on any other day of the week. Moreover, bid-ask spreads will be larger on Fridays.

2.2 Econometric implications
Tests of the unbiasedness hypothesis of the foreign exchange market are typically performed by regressing the relative change in the spot rate onto the relative forward premium,

\[ \frac{s_t - s_{t-1}}{s_t} = a + b \frac{f_t - s_t}{s_t} + \epsilon_t, \quad (33) \]

where \( \epsilon_t \) is the part of the relative change in the spot rate that cannot be predicted with time information. The “true” values of the forward and spot rates are not observable, however, and have to be estimated. Usually, the estimated value is the average of the bid and ask quotes. Substituting these values for \( s_n, s_n, \) and \( f_t \) in (33) and using \( B_t \) instead of \( s_t \) to obtain stationarity yield\(^2\)

\[ \frac{1}{2} (A_t + B_t) - (A_t + B_t) \]

\[ = a^* + \beta^* \frac{1}{2} \frac{1}{B_t} - (A_t + B_t) + \epsilon_t. \quad (34) \]

\(^2\)There is substantial evidence that foreign exchange rates follow a geometric random walk [see the survey in Hodrick (1987), p. 29]. Hence, stationarity; an be obtained by dividing the change in the foreign exchange rate by last period’s level. Because the level of the true exchange rate is unobservable, the bid quote, the ask quote, or a combination of the bid and ask quotes, have to be used instead. We used the bid quote. The choice is Irrelevant as long as the bid-ask spread itself is stationary.
Under the null hypothesis, and assuming that the true forward and spot rates are halfway between their respective bid and ask quotes, the intercept and slope coefficients of the regression, i.e., $\alpha^*$ and $\beta^*$, should be equal to zero and unity, respectively.

We shall first derive a regression equation that can be used to test the unbiasedness hypothesis without imposing any constraints on the location of the true forward and spot rates within the bid-ask spreads. The statistical properties of the resulting estimates are then compared to those obtained from (34).

### 2.2.1 Tests of the unbiasedness hypothesis and the modeling of the bid-ask spread.

For the sake of simplicity, the location of the expected future spot rate and the spot rate within the forward and the spot bid-ask spreads, respectively, is first assumed to be constant over time. Let $\theta$ be a constant such that

$$E[s_T|\phi] = \gamma A_T^f + (1 - \gamma) B_T^f, \quad \forall t. \tag{35}$$

Let $\theta$ indicate the location of the future spot rate within the spot bid-ask spread at time $T$

$$s_T = \theta A_T^f + (1 - \theta) B_T^f. \tag{36}$$

According to Proposition 2, the forward bid and ask commissions must be positive. Consequently, the parameter $\theta$ must be in the interval $[0, 1]$. The parameter $\gamma$, however, is not constrained.

Under the assumption of rational expectations, Equation (35) can be rewritten

$$s_T = (\gamma A_T^f + (1 - \gamma) B_T^f) + \xi_T, \tag{37}$$

where $\xi_T$ is a prediction error, uncorrelated with information publicly available at time $t$. Substituting the expression in (36) for $s_T$, and subtracting the spot bid at time $t$, i.e., $B_t^s$, from (37) yield

$$B_T^s - B_t^s = \theta (B_T^s - A_T^f) + \gamma (A_T^f - B_T^f) + (B_T^f - B_t^f) + \xi_T. \tag{38}$$

To obtain stationarity, the left- and right-hand sides of Equation (38) are scaled by $B_t^s$. This yields

$$\frac{B_T^s - B_t^s}{B_t^s} = \theta \frac{B_T^s - A_T^f}{B_t^s} + \gamma \frac{A_T^f - B_T^f}{B_t^f} + \frac{B_T^f - B_t^f}{B_t^s} + \xi_T. \tag{39}$$

Equation (39) shows that the change in the relative bid spot quote over the period $[t, T]$ can be decomposed into four terms. The first two components are proportional to the bid-ask spreads in the spot market at time $T$, and in the forward market at time $t$, respectively.

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*Though it is not explicitly reflected in the notation, the error term is also divided by $B_t^s$.}
The last two components are the forward premium at time $t$, and the prediction error, respectively. The second and third terms are in the information set at time $t$, and are, thus, uncorrelated with the error term $\xi_t$. The first term, however, will, in general, be correlated with the prediction error $\xi_t$, since the future spot bid-ask spread is likely to be affected by past unexpected changes in the spot price.

Therefore, the unbiasedness hypothesis could be tested by running the regression

$$ \frac{B^*_T - B^*_t}{B^*_t} = \alpha + \theta \frac{B^*_T - A^*_t}{B^*_t} + \beta \frac{A^*_t - B^*_t}{B^*_t} + \beta \frac{B^*_T - B^*_t}{B^*_t} + \xi_t, \quad (40) $$

and testing whether the intercept $\alpha$ and the slope coefficient $\beta$ are equal to zero and unity, respectively. In addition to testing the unbiasedness hypothesis, this regression can also be used to determine the exact location of the true forward and spot rates within the forward and spot bid and ask quotes. A value equal to $1/2$ for the parameter $\gamma$, for example, would indicate that the expected future spot rate is exactly halfway between their respective bid and ask quotes. Unlike (34), the regression equation does not constrain either the spot rate or the forward rate to be halfway between their respective bid and ask quotes. The regression is complicated by the correlation between the error term and the spot bid-ask spread at time $T$. This econometric problem can be solved by using an instrument variables procedure. The choice of appropriate instruments is examined in Section 3.3.2.

### 2.2.2 The inconsistency of the regression parameter estimates when the bid-ask spread is not explicitly modeled.

The two regression equations (34) and (40) can be used to test the unbiasedness hypothesis but only the latter explicitly takes into account the bid-ask spreads in the spot and forward foreign exchange markets. Interesting inferences on the statistical properties of the regression parameter estimates $\alpha^*$ and $\beta^*$ in (34) can be drawn from their comparison with $\alpha$ and $\beta$ in (40). To facilitate the comparison, Equation (40) is rewritten so it has the same dependent variable as (34). This is achieved by subtracting the spot price at $t$ estimated at the average of the bid and ask quotes from (37), and replacing $s_T$ by its value given by Equation (36). These two steps yield

$$ \frac{1}{2}[(A^*_t + B^*_t) - (A^*_t + B^*_t)] $$

$$ = \left( \frac{1}{2} - \theta \right) (A^*_t - B^*_t) + \frac{1}{2}[(A^*_t + B^*_t) - (A^*_t + B^*_t)] $$

$$ + \left( \gamma - \frac{1}{2} \right) (A^*_t - B^*_t) + \xi_t. \quad (41) $$

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Suppose that the future spot rate is in the middle of the spot bid and ask quotes, i.e., $\theta = 1/2$. Dividing by $B_i$ to obtain stationarity, the unbiasedness hypothesis can be tested by running the regression:

$$
\frac{1}{2} (A_i + B_i) - (A_i + B_i) = \alpha + \beta \left( \frac{1}{2} (A_i + B_i) - (A_i + B_i) \right) B_i + \left( \gamma - \frac{1}{2} \right) \frac{(A_i - B_i)}{B_i} + \xi_t. \quad (42)
$$

The comparison of Equations (34) and (42) clearly shows that the ordinary least squares (OLS) estimators of $\alpha^*$ and $\beta^*$ will not converge to $\alpha$ and $\beta$ when $\gamma \neq 1/2$. The inconsistency follows from the measurement error in the explanatory variable used in (34). The error, i.e., $(\gamma - \frac{1}{2}) \frac{(A_i - B_i)}{B_i}$, is proportional to the bid-ask spread in the forward market. The inconsistency is a function of (i) the correlation between the forward premium and the forward bid-ask spread, and (ii) whether $\gamma$ exceeds $1/2$ or not. The sign of the correlation between these two variables is hard to predict, however. Moreover, the value of $\gamma$ is generally unknown. Let us assume, for the sake of analysis, that $\gamma$ exceeds $1/2$. The OLS estimate of the slope coefficient $\beta^*$ will then be biased downward if the error is uncorrelated with the explanatory variable. The bias will worsen if the correlation is positive. The bias might disappear, however, if the correlation is negative.

3. The Empirical Evidence

3.1 The sample

The data on the foreign exchange market, namely, the spot rates, the one-month forward rates, and the one-month eurocurrency interest rates, were obtained from Data Resources Incorporated. Daily observations for the period June 1, 1973 through June 13, 1988 are available for ten currencies, namely, the British pound, the Canadian dollar, the Danish krone, the Dutch guilder, the French franc, the Italian lira, the Japanese yen, the Swiss franc, and the Deutsche mark. The foreign exchange rates, quoted in U.S. dollars, are expressed as bids.

---

*Eurocurrency deposits are foreign-currency denominated deposits in a bank outside the country where the currency is issued as legal under. Standard maturities are for 1, 2, 3, 6, 9, and 12 months. As of the early 1980’s, there were 15 eurocurrencies regularly traded in London. In this study, we use 30-day eurocurrency rates.*
and offers reflecting the New York opening market. The French franc rates used in the empirical section are cross-rates. Cross-rates are calculated from exchange rates with respect to the dollar. The Deutsche mark ask rate in terms of French franc, for instance, is obtained as the ratio of the Deutsche mark ask rate in terms of dollar over the French franc bid rate in terms of dollar. The daily one-month eurocurrency rates are also available for the same period of time. They are quoted London mid-morning, except for the euro-pound which is quoted mid-morning Paris.

3.2 The intraweek patterns of the bid-ask spread in the foreign exchange market

Bid-ask spreads are conjectured to be higher on Fridays on average because of pending government intervention during weekends and the increased adverse selection problem faced by market makers. To test this hypothesis, the bid-ask spread is computed each day as the difference between the ask and the bid quotes for the spot and the forward currency, and also for the eurocurrency rates. Univariate and multivariate statistics are used to test the equality of the bid-ask spread on Friday relative to its Monday, Tuesday, Wednesday, and Thursday counterpart. More specifically, univariate t-statistics are used to test the hypothesis that

$$\{\text{Ask}_{t,j} - \text{Bid}_{t,j}\} = \{\text{Ask}_{t,k} - \text{Bid}_{t,k}\}, \quad j = 1, \ldots, 4, \quad k = s, f, e, \quad (43)$$

where the subscript $j$, with $j = 1, \ldots, 5$, designates the day of the week, starting with day 1 on Monday and ending with day 5 on Friday. The superscript $k$, with $k = s, f, e$, designates the spot currency, the forward currency, and the eurocurrency rate, respectively. Multivariate $F$-statistics are used to test the equality of the bid-ask spread each day of the week,

---

9 The source is Bank of America. The European markets are not closed yet when markets open in New York. The trading volume is substantial at that time. Thin trading is, therefore, unlikely to explain the results reported below.

10 This is consistent with market practice. A bank asked to quote the exchange rate between two currencies not involving the US. dollars would normally calculate this from the two exchange rates with respect to the dollar, although some will occasionally try to undercut. See, for example, Grabbe (1986), p. 64. Banks use U.S. dollar rates because of the size of the dollar market relative to other markets. Even central bank intervention to correct prices of currencies within the E.M.S. is often done through the U.S. dollars market. See Taylor (1982), p. 62. Again this is not surprising: the dollar is by far the most important official reserve currency. Because of the derivative nature of cross-markets and the pivotal role of the dollar in government intervention, bid-ask spreads of cross-rates would reflect to a large extent the information asymmetries that this study focuses on.

11 All eurocurrency rates are quoted on a 360-day basis except for the British pound quoted on a 365-day basis.

12 The source is Bank of America. The foreign exchange and eurocurrency data are non synchronous. The lag is approximately three hours.
The tests are successively performed on the U.S. dollar and the French franc. There are no substantial differences between the two sets of tests and only the latter is reported. Tables 1 and 2 show the empirical evidence for the foreign exchange market and the eurocurrency market, respectively.

Three important conclusions can be drawn from the empirical evidence in these two tables. First, bid-ask spreads in both the forward and the spot foreign exchange markets are significantly larger on Fridays. This result obtains regardless of the currency. The probability levels are generally less than .01. Second, the difference between the bid-ask spread on Friday and its Monday, Tuesday, Wednesday, or Thursday counterpart is slightly larger for the forward currency than for the spot currency. The difference, however, seems less statistically significant for the latter than for the former. Third, unlike what is observed in the foreign exchange market, the bid-ask spreads in the eurocurrency market are not larger on Fridays. Further, no systematic pattern is observed on any other day.

Additional characteristics of the bid-ask spreads are presented in Table 3, which shows, for Fridays, the average forward bid-ask spread, the average forward premium, measured using the average bid and ask quotes, and the correlation between these two variables. All variables are scaled using the spot bid quote. The average size of the forward bid-ask spread relative to the forward premium enables one to assess the importance of the biases that are being modeled. If the bid-ask spread is a small fraction of the forward premium, biases introduced by using a particular combination of bid and ask quotes will be inconsequential. Also, as discussed in Section 2.2.2, the correlation coefficients should indicate the direction of the biases.

Table 3 shows that the forward bid-ask spread, while small in absolute value, is nevertheless sizeable relative to the forward premium. Table 3, A shows that the forward bid-ask spread represents

\[
[\text{Ask}^*_k - \text{Bid}^*_k] = [\text{Ask}^*_{k+1} - \text{Bid}^*_{k+1}] = \cdots = [\text{Ask}^*_n - \text{Bid}^*_n], \quad k = s_f, e. \quad (44)
\]

The difference in the results obtained for the foreign exchange market and the eurocurrency market could be attributed to "rounding." This is unlikely, however. Rounding cannot be an issue in the foreign exchange market since no minimum "uptick" or "downtick" exists. In contrast, in the eurocurrency market, quotes are rounded up to 6.25 basis points (1/16%). If, at an initial bid-ask spread, eurocurrency dealers lose money, they will increase the spread by at least 6.25 basis points. This might correspond to a larger increase than what is needed to reestablish the zero expected profit condition. Consequently, rounding must bias the tests in favor of rejection.
Table 1  
Univariate and multivariate tests of the difference between the bid-ask spread on Friday and on Mondays, Tuesdays, Wednesdays, and Thursdays, respectively (time period: June 1, 1973-June 13, 1988; daily observations)

### A: The spot FF

<table>
<thead>
<tr>
<th>Spot</th>
<th>Sample size</th>
<th>( \Delta_1 ) (( \Delta_1 ))</th>
<th>( \Delta_2 ) (( \Delta_2 ))</th>
<th>( \Delta_3 ) (( \Delta_3 ))</th>
<th>( \Delta_4 ) (( \Delta_4 ))</th>
<th>F-test</th>
<th>p val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF/FL</td>
<td>729</td>
<td>.00077 (4.84)</td>
<td>.00082 (5.24)</td>
<td>.00066 (3.54)</td>
<td>.00072 (4.53)</td>
<td>7.38</td>
<td></td>
</tr>
<tr>
<td>FF/DM</td>
<td>680</td>
<td>.00079 (6.06)</td>
<td>.00073 (5.30)</td>
<td>.00065 (4.07)</td>
<td>.00067 (4.78)</td>
<td>9.77</td>
<td></td>
</tr>
<tr>
<td>FF/SF</td>
<td>753</td>
<td>.00123 (7.29)</td>
<td>.00122 (7.26)</td>
<td>.00122 (6.54)</td>
<td>.00116 (6.89)</td>
<td>17.32</td>
<td></td>
</tr>
<tr>
<td>FF/E</td>
<td>711</td>
<td>.00285 (5.08)</td>
<td>.00315 (5.88)</td>
<td>.00254 (4.08)</td>
<td>.00234 (4.16)</td>
<td>9.28</td>
<td></td>
</tr>
<tr>
<td>FF/CA$</td>
<td>738</td>
<td>.00070 (2.81)</td>
<td>.00116 (5.36)</td>
<td>.00105 (4.45)</td>
<td>.00093 (4.23)</td>
<td>8.04</td>
<td></td>
</tr>
<tr>
<td>FF/yen</td>
<td>729</td>
<td>.000006 (3.92)</td>
<td>.000006 (3.93)</td>
<td>.000004 (2.01)</td>
<td>.000007 (4.60)</td>
<td>6.15</td>
<td></td>
</tr>
<tr>
<td>FF/llira</td>
<td>731</td>
<td>.000001 (3.40)</td>
<td>.000001 (7.74)</td>
<td>.000002 (5.03)</td>
<td>.000002 (4.11)</td>
<td>16.16</td>
<td></td>
</tr>
</tbody>
</table>

### B: The forward FF

<table>
<thead>
<tr>
<th>Forward</th>
<th>Sample size</th>
<th>( \Delta_1 ) (( \Delta_1 ))</th>
<th>( \Delta_2 ) (( \Delta_2 ))</th>
<th>( \Delta_3 ) (( \Delta_3 ))</th>
<th>( \Delta_4 ) (( \Delta_4 ))</th>
<th>F-test</th>
<th>p val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF/FL</td>
<td>729</td>
<td>.00119 (4.81)</td>
<td>.00157 (5.56)</td>
<td>.00111 (4.07)</td>
<td>.00091 (3.47)</td>
<td>8.30</td>
<td></td>
</tr>
<tr>
<td>FF/DM</td>
<td>680</td>
<td>.00109 (4.98)</td>
<td>.00107 (4.77)</td>
<td>.00083 (3.48)</td>
<td>.00073 (3.19)</td>
<td>7.41</td>
<td></td>
</tr>
<tr>
<td>FF/SF</td>
<td>753</td>
<td>.00119 (4.09)</td>
<td>.00159 (6.07)</td>
<td>.00150 (5.69)</td>
<td>.00140 (5.59)</td>
<td>10.58</td>
<td></td>
</tr>
<tr>
<td>FF/E</td>
<td>711</td>
<td>.00443 (4.87)</td>
<td>.00452 (5.07)</td>
<td>.00354 (3.61)</td>
<td>.00277 (3.04)</td>
<td>7.70</td>
<td></td>
</tr>
<tr>
<td>FF/CA$</td>
<td>738</td>
<td>.00095 (2.10)</td>
<td>.00211 (5.48)</td>
<td>.00191 (4.45)</td>
<td>.00141 (3.48)</td>
<td>9.29</td>
<td></td>
</tr>
<tr>
<td>FF/yen</td>
<td>729</td>
<td>.000008 (3.16)</td>
<td>.000011 (4.04)</td>
<td>.000007 (2.66)</td>
<td>.000008 (3.42)</td>
<td>4.35</td>
<td></td>
</tr>
<tr>
<td>FF/llira</td>
<td>731</td>
<td>.000002 (1.78)</td>
<td>.000005 (6.73)</td>
<td>.000002 (2.92)</td>
<td>.000001 (9.99)</td>
<td>14.91</td>
<td></td>
</tr>
</tbody>
</table>

\( \Delta_1 = [\text{Ask}_1 - \text{Bid}_1] - [\text{Ask}_5 - \text{Bid}_5] \), where the subscripts 1 and 5 stand for Monday and Friday, respectively. \( \Delta_2 = [\text{Ask}_2 - \text{Bid}_2] - [\text{Ask}_5 - \text{Bid}_5] \), where the subscripts 2 and 5 stand for Tuesday and Friday, respectively. \( \Delta_3 = [\text{Ask}_3 - \text{Bid}_3] - [\text{Ask}_5 - \text{Bid}_5] \), where the subscripts 3 and 5 stand for Wednesday and Friday, respectively. \( \Delta_4 = [\text{Ask}_4 - \text{Bid}_4] - [\text{Ask}_5 - \text{Bid}_5] \), where the subscripts 4 and 5 stand for Thursday and Friday, respectively. The multivariate F-statistic tests the hypothesis that \( \Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 0 \). T-statistics appear in parentheses. The rates quoted in French francs are cross-rates. They are calculated from the rates quoted to U.S. dollars. Standard abbreviations are used: FF = French franc, FL = Dutch guilder, DM = Deutsche mark, SF = Swiss franc, $ = $ British pound.

a minimum of 26 percent and a maximum of 68 percent of the forward premium when the currencies are expressed relative to the U.S. dollar. Similarly, from Table 3, B, the minimum and maximum values are 41 percent and 144 percent, respectively, when the currencies are expressed relative to the French franc. This evidence suggests that the estimates of the “Fama-like” regression parameters are likely to be biased. Table 3, A displays the correlations between these two variables when the currencies are expressed relative to the U.S. dollar.
Table 3  
Correlation between the forward premium and the difference the forward ask and bid quotes (time period: June 1, 1973-June 13, 1988; observations)

<table>
<thead>
<tr>
<th>Currency</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\rho$</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US$/FL</td>
<td>.00115</td>
<td>.00235</td>
<td>.071</td>
<td>.027</td>
</tr>
<tr>
<td>US$/DM</td>
<td>.00080</td>
<td>.00311</td>
<td>.096</td>
<td>.005</td>
</tr>
<tr>
<td>US$/SF</td>
<td>.00135</td>
<td>.00468</td>
<td>.122</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>US$/C</td>
<td>.00085</td>
<td>-.00195</td>
<td>.144</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>US$/CA$</td>
<td>.00055</td>
<td>-.00089</td>
<td>-.056</td>
<td>.064</td>
</tr>
<tr>
<td>US$/yen</td>
<td>.00131</td>
<td>.00234</td>
<td>-.330</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>US$/lira</td>
<td>.00230</td>
<td>-.00622</td>
<td>-.279</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>US$/FF</td>
<td>.00156</td>
<td>-.00230</td>
<td>-.675</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

A: US$ exchange rate

B: FF exchange rate

$\mu_1$, is the average difference between the forward ask and bid quotes defined as \( (A_i - B_i)/B_i \); $\mu_2$ is the average forward premium, defined as \( \frac{1}{T} (A_i + B_i) - (A_i + B_i)/B_i \); $\rho$ is the correlation between the forward premium and forward bid-ask spread; $p$ is the probability value of the hypothesis that $\rho = 0$. The rates quoted in French francs are cross-rates. They are calculated from the rates quoted in U.S. dollars.

employed by Fama (1984) to test the unbiasedness hypothesis. The GMM estimator, unlike the SUR, does not require the assumption of time-series homoskedasticity and independence.¹⁶

33.1 The traditional tests of the unbiasedness hypothesis. Table 4 shows the results obtained by running the regression specified in (34), i.e., the Fama-like regression. The two regression explanatory variables, namely the unit vector and \( \frac{1}{T} (A_i + B_i) - (A_i + B_i)/B_i \), respectively, are used as the instrumental variables. The U.S. dollar and the French franc results are reported in parts A and B of Table 4, respectively.

¹⁶ The four-week overlap in the prediction errors that follows, from the use of weekly observations on one-month forward rates can be explicitly modeled with a MA(3) errors component. To avoid singularity problems, a Newey-West procedure is used [Newey and West (1987)]. This procedure requires, however, the specification of a maximum lag length over which to calculate the autovariances. In theory, this lag length should increase with sample size at a rate greater than \( T^{1/4} \). The lag length was determined as follows. Fama’s model (1984) was first run on 184 nonoverlapping Friday observations. The model was rejected. Fama’s model was then run on overlapping data and the length increased until the rejection levels obtained with overlapping and nonoverlapping observation were similar. This gave a lag length equal to 5, which, incidentally, is close to \( T^{1/4} \) for T = 738, the total sample size. The determination of the exact lag length in the Newey-West procedure is admittedly arbitrary. Consequently, the relative improvement of a model’s \( x^2 \) over another model’s \( x^2 \) conveys more reliable information than their absolute values.
Tests of the unbiasedness hypothesis, where the forward and spot rates are constrained to be halfway between respective bid-ask quotes (time period: June 1, 1973-June 13, 1988; daily observations)

Table 4

The empirical evidence in Table 4, A is very similar to the results reported by Fama (1984) and Hodrick and Srivastava (1986). The slope coefficients are negative and most are statistically significant at the 5% or 1% level. Four of the intercepts are statistically different from zero. The empirical evidence is, therefore, consistent with a volatile risk premium. As Table 4, B indicates, the results obtained by running the same regression for the French franc instead of the U.S. dollar singularly contrast with the previous findings. The slope coefficients are much closer to unity than in the previous case. Except in one case, the intercepts are not statistically different from zero. This suggests a less volatile risk premium.

To test more formally the unbiasedness hypothesis, the intercepts

\[ \hat{\alpha} \] and \[ \hat{\beta} \] are GMM estimates of the coefficients in the regression

\[
\frac{1}{2} \left( [(A_1 + B)] - (A_1 + B) \right) = \alpha^* + \beta^* \frac{1}{2} \left( [(A_1 + B)] - (A_1 + B) \right) + \epsilon_r
\]

In the case of \( \hat{\alpha}^* \) and \( \hat{\beta}^* \) indicate significance at the 5% and 1% level, respectively, for \( \alpha^* = 0 \). In the case of \( \beta^* \) and \( \hat{\beta}^* \) indicate significance at the 5% and 1% level, respectively, for \( \beta^* = 1 \). All significance levels are based on two-tailed tests. Two instruments are used in the GMM tests of the restrictions that \( \alpha^* = 0 \) and \( \beta^* = 1 \), namely, 1 and \( \frac{1}{2} \left( [(A_1 + B)] - (A_1 + B) \right) \). The rates quoted in French francs are cross-rates. They are calculated from the rates quoted in U.S. dollars.

The empirical evidence in Table 4, A is very similar to the results reported by Fama (1984) and Hodrick and Srivastava (1986). The slope coefficients are negative and most are statistically significant at the 5% or 1% level. Four of the intercepts are statistically different from zero. The empirical evidence is, therefore, consistent with a volatile risk premium. As Table 4, B indicates, the results obtained by running the same regression for the French franc instead of the U.S. dollar singularly contrast with the previous findings. The slope coefficients are much closer to unity than in the previous case. Except in one case, the intercepts are not statistically different from zero. This suggests a less volatile risk premium.

To test more formally the unbiasedness hypothesis, the intercepts
and slope coefficients, i.e., the $a^*$s and $b^*$s, of the regression, are constrained to be equal to zero and unity, respectively. A $\chi^2$ statistic is used to test the constraints jointly for the eight currencies in Table 4, A and the seven currencies in Table 4, B. The number of degrees of freedom is equal to 16 and 14 for the tests reported in parts A and B of Table 4, respectively, since two instruments are used for each currency and no parameter is estimated. The results are presented at the bottom of the two panels. The comparison of the two $\chi^2$ statistics and their associated probability levels indicates that the unbiasedness hypothesis is rejected in both cases but at a higher significance level for the U.S. dollar than for the French franc.

Table 4 shows a much less volatile risk premium in traditional Fama-like tests of the unbiasedness hypothesis for the French franc than for the U.S. dollar. One could attribute this difference to the degree of uncertainty about government intervention. Most European currencies, including the French franc, are tied together through an exchange rate mechanism such as the E.M.S., which decreases uncertainty about government interventions. The currencies in the E.M.S. are frequently devalued or revalued and the timing and direction of the E.M.S. realignments are relatively easy to predict. Because of this, one would expect the risk premium for the French franc to be less volatile than for the U.S. dollar. On the other hand, more uncertainty about government intervention will also generate more serious biases because of the more pronounced asymmetry of the bid-ask spreads induced by insider trading. In this sense, the U.S. dollar risk premium could very well be less volatile than indicated by Table 4.

3.3.2 The asymmetry of the bid-ask spread. The empirical evidence obtained by running the regression defined in Equation (40) is now examined. This regression is used to test (i) the unbiasedness hypothesis, (ii) whether the expected future spot rate is inside the forward bid and ask quotes, and (iii) if so, whether they are halfway between the bid and ask quotes. Four instruments are used, namely, the unit vector? the forward premium; the forward bid-ask spread, and the change in the forward premium over the previous week. The last instrument is selected to capture changes in expectations. The tests are performed on continental European currencies with respect to the French franc. These currencies have either systematically been revalued or devalued at random points in time with respect to the French franc. The direction and size of government interventions are

\[\text{The number of degrees of freedom is equal to the total number of orthogonality conditions less the total number of parameters to be estimated.}\]
relatively easy to predict\textsuperscript{18}. Also, the Swiss franc, though not part of the E.M.S, is nevertheless included in the analysis as the Swiss National Bank attempted to peg it to the Deutsche mark during most of the sample time period. Four models are tested. They differ with respect to the constraints imposed on the parameters $\alpha$, $\beta$, and $\theta$. In particular, models 2 and 3 allow $\gamma$ to be time varying. In other words, the location of the expected future spot rate within the forward bid and ask quotes changes over time. This should reflect changes in the probability of realignment. We assume that changes in the interest rate differential across currencies capture changes in the probability of currency realignment.

Model 1:

$$\alpha = 0, \quad \beta = 1, \quad \theta = \frac{1}{2}.$$  

The parameter $\gamma$, assumed to be constant over time, is the only parameter to be estimated. Model 1 tests the unbiasedness hypothesis with fixed bid and ask commissions.

Model 2:

$$\alpha = 0, \quad \beta = 1, \quad \theta = \frac{1}{2},$$  

$$\gamma_t = \gamma (1 + 100 [(B^*_t - B^0_t) - (B^*_{t-1} - B^0_{t-1})]).$$  

The second model is similar to the first except that the parameter $\gamma$ is assumed to be time varying. The specification reflects the idea that governments defend their currency by increasing the nominal interest rate relative to the foreign interest rate before accepting currency realignments. From the interest rate parity theorem, the difference in interest rates across countries is equal to the forward premium.\textsuperscript{19} Model 2 tests the unbiasedness hypothesis with time-varying bid and ask commissions.

Model 3:

$$\alpha = 0, \quad \theta = \frac{1}{2},$$  

$$\gamma_t = \gamma (1 + 100 [(B^*_t - B^0_t) - (B^*_{t-1} - B^0_{t-1})]).$$  

The third model is similar to the second except that the slope coefficient ($\beta$) is not constrained to be equal to unity, unlike the two

\textsuperscript{18} The analysis could be attempted on other currencies. The empirical evidence in Table 3 suggests that the biases are potentially severe for currencies like the Canadian dollar. Unfortunately, the correlation between the forward premium and the forward bid-ask spread introduces multicollinearity, which generates inefficient estimates of the $\beta$ and $\gamma$ coefficients. Our experience confirmed this. Nevertheless, the more challenging difficulty was to capture variations in the perceived likelihood and direction of government intervention for currencies with respect to the U.S. dollar.

\textsuperscript{19} The reason for the scaling (x100) is that interest rate changes are minuscule. The average and standard deviations are -.0000195, .0092225 (FF/FL); -.0000239, .0083397 (FF/DM); -.0000234, .0117744 (FF/SF); -.5,00000005 .0000210 (FF/lira). Unsuccessful attempts made to estimate a separate scale parameter for each currency.
previous models. Model 3 allows for a variable risk premium with time-varying bid and ask commissions.

Model 4:

\[ \theta = \gamma = 0 \]

The last model corresponds to Fama’s regression on bid quotes, i.e., the two parameters \( \gamma \) and \( \theta \) are constrained to be equal to zero while \( \alpha \) and \( \beta \) are estimated. Model 4 allows for a variable risk premium but imposes fixed and symmetric bid and ask commissions.

A \( \chi^2 \) tests the restrictions that the error of each model is uncorrelated with the instruments.\(^{20,21}\) The regressions are performed for each day of the week separately. The empirical evidence obtained on Fridays is examined first. The results are presented in Table 5.

Higher bid-ask spreads were documented on Fridays in Table 1. The asymmetry of the bid-ask spreads on Fridays is also clearly supported by the results obtained for the parameter \( \gamma \) of model 1. The estimate is high, above 0.5, but not statistically different from 1.00 at the 5% level, for a currency such as the Italian lira, which has systematically been devalued with respect to the French franc over the sample time period. This suggests a higher bid than ask commission. Conversely, the estimates are low and negative, though not statistically different from zero, or currencies such as the Deutsche mark, the Dutch guilder, and the Swiss franc, which have been systematically revalued with respect to the French franc. This suggests a higher ask than bid commission.

The probability of realignments change over time, however. Therefore, more reliable inferences should be drawn from a model that allows the bid and ask commissions to be time varying, such as model 2. The empirical evidence supports this. The estimates of \( \gamma \) tie tighter, i.e., have a lower standard error. The efficiency gains between models 1 and 2 are substantial except for the Italian lira. The other conclusions remain unaltered. The bid-ask spreads are asymmetric with higher ask or bid commissions depending on the direction of change.

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\(^{20}\) In the four models, the parameter \( \theta \) is constrained and set equal to 1/2. The parameter \( \theta \) could not be estimated accurately because an instrument that would simultaneously (i) capture the variation in the future spot bid-ask spread and (ii) be in the information set at the \( t \) could not be found. As Equation (40) indicates, only the value of \( \theta \) at time \( t \mbox{m} \) matters. The value of the parameter \( \theta \) at time \( t \) is irrelevant. A similar remark applies to the intercept, i.e., the \( \alpha \) parameter. In the four models, \( \alpha \) is considered to zero. Attempts were made to estimate the intercept. They were unsuccessful mainly because of convergence problems due to a flat criterion function.

\(^{21}\) The number of degrees of freedom for models 1-4 is equal to 12, 12, 8, and 16, respectively. Regardless of the model, the total number of moment conditions is equal to \( 4 \times 4 = 16 \), i.e., four moment conditions for each of the four currencies. The total number of parameters to be estimated varies across models, however, explaining the difference in the number of degrees of freedom.
### Table 5
Tests of the unbiasedness hypothesis for Friday results with a sample size of 738 (time period: June 1, 1973—June 13, 1986; daily observations)

<table>
<thead>
<tr>
<th>Currency</th>
<th>Reg Pnt.</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL</td>
<td>$\alpha$</td>
<td>-.110*††</td>
<td>(.271)</td>
<td>.120*††</td>
<td>(.066)</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>$\alpha$</td>
<td>-.489*††</td>
<td>(.334)</td>
<td>-.043**††</td>
<td>(.049)</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SF</td>
<td>$\alpha$</td>
<td>-.077††</td>
<td>(.356)</td>
<td>.209**††</td>
<td>(.086)</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lira</td>
<td>$\alpha$</td>
<td>.660**†</td>
<td>(.142)</td>
<td>.655**†</td>
<td>(.139)</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2$ test</td>
<td>$\chi^2$</td>
<td>22.597</td>
<td>(.031)</td>
<td>20.956</td>
<td>(.051)</td>
</tr>
</tbody>
</table>

| Estimates of the parameters in the equation |

$$\frac{B_t - B_{t-1}}{B_t} = \alpha + \beta \frac{B_t - A_{t-1}}{B_t} + \gamma A_{t-1} - B_{t-1} + \delta \frac{B_t - B_{t-1}}{B_t} + \varepsilon_t$$

are reported under the following constraints. Degrees of freedom (d.o.f.) of the corresponding $\chi^2$ test are in parentheses. Model 1: $\alpha = 0, \theta = \frac{1}{2}$, and $\beta = 1$ (12 d.o.f.). Model 2: $\alpha = 0, \theta = \frac{1}{2}, \beta = 1$, and $\gamma = \gamma(1 - 100((B_t - B_o) - (B_{t-1} - B_{t-1})))$ (12 d.o.f.). Model 3: $\alpha = 0, \theta = \frac{1}{2}$, and $\gamma = \gamma(1 - 100((B_t - B_o) - (B_{t-1} - B_{t-1})))$ (8 d.o.f.). Model 4: $\theta = \gamma = 0$; corresponds to the regression of Table 4 on bid quotes (16 d.o.f.). Four instrumental variables are used, namely a vector of $A_{t-1} - B_o$, $(B_t - B_o)/B_t$, and $100((B_t - B_o) - (B_{t-1} - B_{t-1}))$. In the case of $\delta$, * and ** indicate significance at the 5% and 1% level, respectively, for $\alpha = 0$. In the case of $\gamma$, * and ** indicate significance at the 5% and 1% level, respectively, for $\gamma = 0$. * and ** indicate significance at the 5% and 1% level, respectively, for $\gamma = .5$. † and ‡ indicate significance at the 5% and 1% level, respectively, for $\gamma = 1$. All significance levels are based on two-tailed tests. The rates quoted in French francs are cross-rates. They are calculated from the rates quoted in U.S. dollars.
of the currency relative to the French franc. As the $x^2$ statistic and its associated probability value indicate, the restrictions, i.e., $\alpha = 0$, $\beta = 1$, and $\theta = 1/2$, are rejected by the data.

The joint null hypothesis that the forward rate is an unbiased predictor and that the future spot rate is halfway between the future spot bid and ask quotes is rejected in both models 1 and 2. The third model allows for a time-varying risk premium by relaxing the constraint on the $\beta$ coefficient. Three important results emerge from the estimation. First, the bid-ask spreads are asymmetric. Second, the $\beta$'s are all above .50. Third, unlike the two previous models, the restrictions, i.e., $\alpha=0$ and $\theta=1/2$, are not rejected by the data.

These results are even more interesting when compared to those obtained for model 4, where the restrictions $\theta=\gamma=0$ are imposed on the regression parameters. This model corresponds to a Fama-like regression on bid quotes. The comparison of models 3 and 4 enables one to assess the importance of the bias induced by the choice of a particular combination of bids and asks, here the bid. The range of the $\beta$ estimates is .68-.82 for model 3 versus -.03-.76 for model 4. The bias is the largest for the Italian lira. The shift in the slope coefficients toward unity, as one moves from model 4 to model 3, suggests that the risk premium is less volatile than what is typically concluded. Finally, the $x^2$ statistic indicates that the restrictions $\alpha = 0$ and $\beta = 1$ are rejected by the data for model 4.

The empirical evidence for the other days of the week, while not reported, can be summarized as follows. An asymmetric bid-ask spread is also found on the other days of the week. The results obtained for model 1 show that the coefficient $\gamma$ is not statistically different from zero, for most currencies except the Italian lira. The standard error estimates of $\gamma$ are large in general, however. A substantial increase in efficiency is observed when the parameter $\gamma$ is allowed to be time varying as in model 2. Also, once the constraint on the $\beta$ coefficient is relaxed, as in model 3, the difference between the estimates of $\gamma$ obtained for the Italian lira and the other currencies disappears, unlike the results obtained on Fridays. $\gamma$ is not statistically different from zero for all the currencies including the Italian lira. This result suggests that over that particular time period, the market expected the average change in the spot minus the forward rate, i.e., the negative

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22 Even after allowing the parameter $\gamma$ to vary, there is still a significant difference between the estimates for the Italian lira and the other currencies. This probably indicates that if governments do not adjust interest rates, market participants consider that there is still a positive probability of realignment.

23 This coincides with the finding in Table 3 that the size of the forward bid-ask spread relative to the forward premium is the largest with the Italian lira.

24 The estimation results for Monday through Thursday can be obtained from the authors.
of the forward premium, to be negative, independently of any consideration on pending government intervention. Stated differently, the market expected the differential between the French and foreign interest rates to narrow.\textsuperscript{25}

The model developed in this article assumes that the superior information, $\delta_{t+1}$, has a limited life of only one period, i.e., a day. The asymmetry of the bid-ask spread on days other than Friday indicates the value of the superior information extends beyond a day and potentially spans a week, i.e., the Monday trades predict the realignments to occur during the subsequent weekend. If information has value beyond one period, the model in this article has to be changed to reflect optimal revelation of information over time by better-informed traders, as in Kyle (1985).

4. Conclusion

In this article, the bid-ask spread in the spot and forward foreign exchange markets when some traders have superior information about government intervention was investigated. We examined four continental European currencies with respect to the French franc. Evidence of higher and asymmetric bid-ask spreads on Fridays was presented, reflecting market makers’ reaction when confronted with better-informed traders. The unbiasedness hypothesis of the forward rate as a predictor of the future spot rate continued to be rejected. Though the slope coefficients were estimated more precisely, they still differed from their value under the unbiasedness hypothesis, i.e., unity. However, the slope coefficients are closer to unity than when the bid-ask spread is modeled. This suggested a less volatile risk premium. While smaller, spreads were also asymmetric on other days of the week. The asymmetry was less pronounced than on Fridays, however. This finding suggested that superior information has value beyond one day, up to a whole week.

Also, in the article, a less volatile risk premium is documented in traditional tests of the unbiasedness assumption when run on currencies with respect to the French franc than in tests of currencies with respect to the U.S. dollar. We conjectured that this volatility differential can be attributed to a different degree of uncertainty about government intervention. Higher uncertainty about government intervention, however, will increase the asymmetry of the bid-ask spreads. If so, the latter will lead to an aggravation of the biases for currencies with respect to the U.S. dollar. The past rejections of the unbiasedness hypothesis, which have been attributed totally to the

\textsuperscript{25} This explanation is consistent with the results reported in note 19.
volatility of the risk premium, might in part be due to the biases caused by bid-ask spreads. The risk premium might, therefore, be less volatile than what has typically been believed. In this article, evidence is provided that this is correct for certain currencies with respect to the French franc. The confirmation for currencies with respect to the U.S. dollar is, however, complicated. It would require finding variables that capture changes in the market’s assessment of the level, direction, and probability of government intervention.

The assumption of risk neutrality in the market microstructure model of the paper is inconsistent with the empirical findings. Consequently, further modeling of the bid-ask spread in the foreign exchange market will have to consider risk aversion, as in Subrahmanyan (1991) or Bossaerts and Hughson (1991). The rejection of risk neutrality is unfortunate, because it could have justified the extraction of skewness in probability assessments of subsequent spot rate changes from foreign currency option prices. This now becomes impossible, since the risk-neutral and true distributions of the underlying spot rates will differ under risk aversion.

References


