Yukawa Interactions and Supersymmetric Electroweak Baryogenesis

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We analyze the quantum transport equations for supersymmetric electroweak baryogenesis including previously neglected bottom and tau Yukawa interactions and show that they imply the presence of a previously unrecognized dependence of the cosmic baryon asymmetry on the spectrum of third generation quark and lepton superpartners. For fixed values of the CP-violating phases in the supersymmetric theory, the baryon asymmetry can vary in both magnitude and sign as a result of the squark and slepton mass dependence. For light, right-handed top and bottom quark superpartners, the baryon number creation can be driven primarily by interactions involving third generation leptons and their superpartners.

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If the Universe was matter-antimatter symmetric at the end of the inflationary epoch, then the microphysics of the subsequently evolving cosmos must have dynamically generated the cosmologically observed baryon asymmetry (the baryon to entropy density ratio \( n_B/s \)).

\[
8.36 \times 10^{-11} < n_B/s < 9.32 \times 10^{-11} \quad (95\% \text{C.L.}) \quad (1)
\]

The standard model (SM) of particle physics and the standard big bang cosmological model contain all the necessary ingredients (Sakharov criteria [2]) for successful baryogenesis: baryon number violation, charge conjugation (C) and charge conjugation-parity (CP) violation, and departures from thermal equilibrium. However, fundamental symmetry tests and collider constraints indicate that generating the observed \( n_B/s \) requires physics beyond the SM (see, e.g., [3]).

Among most widely studied viable possibilities is electroweak baryogenesis (EWB), which is testable with low-energy searches for permanent electric dipole moments and high-energy studies at the Large Hadron Collider (LHC) [4]. In this scenario, electroweak symmetry breaking (EWSB)—a cosmological transition in which \( SU(2)_L \) is broken at a temperature \( T \sim 100 \text{ GeV} \)—proceeds via a strong, first order phase transition during which bubbles of broken electroweak symmetry nucleate and expand in a background of unbroken symmetry. Particle-antiparticle asymmetries generated by CP-violating interactions at the bubble wall induce a nonzero density of left-handed fermions, \( n_{\text{left}} \), that diffuses into the unbroken background where baryon number violating \( SU(2)_L \) sphaleron (electroweak sphaleron) transitions convert it into the baryon number. The expanding bubbles capture the nonvanishing baryon number and freeze it in by quenching the sphalerons, leading to \( n_B \neq 0 \) in the bubble interior. The baryon number is proportional to \( n_{\text{left}} \), which in turn depends on CP-violating interactions and chemical potential equilibrating reactions such as Yukawa and nonperturbative \( SU(3)_c \) transitions (strong sphalerons).

In this Letter, we reanalyze \( n_{\text{left}} \) in the context of the minimal supersymmetric standard model (MSSM) and observe new features which have been missed in previous works (see, e.g., [5–9] and references therein): (i) Yukawa interactions between bottom quarks, Higgs bosons, and their superpartners cannot be neglected in EWB, even if the ratio of the vacuum expectation values of the two MSSM Higgs doublets, \( \tan\beta \equiv v_u/v_d \), is mildly larger than unity (a parameter region favored by current experimental constraints). This typically results in a qualitative change from the standard picture: the first two generations of quarks and squarks decouple in EWB if the first two generations develop a CP-violating asymmetry mostly through strong sphalerons. (ii) The MSSM prediction for \( n_B/s \) can vary in magnitude and sign as the masses of the third generation sfermions are varied, even when the dominant source of CP violation is proportional to a single phase with a fixed sign. (iii) There exist parameter regions in which left-handed (LH) leptons drive the baryon number production, unlike the traditional situations in which EWB proceeds mainly through the interactions of the LH quarks with electroweak sphalerons. This occurs in the large \( \tan\beta \) region, in which \( \tau \) Yukawa interactions are significant. Unlike standard thermal leptogenesis scenarios (see, e.g., [3]), this new scenario does not require the participation of a right-handed neutrino sector.

In what follows, we first present the computational framework and analytic intuition for our main results. We subsequently give the full numerical results and discuss their implications. Although our discussion is framed within the MSSM, many of our conclusions are general and are likely to apply to EWB scenarios in extensions of the MSSM.

Framework and analytic intuition.—The current density \( j^A_p \) for each particle species \( p \) satisfies a quantum Boltzmann equation (QBE) of the form \( \partial_t j^A_p = S^p_c + S^p_{cv} \), where \( S^p_c \) and \( S^p_{cv} \) are, respectively, CP-conserving and CP-violating source terms which de-
pend on the MSSM interactions and chemical potentials. $S_p^{CPV}$ includes the terms that push the system toward chemical equilibrium, while $S_p^{CPV}$ contains the effects of CP-violating interactions involving the expanding bubble. We have developed numerical solutions to the full set of QBEs for all MSSM particle species chemical potentials, including contributions to $S_p^{CPV}$ from previously neglected Yukawa and triscalar interactions as well as “supergauge” interactions involving gauginos, particles, and sparticles.

The full numerical results, given at the end of this Letter, can be understood by considering an analytic solution which typically is valid in the limit of large tan $\beta$ and superequilibrium (see below). In this regime, there exists a hierarchy of time scales in the phase transition dynamics that implies a set of simple relations between particle chemical potentials and densities: $\tau_{\text{diff}}$, associated with the diffusion of particle densities ahead of the advancing bubble wall; a set of time scales $\tau_{\text{eq}}$, associated with different interactions that move the plasma toward chemical equilibrium or zero chiral charge; and $\tau_{\text{EW}}$, associated with the conversion of $n_l$ into baryon number by the electroweak (EW) sphalerons. Typically $\tau_{\text{eq}} \ll \tau_{\text{diff}} \ll \tau_{\text{EW}}$, implying that the dynamics of the first and second generation (s)fermions largely decouple from those of the third generation, which then become the dominant source of $n_B/\epsilon$.

We begin with the largest time scales. The electroweak sphaleron time scale is $\tau_{\text{EW}} \sim \Gamma_{\text{EW}}^{-1} \sim 10^8/T$, since $\Gamma_{\text{EW}} = 6\kappa\alpha_w^2 T$, with $\kappa \approx 20$ [10] and $\alpha_w$ the $SU(2)_L$ fine structure constant. The diffusion time depends on an effective diffusion constant for the plasma, $D \approx 50/T$ [11,12], and the velocity of the advancing wall, $v_w \sim 0.05$ [13]; $\tau_{\text{diff}} \equiv D/v_w^2 \sim 10^4/T$ [14].

The following reactions determine $\tau_{\text{eq}}$: (a) for third generation fermions, Higgs scalars, and their superpartners, Yukawa interactions associated with the decay, absorption, and scattering of particles within the thermal plasma; (b) strong sphalerons that favor a relaxation of chiral charge to zero; (c) supergauge processes involving spontaneous emission and absorption of gauginos, such as $q + \tilde{V} \leftrightarrow \tilde{q}$. For example, the Yukawa-induced equilibration time scale for third generation left-handed quarks ($q$), right-handed top squarks ($\tilde{t}$), and Higgsinos ($\tilde{h}$), driven by the scattering $q + \tilde{t} \leftrightarrow \tilde{h}$ is numerically $\tau_{\text{eq}} \approx 20/Y^2 T$ for $m_{\tilde{h}} = 200$ GeV and $m_{\tilde{t}} = 100$ GeV (Y is the top Yukawa coupling). Since $Y_t \approx 1$, $\tau_{\text{eq}} \ll \tau_{\text{diff}}$, which in turn implies the approximation $\mu_q = \mu_{\tilde{h}} = \mu_{\tilde{t}} = 0$ on $\tau_{\text{diff}}$ time scales.

Similarly, the time scale for strong sphaleron-induced relaxation of the total chiral charge, $N_3 \equiv \Sigma_j (2\mu_{q_j} - \mu_{u_j} - \mu_{d_j})$, where $q_j$, $u_j$, and $d_j$ denote the left-handed quark doublet and right-handed quark singlets of generation $j$, is $\tau_{\text{eq}} \sim \Gamma_{\text{ss}}^{-1} \sim 300/T$, since $\Gamma_{\text{ss}} = 6\kappa' (8/3)\alpha_s^4 T$ with $\alpha_s$ being the strong coupling and $\kappa' \sim O(1)$ [15]. Hence, $\tau_{\text{eq}} \ll \tau_{\text{diff}}$, leading to the condition $N_3 = 0$ on $\tau_{\text{diff}}$ time scales.

Finally, when the masses of gauginos ($\tilde{V}$) are sufficiently light, supergauge processes can lead to chemical equilibration between SM particles and their superpartners, a situation we denote as “superequilibrium” defined mathematically as $\mu_\mu + m_{\tilde{h}} - m_{\tilde{t}} = 0$, $\mu_\mu - m_{\tilde{h}} - m_{\tilde{t}} = 0$, $\mu_\ell - m_{\tilde{h}} - m_{\tilde{t}} = 0$, (3) where $\ell$ is the left-handed lepton; the difference of the sign of $\mu_{\tilde{h}}$ in Eq. (3) follows from hypercharge conservation.
Adding the first two implies that $2 \mu_q - \mu_T - \mu_b = 0$, such that the third generation contribution to $N_5$ vanishes. As previously noted, on the time scale of $\tau_{\text{diff}}$, we have $N_5 = 0$. Moreover, because the first and second generation Yukawa couplings (diagonal or off diagonal in gauge eigenstate basis) are tiny compared to the diagonal values for the third generation, there exist no significant interactions to generate nonvanishing first and second generation quark densities. Consequently, the first two generation densities approximately vanish for large $\tan \beta$.

We now use the approximate conservation of baryon and lepton number on the $\tau_{\text{diff}}$ time scale to relate the total fermion plus sfermion densities to those for the Higgs plus Higgsinos. Since the first and second generation (s)quark densities vanish, the conservation of the baryon number in terms of chemical potentials is $k_Q \mu_Q = -k_T \mu_T - k_B \mu_B$. Using $\mu_T + \mu_B = 2 \mu_Q$ as implied by Eqs. (3) and (2), we obtain

$$\mu_Q = \frac{k_b - k_T}{k_Q + k_B + k_T} \mu_H \rightarrow Q = \frac{k_Q}{k_H} \frac{k_B - k_T}{k_Q + k_B + k_T} H.$$  

(4)

Similarly, applying lepton number conservation and Eq. (2) to third generation leptons implies

$$L = \frac{k_l}{k_H} \frac{k_{\tau}}{k_L + k_{\tau}} H,$$  

(5)

where $L$ ($\tau$) denotes the third generation left-handed (charged right-handed) lepton supermultiplet density. Using the approximate vanishing of first and second generation fermion densities, we obtain the total left-handed fermion density

$$n_\text{left} \approx n_q + n_{\ell} = \frac{k_q}{k_H} Q + \frac{k_{\ell}}{k_L} L$$

$$\approx \left[ \frac{k_q}{k_H} \left( \frac{k_b - k_T}{k_Q + k_B + k_T} \right) + \frac{k_{\ell}}{k_L} \left( \frac{k_{\tau}}{k_{L + k_{\tau}}} \right) \right] H.$$  

(6)

Equation (6) is the key analytic result for the large $\tan \beta$ superequilibrium regime. It relates the nonvanishing Higgs supermultiplet density induced by CP-violating transport dynamics to $n_\text{left}$ via the analytical weights $k_p$ for third generation SM fermions, Higgs bosons, and their superpartners. The first and second terms on the right-hand side correspond to the contributions from third left-handed generation quarks and leptons, respectively.

The dependence of these contributions differs strikingly from what has appeared previously in the literature: $n_\text{left} \approx 5Q + 4T$, with, e.g., $k_b - k_T \rightarrow k_B - 9k_T$ and $k_Q + k_B + k_T \rightarrow 9k_Q + k_B + 9k_T$ in the numerator and denominator, respectively, of Eq. (4). These differences arise from (i) the contributions from the LH third generation (s)lepton density engendered when $Y_\tau$ is sufficiently large, and (ii) the $Y_B$-induced bottom quark superequilibrium conditions of Eq. (3). As discussed above, including the latter implies vanishing third generation contribution to $N_5$ and a corresponding suppression of the first and second generation quark densities. Using our full numerical solutions, we have verified that in the regime of small $Y_b$ and $Y_\tau$, one recovers the previously identified dependence of $n_\text{left}$ on $Q$ and $T$.

The presence of the statistical weights in Eq. (6) implies a strong dependence of $n_B/s$ on the thermal masses ($m_f$) of the third generation squarks and sleptons, as one may observe from the expression for the $k_p$ [7]:

$$\frac{k_p(m_p/T)}{k_p(0)} = \frac{c_{FR}}{\pi^2} \int_{m_p/T}^{\infty} dx \frac{x e^x}{(e^x - 1)^2} \sqrt{x^2 - m_p^2/T^2},$$  

(7)

where $c_{FR} = 6(3)$ and the $(+ (-))$ sign is for fermions (bosons). For a fixed right-handed (RH) top squark mass $m_{\tilde{t}}$, there exists a regime for $m_{\tilde{b}} = m_{\tilde{t}}$ in which $k_B = k_T$ in Eq. (6), corresponding to a nearly vanishing quark contribution to $n_\text{left}$. Indeed, this is the precise region in which the advertised sign flip can occur depending on the magnitude of sparticle masses. Furthermore, if the quark contributions are small, for sufficiently light $\tilde{t}$, EWB may actually be driven by the (s)lepton sector of the MSSM. This possibility does not arise under the previously studied assumptions of negligible $Y_b,\tau$ and represents a qualitatively new class of supersymmetric baryogenesis scenario.

Numerical solution and $n_B/s$.—Before turning to full numerical examples, we first isolate the essential physics with an approximate, analytic solution. In the regime for which $Q$ and $L$ are proportional to $H$ as in Eqs. (4) and (5), it is convenient to combine the QBEs into a single equation for $H$. Eliminating all terms containing the fast Yukawa and strong sphaleron rates, we find

$$\partial_{\lambda} |_{\tilde H} = -\Gamma H + \frac{S_{CP}^PV + S_{CPV}^V - S_{CPV} - S_{CP}^V}{1 + K_T + K_L - K_B},$$  

(8)

where $\Gamma = (\Gamma_h + \Gamma_T + \Gamma_B + \Gamma_\tau)/(1 + K_T + K_L - K_B)$ with $K_p \equiv H/P$, the $\Gamma_p$ being chiral relaxation transport coefficients that vanish outside the bubble [7], and where the $S_{CPV}$ denote the CP-violating source terms arising from the scattering of superpartners $\tilde p$ from the spacetime dependent Higgs vacuum expectation values. The diffusion ansatz allows one to express the left-hand side of Eq. (8) in terms of the density $H$: $\partial_{\lambda} |_{\tilde H} = H - 3D^2 H$, where $D$ is the effective diffusion constant introduced earlier.

Here, we will rely on the popular Higgsino CP-violating source (e.g., [5,6,9]) in order to illustrate the impact of the third generation sfermion masses and defer a study of possibly important $S_{CP}^V$ and $S_{CP}^{PV}$ contributions. In order to evade stringent electric dipole moment constraints in the large $\tan \beta$ regime [18], we allow independent relative phases $\phi_{\mu_i} \equiv \text{Arg}(\mu_i M_b^+)$ ($i = 1, 2$), where $b$ is the supersymmetry-breaking Higgs mass parameter. We also work in the resonant “bino-driven” regime [19], and choose $|\mu| = |M_1| = 120$ GeV and $|M_2| = 250$ GeV, computing $S_{CPV}^V$ and $\tilde \Gamma$ from the expressions in Ref. [7] and a bubble wall profile given in [6] for $\tan \beta = 20$. We consider a scenario with a RH top squark with $T = 0$ mass
\[ m_{\tilde{b}_1} = 122 \text{ GeV} \] as needed to obtain a strong, first order phase transition [20]. All first and second generation sfermions and third generation LH sfermions have masses equal to 1 TeV.

The results for \( n_B/s \) as a function of the \( T = 0 \) RH bottom squark mass, \( m_{\tilde{b}_1} \), in units of the observed value for \( \sin \phi_{\mu,1} = -1 \) are given in Fig. 1. The solid and dashed curves give the results obtained with the complete numerical solution of the QBEs and the large \( \tan \beta \) analytic solution embodied in Eqs. (6) and (8), respectively, for two representative RH tau slepton mass: \( m_{\tilde{\tau}_1} = 90 \text{ GeV} \) and 1 TeV. The dotted curve shows the numerical result obtained when bottom and tau Yukawa interactions are neglected (different treatments of the \( CP \)-violating sources and diffusion can lead to smaller \( |n_B/s| \) [9]).

The impact of including realistic bottom and tau Yukawa interactions in the large \( \tan \beta \), superequilibrium regime is striking, as seen in Fig. 1. Neglecting \( Y_{b,\tau} \) generally leads to a baryon asymmetry that is larger in magnitude, positive, and relatively insensitive to \( m_{\tilde{\chi}_1^0} \). The close agreement between the solid and dashed curves indicates our foregoing analysis captures the primary features of the dynamics in this regime. As expected, \( n_B/s \) is largest in magnitude when \( m_{\tilde{\chi}_1^0} \) is heavy and decreases as \( m_{\tilde{\chi}_1^0} \) is decreased, reflecting the growing importance of sbottoms as they become light and the greater cancellation between \( k_T \) and \( k_b \) in Eq. (6). For heavy RH staus, the magnitude of the (s)lepton contribution to \( n_{\tilde{\nu}_L} \) is small, and so \( n_B/s \) vanishes when \( m_b \sim m_j \) and \( k_b \sim k_f \). For light staus, the (s)lepton contribution becomes important, and for either very heavy or very light sbottom, this contribution can change \( n_B/s \) by a factor of 2. (Because \( n_B/n_{\tilde{\nu}_L} < 0 \), very light \( b \) and \( \tau \) lead to negative \( n_B/s \), and for large \( m_{\tilde{\chi}_1^0} \), \( n_B/s \) is positive.)

Deviations from Eq. (6) can occur for small \( \tan \beta \) or if superequilibrium is not valid. The region of \( \tan \beta \leq 10 \) corresponds to an interpolation between the upper dashed curve and the dotted curve in Fig. 1, reflecting the larger third generation quark and smaller leptonic components to \( n_{\tilde{\nu}_L} \). We have estimated that the rate for Cabibbo-suppressed flavor-changing \( 2 \to 2 \) scattering is much less than \( \tau_{\text{diff}}^{-1} \) and may be neglected. We have also made (standard) assumptions regarding the trilinear scalar couplings which affect the decoupling of the light generations. Exploration of these and other effects will appear in forthcoming publications [12].

Summary.—During the upcoming era of more sensitive electric dipole moment searches and LHC studies, it is crucial to explore testable EWB scenarios such as MSSM baryogenesis [4]. We have shown in this context that the bottom Yukawa coupling is important even for moderate values of \( \tan \beta \), which leads to dramatic changes in the basic physical EWB mechanism and the associated MSSM parameter space constraints. We also showed that the magnitudes of the sparticle masses can change the sign of the baryon asymmetry, and we identified a new lepton driven supersymmetric baryogenesis scenario which does not involve RH (s)neutrinos.

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[9] D. Chung et al. (to be published).