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Measurements of Solid Spheres Bouncing Off Flat Plates

1 Introduction

Recent years have seen a substantial increase of interest in the flows of granular materials whose rheology is dominated by the physical contact between particles and between particles and the containing walls. Considerable advances in the theoretical understanding of rapid granular material flows have been made by the application of the statistical methods of molecular gas dynamics (e.g., Jenkins and Savage (1983), Lun et al. (1984)) and by the use of computers simulations of these flows (e.g., Campbell and Brennen (1985), Walton (1984)). Experimental studies aimed at measurements of the fundamental rheology properties are much less numerous and are understandably limited by the great difficulties involved in trying to measure velocity profiles, solid fraction profiles, and fluctuating velocities within a flowing granular material. Nevertheless, it has become clear that one of the most severe problems encountered when trying to compare experimental data with the theoretical models is the uncertainty in the material properties governing particle/particle or particle/wall collisions. Many of the theoretical models and computer simulations assume a constant coefficient of restitution (and, in some cases, a coefficient of friction).

The purpose of the present project was to provide some documentation for particle/wall collisions by means of a set of relatively simple experiments in which solid spheres of various diameters and materials were bounced off plates of various thickness and material. The objective was to provide the kind of information on individual particle/wall collisions needed for the theoretical rheological models and computer simulations of granular material flows: in particular, to help resolve some of the issues associated with the boundary condition at a solid wall. For discussion of the complex issues associated with dynamic elastic or inelastic impact, reference is made to Goldsmith (1960) and the recent text by Johnson (1985).

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2 Normal Impact Between Spheres and Uniform Plates

Most of the literature (e.g., Goldsmith (1960), Johnson (1985)) concentrates on the impact of a sphere with large blocks. Yet the walls of most containers of granular material flows are plates which can often be thinner than one particle diameter. Hence, the first set of tests was carried out to determine the influence of the ratio of the particle diameter to the plate thickness on the coefficient of restitution resulting from a normal impact. The solid spheres used were precision ball bearings (steel or bronze) and glass spheres ranging in diameter from 0.317 to 2.54 cm. These were dropped from various heights onto plates of lucite or aluminum (ranging in thickness from 0.317 to 3.81 cm). Prior to release the spheres were suspended at the end of a tube by means of a small vacuum applied to the tube. A quick opening valve relieved this vacuum and released the particle in a precise and repeatable manner and without any significant rotational velocity.

The sphere trajectories and the height of the rebound were determined from multiple exposure photographs made using stroboscopic light; the strobe frequency was adjusted to achieve maximum resolution which occurred when each image of the sphere was separated by a maximum of one or two sphere diameters. Aerodynamic forces were determined to be small (at least outside the immediate moment of impact). Hence, for example, in the case of normal impacts, the coefficients of restitution were calculated from the ratio of the release height to the maximum height achieved after the first (or subsequent) rebounds.

The first question which arose concerned the effect of the structure supporting the plates upon the results. Two sides of the rectangular plates were solidly clamped to a relatively rigid support structure. By dropping the sphere to impact the plate at various distances from either of the clamped edges, it was determined that the coefficient of restitution decreased with distance of the impact point from the clamped edge until, at a certain critical distance, it became independent of the distance from the support. An example of this dependence is shown in Fig. 1 in which data for steel ball bearings impacting a 1.27-cm thick lucite plate are presented.

The critical distance was a linear function of the ball diameter but was independent of the plate thickness at least for the two thicknesses investigated. As seen in Fig. 2, the

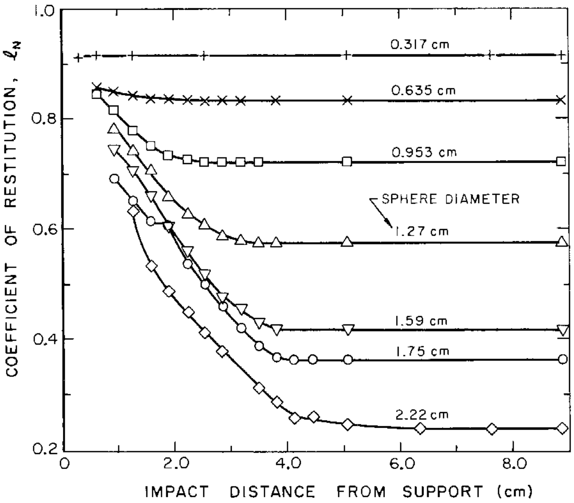

Fig. 1 Coefficient of restitution as a function of the distance of the impact point from the clamping support for various steel spheres impacting a 1.27-cm thick lucite plate from a drop height of 63.5 cm

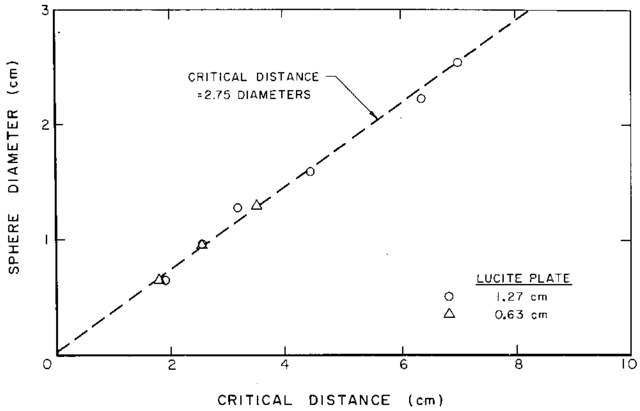

Fig. 2 Critical distance from support plotted versus sphere diameter for the steel sphere/lucite plate experiments

critical distance was 2.75 sphere diameters for the steel ball/lucite plate system. In theory, the rebound of the sphere can only be affected by the proximity of a plate support if there is sufficient time during contact for a wave to travel through the plate from the point of impact to the support and return to the impact region. Dilational, shear, and Rayleigh surface waves will be generated by impact, their speeds being given by

$$c = k[E_p/\rho_p]^{1/2} \tag{1}$$

where, assuming Poisson's ratio for the plate material to be 0.3, k is roughly 1.16, 0.62, and 0.57 for the three types of wave, respectively. The plate modulus of elasticity is E_p and its density is ρ_p; for lucite (polymethyl methacrylate) we assume a value of E of 2.6×10^9 kg/m sec^2 and a specific gravity of 1.18, though the former can only be considered quite approximate.

The Hertzian contact time, T_c, for a sphere on a semi-infinite block will be used to estimate the contact time with a finite plate and is given (Goldsmith (1960)) by

$$T_c = 3.21[(1-\nu_s^2) + (1-\nu_p^2)E_s/E_p]^{2/5} a \Big/ \left(\frac{E_s}{\rho_s}\right)^{2/5} (v_{N1})^{1/5} \tag{2}$$

where a is the sphere radius, v_{N1} is its impact velocity, E_s, ρ_s, ν_s are the modulus of elasticity, the density and Poisson's ratio for the sphere material, and ν_p is Poisson's ratio for the plate. The following values are assumed for the steel sphere/lucite plate system: $\nu_p = 0.3$, $\nu_s = 0.208$, $E_s = 2\times10^{11}$ kg/m sec^2. Moreover, due to the fifth power, the dependence on v_{N1} is not very strong and we assume a drop height of 60 cm so that $v_{N1} = 28.7$ cm/sec. Consequently, for each of the three types of wave the critical distance from the impact point to the support (for the steel sphere/lucite plate system) should be

$$\frac{cT_c}{2} = 11.14\ k\ a. \tag{3}$$

Note that the linear dependence of the critical distance on the sphere size is reproduced by the present experiments. Furthermore, the critical distance predicted by the theory would be 6.46, 3.45, or 3.18 sphere diameters for the dilational, shear, and Rayleigh waves, respectively. The difference between this and the observed critical distance of 2.75 sphere diameters may be due to inaccuracies in the lucite material properties. The agreement between theory and observation may be as good as could be expected and suggests that it is the reflection of the shear or Rayleigh waves rather than the dilational wave which creates the effect of the supporting system.

As a result of the foregoing investigation, the effect of the plate support system could be eliminated by ensuring that for each sphere/plate system, the impact point occurred beyond the critical distance from the support. Under these conditions, coefficients of restitution for normal impacts, e_N, were obtained for a wide range of sphere diameters, plate thicknesses ($2b$), and impact velocities (drop heights). Leaving aside, for the moment, the issue of the dependence on impact velocity, it was immediately apparent that the coefficient of restitution was almost solely a function of the ratio of the sphere diameter, $2a$, to the plate thickness, $2b$, irrespective of the separate values of these dimensions. This almost universal functional dependence is demonstrated in Fig. 3 for the steel sphere/lucite plate system and in Fig. 4 for experiments with Pyrex glass spheres and lucite plates. Data for the entire range of sphere diameters and plate thicknesses are presented in both figures.

It is important to emphasize that the decrease in e_N with increasing a/b does not necessarily imply an increased loss in kinetic energy of the whole system. Instead, with decreasing plate thickness, an increasing amount of kinetic energy is transferred to the kinetic energy of the plate at the instant of departure. Zener (1941) (see also Goldsmith (1960)) has analyzed the normal impact of spheres with flat plates and obtained an expression for the kinetic energy imparted to the plate upon departure. This is then converted to yield an "effective" coefficient of restitution for the sphere assuming no loss in the total kinetic energy. The result is a coefficient of restitution which is a function only of the impact parameter, λ, defined as

$$\lambda = \left(\frac{\Pi\rho_s}{\rho_p}\right)^{3/5} \frac{1}{4\sqrt{3}} \left(\frac{a}{b}\right)^2 \left[\frac{v_{N1}^2\rho_p(1-\nu_p^2)}{E_p}\right]^{1/10} \times \left[1 + \frac{E_p(1-\nu_s^2)}{E_s(1-\nu_p^2)}\right]^{-2/5}. \tag{4}$$

Note that consistent with our experiments, it follows that e_N is a function only of a/b for given materials. Zener's results are included in Figs. 3 and 4 assuming the previously quoted values for ν and the material properties of steel and lucite; for glass the values $\nu = 0.22$, $E = 6.2\times10^{10}$ kg/m sec^2 and $\rho = 2.24$ gm/cm^3 were employed. The theory and experiments agree very well. The slightly lower experimental values of e_N can be ascribed to other energy loss mechanisms not included in the theory, in particular, to the energy required for plastic deformation. It follows that although the effective coefficient of restitution is quite small, most of the apparent loss of kinetic

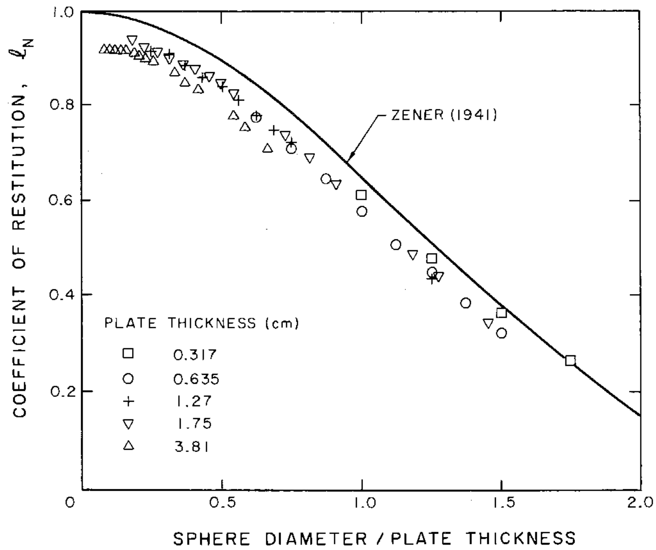

Fig. 3 Coefficients of restitution for normal collisions of steel spheres (diameters varying from 0.317 to 2.54 cm) impacting lucite plates of various thicknesses as indicated. Also shown is the theoretical result of Zener (1941).

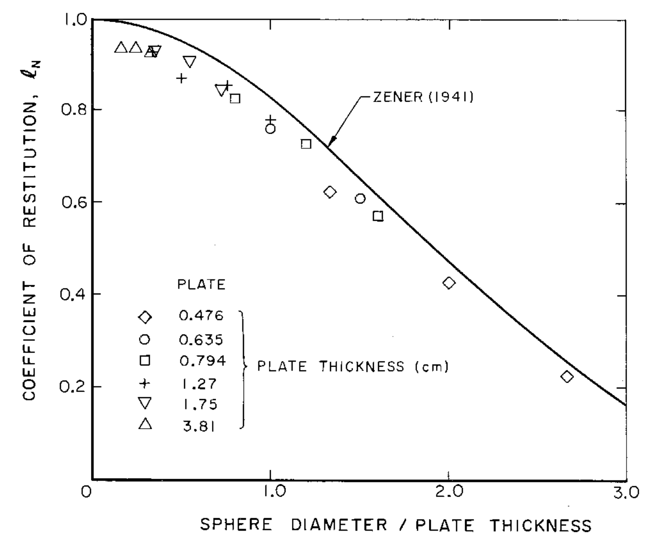

Fig. 4 Coefficients of restitution for normal collisions of Pyrex glass spheres (diameters varying from 0.635 to 1.27 cm) impacting lucite place of various thicknesses as indicated. Also shown is the theoretical result of Zener (1941).

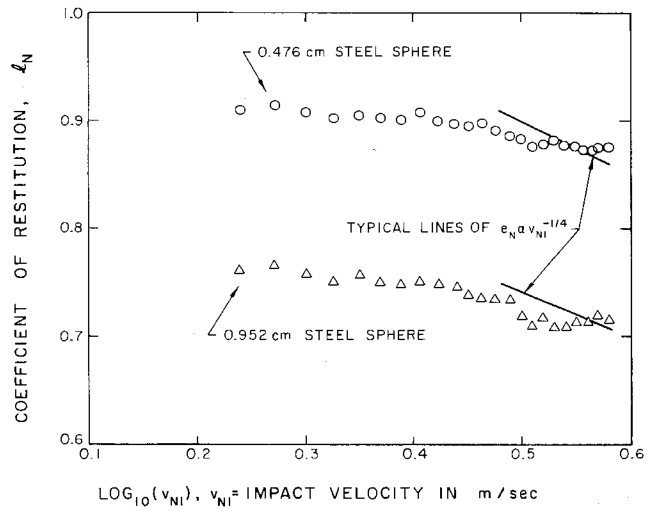

Fig. 5 Variation of coefficient of resitution with normal incident velocity for steel spheres impacting a 1.27-cm lucite plate

energy is converted into motion of the plate and in a granular material flow could be subsequently recovered by a subsequent collision of a particle with the wall. Indeed, the comparisons in Figs. 3 and 4 suggest that the actual energy loss may even be smaller for larger a/b than for small a/b.

Finally, we should comment on the influence of impact velocity. Previous experiments and theory (Johnson (1985)) on the impact of spheres with large blocks of material have shown that the coefficient of restitution is essentially unity until the critical impact velocity causing plastic deformation is exceeded. This critical velocity is very small for most materials and is exceeded in all the circumstances of the present experiments. At significantly higher velocities fully plastic deformation occurs, and when this is reached, experiment and theory indicate a coefficient of restitution which decreases like $(v_{N1})^{-1/4}$ with increasing impact velocity, v_{N1}. In Fig. 5, the results for one specific system, one plate thickness, and two sizes of sphere are plotted against v_{N1} for velocities ranging from 1.7 to 3.8 m/sec. Clearly the impacts lie between the critical velocity for plastic deformation and the fully plastic deformation velocity. The dependence on impact velocity is weak at the lower velocities; at higher velocities it may be approaching the $(v_{N1})^{-1/4}$ dependence. Some granular material flows will involve impact velocities substantially smaller than 1.7 m/sec; however, it was difficult to obtain accurate data at the very small drop heights involved.

It may be useful to observe that the phenomena described provides a full explanation of a demonstration which Profs. D. Shield and J. K. Knowles brought to our attention. This demonstration involves ball bearings of several sizes and lucite plates of several thicknesses which are supported by two wooden bars. When the balls are dropped on the thicker plate, the largest ball exhibits a substantial rebound while the smallest ball hardly bounces at all. On the other hand, when they are dropped onto the thinner plate the reverse is observed. This occurs because of two competing effects, one of which dominates with the thinner plates and the other with the thicker plates. The first effect is that the larger the ball bearing the closer it is to the support in terms of ball diameters and therefore the larger the coefficient of restitution. This first effect dominates for the larger plates because (as seen in Fig. 3) the coefficient of restitution only depends weakly on the plate thickness when the ratio of the ball diameter to plate thickness is small. The other effect is that the effective coefficient of restitution decreases quite rapidly with increasing ball diameter to plate thickness once the latter ratio exceeds about 0.5. Hence, for the thinner plates this effect overwhelms the support proximity effect and causes the smaller ball bearings to have a larger rebound.

One other result which follows from the present measurements and is essentially a restatement of the Shield-Knowles demonstration is that a plot of the coefficient of restitution against the sphere diameter for a plate of a given thickness, with supports a fixed distance apart, would exhibit a minimum. We initially produced such plots and were puzzled by their form until the effect of the support became clear. It is interesting to note that Koller and Busenhart (1986) observed a minimum in the coefficients of restitution for spheres impacting spherical shells. Those authors emphasize that a spherical shell will tend to rebound whereas a flat plate will not (their supports are well beyond any critical distance). In this respect a spherical shell may be phenomenologically similar to a flat plate with supports at less than the critical distance from the impact point.

3 Oblique Impact Between Solid Spheres and Uniform Plates

As Johnson (1985) indicates in his review, oblique collisions with plastic deformation and the possibility of microslip at the contact surfaces involve complicated mechanics at the microscale. This makes prediction of appropriate macroscopic collision characteristics quite difficult. Most of the theoretical or computational models of granular material flow assume (1) a "normal" condition consisting of a coefficient of restitution, e_N, relating the normal component of the departure velocity to the normal component of the approach velocity and (2) a "tangential" condition. Several asympotic or simplified "tangential" conditions have been employed in theoretical and computational models. One possibility is to assume that slip exists between the contact surfaces during the entire collision process; one might then relate the normal and tangential impulses by a friction coefficient. Alternatively, slip may cease during contact in which case an appropriate tangential condition would be to equate the departure tangential velocities of the two contacting surfaces.

Further discussion of these possibilities and the variation with incidence angle is given in the recent review by Brach (1988). Brach also provides a most useful summary of the existing data on oblique impact. The present experiments were carried out to obtain further information on the appropriate tangential condition.

The present oblique experiments used a 0.317-cm aluminum plate set at a series of angles, θ, to the horizontal (up to 70 deg in increments of 5 deg). Thus, θ is the angle of the trajectory to a target surface normal. Glass and steel balls of different sizes were dropped from various heights to impact the plate obliquely. The entire trajectory was photographed, and both components of the impact and departure velocities of the center of the spheres were ascertained from these trajectories. These velocities were used to calculate a "normal coefficient of restitution," e_N, defined as $-v_{N2}/v_{N1}$, where v_{N1} and v_{N2} are the components of the approach and departure velocities normal to the plate (positive into plate), and a "tangential coefficient of restitution," e_T, defined as v_{T2}/v_{T1} where v_{T1} and v_{T2} are the corresponding components tangential to the plate. Note that these are all translational velocities of the center of the spheres: The rotational velocities of the spheres before impact were zero and this fact is used later in discussing velocities of the contact point.

An alternative way to present this data is to plot the total coefficient of restitution e which represents the total loss of translational kinetic energy where

$$e=[e_N^2\cos^2\theta+e_T^2\sin^2\theta]^{1/2} \tag{5}$$

and a coefficient f, defined as the ratio of tangential to normal impulse of the collision so that

$$f=(1-e_T)\tan\theta/(1+e_N). \tag{6}$$

Following Brach (1988) the quantity f is termed the impulse ratio. The normal and tangential impulses acting through the contact point are denoted by J and K, respectively, so that

$$J=m(v_{n1}-v_{n2});\ K=m(v_{T1}-v_{T2}). \tag{7}$$

Furthermore,

$$Ka=mk^2(w_1-w_2) \tag{8}$$

where κ is the radius of gyration of the sphere and w_1, w_2 are the rotational velocities before and after impact. Also note, for future purposes, that the tangential velocity of the contact point upon departure, v_{Tc2}, is given by $v_{Tc2}=v_{T2}+aw_2$, and it follows that since $k^2/a^2=2/5$ for a sphere, and since $w_1=0$ in the present experiments then

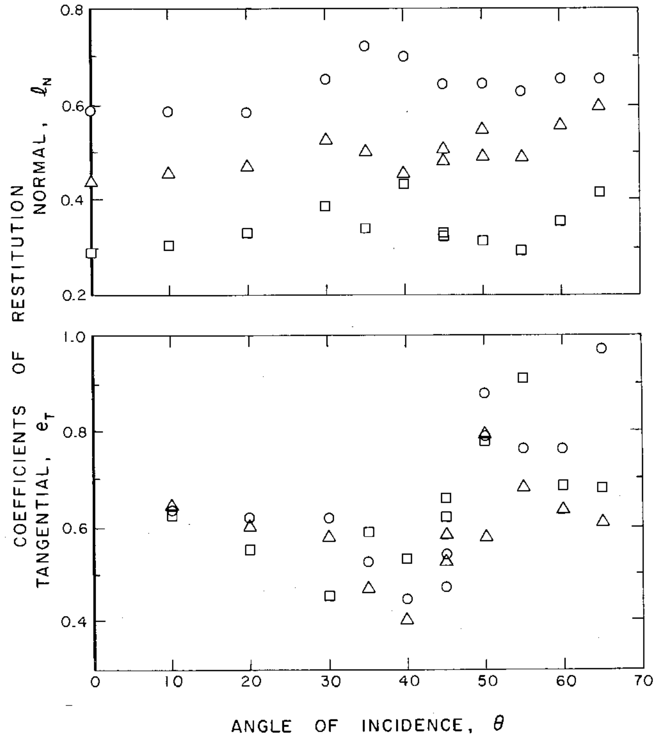

Fig. 6 Normal and tangential coefficients of restitution, e_N and e_T, for Pyrex glass spheres obliquely impacting a 0.317-cm thick aluminum plate: ○ = 0.635-cm sphere, ▽ = 0.953-cm sphere, □ = 1.27-cm sphere

$$\frac{v_{Tc2}}{v_{N1}}=\tan\theta\left\{\frac{7}{2}\frac{v_{T2}}{v_{T1}}-\frac{5}{2}\right\}=\tan\theta\left\{\frac{7}{2}e_T-\frac{5}{2}\right\}. \tag{9}$$

Consider first the simple theory described by Brach (1988) in which tangential deformations are neglected. Then for incidence angles, θ, less than a certain critical value, θ_c, slip will cease prior to departure so that $v_{Tc2}=0$ and from the relations (6) and (9).

$$e_T=5/7 \text{ and } f=\frac{2\tan\theta}{7(1+e_N)} \tag{10}$$

On the other hand, for $\theta>\theta_c$, slip will continue throughout contact and the ratio f should be equal to a Coulombic friction coefficient, μ, so that

$$e_T=1-\mu(1+e_N)\cot\theta \quad \text{and} \quad f=\mu. \tag{11}$$

Thus, as θ increases, f should first increase from 0 according to equation (10) until the critical value of $\theta_c\approx\arctan(7(1+e_N)\mu/2)$ is reached. For larger values of θ, f should remain relatively constant or, as Brach observes, may decrease due to a decrease in μ with increasing slip velocity. In the context of the present tests, experiments were carried out to measure the Coulombic coefficient of friction between the glass spheres and the aluminum plates. An average value of $\mu=0.23$ resulted when the target was clean and polished. Values as high as $\mu=0.3$ resulted when the aluminum surface was dirtier. Since e_N varied from 0.3 to 0.7 (see Fig. 6), the corresponding critical values, θ_c, were in the range of 45 deg to 55 deg.

A more detailed analysis should include the deformations and velocities associated with the elasticity of both surfaces. Then, in the case when slip has ceased during contact, the velocity of the contact point is no longer zero upon departure, but is equal to the time rate of change of the tangential deformation. Maw, Barber, and Fawcett (1976 and 1981) have analyzed this problem and shown both theoretically and experimentally that the appropriate tangential condition

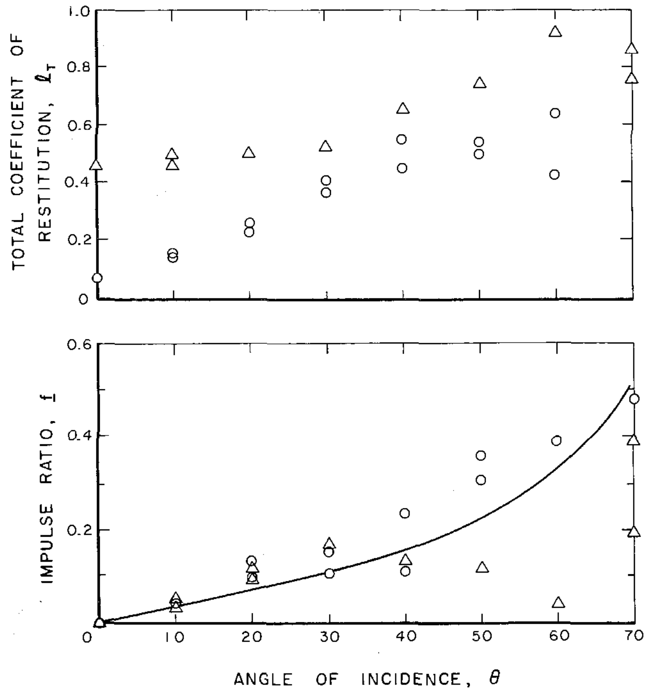

Fig. 7 Normal and tangential coefficients of restitution, e_N and e_T, for steel spheres obliquely impacting a 0.317-cm thick aluminum plate: △ = 0.317-cm sphere, ○ = 0.953-cm sphere

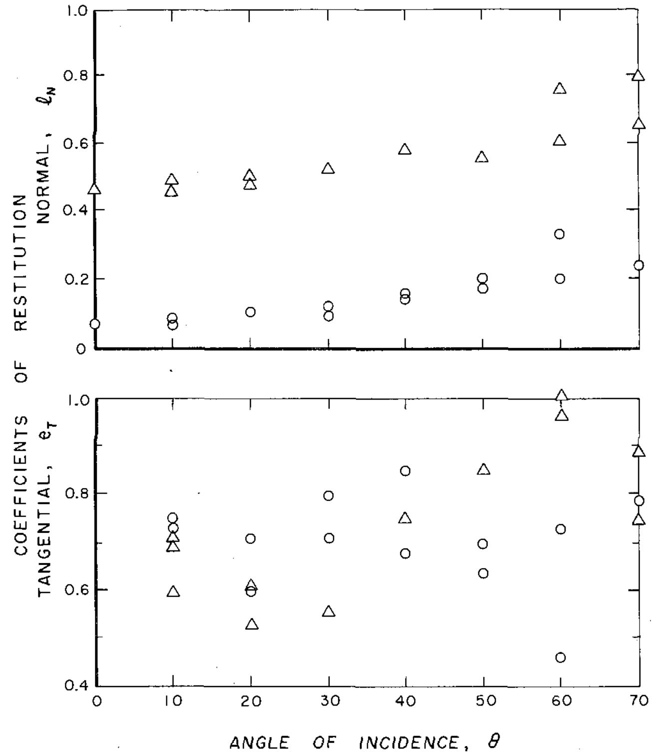

Fig. 8 Total coefficient of resitution and impulse ratio, f, corresponding to the data of Fig. 6.

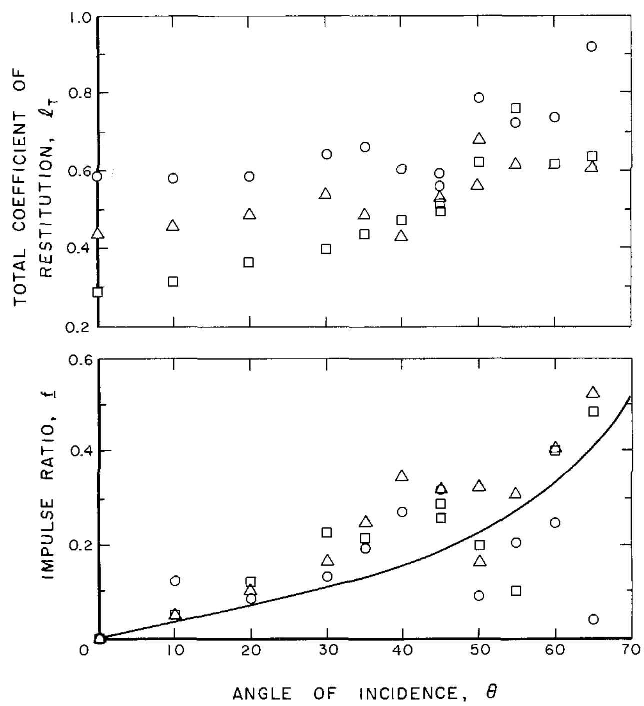

Fig. 9 Total coefficient of restitution and impulse ratio corresponding to the data of Fig. 7.

depends on the material properties (including a Coulomb friction coefficient, μ, at the contact interface and a parameter, κ, which is a ratio of elastic moduli of the two materials) and on the incidence angle. Specifically they demonstrate that the phenomenon is controlled by a nondimensional incidence angle $(\kappa/\mu)\tan\theta$ where θ is the angle of the trajectory of the contact point to a normal to the surface. When $(\kappa/\mu)\tan\theta$ is less than a value of the order of 4 to 6, tangential elasticity causes the tangential relative velocity of the contact point upon departure to be negative (opposite direction to the incident tangential velocity). One must exceed this critical value of 4 to 6 for the departure tangential velocity to become positive. When κ is of order unity, the critical angles resulting from the Maw, Barber, and Fawcett analysis do not differ greatly from those of the simpler analysis described previously.

In the present investigation, preliminary experiments revealed that although the data for e_N was quite consistent and repeatable, the data for e_T was very scattered. This scatter was reduced but not eliminated by carefully cleaning all the surfaces with acetone prior to every test. We concluded that the scatter was at least in part caused by lack of repeatability in the frictional conditions existing at the contact surface. Data for 0.635-cm, 0.953-cm, and 1.27-cm glass spheres and for 0.317-cm and 0.953-cm steel spheres are presented in Figs. 6 and 7, respectively. The variation of the normal coefficient, e_N, with incidence angle is not great in any of this data. In general, however, e_N does appear to increase a little with incidence angle. In the case of the glass spheres, there is a slight local maxima at an incidence angle of 30 deg – 40 deg. The data for the steel spheres displayed a substantial increase in scatter at the larger incidence angles.

The corresponding data for the tangential coefficient, e_T, is shown in Figs. 6 and 7. The data for the glass spheres shows more variation with incidence angle than occurred with the normal coefficient. It is also striking that the scatter in the data increases markedly above an incidence angle of about 40 deg. The same was true of the data for the steel spheres which showed substantial increase in the scatter above about 50 deg.

The data of Figs. 6 and 7 was also used to produce the graphs of total coefficient of restitution, e, and impulse ratio, f, included as Figs. 8 and 9. Both sets of data show a consistent increase in e with increasing incidence angle, a trend which appears consistent with previous data (Brach, 1988). The data for the impulse ratio show that f increases systematically with θ until an incidence angle of about 45 deg (Fig. 8) or about 35 deg (Fig. 9) is reached. These values are

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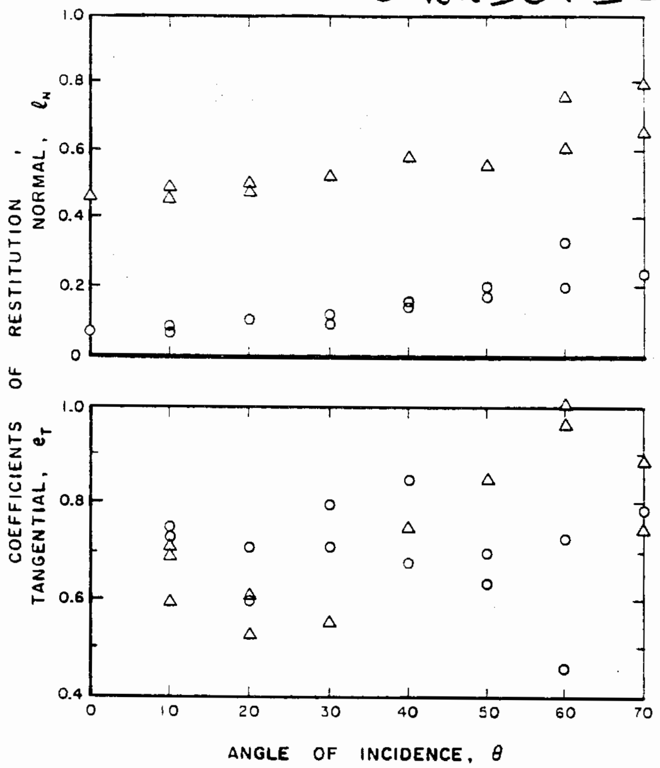

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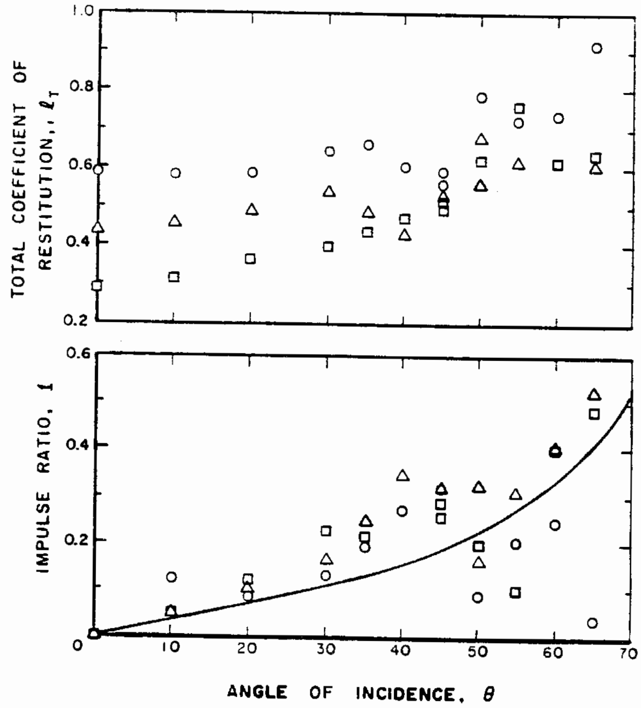
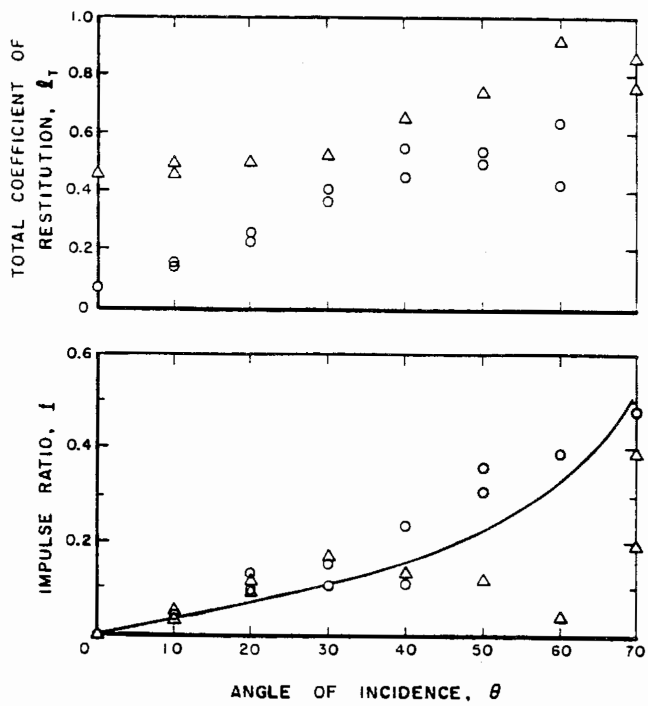


Fig. 8 Total coefficient of resitution and impulse ratio, *f*, corresponding to the data of Fig. 6.



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It should, however, also be noted that the data above the critical incidence angles of about 45 deg and about 35 deg is very scattered and inconsistent. Because of this scatter it is not possible to say whether it corresponds to a constant value of $f = \mu$ or not. Indeed, the data do suggest that collisions at angles greater than the critical may be very sensitive to variations in the effective friction of the surfaces in the vicinity of the contact points.

In the case of the glass spheres (Figs. 6 and 8), the critical incidence angle is particularly evident in the data of e_T (Fig. 6). Note that for subcritical incidence angles of less than about 45 deg, e_T is smaller than the value of 5/7 (0.71) which would occur if $v_{cT2} = 0$. Values of e_T less than 5/7 imply a negative v_{cT2} and therefore indicate that the tangential elasticity played a significant role in determining the rebound conditions for subcritical incidence angles. Some of the data for the steel spheres (Fig. 7) show a similar trend, but the evidence is less conclusive in that case.

4 Acknowledgments

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