THE SPACE-DISTRIBUTION OF THE PHOTO-ELECTRONS EJECTED BY X-RAYS

By E. C. Watson

Abstract

None of the theories which have heretofore been proposed to account for the apparent emission of x-ray electrons from the atom over a wide range of angles instead of in one definite direction is entirely satisfactory. Since the scattering of the electrons which takes place in neighboring atoms has not been completely eliminated in any of the experimental work, the Rutherford theory of nuclear scattering is applied to the problem. This theory together with the assumption that the electrons all start out from the parent-atom in the same direction can be made to explain in a satisfactory way all the details of the observed space-distribution. It gives the amount of the spread quantitatively; it gives the form of the distribution curves; it explains the difference between the lateral and longitudinal distributions; it explains the dependence of the amount of the spread upon the nature of the atoms from which the electrons are ejected and the frequency of the x-rays which do the ejecting; and it accounts for the disagreement among various observers regarding this dependence.

Incidentally it is pointed out that the scattering of 17 kv electrons by hydrogen nuclei is many times greater than can be accounted for on the classical theory.

THE problem of the nature of the force exerted upon an electron by a field of radiation is a fundamental one. Much light is thrown upon it by studies of the directions in which photo-electrons are ejected by x-rays. Experiments of this kind have recently been performed by C. T. R. Wilson[1] in England, Auger[2] in France, Bothe[3] and Kirchner[4] in Germany, and by Bubb[5], Loughridge[6], and the writer[7] in this country. These experiments show that while the most probable direction of ejection is exactly that of the electric vector of the incident radiation except for a forward component, there is apparently a very considerable spread in the directions of the individual electrons. Theories to account for this apparent emission from the atom over a wide range of angles instead of in one definite direction have been given by Bubb,[8] Bothe,[9] Auger and Perrin,[10] Wentzel,[11] Beck,[12] and Oppenheimer,[13] but none of them is entirely satisfactory in explaining all the

[1] C. T. R. Wilson, Proc. Roy. Soc. **A104,** 1 and 192 (1923).
[2] P. Auger, Comptes rendus **178**, 929, 1535 (1924); Jour. de Phys. et Rad. **8,** 85 (1927).
[3] W. Bothe, Zeits. f. Physik **26,** 59 (1924).
[4] F. W. Bubb, Phys. Rev. **23,** 137 (1924); Nature **112,** 363 (1923).
[5] D. H. Loughridge, Phys. Rev. **26**, 697 (1925) and **30**, 488 (1927)
[6] F. Kirchner, Phys. Zeits. **27,** 385 and 799 (1926); also Ann. d. Physik **83,** 521 (1927).
[7] E. C. Watson, Phys. Rev. **30,** 479 (1927).
[8] F. W. Bubb, Phil. Mag. **49,** 824 (1925).
[9] W. Bothe, Zeits. f. Physik **26,** 74 (1924).
[10] P. Auger and F. Perrin, Comptes rendus **180,** 1742 (1925), Jour. de Phys. et Rad. **8,** 93 (1927).
[11] G. Wentzel, Zeits. f. Physik **40,** 574 (1927).
[12] G. Beck, Zeits. f. Physik **41,** 443 (1927).
[13] J. R. Oppenheimer, Zeits. f. Physik **41,** 291 (1927).

now known experimental facts. The purpose of this paper is to point out that nuclear scattering of the sort postulated by Rutherford with such brilliant success in the case of α-particles has not been sufficiently considered in this connection and that it may account for the whole effect.[14]

I. The Experimental Situation

The experiments bearing upon this question have been of two general types: those which make use of unpolarized x-rays and study the distribution of electrons about the direction of the x-ray beam (this is usually called the *longitudinal distribution*) and those which use polarized x-rays and study the distribution about the direction of the electric vector in a plane perpendicular to the x-ray beam (this is called the *lateral distribution*).

The expansion-chamber technique of C. T. R. Wilson furnishes what is unquestionably the most powerful method which has been used in studies of this kind. It has been employed by Auger and Loughridge to study the longitudinal distribution and by Bubb and Kirchner to determine the lateral distribution.

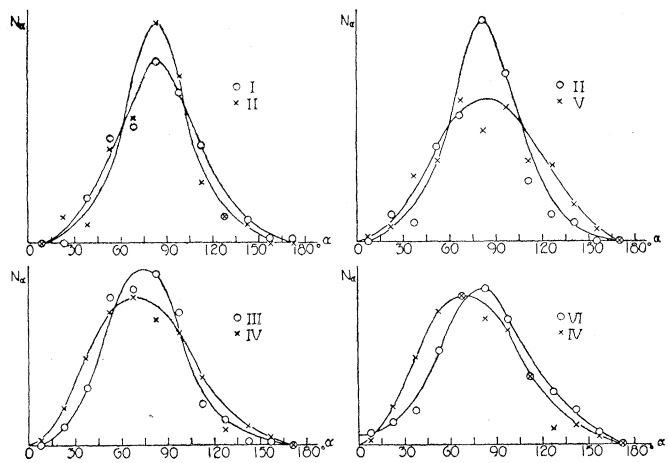

Fig. 1. Longitudinal distribution of photo-electrons ejected by x-rays as found by Auger.
I. K electrons of oxygen or nitrogen ejected by 15 kv primary x-rays.
II. K electrons of oxygen or nitrogen ejected by 20 kv primary x-rays
III. K electrons of argon ejected by 80 kv primary x-rays
IV. L electrons of xenon ejected by 80 kv primary x-rays
V. K electrons of krypton ejected by 22 kv primary x-rays
VI. K electrons of xenon ejected by 45 kv primary x-rays

Auger's results are shown in Fig. 1 in which the number of photo-electron tracks is plotted as a function of the angle of ejection measured from the forward direction of the x-ray beam for a number of experimental conditions. As heterogeneous x-radiation from a Coolidge tube was used the exact frequency of the rays which are most effective in each case is not known, but

[14] See also E. C. Watson, Phys. Rev. **29**, 752 (1927) and Proc. Nat. Acad. **13**, 584 (1927).

it is clear that the amount of the spread (i.e. the fraction of the whole number of tracks which start out at angles other than the most probable one) depends, to some extent at least, upon both the frequency of the incident x-rays and the nature of the gas in the expansion-chamber. This is in agreement with the results obtained by Bothe using a point-discharge ion-counter and those of Seitz[15] who studied the ionization on the two sides of very thin metallic films.

Loughridge, using zirconium-filtered radiation from a tube with molybdenum target driven at 40 kv, found that the amount of the spread was practically the same when argon or air was introduced into the expansion-chamber, but that it was less in hydrogen.

Kirchner, on the other hand, interprets his results on the lateral distribution as proving that the lateral spread is independent of both the frequency of the incident x-rays and the nature of the gas in the expansion-chamber. His results together with those previously obtained by Bubb are shown in Fig. 2 which is taken from Kirchner's paper. Here again the number of tracks is plotted against the direction of emission, the direction of the electric vector being at 90°. Heterogeneous radiation from an x-ray tube driven at approximately 50 kv was used in these experiments, but Kirchner finds nearly the same distribution for the tracks which are shorter than 15 mm that he does for those longer than 15 mm. This fact may not be significant, however, as the length of the track depends upon other factors than the energy of ejection and consequently the average energy of ejection of the longer tracks may not be much greater than that of the shorter tracks. Thus Nuttall and Williams[16] using strictly monochromatic x-rays found that the length of tracks varied as much as 30% each side of the mean. The results of Auger and Bothe which do show a variation with frequency are therefore the more convincing.

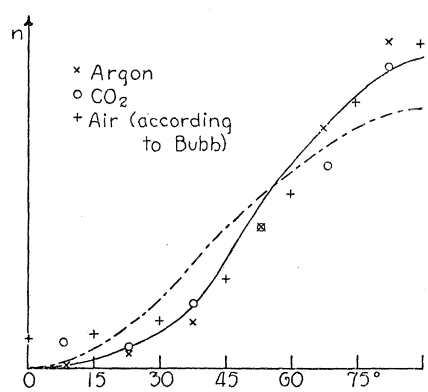

Fig. 2. Lateral distribution of photo-electrons ejected by x-rays as given by Kirchner.

Moreover as regards the dependence of the spread upon the nature of the gas in the expansion-chamber, Kirchner's results are not as convincing as are those of Auger, Bothe, and Loughridge, because the range of molecular weights used in his experiments was not so large. We conclude therefore that the evidence favors a variation in spread both with the nature of the gas and the frequency of the incident x-rays.

The results of Loughridge on air and hydrogen are of particular interest in this connection. They show that the spread is determined not so much by

[15] W. Seitz, Ann. d. Physik **73**, 183 (1924); Phys. Zeits. **25**, 546 (1924) and **26**, 610 (1925).
[16] J. M. Nuttall and E. J. Williams, Phil. Mag. **2**, 1109 (1926).

the nature of the atom from which ejection takes place as by the sort of molecules which surround it; for a simple calculation based upon the fact that the absorption of x-rays is proportional to the fourth-power of the atomic number shows that when hydrogen is the gas in the expansion-chamber practically all the electrons are ejected from the oxygen atoms of the water-vapor which must always be present. Consequently the difference between the spreads in air and hydrogen cannot be due to an appreciable difference in the atomic number of the atom from which the electron is ejected.

The dotted line in Fig. 2 is an attempt to fit the experimental points by a sine-square function, while the solid curve is a sine-cube function. No such functions can be made to fit the facts satisfactorily, however, because the experimental curves do not fall to zero at 0°. This is in marked contrast with the curves for the longitudinal distribution which do fall to zero at both 0° and 180° (as of course they must because the ordinate for any abscissa α is the number of tracks which start out through the solid angle between α and $\alpha+d\alpha$ to the direction of the x-ray beam and this solid angle becomes zero when α equals 0° and 180°). The failure of the curves for the lateral distribution to fall to zero may be due to lack of complete polarization of the incident x-rays, but this is unlikely as the non-polarized portion of the x-ray beam was carefully corrected for by both Bubb and Kirchner.

A comparison of the curves for the lateral distribution obtained by Bubb and Kirchner with those of the longitudinal distribution obtained by Auger and Loughridge shows that the amount of the spread is greater in the former than it is in any of the latter. Exact data upon this point are given in Tables II and III, below.

Finally it must be pointed out (and this is the consideration which makes the writing of this paper necessary) that the results of the expansion-chamber experiments are not as unambiguous as they are usually supposed to be. Thus it seems to have been universally assumed that these experiments give the actual directions of the electrons as they leave the individual atoms. This is not strictly true. The x-ray when it ejects an electron ionizes the atom and a water droplet condenses upon it. The diameter of this droplet was found by measurement upon the plates obtained by Loughridge to range from 0.5 to 0.9 mm.[17] Thus the space around the atom from which the electron is ejected is obscured and any change in the direction of motion of the electron which takes place in this distance cannot be detected. This fact has heretofore been thought to be of no significance because calculations of the amount of scattering to be expected in this distance were not made and simple scrutiny of the tracks themselves (which are often apparently quite straight for comparatively large distances) leads one to believe that no appreciable

[17] Mr. E. J. Williams has kindly written me that in his experiments the average diameter of the initial droplet is only about 0.1mm. In this case however the distance between droplets is probably larger. In any case it seems to the writer that it would be exceedingly difficult, if not impossible, to make exact measurements upon the direction of the tracks down to less than 0.5 mm from the atom from which the electron is ejected.

change in direction can take place in so small a distance. The calculations to be presented in this paper indicate, however, both that a considerable scattering will take place in a distance of the order of 0.5 mm and that the apparent straightness of the tracks is very deceptive.

II. Discussion of Previous Theories

Theories of the space-distribution have been of two kinds; those which account for the spread by compounding the momentum (assumed random in direction) of the electron in its atomic orbit just before ejection with the momentum imparted by the ejecting radiation (Bubb and Bothe), and those which deduce a probability function for the angle of ejection from general postulates, such as those of symmetry (Auger and Perrin) or those of the new quantum mechanics (Wentzel, Beck, and Oppenheimer).

Theories of the first kind seem to the writer to be definitely invalidated by the experimental results just given. Such theories demand that the spread be very great when the energy of the electron in its atomic orbit is only a little less than the energy of the incident quantum. The results of Auger show that this is not the case. Thus curve VI in Fig. 1 (for which the orbital energy $h\nu_0$ is 35 kv and the incident energy $h\nu$ is 45 kv) is practically no wider than curve IV (for which $h\nu_0 = 5$ kv and $h\nu = 80$ kv); nor is curve V (for which $h\nu_0 = 14$ kv and $h\nu = 22$ kv) any wider than is to be expected from considerations to be given later. The difficulty cannot be avoided by recourse to the new quantum mechanics which allows the moment of momentum of the K-electrons to be zero, as the writer[18] has recently shown that the spread is certainly not markedly greater for the L_{III} electrons of gold ejected by the $K\alpha$ rays of molybdenum ($h\nu_0 = 12$ kv and $h\nu = 17$ kv) than it is for the N and O electrons whose orbital energies are negligible.

All of the theories of the second kind so far proposed have led to a lateral distribution which is proportional to the cosine-square of the angle of ejection measured from the direction of the electric vector. There are four objections to such a distribution: (1) while Kirchner has succeeded in bringing his latest experimental results into fair agreement with this distribution, his earlier curves as well as those obtained previously by Bubb show a much sharper maximum than is given by the cosine-square law (this is shown in Fig. 2 where, as stated above, the dotted curve represents the cosine-square distribution); (2) the curves of the longitudinal distribution show a sharper maximum than is given by the corresponding cosine-cube law; (3) as has previously been pointed out the experimental curves do not fall to zero at 90° from the direction of the electric vector; and (4) such theories cannot without additional assumptions account for the variations in spread which Bothe, Auger, and Loughridge found when they changed the frequency of the incident x-rays and the nature of the gas in the chamber.

We conclude therefore that none of the theories so far proposed is entirely successful in accounting quantitatively for the observed space-distribution. Since the theoretical distributions are if anything already too broad, the

[18] E. C. Watson, Proc. Nat. Acad. **13**, 659 (1927).

agreement becomes still worse when allowance is made for the scattering in neighboring atoms. It therefore seemed worth while to calculate what the effect of scattering alone would be.[19]

III. Scattering in Neighboring Atoms

The Rutherford theory of nuclear scattering applied to β-rays leads to the equation[20]

$$\rho_\phi = \pi n t (Ze^2/2T)^2 \cot^2(\phi/2) \tag{1}$$

where ρ_ϕ is the fraction of a beam of β-rays of charge e and kinetic energy T scattered through an angle equal to or greater than ϕ by a thickness t of scattering material containing n nuclei per cc of charge Ze.

This formula[21] (when modified by the inclusion of a relativity correction[22] worked out by C. G. Darwin for the change in mass of the electron with velocity in its orbit around the nucleus) has been completely verified in the case of the scattering of β-rays from radium E by Chadwick and Mercier[23] and for 30-77 kv cathode-rays by Schonland[24] provided thin enough foils are used so that Wentzel's criterion[25] for "single" scattering holds. It may therefore be used to calculate the fraction of the whole number of tracks which experience deflections greater than any given amount in going a distance of the order of 0.4 mm in the expansion-chamber experiments of Auger, Bubb, Kirchner, and Loughridge, on the assumption that all the electrons start out initially in the same direction. This fraction we shall call simply the "amount of the spread."

[19] That scattering in neighboring atoms might be made to account for the whole of the observed spread was suggested by the writer's results on the longitudinal distribution obtained with exceedingly thin metallic films. These showed that, in this case at least, nuclear scattering was the principal factor in bringing about the spread and that the ratio of the number of electrons leaving in a forward or backward direction to the number leaving at right angles to the x-ray beam became smaller as the thickness of the film decreased. This made plausible the assumption that for an infinitesimally thin film all the electrons would be ejected in a direction a little forward of normal to the x-ray beam.

[20] E. Rutherford, Phil. Mag. **21,** 669 (1911). See also J. A. Crowther and B. F. J. Schonland, Proc. Roy. Soc. **A100,** 526 (1922).

[21] For cathode-rays of small velocity Davisson [(Phys. Rev. **21,** 637 (1923) and **22,** 242 (1923)] has shown that equation (1) must be modified to take into account the screening effect of the outer electrons in the scattering atom. The velocities of the electrons in all the experiments on the space-distribution are, however, large enough so that corrections of this sort are negligible.

[22] For electrons ejected by the $K\alpha$ rays of molybdenum this correction is negligible except at large angles of scattering. For higher velocity electrons it may multiply ρ_ϕ by a factor of 2 or 3. Since, however, the exact frequencies of the ejecting x-rays are not known in the experiments of Auger, Bubb, and Kirchner, this correction cannot be made accurately and it is therefore omitted in the calculations which follow. In any case it always operates to increase the scattering.

[23] J. Chadwick and P. H. Mercier, Phil. Mag. **50,** 208 (1925).

[24] B. F. J. Schonland, Proc. Roy. Soc. **A113,** 87 (1926).

[25] G. Wentzel, Ann. d. Physik **69,** 335 (1922). See also Chadwick and Mercier, loc. cit. and Schonland, loc. cit.

Before applying equation (1) to this calculation, however, it is necessary to determine whether Wentzel's criterion for "single" scattering holds for distances of 0.4 mm in air, argon, hydrogen, etc.

a. *Wentzel's criterion for "single" scattering.* Wentzel's criterion for "single" scattering may be simply stated as follows: the scattering will be "single" provided the angle of scattering ϕ is at least some small multiple of ω, where ω is given by the equation

$$\cot(\omega/2) = 2TR/Ze^2 \tag{2}$$

where $R = (2/\pi nt)^{1/2}$ and T, Z, e, n, and t have the same meaning as above.

The smallest value of the ratio $\phi/4\omega$ for which the scattering will be certainly "single" has been made precise by experiments on the scattering of both α-and β-rays by thin foils. The smallest value of $\phi/4\omega$ in the α-ray experiments of Chadwick was 7, and the scattering was certainly "single." In the β-ray experiments of Crowther and Schonland for which $\phi/4\omega$ varied from 3.1 to 1.7, the value of ρ_ϕ was in general several times that given by equation (1) and the scattering has been shown to have been mainly "plural,"[26] while in the β-ray experiment of Chadwick and Mercier in which the scattering was certainly "single," $\phi/4\omega$ ranged from 4.5 to 7.5. The most precise evaluation of $\phi/4\omega$ for electrons has, however, been made by Schonland. With aluminum foil and both 59,900 and 77,300 volt electrons he obtained a linear relation between ρ_ϕ and t until, as the thickness increased, $\phi/4\omega$ became less than 3, when ρ_ϕ began to increase much more rapidly. This shows that the scattering will be "single" if $\phi/4\omega$ is greater than 3, but that it will be "plural" or "multiple" if $\phi/4\omega$ is less than 3.

For 15 and 24 kv electrons scattered in going a distance of 0.04 cm in argon, air, and hydrogen under standard conditions equation (2) gives the values of $\phi/4\omega$ in Table I.

TABLE I. *Values of* $\phi/4\omega$

Angle	$\phi/4\omega$ for 15 kv electrons			$\phi/4\omega$ for 24 kv electrons		
	argon	air (oxygen)	hydrogen	argon	air (oxygen)	hydrogen
90°	1.7	4	32	2.8	6.4	50
80°	1.5	3.6		2.5	5.6	
70°	1.3	3.1		2.2	5.0	
60°	1.1	2.7		1.8	4.2	
50°	1.0	2.2		1.5	3.5	
40°	0.8	1.8	14	1.2	2.8	22

This shows that the scattering in both air and argon is mainly "plural" even for distances of 0.04 cm. Such a result is very surprising in view of the apparent straightness of the tracks themselves, particularly as the number of right-angle bends in a distance of a whole cm has actually been counted by

[26] Plural scattering seems to be the preferred translation of the German "mehrfachstreuung." It is used to denote the intermediate case between single and multiple scattering in which the number of deflections experienced by a single particle is neither one nor large enough so that a Gaussian distribution may be assumed.

C. T. R. Wilson[27] and found to be in good agreement with equation (1). Since Wentzel's criterion for "single" scattering in thin metal foils should apply equally well to scattering in gases, this discrepancy must mean either that in the expansion-chamber experiments only the comparatively straight portions of the tracks are recognized as such, or that because of the appreciable size of the water droplets which make up the tracks many of the bends are obscured. Since the scattering is "plural" even for a distance which is less than the width of the tracks, it is not impossible that a series of droplets which seem to form a perfectly straight track may be due to an electron which actually followed a somewhat zigzag path.[28]

In view of this uncertainty in the interpretation of the experimental results, we shall assume, except where there is unambiguous evidence to the contrary, that the scattering takes place in accordance with equation (1). Furthermore, since in the cases which we shall consider the scattering is "plural" according to Wentzel's criterion, we shall multiply the value of ρ_ϕ obtained from equation (1) by a factor of 2 or 3. This is necessitated by the results of Crowther and Schonland which show that when $\phi/4\omega$ lies between 3 and 1.7 the scattering is between 2 and 3 times that given by equation (1).

b. *Lateral distribution.* Equation (1) was derived for the case of the scattering of a narrow beam of electrons traversing a thin lamina of the scattering material. If the photo-electrons are all ejected in the direction of the electric vector, this equation can be applied directly[29] to the calculation of the lateral distribution on the assumption that it is due entirely to nuclear scattering. The only modification necessary is caused by the fact that in this case electrons will be ejected in both the positive and negative directions of the electric vector and since the cot $\phi/2$ falls to zero at 180° from the direction of emission instead of at 90°, electrons ejected at angles between 90° and 180° will add themselves to those emitted between 90° and 0°

TABLE II. *Lateral spread*

	$\rho_{90°}$	$\rho_{80°}$	$\rho_{70°}$	$\rho_{60°}$	$\rho_{50°}$	$\rho_{40°}$
Observed	1.2%	1.9%	3.2%	5.1%	9.7%	19%
Calculated	1.1%	1.5%				
Calculated×2			4.4%	6.4%	10%	
Calculated×3						23%

[27] C. T. R. Wilson, Proc. Roy. Soc. **A104,** 205 (1923).

[28] Several discrepancies now outstanding between experiment and theory will be explained if the actual path of the electron is appreciably longer than the more or less straight track which envelopes it. Thus both C. T. R. Wilson (Proc. Roy. Soc. **A104,** 195 (1923) and Williams (Nature **119,** 489 (1927) in their expansion-chamber experiments found that the ionization per cm along the track was greater than that given by the theory of J. J. Thompson for the ionization per cm along the electron path. This difference disappears if the actual path length is greater than the distance measured along the track.

[29] There is some uncertainty as to whether the experiments of Bubb and Kirchner give as a function of ϕ the number of tracks which go out through solid angle $2\pi \sin \phi \, d\phi$ or their projection upon the plane normal to the x-ray beam. If the latter, the calculated values of ρ_ϕ will be smaller than those given in Table II.

respectively. If the composite curve of Bubb's and Kirchner's results (Fig. 2) is corrected for this effect and the fractions (ρ_ϕ) of the tracks which start out at angles greater than 90°, 80°, 70°, etc., are determined from the corrected curve the values given in the first row of Table II are obtained. The calculated values are for argon using equation (1) and putting $n=2.7\times10^{19}$, $t=0.04$ cm, $Z=18$, $T=23{,}000$ volts $=77\times4.774\times10^{-10}$ ergs.[30]

The close agreement between the calculated and experimental values shows that scattering in neighboring atoms is sufficient to account for the whole *magnitude* of the spread without unreasonable values of t and T being assumed, and the fair agreement for all values of the angle of scattering between 40° and 90° even when integral multipliers are used proves that the theory also gives the *shape* of the distribution very exactly.

To obtain the same agreement for the distribution in air and CO_2, either larger values of t or smaller values of T must be assumed. It is not necessary to modify either very largely, however, as the molecules of air being diatomic and those of CO_2 being triatomic are nearly as effective in scattering as are the monatomic molecules of argon. That this is the case is shown by some unpublished results of Mr. C. D. Anderson in this laboratory who counted roughly the number of right-angle bends in the first cm of path on the plates of Loughridge and found that 11% of the tracks in air showed such bends and 14% in argon. Moreover since heterogeneous x-rays were used in Bubb's and Kirchner's experiments, the frequencies of the x-rays which were most effective in ejecting electrons were probably different in the various gases. These considerations are sufficient to account for the fact that Kirchner found no difference in the spread for these gases.

c. *Longitudinal distribution.* In the case of the longitudinal distribution equation (1) must be further modified. If we assume that each electron leaves the atom exactly in the direction of the electric vector of the ejecting radiation, then electrons ejected by unpolarized x-rays will start out at random in a plane containing the electric vector and perpendicular to the direction of the x-ray beam. This means that in discussing the longitudinal distribution we must treat the scattering as from a plane instead of from a line. This may easily be done as follows:

Differentiating equation (1) we find that the number of electrons scattered through solid angle $2\pi \sin\phi d\phi$ is

$$\pi nNt(Ze^2/2T)^2 \cot(\phi/2) \operatorname{cosec}^2(\phi/2)d\phi \tag{3}$$

where N is the total number of electrons and ϕ is the angle of scattering

[30] The exact value of T which should be used in this calculation is uncertain, since as has already been said the frequencies of the x-rays which are most effective in ejecting electrons in the experiments of Bubb and Kirchner are not known. The value used here is justified by Kirchner's observation that in his experiments the majority of the tracks were less than 12 mm long (corresponding to a value of T of about 23 kv), even though his x-ray tube was driven at 50–60 kv, and by Loughridge's observation that in argon the tracks were considerably shorter when heterogeneous radiation from a tungsten-target tube driven at 40 kv was used than they were for the zirconium-filtered radiation from a molybdenum-target tube driven at the same voltage (T for molybdenum $K\alpha$ rays $=15$ kv.)

measured from the direction of the electric vector. The number scattered through unit solid angle will therefore be

$$\frac{\pi nNt(Ze^2/2T)^2 \cot(\phi/2)\operatorname{cosec}^2(\phi/2)d\phi}{2\pi \sin\phi d\phi} = nNt\left(\frac{Ze^2}{2T}\right)^2 \frac{1}{(1-\cos\phi)^2}$$

But with unpolarized x-rays the direction of the electric vector will be at random in a plane perpendicular to the direction of the x-ray beam. If then θ represents the angle which any specified direction (in which we wish to calculate the scattering) makes with this plane and ω is the azimuth of that direction measured from the direction of the electric vector, we have

$$\cos\phi = \cos\theta\cos\omega$$

and therefore the number of electrons scattered through unit solid angle at an angle θ to the plane will be

$$\frac{nNt}{2\pi}\left(\frac{Ze^2}{2T}\right)^2 \int_0^{2\pi} \frac{d\omega}{(1-\cos\theta\cos\omega)^2} = nNt\left(\frac{Ze^2}{2T}\right)^2 \frac{1}{\sin^3\theta}$$

and the number scattered through solid angle $2\pi\cos\theta d\theta$ will be given by

$$dN = 2\pi nNt\left(\frac{Ze^2}{2T}\right)^2 \frac{\cos\theta}{\sin^3\theta}d\theta = 2\pi nNt\left(\frac{Ze^2}{2T}\right)^2 \cot\theta\operatorname{cosec}^2\theta d\theta. \tag{4}$$

Integrating we get the fraction scattered at angles equal to or greater than θ to be

$$\rho_\theta = \pi nt(Ze^2/2T)^2 \cot^2\theta \tag{5}$$

The distribution in this case is therefore a function of the whole-angle instead of the half-angle as in the case of the lateral distribution. The coefficient $\pi nt(Ze^2/2T)^2$ is moreover the same as that in equation (1) and therefore, since $\cot\theta$ is always less than $\cot(\theta/2)$, the spread of the longitudinal distribution will be less than that of the lateral under similar experimental conditions.[31]

It is possible also to choose reasonable values of t and T such that equation (5) will give quantitatively both the magnitude and shape of the longitudinal

[31] Since the most probable direction of ejection of the electrons is not exactly in the plane of the electric vector, but instead is a little forward of it, we should have treated instead of scattering from a plane, scattering from the surface of a cone whose axis is the direction of the x-ray beam and whose semi-angle is a little less than 90°, the exact angle depending upon the frequency of the incident x-rays. A treatment of this case, similar to that given above for the case of scattering from a plane, leads to the equation

$$dN = \frac{\pi}{2}nNt\left(\frac{Ze^2}{2T}\right)^2\left[\frac{1}{\sin^2(\alpha+\beta)/2} + \frac{1}{\sin^2(\alpha-\beta)/2}\right]\frac{\sin\alpha\, d\alpha}{\cos\beta - \cos\alpha}$$

where α is the angle of scattering measured now from the forward direction of the x-ray beam and β is the semi-angle of the cone. This equation reduces to equation (3) when $\beta=0$ and to equation (4) when $\beta=90°$ as it should. In the experimental work to be discussed, however, β is in all cases close to 90° and consequently equation (5) gives the spread with sufficient accuracy.

spread. Thus in Table IIIA the experimental values of ρ_θ obtained by Loughridge for electrons ejected in argon by the $K\alpha$ rays of molybdenum are compared with corresponding values calculated from equation (5) putting $n=2.7\times10^{19}$, $t=0.04$ cm, $Z=18$, and $T=14.1$ kv. Similarly, in Table IIIB the spreads obtained from curve I of Fig. 1 are compared with values obtained from equation (5) when $n=2.7\times10^{19}$, $t=0.08$ cm, $Z=8$, and $T=12$ kv; and Table IIIC compares values obtained from cuve II of Fig. 1 with those calculated for 15 kv electrons in oxygen.[32]

TABLE III. *Longitudinal spread.*

	$\rho_{90°}$	$\rho_{80°}$	$\rho_{70°}$	$\rho_{60°}$	$\rho_{50°}$	$\rho_{40°}$
	A. 14 kv electrons in argon (Loughridge)					
Observed	0%	0.15%	0.8%	2.2%	5.6%	12%
Calculated×2	0%	0.18%	0.75%	1.9%		
Calculated×3					6.0%	12%
	B. 12 kv electrons in oxygen (Auger I.)					
Observed	0%	0.08%	0.65%	2.3%	6.3%	13.5%
Calculated	0%	0.09%				
Calculated×2			0.8%	2.0%		
Calculated×3					6.6%	13.2%
	C. 15 kv electrons in oxygen (Auger II.)					
Observed	0%	0%	0.1%	1.0%	3.8%	9.4%
Calculated	0%	0.06%	0.26%			
Calculated×2				1.3%		
Calculated×3					4.2%	8.4%

The factors 2 and 3 are introduced only where Wentzel's criterion and the experimental results of Crowther and Schonland indicate that they should be. The improvement in the agreement between theory and experiment which is brought about by their use is a further argument that the scattering is "plural." If however the introduction of these factors is objected to on the ground that Wentzel's criterion cannot be applied in gases and that C. T. R. Wilson's count of the number of right-angle bends along the tracks was sufficiently accurate to show that the scattering is "single," the theory of nuclear scattering can still be made to fit fairly well (quite well in the case of argon) by choosing larger values of t and smaller values of T which are still consistent with the experimental facts.

Finally the variations in the amount of the spread with the frequency of the incident x-rays and the nature of the gas, which Bothe[33] and Auger found, take place in the way demanded by the coefficient $\pi nt(Ze^2/2T)^2$. Thus curves I and II of Fig. 1 show that when the voltage on the x-ray tube is raised from 15 to 20 kv, other conditions remaining the same, the spread decreases in the way required by the theory. Again if curves III and IV are compared it is seen that when argon is replaced by xenon, the voltage being kept constant, the curve broadens as it should. That the effect is not larger is probably due

[32] The exact values of T to be used in these last two cases are again uncertain. The values used (12 kv and 15 kv) are for reasons already given (see footnote 30) less than the voltages actually applied to the x-ray tube.

[33] According to Wentzel's criterion the scattering was also "plural" in the experiments of Bothe except at large angles.

to the fact that only small quantities of argon and xenon were introduced, the gas in the expansion-chamber being principally hydrogen. The same effect is shown by a comparison of curves II and V. The fact that Loughridge found no difference between the spread in air and in argon has already been explained in the discussion of the lateral distribution. That the variations with voltage are not larger may be due to the effect of the relativity correction to which reference has already been made. It is more probable, however, that even when the voltage on the x-ray tube is high the majority of the electrons are ejected by x-rays of low energy because their absorbability is so much greater. Experiments of the sort made by Nuttall and Williams with strictly monochromatic x-rays will be necessary to settle this point.

One apparently serious difficulty for the theory still remains, however. Loughridge found the spread in hydrogen only a little less than in argon and the theory of nuclear scattering here given demands that it be several hundred times less. This is too great a discrepancy to be accounted for by simple inadequacy in the number of tracks counted. It can only mean that either the theory is wrong or the scattering in hydrogen is actually much greater than that predicted by the classical theory. Fortunately the order of magnitude of the scattering in hydrogen can be ascertained independently of the space-distribution by studying the sharp bends in the tracks themselves and Dr. Loughridge very kindly allowed Mr. Anderson to count the number of tracks on his plates which showed sharp bends of 90° or more within the first cm of their range. In hydrogen 18 out of 190 tracks (9.5%) showed such bends, while in argon only 12 out of 85 (14%) did so. This means that the nuclear scattering in hydrogen is actually only a little less than it is in argon, and consequently it is quite sufficient to account for the observed space-distribution. The difficulty thus turns out to be an argument for, rather than against, the theory here presented.

The fact that the nuclear scattering in hydrogen is actually many times greater than was predicted by the theory probably means that the inverse-square law of force between the nucleus and the electron breaks down when the distance of approach becomes sufficiently small. A similar effect was found by Rutherford[34] and Chadwick and Bieler[35] in the case of close collisions between α-particles and hydrogen nuclei. This is, however, so far as the writer knows, the first time such an effect has been observed for collision between electrons and nuclei and a further study of it would seem to offer further possibilities for determining the exact limits of the inverse-square law and perhaps also of studying the structure of the electron and proton, or of determining their magnetic moments.[36]

IV. Scattering in Getting Out of the Parent-Atom

An elementary calculation of the probability of an electron having its direction changed in getting out of the parent-atom, based upon the assump-

[34] E. Rutherford, Phil. Mag. **37,** 537 (1919).

[35] J. Chadwick and E. S. Bieler, Phil. Mag. **42,** 923 (1921).

[36] See in this connection E. Rutherford and J. Chadwick, Phil. Mag. **4,** 605 (1927).

tions (1) that the impulse received by the electron is sensibly instantaneous, (2) that the field of the nucleus of the parent-atom acts upon the electron in exactly the same way as do the fields of the nuclei of neighboring atoms through which it passes, (3) that the electron originates in a circular Bohr orbit whose radius is given by the simple Bohr theory, and (4) that the orientation of the electron with respect to the nucleus at the moment of ejection is a random one, leads to the following approximate formula, when the absorption frequency ν_0 of the electron in the atom is small compared with the frequency ν of the ejecting x-ray, namely

$$\rho_\phi = (\nu_0/4\nu) \cot^2 (\phi/2) \tag{6}$$

where ρ_ϕ is as before the fraction of a large number of electrons which will have their direction changed by an amount equal to or greater than ϕ. This equation is of the same form as that for the scattering in neighboring atoms, but the amount of the spread given by it is, in the case of the lighter atoms, much smaller than that already considered. In the case of argon and the $K\alpha$ rays of molybdenum, however, $\rho_{40°}$ comes out about 6%, which would have to be taken into account if the effect were a real one. However, in the neighborhood of an absorption edge, i.e. when ν_0 is only a little less than ν, this scattering in getting out of the atom should be large. The results of Auger show that such is not the case. This may mean that a particular orientation of the atom with respect to the direction of the ejecting x-ray is necessary for photo-electric ejection, or that the ejection is always radial, but the theoretical considerations involved are too uncertain to warrant any definite interpretations.

V. Summary and Conclusion

The results of this investigation may be summarized as follows: The Rutherford theory of nuclear scattering together with the assumption that the electrons all start out from the parent-atom in the same direction can be made to account satisfactorily for all the details of the observed space-distribution of the photo-electrons ejected by x-rays. It gives the amount of the spread quantitatively; it gives the form of the distribution curves; it explains the difference between the lateral and longitudinal distributions both as regards the amount of the spread and as regards the falling of the curves to zero; it explains the dependence of the amount of the spread upon the nature of the atoms from which the electrons are ejected and the frequency of the x-rays which do the ejecting; and it accounts for the disagreement among the various observers regarding this dependence.

We conclude therefore that the reality of the space-distribution has not been established and that probably all the photo-electrons ejected by x-rays of a given frequency from the same kind of atom are ejected in one and the same direction with respect to the electric vector.

It should be stated, however, that while nuclear scattering thus seems *adequate* to account for the whole of the space-distribution, it is not necessarily the only effect which is present. If the experimental curves should on

further investigation turn out to be actually not more sharp than a cosine-square curve (and the experiments are probably as yet neither complete nor accurate enough to be sure that this is not the case) the cosine-square distribution *plus nuclear scattering* would also be satisfactory in explaining the facts; for the effect of even a large amount of scattering upon a cosine-square distribution is small except at 90°, but would be large enough to explain both the failure of the curves of the lateral distribution to fall to zero and the dependence of the spread upon the nature of the gas and the frequency of the x-rays. In any case nuclear scattering will be present and must be taken into account.

VI. Acknowledgments

This paper would have been impossible if the writer had not had free access to the important experimental results of Dr. D. H. Loughridge and Mr. C. D. Anderson before they were published and he desires to express his sincere thanks to them both for putting these results completely at his disposal and for freely discussing with him many of the details of the work. The constructive criticism of Dr. I. S. Bowen also played an important part in the paper and the writer gratefully acknowledges his help.

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Note added in proof: Since the manuscript of this paper was sent to the Editors, the arguments against scattering's having any appreciable effect upon the space-distribution have been published by Kirchner [Ann. d. Physik, **84,** 899, (1927)]. As all these arguments have already been discussed only two remarks are called for: (1) The writer still holds that scattering is *adequate* to account for the whole of the space-distribution observed by Bubb, Kirchner, Auger, and Loughridge. This is best shown in the calculations based upon the results of Loughridge in which none of the factors entering into the calculation except t is uncertain and a very conservative value for it is used. Moreover, in the case of Kirchner's own published results, if the relatively large number of tracks which appear to start out at nearly 90° from the direction of the electric vector or if the departures from the cosine-square distribution which he observes in his most recent work [Ann. d. Physik, **83,** 521 (1927)] are to be explained as due to nuclear scattering—and there seems to be no other explanation—then the scattering is adequate to account for the whole of the distribution, for the reason given in the next to last paragraph of this paper. (2) While the distribution curves used by the writer may be statistically less accurate than the corresponding integral curves plotted by Kirchner, they are also far more sensitive than the integral curves. Thus, on the scale which Kirchner uses, a cosine-cube curve can scarcely be distinguished from a cosine-square curve. That the variations shown by the differential curves are real is argued by the fact that they are systematic and by the results of Bothe and Seitz.

This paper is frankly an attempt to push the scattering idea as far as possible. If Williams [Nature, **121,** 134 (1928)] has succeeded in completely eliminating its effect from his results, the arguments here presented are still of value in explaining the variations observed by others and in focusing attention sharply upon the ambiguities in the expansion-chamber results.