Search for the decay $B^+ \to K_S^0 K^0\pi^+$. 

Charmless decays of $B$ mesons to final states with even numbers of strange quarks or antiquarks, such as $B^+ \rightarrow K^0_S K^0_S \pi^+$ [1], are suppressed in the standard model. Such decays proceed mainly via the $b \rightarrow d$ loop (penguin) transition. Hadronic $b \rightarrow d$ penguin transitions have been observed in the decays $B^0 \rightarrow K^0_S 0$ and $B^+ \rightarrow K^0_S K^+$ [2,3], and their effects have also been seen through direct CP violation in charmless $B$ decays, such as $B^0 \rightarrow \pi^+ \pi^-$ [4,5] and $B^0 \rightarrow \pi^+ \pi^- \pi^0$ [6,7]. In contrast to $B^0, B^+$ mixing, which is a $b \rightarrow d$ process with a change of beauty-flavor quantum number of $\Delta F = 2$, little experimental information exists on $\Delta F = 1 b \rightarrow d$ decay amplitudes. There is still potential for new physics effects to be uncovered in these decays.

The decay $B^+ \rightarrow K^0_S K^0_S \pi^+$ has not yet been observed. The upper limit on the branching fraction at 90% confidence level (CL) is $3.2 \times 10^{-6}$ [8]. A model based on the factorization approximation, which uses heavy-quark and chiral symmetries, predicts a nonresonant branching fraction for $B^+ \rightarrow K^0_S K^0_S \pi^+$ of order $10^{-6}$ [9]. Decays via intermediate resonant states can also lead to the $K^0_S K^0_S \pi^+$ final state. This motivates an inclusive analysis incorporating both nonresonant and resonant modes. Based on the measured branching fraction $B(B^+ \rightarrow f_2(1270) \pi^+) = (8.2 \pm 2.5) \times 10^{-6}$ [10–12], the product branching fraction for $B^+ \rightarrow f_2(1270) \pi^+$ with $f_2(1270) \rightarrow K^0_S K^0_S$ should be around $10^{-7}$. Similarly, $B^+ \rightarrow f_0(980) \pi^+$ and $B^+ \rightarrow K^+ + (892)K^0$ decays could contribute to the $K^0_S K^0_S \pi^+$ channel. The branching fraction for $B^+ \rightarrow K^+ + (892)K^0$ is predicted to be of order $10^{-6}$ or less [13–18].

Another motivation comes from the recent observation of $B^+ \rightarrow K^+ K^- \pi^+$ by BABAR, with an inclusive branching fraction of $B(B^+ \rightarrow K^+ K^- \pi^+) = [5.0 \pm 0.5(\text{stat}) \pm 0.5(\text{syst})] \times 10^{-6}$ [19]. An unexpected peak seen near 1.5 GeV/$c^2$ in the $K^+ K^-$ invariant-mass spectrum, which we dub the $f_K(1500)$, accounts for approximately half of the total event rate. If the decay of the $f_K(1500)$ follows isospin symmetry, then equal rates would be expected to $K^+ K^-$ and to $K^0 K^0$. If the $f_K(1500)$ has even spin, then $f_K(1500) \rightarrow K^0 K^0$ decays would result in 50% $K_S^0 K_S^0$ and 50% $K_S^0 K_S^0$ final states, whereas if the $f_K(1500)$ has odd spin, then the $K_S^0 K_S^0$ final state is forbidden by Bose symmetry. Observation of the decay $f_K(1500) \rightarrow K_S^0 K_S^0$ in $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ could therefore provide information on the spin or the quark content of the $f_K(1500)$ and could help to elucidate the relationship between this state and similar unexplained structures seen in $B^+ \rightarrow K^+ K^- K^+$ decays [20,21]. Structures in the $K_S^0 K_S^0$ mass spectrum have also been observed in two-photon [22] and electron-proton collisions [23].

We report a search for the decay $B^+ \rightarrow K^0_S K^0_S \pi^+$. The analysis is based on data collected at the PEP-II asymmetric-energy $e^+ e^-$ collider [24] at SLAC. The data sample consists of an integrated luminosity of 423.7 fb$^{-1}$ recorded at the $Y(4S)$ resonance (on-peak) and 43.9 fb$^{-1}$ collected 40 MeV below the resonance (off-peak). The on-peak data sample contains $(465.1 \pm 5.1) \times 10^6 B \bar{B}$ pairs [25].

The BABAR detector is described in detail elsewhere [26]. Charged particles are detected and their momenta measured with a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) located inside a 1.5 T solenoidal magnet. Surrounding the DCH is a detector of internally reflected Cherenkov radiation (DIRC), designed for charged particle identification. Energy deposited by electrons and photons is measured by a CsI(Tl) crystal electromagnetic calorimeter (EMC). Muons and long-lived neutral hadrons are identified in the flux return of the solenoid instrumented with resistive plate chambers and limited streamer tubes.

We reconstruct a $B^+ \rightarrow K^0_S K^0_S \pi^+$ candidate by combining a pair of $K_S^0$ mesons and a charged pion. A $K_S^0 \rightarrow \pi^+ \pi^-$ candidate is formed from a pair of oppositely charged tracks with an invariant mass that lies within 15 MeV/$c^2$ of the nominal $K_S^0$ mass [11], which corresponds to 5 times the $K_S^0$ mass resolution. We require the ratio of measured $K_S^0$ lifetime and its uncertainty to be greater than 20, the cosine of the angle between the line connecting the $B$ and $K_S^0$ decay vertices and the $K_S^0$ mo-
momentum vector to be greater than 0.999, and the $K_S^0$ vertex probability to be greater than $10^{-6}$. Charged pions coming from the $B$ decay are identified with the energy loss ($dE/dx$) information from the SVT and DCH, and the Cherenkov angle and the number of photons measured by the DIRC. The efficiency for pion selection is approximately 76% including geometrical acceptance, while the probability for misidentification of kaons as pions is less than 15%, up to a momentum of 4 GeV/$c$. We require pion candidates not to be consistent with the electron hypothesis, based on information from the $dE/dx$, the shower shape in the EMC, and the ratio of the shower energy and track momentum.

Continuum $e^+ e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events are the dominant background. To discriminate this type of event from signal we use a boosted decision tree (BDT) [27] that combines five discriminating variables. The first of these is the ratio of $L_2$ to $L_0$, with $L_j = \sum_i p_i |\cos \theta_i|$, where $\theta_i$ is the angle, with respect to the $B$ thrust axis, of the track or neutral cluster $i$, and $p_i$ is its momentum. The sum excludes the daughters of the $B$ candidate and all quantities are calculated in the $e^+ e^-$ center-of-mass (CM) frame. The other four variables are the absolute value of the cosine of the angle between the $B$ direction and the beam ($z$ axis), the magnitude of the cosine of the angle between the $B$ thrust axis and the $z$ axis, the product of the $B$ candidate’s charge and the flavor of the recoiling $B$ as reported by a multivariate tagging algorithm [28], and the proper time difference between the decays of the two $B$ mesons divided by its uncertainty. The BDT is trained using off-peak data as well as simulated signal events that pass the selection criteria. We make a requirement on the BDT output (BDT$_{\text{out}}$) such that approximately 96% of the signal is retained and 60% of the continuum background is rejected.

In addition to BDT$_{\text{out}}$, we distinguish signal from background events using two kinematic variables: the beam-energy-substituted mass $m_{\text{ES}} = \sqrt{s}/4 - p_B^2$ and $\Delta E = E_B - \sqrt{s}/2$, where $\sqrt{s}$ is the total $e^+ e^-$ CM energy and ($E_B$, $p_B$) is the four-momentum of the $B$ candidate measured in the CM frame. We select signal candidates that satisfy $5.250 < m_{\text{ES}} < 5.286$ GeV/$c^2$ and $|\Delta E| < 0.1$ GeV. This region includes a sufficiently large range of $m_{\text{ES}}$ below the signal peak to allow properties of the continuum distribution to be determined in the maximum likelihood fit.

Another source of background arises from $B^+ \rightarrow \bar{D}^0(\rightarrow K_S^0 K_{L}^0)\pi^+$ decays, where the final state particles are identical to the signal. We reduce this background by rejecting any event containing a signal candidate with a $K_S^0 K_{L}^0$ invariant mass in the range $1.82 < M_{K_S^0 K_{L}^0} < 1.90$ GeV/$c^2$.

The efficiency for signal events to pass the selection criteria is 28%, determined with a Monte Carlo (MC) simulation in which decays are generated uniformly in three-body phase space. We find that approximately 9% of the selected $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ events contain more than one candidate, in which case we choose that with the highest $B$-vertex probability. We have checked that this procedure does not bias the fit variables. In about 2% of the signal events, the $B$ candidate is misreconstructed because one of its daughter tracks is replaced by a track from the rest of the event. Such events are considered to be a part of the signal component.

We study possible residual backgrounds from $B\bar{B}$ using MC event samples. These backgrounds arise from decays with similar kinematic properties to the signal or because particles get lost to, or attached from, the rest of the event in the process of reconstruction. The $B\bar{B}$ background modes can conveniently be divided into two categories, based on their shapes in $m_{\text{ES}}$ and $\Delta E$. The first category ($B\bar{B}_1$) contains only $B^+ \rightarrow K_S^0 K_{L}^0 K^+$ decays, which peak in $m_{\text{ES}}$ around the $B$ mass and in $\Delta E$ near $-0.06$ GeV. The second category ($B\bar{B}_2$) contains the remaining $B\bar{B}$ backgrounds and is mainly combinatorial.

We perform an unbinned extended maximum likelihood fit to the candidate events using three input variables: $m_{\text{ES}}$, $\Delta E$, and BDT$_{\text{out}}$. For each category $j$ (signal, continuum background, $B\bar{B}_1$, or $B\bar{B}_2$), we define a probability density function $P_j$ (PDF), and evaluate it for each event $i$:

$$P_j = P_j(m_{\text{ES}}, \Delta E) \cdot P_j(\text{BDT}_\text{out}).$$

The signal, continuum background, and $B\bar{B}_2$ background exhibit negligible correlations between $m_{\text{ES}}$ and $\Delta E$, and so the PDF is further factorized:

$$P_j(m_{\text{ES}}, \Delta E) = P_j(m_{\text{ES}}) \cdot P_j(\Delta E).$$

The extended likelihood function is

$$L = \prod_k e^{-n_k} \prod_i \left[ \sum_j n_j P_j \right].$$

where $n_j(n_k)$ is the yield for event category $j(k)$. The signal $m_{\text{ES}}$ distribution is parametrized with the sum of a Gaussian and a Crystal Ball function [29] while the $\Delta E$ distribution is parametrized with a modified Gaussian function with different widths on each side, as well as with additional tails that can be different on each side. We fix the shape parameters to the values obtained from the $B^+ \rightarrow K_S^0 K_{L}^0 \pi^+$ phase-space MC sample. The continuum background $m_{\text{ES}}$ shape is described by an empirical threshold ARGUS function, $x\sqrt{1-x^2} \exp[-\xi(1-x^2)]$, with $x = 2m_{\text{ES}}/\sqrt{s}$ and $\xi$ a free parameter [30], while the continuum $\Delta E$ shape is modeled with a linear function. We describe the $m_{\text{ES}}$ and $\Delta E$ shapes for the $B\bar{B}_1$ sample with a two-dimensional histogram determined from MC events, which accounts for correlations between these variables. One-dimensional histograms are used to describe the $m_{\text{ES}}$ and $\Delta E$ distributions for the $B\bar{B}_2$ sample. The BDT$_{\text{out}}$ distributions for all components are described by one-dimensional histograms. These are obtained from MC.
events for signal and the $B\bar{B}$ background categories. The continuum background BDT$_{out}$ shape is determined from a combination of off-peak data and on-peak data in a continuum-dominated sideband of $m_{ES}$, independent of the signal region, from which the expected $B\bar{B}$ backgrounds have been subtracted.

The free parameters of our fit are the yields of the signal, continuum, and two $B\bar{B}$ background categories, together with the $\xi$ parameter of the continuum $m_{ES}$ shape and the slope of the continuum $\Delta E$ shape.

We test the fitting procedure by applying it to ensembles of simulated experiments where events are generated from the PDF shapes as described above for all four categories of events. We repeat the exercise with $q\bar{q}$ events generated from the PDF while signal events are randomly extracted from the MC samples. The $B\bar{B}$ background events are either generated from PDF shapes or drawn from MC samples. In all cases, these tests confirm that our fit performs as expected. No bias is found for the value of the signal yield observed in the data.

The fit to 16 739 candidate events gives a signal yield of $15 \pm 15$ events, where the error is statistical only. The fit returns yields for the continuum, $BB_1$ and $BB_2$ background categories of $15 \pm 140$, $89 \pm 25$, and $1140 \pm 70$ events, respectively. These are somewhat larger than the expected values for the first and last categories and smaller for the second, a pattern that can be explained by the correlations between these yields.

The results of the fit are shown in Fig. 1. In these plots the continuum background contribution has been suppressed by applying a requirement on the ratio of the signal likelihood to the sum of the signal and continuum likelihoods, calculated without use of the plotted variable. The value of this requirement for each plot rejects about 97% of the continuum background while retaining 63%–71% of the signal, depending on the variable.

We determine the inclusive branching fraction for $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ by dividing the observed signal yield by the reconstruction efficiency, the number of $B\bar{B}$ events in the data sample, and the square of the daughter branching fraction $\mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-) = 0.6920 \pm 0.0005$ [11]. We assume equal decay rates of $\Upsilon(4S)$ into $B^+ B^-$ and $B^0 \bar{B}^0$ pairs. The value obtained is $\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 \pi^+) = (2.5 \pm 2.4) \times 10^{-7}$, where the error is statistical only. The statistical significance of the signal is 1.1$\sigma$, which is calculated as $\sqrt{-2 \ln(L_0/L_{max})}$, where $L_{max}$ denotes the likelihood with the nominal signal yield of 15 events and $L_0$ denotes the likelihood with the signal yield fixed at zero.

There is a significant dependence of the selection efficiency on the kinematics of the $K_S^0 K_S^0 \pi^+$ final state. The nominal efficiency is calculated by assuming a phase-space distribution of $K_S^0 K_S^0 \pi^+$ events. Since we do not know the true distribution, a systematic uncertainty of 24% is evaluated from the RMS variation of the efficiency across the $K_S^0 K_S^0 \pi^+$ Dalitz plot. Smaller systematic uncertainties on the fitted yield arise from uncertainties in the PDF shapes (4 events), including possible differences between data and MC simulations, which are studied using a control sample of $B^0 \rightarrow D^- (\rightarrow K_S^0 \pi^-) \pi^+$ events. We assign an uncertainty of 2% to events to account for fit bias. Other uncertainties on the efficiency arise from charged particle reconstruction (0.4%), particle identification (1.4%), and the $K_S^0$ selection (1.8%). The uncertainty on the number of $B\bar{B}$ pairs is 1.1%.

The systematic uncertainties are added in quadrature to give a total of 38%. Hence the inclusive branching fraction $\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 \pi^+) = (2.5 \pm 2.4 \pm 0.9) \times 10^{-7}$, where the first (second) error is statistical (systematic).

Since our result is consistent with no signal, we determine a 90% CL upper limit on the branching fraction ($\mathcal{B}_{UL}$). This limit is calculated by integrating the likelihood in the physical region such that $\int_0^{\mathcal{B}_{UL}} \mathcal{L}(x)dx/\int_0^{\infty} \mathcal{L}(x)dx = 0.9$, where $\mathcal{L}(x)$ is the likelihood function for the signal yield $x$. We have confirmed that the statistical uncertainties from the fit are Gaussian, to a good approximation. We therefore assume a Gaussian behavior for the overall likelihood, with a width calculated from the sum in quadrature of the statistical and systematic uncertainties. Our result is $\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 \pi^+) < 5.1 \times 10^{-7}$ at 90% CL.

The lack of signal in this decay mode contrasts with that observed for $B^+ \rightarrow K^+ K^- \pi^+$ [19]. This result disfavors models in which the $f_X(1500)$ has even spin and decays with isospin symmetry. If the $f_X(1500)$ is confirmed to

![FIG. 1 (color online). Projections of candidate events with the fit results overlaid. From left to right are shown the projections onto the $m_{ES}$, $\Delta E$, and BDT$_{out}$ variables. The points show the data and the solid (blue) curves show the total fit result. The dotted (red) curves show the continuum background, the dashed (green) curves show the total background, and the dash-dotted (black) curves show the signal distributions.](051101-6)
have even spin in future measurements, this may indicate a non-\(q\bar{q}\) nature of this state.

In conclusion, with a data sample of 423.7 fb\(^{-1}\), we have performed a search for the decay \(B^+ \rightarrow K_S^0 K^0 \pi^+\). We observe no significant signal and set a 90% confidence level upper limit on the branching fraction of \(5.1 \times 10^{-7}\). This result provides useful information for the understanding of low energy spectroscopy.

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[1] Inclusion of the charge conjugate mode is implied.