Memoryless Relay Strategies for Two-Way Relay Channels

Tao Cui, Tracey Ho, *Member, IEEE*, and Jörg Kliewer, *Senior Member, IEEE*

***Abstract*—We propose relaying strategies for uncoded two-way relay channels, where two terminals transmit simultaneously to each other with the help of a relay. In particular, we consider a memoryless system, where the signal transmitted by the relay is obtained by applying an instantaneous relay function to the previously received signal. For binary antipodal signaling, a class of so called absolute (abs)-based schemes is proposed in which the processing at the relay is solely based on the absolute value of the received signal. We analyze and optimize the symbol-error performance of existing and new abs-based and non-abs-based strategies under an average power constraint, including abs-based and non-abs-based versions of amplify and forward (AF), detect and forward (DF), and estimate and forward (EF). Additionally, we optimize the relay function via functional analysis such that the average probability of error is minimized at the high signal-to-noise ratio (SNR) regime. The optimized relay function is shown to be a Lambert W function parameterized on the noise power and the transmission energy. The optimized function behaves like abs-AF at low SNR and like abs-DF at high SNR, respectively; EF behaves similarly to the optimized function over the whole SNR range. We find the conditions under which each class of strategies is preferred. Finally, we show that all these results can also be generalized to higher order constellations.**

***Index Terms*—Two-way channel, wireless relay networks, functional analysis.**

I. Introduction

TWO-WAY communication is a common scenario where two parties simultaneously transmit information to each other. The two-way channel was first considered by Shannon [3], who derived inner and outer bounds on the capacity region. Recently, the two-way relay channel (TWRC) has drawn renewed interest from both academic and industrial communities [4]–[10] due to its potential application to cellular networks and peer-to-peer networks. AF and DF protocols for one-way relay channels are extended to the half-duplex Gaussian TWRC in [6] and the general full-duplex discrete TWRC in [5]. In [7], network coding is used to increase the sum-rate of two users. With network coding, each node in a network is allowed to perform algebraic operations on received packets instead of only forwarding or replicating received packets. Most of these works [5]–[7] focus on capacity bounds for strategies similar to those for one-way relay channels [11]. Furthermore, physical layer network coding (PNC) is considered in [8] for two-way AWGN relay channels. Also, two partial detect and forward (PDF) schemes are proposed in [10] for distributed space time coding to achieve diversity in two-way relay fading channels with multiple relays. These two works [8], [10] propose new relaying strategies without addressing their optimality.

In this paper we consider an uncoded scenario with memoryless relays, which is beneficial in those situations when the relay is under a strict complexity or latency constraint. The former case applies, e.g., if the relay is part of a sensor network with battery powered nodes, where the complexity for relaying the partner nodes' data must be kept small. Also, minimizing the end-to-end delay in networked communication is important in real-time applications with feedback, where typically a bidirectional unicast session is established.

In particular, in the following work we analyze and optimize the symbol error probability at each receiver without considering the effect of any end-to-end channel coding that may be applied. We first derive the symbol error probabilities for existing amplify and forward (AF) and detect and forward (DF) schemes for TWRCs using binary antipodal signaling. Noting the performance limitations of these existing schemes, we develop a number of new schemes. We classify both existing and new schemes into two categories: absolute (abs)-based schemes, where the relay transmits an instantaneous function of the *absolute value* of the received signal, and non-abs-based schemes where the sign of the received signal is preserved by the instantaneous relay function. The advantage of abs-based schemes is that for binary antipodal signaling at the terminals the relay performs a constellation compression such that the transmitted signal from the relay is again an antipodal signal with only two constellation points. In fact, the abs-based scheme bears resemblance to network coding where the relay performs an XOR on the decoded data from the terminals [12]. However, in an abs-based scheme the relay receives the real-valued sum of the data from the two terminals plus noise on the physical layer, whereas in network coding the addition is performed over a finite field on the network layer. In contrast to abs-based schemes, in the case of binary antipodal signaling non-abs-based schemes require the relay to transmit four constellation points, which may lead to a larger transmit power and higher decoding complexity. However, as we will see, the relative performance of abs- and non-abs-

Paper approved by G.-H. Im, the Editor for Equalization and Multicarrier Techniques of the IEEE Communications Society. Manuscript received April 28, 2008; revised October 12, 2008 and December 24, 2008.

T. Cui and T. Ho are with the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125, USA (e-mail: {taocui, tho}@caltech.edu).

J. Kliewer is with the Klipsch School of Electrical and Computer Engineering, New Mexico State University, Las Cruces, NM 88003, USA (e-mail: jkliewer@nmsu.edu).

This work has been supported in part by DARPA grant N66001-06-C-2020, Caltechs Lee Center for Advanced Networking, the Okawa Foundation Research Grant, NSF grant CCF-0830666, and a gift from Microsoft Research. This paper has been presented in part at the IEEE International Conference on Communications, May 2008, Beijing, China [1], and in part at the Information Theory and Applications Workshop, Jan. 2008, San Diego, CA, USA [2].

Digital Object Identifier 10.1109/TCOMM.2009.10.080222

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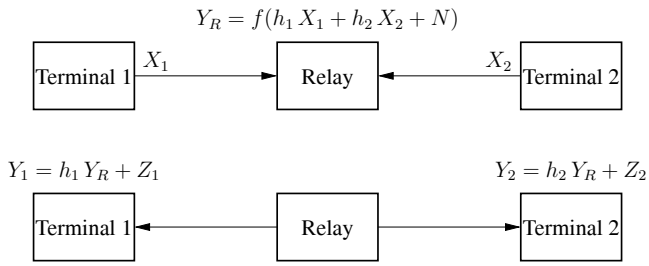

Fig. 1. Two-way relay channel.

based schemes depends on the characteristics of the channels between terminals and relay.

Specifically, the abs-based schemes include an abs-based AF (AAF) scheme, an abs-based DF (ADF) scheme and a novel estimate and forward (EF) strategy by extending the EF scheme in [13] for the one-way relay channel to TWRCs, all of which can substantially outperform existing schemes. Besides characterizing the performance of different schemes, we also optimize the relay strategy within the class of abs-based strategies via functional analysis, where the solution minimizes the average probability of error at the terminals[1] over all possible relay functions at high SNR, and generally outperforms all other strategies we consider. This approach can be seen as a generalization of the result from [14] for the one-way case. The optimized relay function is shown to be a Lambert W function parameterized on the noise power and the transmission energy. Interestingly, the optimized function looks like the AAF scheme at low SNR and like the ADF scheme at high SNR. The EF strategy leads to a relay function which is similar in shape compared to the optimized function in all SNRs. We also prove that DF performs better than ADF if the two-way channel is very asymmetric or the relay has greater power than the two terminals, while ADF performs better than DF in less asymmetric channels or when the relay has roughly the same power as the terminals. These results will also be generalized to higher order constellations at the terminals such as quadrature amplitude modulation (QAM).

This paper is an expanded version of work presented in [1], [2].

Notation: In the following, $p_X(x)$ denotes the probability density function (pdf) of a random variable X, and $\mathcal{G}(x, \sigma^2) \triangleq \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$ denotes the pdf of a normal random variable X with mean 0 and variance σ^2. $Q(\cdot)$ represents the Q-function.

II. System Model

The system model is illustrated in Fig. 1, where the X_i are the transmitted symbols from some given constellation at terminal i, $i = 1, 2$, Y_i are the received symbols at the terminals, and Y_R is the transmitted symbol at the relay. Communication takes place in two phases. In the multiple-access (MAC) phase, both terminals simultaneously send a block of data symbols to the relay which generates $Y_R = f(h_1 X_1 + h_2 X_2 + N)$ with the relay function $f(\cdot)$. Here, h_1 and h_2 represent deterministic attenuation factors for the terminal-to-relay and relay-to-terminal channels, which could for example represent a single realization of a fading process. Throughout this paper, we assume that $h_1 \geq h_2 \geq 0$ without loss of generality. The quantity N represents the additive white Gaussian noise (AWGN) at the relay with mean zero and variance σ_r^2. In the broadcast phase, the relay transmits Y_R to both terminals 1 and 2, where Z_i is the AWGN at terminal i with mean zero and variance $\sigma_{s_i}^2$. The discrete-time model for the TWRC can therefore be written as

$$Y_i = h_i f(h_1 X_1 + h_2 X_2 + N) + Z_i, \quad i = 1, 2. \quad (1)$$

For the sake of brevity we also define the received signal at the relay as $U = h_1 X_1 + h_2 X_2 + N$. Since each terminal knows what it has sent to the relay in the MAC phase, it can recover the information from the other terminal based on the received Y_i and its own *a priori* symbols X_i. In addition, we impose an average power constraint on X_i: $E\{|X_i|^2\} \leq P_s, \quad i = 1, 2$, as well as on the output of the relay: $E\{|f(h_1 X_1 + h_2 X_2 + N)|^2\} \leq P_r$.

We assume for notational simplicity that the noise variance at the two terminals is the same, i.e., $\sigma_{s_1}^2 = \sigma_{s_2}^2 = \sigma_s^2$; extensions to the more general case are straightforward. Also, it is assumed that the terminals and the relay know h_1 and h_2, which may be obtained by using channel estimation at the relay or the feedback channel from the two terminals, see e.g., [15]. Further, we assume that the two terminals are perfectly synchronized and compensate for channel rotation prior to transmission. Under these assumptions, the channel coefficients h_1 and h_2 are used as real-valued attenuation factors. Alternatively, the synchronization approach from [16] could be applied at the terminals where pilot symbols are used to estimate the phase differences between the two terminal signals in the signal received from the relay.

We focus on symbol error probability as a performance metric: each terminal is assumed to perform a hypothesis test to decide which symbol was transmitted by the other terminal; we do not consider the effect of any end to end channel coding that may be applied. Note that (1) both applies to a half duplex system with two time slots, where the transmission from one terminal to the other takes place in a multiple-access and a broadcast time slot, or a full duplex system.

III. Relay Strategies for the BPSK Case

We begin by considering BPSK; an extension to higher order constellations is given in Section V. Each terminal transmits $X_i = \pm\sqrt{P_s}$. We consider two classes of relay strategies: absolute value strategies, where the relay transmits a non-decreasing function of $|U|$, and non-absolute value strategies, where the relay transmits an odd non-decreasing function of U.

We first show that the error probability is minimized if the terminals employ threshold detection as follows.

- For non-abs-based strategies: If $x_i = \sqrt{P_s}$ has been sent in the MAC phase then terminal i decides on $\sqrt{P_s}$ if $y_i \geq v_i$ and on $-\sqrt{P_s}$ otherwise, where v_i is its detection threshold and y_i is the value of its received symbol Y_i.

[1]An alternative objective would be to minimize the maximum of the two terminals' error probabilities, which gives the same result in high SNR, but is in general more complicated to work with mathematically.

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$$P_e^{(1)} = \frac{1}{2} + \frac{1}{2}\int_0^w A(u)du \underbrace{\left[\int_{-\infty}^{v_1} \mathcal{G}\left(y - h_1 b, \sigma_s^2\right) dy\right]}_{C(v_1,b)} + \frac{1}{2}\int_w^{+\infty} A(u)du \underbrace{\left[\int_{-\infty}^{v_1} \mathcal{G}\left(y - h_1 a, \sigma_s^2\right) dy\right]}_{D(v_1,a)}$$
$$+ \frac{1}{2}\int_0^w B(u)du \underbrace{\left[\int_{-\infty}^{v_1} \mathcal{G}\left(y + h_1 b, \sigma_s^2\right) dy\right]}_{E(v_1,b)} + \frac{1}{2}\int_w^{+\infty} B(u)du \underbrace{\left[\int_{-\infty}^{v_1} \mathcal{G}\left(y + h_1 a, \sigma_s^2\right) dy\right]}_{F(v_1,a)}. \quad (2)$$

Likewise, if $x_i = -\sqrt{P_s}$ has been sent, then terminal i decides on $\sqrt{P_s}$ if $y_i \geq -v_i$ and on $-\sqrt{P_s}$ otherwise.

- For abs-based strategies: each terminal decides for either ($X_1 = \sqrt{P_s}$, $X_2 = \sqrt{P_s}$) or ($X_1 = -\sqrt{P_s}$, $X_2 = -\sqrt{P_s}$) depending on what the terminal has previously sent to the relay, if the received signal exceeds a threshold value v_i. Otherwise, if the received signal is smaller than the threshold v_i it decides for either ($X_1 = \sqrt{P_s}$, $X_2 = -\sqrt{P_s}$) or ($X_1 = -\sqrt{P_s}$, $X_2 = \sqrt{P_s}$).

Theorem 1: When each terminal transmits $\sqrt{P_s}$ and $-\sqrt{P_s}$ with equal probability, for any given non-abs-based relay function $f(U)$ or abs-based relay function $f(|U|)$ where f is a non-decreasing function of U or $|U|$, respectively, threshold detection at the terminals minimizes the probability of error.

The proof is given in the Appendix.

A. Non-Abs-Based Strategies

The average probability of error at terminal 1 is

$$\begin{aligned} P_e^{(1)} =& \frac{1}{4}\Big(\Pr(y_1 < v_1 | x_1 = x_2 = \sqrt{P_r}) \\ &+ \Pr(y_1 > v_1 | x_1 = \sqrt{P_r}, x_2 = -\sqrt{P_r}) \\ &+ \Pr(y_1 < -v_1 | x_1 = -\sqrt{P_r}, x_2 = \sqrt{P_r}) \\ &+ \Pr(y_1 > -v_1 | x_1 = x_2 = -\sqrt{P_r})\Big) \\ =& \frac{1}{2} + \frac{1}{2}\int_{-\infty}^{+\infty} \Big(\mathcal{G}\left(u - (h_1+h_2)\sqrt{P_s}, \sigma_r^2\right) \\ &- \mathcal{G}\left(u - (h_1-h_2)\sqrt{P_s}, \sigma_r^2\right)\Big) \\ &\times \left[\int_{-\infty}^{v_1} \mathcal{G}\left(y - h_1 f(u), \sigma_s^2\right) dy\right] du. \end{aligned} \quad (3)$$

The average probability of error at terminal 2 is given by interchanging subscripts $_1$ and $_2$.

1) Amplify-and-Forward: We analyze the performance of amplify and forward [6], where a linear function $f(\cdot)$ is used. To satisfy the average power constraint at the relay, $f(\cdot)$ is equal to $f(u) = \sqrt{\frac{P_r}{(h_1^2+h_2^2)P_s+\sigma_r^2}}u$, which yields an output at terminal i according to

$$Y_i = h_i\sqrt{\frac{P_r}{(h_1^2+h_2^2)P_s+\sigma_r^2}}(X_1+X_2) + \left(h_i\sqrt{\frac{P_r}{(h_1^2+h_2^2)P_s+\sigma_r^2}}N + Z_i\right),\ i = 1,2. \quad (4)$$

Therefore, given x_1 and x_2 were transmitted, the conditional pdf of the output Y_i is

$$p_{Y_i|X_1,X_2}(y_i|x_1,x_2) = \mathcal{G}\left(y_i - h_i\sqrt{\frac{P_r}{(h_1^2+h_2^2)P_s+\sigma_r^2}}(x_1+x_2), \frac{h_i^2 P_r \sigma_r^2}{(h_1^2+h_2^2)P_s+\sigma_r^2} + \sigma_s^2\right), \quad (5)$$

where $\mathcal{G}(x,\sigma^2)$ is defined at the end of Section I. Given x_i, we observe from (5) that terminal i's decoding threshold is $v_i = h_i\sqrt{\frac{P_r}{(h_1^2+h_2^2)P_s+\sigma_r^2}}x_i$. Therefore, the average probability of error at terminal i, $i = 1,2$ is

$$P_e^{(i)} = Q\left(\sqrt{\frac{h_i^2 P_r P_s}{h_i^2 P_r \sigma_r^2 + (h_1^2+h_2^2)P_s\sigma_s^2 + \sigma_r^2\sigma_s^2}}\right). \quad (6)$$

2) Detect-and-Forward: In DF the relay performs hard decisions and maps each decision region to a fixed value that it transmits, i.e.,

$$f(u) = \begin{cases} a, & \text{if } u \geq w, \\ b, & \text{if } w > u \geq 0, \\ -f(-u), & \text{otherwise}, \end{cases} \quad (9)$$

The error probability at the terminals is optimized over the relay threshold w, relay transmit values a and b, and the terminal detection thresholds v_1 and v_2, subject to the average power constraint at the relay. Substituting (9) into (3), the average probability of error at terminal 1 can be written as (2) at the top of this page, where

$$\begin{aligned} A(u) &\triangleq \mathcal{G}\left(u-(h_1+h_2)\sqrt{P_s}, \sigma_r^2\right) - \mathcal{G}\left(u-(h_1-h_2)\sqrt{P_s}, \sigma_r^2\right), \\ B(u) &\triangleq \mathcal{G}\left(u+(h_1+h_2)\sqrt{P_s}, \sigma_r^2\right) - \mathcal{G}\left(u+(h_1-h_2)\sqrt{P_s}, \sigma_r^2\right). \end{aligned} \quad (10)$$

Taking the partial derivative of $P_e^{(1)} + P_e^{(2)}$ with respect to w and setting this to zero, we obtain

$$\frac{\partial(P_e^{(1)} + P_e^{(2)})}{\partial w} = A(w)\left(C(v_1,b) - D(v_1,a)\right) + B(w)\left(E(v_1,b) - F(v_1,a)\right) + \frac{\partial P_e^{(2)}}{\partial w} = 0. \quad (11)$$

As the optimal solution of w in (11) depends on a, b, v_1, v_2 in a complicated way, it is hard to solve (11) directly. One way to approximate the optimal solution is to use an iterative method. At the beginning of the k-th iteration, assuming that $w^{(k)}$ is given ($w^{(0)} = h_1\sqrt{P_s}$), we can optimize $a^{(k)}, b^{(k)}, v_1^{(k)}, v_2^{(k)}$ as follows. When $w^{(k)}, a^{(k)}, b^{(k)}$ are given, $v_1^{(k)}, v_2^{(k)}$ can be written as a function of $a^{(k)}, b^{(k)}$ by minimizing the average error probability. Finally, we perform a two dimensional

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$$\begin{aligned} g(u) &= E\{h_1x_1 + h_2x_2|u\} \\ &= \frac{\sinh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)e^{-\frac{(h_1+h_2)^2P_s}{2\sigma_r^2}}(h_1+h_2) + \sinh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)e^{-\frac{(h_1-h_2)^2P_s}{2\sigma_r^2}}(h_1-h_2)}{\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)e^{-\frac{(h_1+h_2)^2P_s}{2\sigma_r^2}} + \cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)e^{-\frac{(h_1-h_2)^2P_s}{2\sigma_r^2}}}\sqrt{P_s} \end{aligned} \tag{7}$$

$$\begin{aligned} G(f) &= P_e^{(1)} + P_e^{(2)} \\ =&1 + \frac{1}{2}\int_{-\infty}^{+\infty}\left(\mathcal{G}\left(u-(h_1+h_2)\sqrt{P_s},\sigma_r^2\right) - \mathcal{G}\left(u-(h_1-h_2)\sqrt{P_s},\sigma_r^2\right)\right)\left[\int_{-\infty}^{v_1}\mathcal{G}\left(y-h_1f(u),\sigma_s^2\right)dy\right]du \\ &+ \frac{1}{2}\int_{-\infty}^{+\infty}\left(\mathcal{G}\left(u-(h_2+h_1)\sqrt{P_s},\sigma_r^2\right) - \mathcal{G}\left(u-(h_2-h_1)\sqrt{P_s},\sigma_r^2\right)\right)\left[\int_{-\infty}^{v_2}\mathcal{G}\left(y-h_2f(u),\sigma_s^2\right)dy\right]du. \end{aligned} \tag{8}$$

search over $a^{(k)}, b^{(k)}$. Then, $w^{(k+1)}$ can be obtained from (11) by using $a^{(k)}, b^{(k)}, v_1^{(k)}, v_2^{(k)}$. The process repeats until convergence or the maximum number of iterations is achieved. From our experiments we find that less than five iterations are required before convergence. Even though this process does not guarantee convergence to the global minimum, we find that it works well in our experiments.

Alternatively, at high SNR, when $w = h_1\sqrt{P_s}$, we have $A(w) = 0$ and the other Q function and Gaussian terms in (11) tend to 0. A suboptimal solution to (11) can be approximated with this w. By substituting $w = h_1\sqrt{P_s}$ into (2), taking the partial derivative of (2) with respect to v_1, and setting the resulting equation to zero we obtain the thresholds

$$v_1 = \frac{h_1(a+b)}{2}, \quad v_2 = \frac{h_2(a-b)}{2}. \tag{12}$$

We can then derive the optimal a and b subject to the power constraint at the relay by substituting (12) into (2). From the resulting expression, by discarding small terms at high SNR we can then show that $P_e^{(1)} + P_e^{(2)}$ can be approximated[2] as

$$\begin{aligned} P_e^{(1)} + P_e^{(2)} \approx\ & Q\left(\frac{h_1(a-b)}{2\sigma_s}\right) + Q\left(\frac{h_2(a+b)}{2\sigma_s}\right) \\ &+ Q\left(\frac{h_2\sqrt{P_s}}{\sigma_r}\right)\left(1 + \frac{1}{2}Q\left(\frac{h_2(3b-a)}{2\sigma_s}\right)\right). \end{aligned} \tag{13}$$

To find the optimal a, b, we need to minimize (13) subject to $a^2 + b^2 = 2P_r$. Whether the first two terms or the third term dominates depends on the relative values of $P_r, P_s, \sigma_r, \sigma_s, h_1$ and h_2. If we optimize the first two terms of (13), we find that

$$\frac{b}{a} = \frac{h_1 - h_2}{h_1 + h_2}, \quad a^2 + b^2 = 2P_r. \tag{14}$$

Substituting (14) back into (13), we obtain

$$\begin{aligned} P_e^{(1)} + P_e^{(2)} \approx\ & 2Q\left(\sqrt{\frac{P_r}{h_1^2+h_2^2}}\frac{h_1h_2}{\sigma_s}\right) + Q\left(\frac{h_2\sqrt{P_s}}{\sigma_r}\right) \\ &\times\left(1 + \frac{1}{2}Q\left(\sqrt{\frac{P_r}{h_1^2+h_2^2}}\frac{h_2(h_1-2h_2)}{\sigma_s}\right)\right). \end{aligned} \tag{15}$$

[2]Actually $\max(P_e^1, P_e^2)$ dominates, which means that at high SNR optimizing $P_e^{(1)} + P_e^{(2)}$ yields the same function as optimizing $\max(P_e^1, P_e^2)$.

Note that (14) agrees with the straightforward DF, where the relay first finds a point from the set $\{-h_1 - h_2, -h_1 + h_2, h_1 - h_2, h_1 + h_2\}$ with the minimum Euclidean distance from the received signal and then transmits a scaled version of this point.

If we optimize the third term of (13), we find that

$$a = \sqrt{\frac{9P_r}{5}}, \quad b = \sqrt{\frac{P_r}{5}}. \tag{16}$$

Substituting (16) back into (13), we obtain

$$P_e^{(1)} + P_e^{(2)} \approx Q\left(\sqrt{\frac{P_r}{5}}\frac{h_1}{\sigma_s}\right) + Q\left(\sqrt{\frac{4P_r}{5}}\frac{h_2}{\sigma_s}\right) + \frac{5}{4}Q\left(\frac{h_2\sqrt{P_s}}{\sigma_r}\right). \tag{17}$$

Note that (16) corresponds to the uniform constellation where the distances between any two adjacent constellation points are identical. Comparing (15) with (17), we find that when $\sqrt{\frac{5P_s\sigma_s^2}{P_r\sigma_r^2}} < \frac{h_1}{h_2} < 2$ we should choose (16), which means that the first two terms in (13) dominate; otherwise, (14) is preferred which means the third term in (13) dominates.

When $h_1 = h_2$, (14) leads to

$$a = \sqrt{2P_r}, \quad b = 0, \tag{18}$$

where the relay decodes only three points as $h_1 - h_2 = 0$.

Numerical simulations reveal that when h_1/h_2 is close to one, (18) performs better than both (14) and (16) where a performance close to the optimal solution is obtained. As h_1/h_2 increases, (14) and (16) outperform (18) at high SNR. But (18) still performs better than (14) and (16) at low SNR, where removing a constellation point results in power savings and performance improvements.

3) Estimate-and-Forward: In this strategy the relay transmits a scaled version of the MMSE estimate of $h_1X_1 + h_2X_2$ given its observation u, i.e., we consider a function $g(u)$ in (7) shown at the top of this page, and set the relay function $f(u)$ to be a scaled version of $g(u)$ to satisfy the power constraint. We find that $g(u)$ in (7) is close to the straightforward DF (14) at high SNR.

4) Optimized Relay Function: The optimal relay function minimizes the sum of average probabilities of both terminals subject to the average power constraint, i.e., (8) at the top of this page. The optimal relay function is the solution of the problem (19) shown at the top of next page.

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$$\min_{f,v_1,v_2} G(f)$$
$$\text{subject to } \int_0^{+\infty} \mathcal{G}\left(u-(h_1+h_2)\sqrt{P_s},\sigma_r^2\right) f^2(u)du + \int_0^{+\infty} \mathcal{G}\left(u-(h_1-h_2)\sqrt{P_s},\sigma_r^2\right) f^2(u)du$$
$$+ \int_0^{+\infty} \mathcal{G}\left(u+(h_1+h_2)\sqrt{P_s},\sigma_r^2\right) f^2(u)du + \int_0^{+\infty} \mathcal{G}\left(u+(h_1-h_2)\sqrt{P_s},\sigma_r^2\right) f^2(u)du = 2P_r. \tag{19}$$

$$P_e = \frac{1}{2} + \frac{1}{2}\int_0^{+\infty} \left(\mathcal{G}\left(u-2\sqrt{P_s},\sigma_r^2\right) + \mathcal{G}\left(u+2\sqrt{P_s},\sigma_r^2\right) - 2\mathcal{G}\left(u,\sigma_r^2\right)\right)\left[\int_{-\infty}^{v} \mathcal{G}\left(y-\beta\left(u-C\right),\sigma_s^2\right)dy\right]du. \tag{20}$$

$$\begin{aligned} P_e &= \frac{1}{2} + \frac{1}{4}\int_{-\infty}^{+\infty} \left(\mathcal{G}\left(u-2\sqrt{P_s},\sigma_r^2\right) + \mathcal{G}\left(u+2\sqrt{P_s},\sigma_r^2\right) - 2\mathcal{G}\left(u,\sigma_r^2\right)\right)\int_{-\infty}^{v} \mathcal{G}\left(y-f(u),\sigma_s^2\right)\,du\,dy \\ &= \frac{1}{2} + \frac{1}{2}\int_0^{w} \left(\mathcal{G}\left(u-2\sqrt{P_s},\sigma_r^2\right) + \mathcal{G}\left(u+2\sqrt{P_s},\sigma_r^2\right) - 2\mathcal{G}\left(u,\sigma_r^2\right)\right)du \\ &\qquad \times \int_{-\infty}^{v} \left(\mathcal{G}\left(y+\sqrt{P_r},\sigma_s^2\right) - \mathcal{G}\left(y-\sqrt{P_r},\sigma_s^2\right)\right)dy. \end{aligned} \tag{21}$$

To solve the functional optimization problem (19), we first fix v_1 and v_2 and derive the relay function as a function of v_1 and v_2 via the Lagrange dual. Then the relay function is substituted into the objective function and the resulting equation is minimized over v_1 and v_2 by performing a line search around v_1 and v_2 in the optimal DF strategy. Since we do not have a convex optimization problem, the obtained solution may be a local optimum. The closed-form solution of (19) is hard to obtain. Nevertheless, we plot the optimized non-abs-based relay function at different SNRs and with different h_1 and h_2 in Fig. 2.

B. Abs-Based Strategies

In this subsection, we consider abs-based strategies, where in particular, we will provide detailed derivations for the special case $h_1 = h_2 = 1$. The derivations for the general case $h_1 > h_2$ are analogous and will only be briefly discussed due to space limitations. As starting point for the following discussions, we note that generally for abs-based schemes the average error probability at each terminal i can be written as

$$P_e = \frac{1}{2}\Pr(y > v_i | x_1 \neq x_2) + \frac{1}{2}\Pr(y < v_i | x_1 = x_2). \tag{22}$$

For $h_1 = h_2$, we have $v_1 = v_2 = v$.

1) Abs-Based Amplify-and-Forward: In this scheme, the relay first takes the absolute value of the received signal and then subtracts a positive constant C from the resulting signal, i.e.,

$$f(u) = \beta\left(|u| - C\right), \tag{23}$$

where β is a coefficient to maintain the average power constraint at the relay. From (22), the average error probability at terminal 1 for $h_1 = h_2 = 1$ can be written as (20) at the top of this page. The optimal solution is given by minimizing (20) with respect to both v and C, which is done numerically since an analytical solution is hard to obtain. The optimal solution depends on the SNR values, but we have observed experimentally that the optimal threshold is very close to zero. So, a simple solution, in particular if the SNR is not accurately known, is to set $v = 0$ and $C = h_1\sqrt{P_s}$ or $C = h_1\sqrt{P_s} + \sigma_r/\sqrt{2}$.

2) Abs-Based Detect-and-Forward: In ADF, the relay performs hard decisions, based on the absolute value of the received signal, to decide whether $2\sqrt{P_s}$, 0, or $-2\sqrt{P_s}$ is received. The relay does not actually detect x_1 and x_2, but only the mixture $h_1x_1 + h_2x_2$. To satisfy the relay's average power constraint, $\sqrt{P_r}$ and $-\sqrt{P_r}$ are transmitted, i.e.,

$$f(u) = \begin{cases} \sqrt{P_r}, & \text{if } |u| \geq w, \\ -\sqrt{P_r}, & \text{otherwise,} \end{cases} \tag{24}$$

where w is a threshold which will be determined below. Note that a related detect-and-forward scheme for the TWRC is already proposed in [8] as physical layer network coding. In the following, we extend this work by providing a detailed analysis of the end-to-end error probability.

For the case $h_1 = h_2 = 1$ the average error probability at each terminal (22) can be written as (21) at the top of this page. Eq. (21) has the nice property that the optimization with respect to w and v is separated. By minimizing (21) over w and v we obtain the optimal w as

$$w = \sqrt{P_s}\left(1 + \frac{\sigma_r^2}{2P_s}\log\left(1 + \sqrt{1 - e^{-4P_s/\sigma_r^2}}\right)\right), \tag{25}$$

and the optimal v as $v = 0$, which gives

$$P_e = \frac{1}{2} + \frac{1}{2}\left(Q\left(\frac{2\sqrt{P_s} - w}{\sigma_r}\right) + 2Q\left(\frac{w}{\sigma_r}\right) - Q\left(\frac{2\sqrt{P_s} + w}{\sigma_r}\right) - 1\right)\left(1 - 2Q\left(\frac{\sqrt{P_r}}{\sigma_s}\right)\right). \tag{26}$$

When $\sigma_r^2 \to 0$ the optimal w converges to $\sqrt{P_s}$. Note that due to the separation of w and v in (21), the optimal w also minimizes the error probability of detection at the relay. When $h_1 > h_2$, we obtain $w = h_1\sqrt{P_s}$ at high SNR.

3) Abs-Based Estimate-and-Forward: In this strategy the relay transmits its minimum mean squared error (MMSE) estimate of $|h_1x_1 + h_2x_2|$. We first address the case $h_1 = h_2 = 1$

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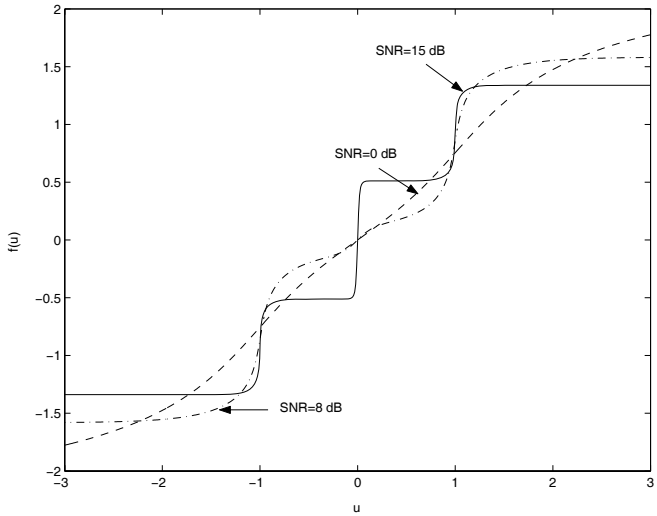
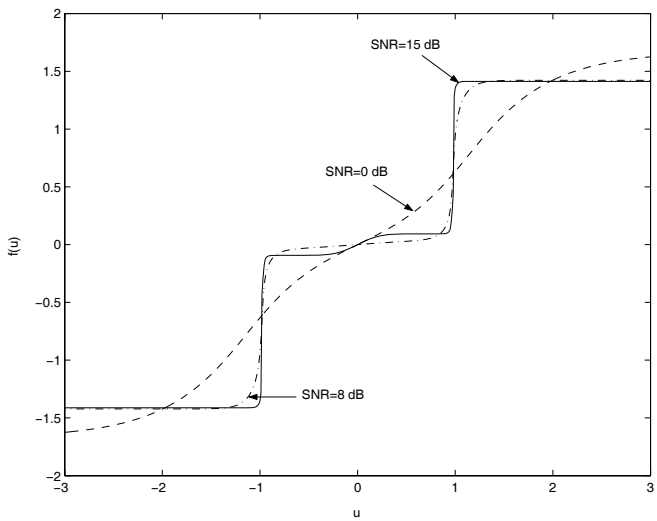


(a) $h_1 = 1$ and $h_2 = 0.5$



(b) $h_1 = 1$ and $h_2 = 0.8$

Fig. 2. The optimized non-abs-based relay function at different SNRs and with different h_1 and h_2.

and derive the MMSE estimator

$$g(u) = E\left\{|x_1 + x_2| \middle| u\right\} = \frac{2\sqrt{P_s}\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right)}{e^{2P_s/\sigma_r^2} + \cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right)}. \quad (27)$$

The relay function $f(u)$ is then a scaled version of $g(u) - C$, i.e.,

$$f(u) = \begin{cases} \beta\left(g(u) - C\right), & \text{if } u \geq 0, \\ f(-u), & \text{otherwise}, \end{cases} \quad (28)$$

where C is a constant as in AAF and $\beta \geq 0$ is a scaling factor to satisfy the average power constraint $E\{f^2(u)\} = P_r$. Optimization of the terminal decoding thresholds is similar to that for AAF. Analogous to the above derivation, for $h_1 > h_2$ we obtain $g(u)$ as (29) at the top of next page.

4) Optimized Relay Strategy: In this section, we optimize the average probability of error over even functions $f(\cdot)$ at the relay. Our approach generalizes the result from [14] for the one-way case. For $h_1 = h_2$ the average probability of error can be obtained from (22) as (30) at the top of next page, which holds since $B(u)$ is an even function in u. Let

$$D(u) \triangleq \mathcal{G}\left(u + 2\sqrt{P_s}, \sigma_r^2\right) + \mathcal{G}\left(u - 2\sqrt{P_s}, \sigma_r^2\right) + 2\mathcal{G}\left(u, \sigma_r^2\right). \quad (31)$$

Our optimization problem is

$$\min_{f,v} \; H(f) = \int_0^{+\infty} B(u)A(f)du, \text{ s.t. } \frac{1}{2}\int_0^{+\infty} D(u)f^2(u)du \leq P_r, \quad (32)$$

which can be solved by considering the Lagrangian

$$\phi(\lambda, f) = H(f) + \frac{\lambda}{2}\left(\int_0^{+\infty} D(u)f^2(u)du - 2P_r\right), \quad (33)$$

where $\lambda \geq 0$ is the Lagrange multiplier of the average power constraint. Differentiating $\phi(\lambda, f)$ with respect to $f(u)$ for each u and setting the result to zero, we obtain after rearranging

$$\frac{\mathcal{G}\left(f(u) - v, \sigma_s^2\right)}{f(u)} = \lambda\frac{D(u)}{B(u)}. \quad (34)$$

Since $\lambda > 0, D(u) > 0$, and if $|u| \geq w$ we have $B(u) \geq 0$ (and $B(u) < 0$ otherwise), we obtain

$$\begin{cases} f(u) \geq 0, & \text{if } |u| \geq w, \\ f(u) < 0, & \text{otherwise}, \end{cases} \quad (35)$$

where w is the relay hard decision threshold defined in (25).

Lemma 1: For $f(u)$ satisfying

$$\begin{cases} f(u) \geq v, & \text{if } |u| \geq w, \\ f(u) < v, & \text{otherwise}, \end{cases} \quad (36)$$

$P_e(f)$ in (30) is a strictly convex function in f (when considering functions that differ on a set of non-zero measure).

Proof: Let f and g be two functions satisfying (36), and let $\lambda \in [0, 1]$ and $\gamma = 1 - \lambda$. Clearly, $\lambda f + \gamma g$ also satisfies (36). Then,

$$\frac{\partial^2 A(f)}{\partial f^2} = \frac{1}{2\sigma_s^2}\left(f(u) - v\right)\mathcal{G}\left(v - f(u), \sigma_s^2\right), \quad (37)$$

is nonnegative if $f(u) \geq v$ and negative otherwise. Since $B(u)\frac{\partial^2 A(f)}{\partial f^2}$ is nonnegative for $|u| \geq w$ and positive otherwise, we have

$$\begin{aligned} P_e(\lambda f + \gamma g) =& \frac{1}{2} + \frac{1}{2}\int_0^{+\infty} B(u)A(\lambda f + \gamma g)du \\ \leq& \lambda P_e(f) + \gamma P_e(g). \end{aligned}$$

■

If $v = 0$, then (34) can be further simplified to be

$$\frac{e^{-\left(f(u)/\sqrt{2\sigma_s^2}\right)^2}}{f(u)/\sqrt{2\sigma_s^2}} = \lambda 2\sqrt{\pi}\sigma_s^2\frac{\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right) + e^{2P_s/\sigma_r^2}}{\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right) - e^{2P_s/\sigma_r^2}}, \quad (39)$$

which can be solved to obtain the following expression for $f(u)$ in (38) at the top of next page. Here, $W(\cdot)$ denotes the Lambert W function, defined by $W(x)e^{W(x)} = x$, and λ is such that the power constraint is satisfied with equality.

Note that $f(u)$ in (38) is derived from the Lagrange dual without any assumption on the convexity of the problem, which may not be a true optimal solution. However, (38) indeed satisfies (35), which means that it is optimal within

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$$g(u) = \frac{|h_1+h_2|\sqrt{P_s}e^{-\frac{(h_1+h_2)^2 P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}{e^{-\frac{(h_1+h_2)^2 P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)+e^{-\frac{(h_1-h_2)^2 P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)} + \frac{|h_1-h_2|\sqrt{P_s}e^{-\frac{(h_1-h_2)^2 P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}{e^{-\frac{(h_1+h_2)^2 P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)+e^{-\frac{(h_1-h_2)^2 P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}. \tag{29}$$

$$P_e(f) = \frac{1}{2}+\frac{1}{2}\int_0^{+\infty}\underbrace{\left(\mathcal{G}\left(u+2\sqrt{P_s},\sigma_r^2\right)+\mathcal{G}\left(u-2\sqrt{P_s},\sigma_r^2\right)-2\mathcal{G}\left(u,\sigma_r^2\right)\right)}_{\triangleq B(u)}\underbrace{\left[\int_{-\infty}^{v}\mathcal{G}\left(y-f(u),\sigma_s^2\right)dy\right]}_{\triangleq A(f)}du, \tag{30}$$

$$f(u) = \begin{cases} \sqrt{\sigma_s^2 W\left(\frac{1}{2\pi\lambda^2\sigma_s^4}\left[\frac{\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right)-e^{2P_s/\sigma_r^2}}{\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right)+e^{2P_s/\sigma_r^2}}\right]^2\right)}, & \text{if } u \geq w, \\ -\sqrt{\sigma_s^2 W\left(\frac{1}{2\pi\lambda^2\sigma_s^4}\left[\frac{\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right)-e^{2P_s/\sigma_r^2}}{\cosh\left(\frac{2\sqrt{P_s}u}{\sigma_r^2}\right)+e^{2P_s/\sigma_r^2}}\right]^2\right)}, & \text{if } w > u \geq 0, \\ f(-u), & \text{if } u < 0. \end{cases} \tag{38}$$

the class of functions satisfying (35). By Lemma 1 and $f^2(u)$ being convex in $f(u)$, the set of functions satisfying (35) and the power constraint of (32) is a convex function set. The optimization under the constraint (35) is thus convex and there is no duality gap. Therefore, (38) is the optimal solution when $v = 0$, which can be achieved in the high SNR regime as shown in the following.

At high SNR, since

$$\lim_{\sigma_r^2\to 0}\frac{\mathcal{G}\left(u+2\sqrt{P_s},\sigma_r^2\right)+\mathcal{G}\left(u-2\sqrt{P_s},\sigma_r^2\right)-2\mathcal{G}\left(u,\sigma_r^2\right)}{\mathcal{G}\left(u+2\sqrt{P_s},\sigma_r^2\right)+\mathcal{G}\left(u-2\sqrt{P_s},\sigma_r^2\right)+2\mathcal{G}\left(u,\sigma_r^2\right)} = \begin{cases} 1, & \text{if } |u| > w, \\ -1, & \text{if } w > |u|, \end{cases} \tag{40}$$

from (34) we obtain

$$f(u) = \begin{cases} C_1, & \text{if } |u| > w, \\ -C_2, & \text{if } w > |u|, \end{cases} \tag{41}$$

where $C_1, C_2 > 0$ are constants. Substituting (41) back into (34), we find that

$$\frac{\mathcal{G}\left(C_1 - v, \sigma_s^2\right)}{C_1} = \lambda = \frac{\mathcal{G}\left(C_2 + v, \sigma_s^2\right)}{C_2}, \tag{42}$$

which gives

$$v = \frac{\log C_1 - \log C_2}{C_1 + C_2}\sigma_s^2 + \frac{C_1 - C_2}{2} \xrightarrow[\sigma_s^2\to 0]{} \frac{C_1 - C_2}{2}. \tag{43}$$

Substituting (43) into (42), we obtain $C_1 = C_2 = C$, which corresponds to ADF. Hence, λ can be approximated as

$$\lambda = \frac{\mathcal{G}\left(C, \sigma_s^2\right)}{C}. \tag{44}$$

Substituting (41)-(44) into (33) and using (26), the dual problem then becomes

$$\min_{C,v} \quad Q\left(\frac{C}{\sigma_s}\right) + \frac{\mathcal{G}\left(C,\sigma_s^2\right)}{C}\left(C^2 - P_r\right). \tag{45}$$

Note that at high SNR $Q\left(\frac{C}{\sigma_s}\right)$ can be approximated as $\frac{\sigma_s}{\sqrt{2\pi}C}e^{-\frac{C^2}{2\sigma_s^2}}$, which decreases faster than $\mathcal{G}\left(C,\sigma_s^2\right) = \frac{1}{\sqrt{2\pi}\sigma_s}e^{-\frac{C^2}{2\sigma_s^2}}$. Therefore, the minimum of (45) is attained at $v = 0$, $C_1 = C_2 = C = \sqrt{P_r}$ when $\sigma_s^2 \to 0$ and $\sigma_r^2 \to 0$. By substituting $v = 0$ and $C_1 = C_2 = C = \sqrt{P_r}$ into (41) and (44), we obtain f^* and λ^*, which gives $\min_f \phi(\lambda^*, f) = G(f^*)$ at high SNR. Therefore, there is no duality gap at high SNR and the optimal solution converges to (41), which is equivalent to the ADF strategy. In general, the optimal v varies with SNR.

For the case $h_1 > h_2$, minimizing the sum of error probabilities of both terminals can be approximated by minimizing the error probability of terminal 2 at high SNR, which gives (46) at the top of next page.

Remarks:

- As seen above, $f(u)$ in (38) is optimal when the two terminals' detection thresholds are set to zero. Our experiments show that this relay function outperforms the other strategies in both high and low SNR regimes. A way to optimize jointly over $f(u)$ and v is to solve (34) for $f(u)$ which depends on both v and λ. For a given v, we can find λ by satisfying the average power constraint. Finally, v can be found by substituting the resulting function into $H(f)$ and optimizing over v. The optimized function using this approach performs better than (38) but is more difficult to implement.

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$$f(u)=\begin{cases}\sqrt{\sigma_s^2 W\left(\frac{1}{2\pi\lambda^2 h_1^2\sigma_s^4}\left[\frac{e^{-\frac{(h_1+h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)-e^{-\frac{(h_1-h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}{e^{-\frac{(h_1+h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)+e^{-\frac{(h_1-h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}\right]^2\right)}, & \text{if } u\geq w,\\ -\sqrt{\sigma_s^2 W\left(\frac{1}{2\pi\lambda^2 h_1^2\sigma_s^4}\left[\frac{e^{-\frac{(h_1+h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)-e^{-\frac{(h_1-h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}{e^{-\frac{(h_1+h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1+h_2)\sqrt{P_s}u}{\sigma_r^2}\right)+e^{-\frac{(h_1-h_2)^2P_s}{2\sigma_r^2}}\cosh\left(\frac{(h_1-h_2)\sqrt{P_s}u}{\sigma_r^2}\right)}\right]^2\right)}, & \text{if } w>u\geq 0,\\ f(-u), & \text{if } u<0.\end{cases} \tag{46}$$

- The optimized relay function can be considered as the solution of instantaneous waterfilling in the signal space in contrast to waterfilling in the spectral or time domain [17].
- Above, we have derived the error probabilities for various strategies with fixed h_1 and h_2. To obtain the performance in fading channels, we integrate the obtained error probabilities over the joint pdf of h_1 and h_2. Except for the optimized relay function for non-abs strategies, we give closed-form expression for the other cases at least in high SNR, which do not have to be re-optimized for different h_1 and h_2.

IV. Comparison Between Two Classes of Strategies

The average error probability of non-abs DF can be approximated by applying Chernoff bound-type arguments to (15) and (17), which gives

$$P_e^{(1)}+P_e^{(2)}\approx\begin{cases}\frac{5}{8}e^{-\frac{h_2^2P_s}{2\sigma_r^2}}, & \text{if } 2>\frac{h_1}{h_2}>\sqrt{\frac{5P_s\sigma_s^2}{P_r\sigma_r^2}},\\ e^{-\frac{h_1^2h_2^2P_r}{2(h_1^2+h_2^2)\sigma_s^2}}+\frac{1}{2}e^{-\frac{h_2^2P_s}{2\sigma_r^2}}, & \text{otherwise.}\end{cases} \tag{47}$$

Likewise, we can approximate the average error probability of ADF for $h_1>h_2$ by using Chernoff bounds on (21) (and the corresponding expression for terminal 2) according to

$$P_e^{(1)}+P_e^{(2)}\approx\frac{1}{2}\left(e^{-\frac{h_1^2P_r}{2\sigma_s^2}}+e^{-\frac{h_2^2P_r}{2\sigma_s^2}}\right)+e^{-\frac{h_2^2P_s}{2\sigma_r^2}}. \tag{48}$$

In the following, we consider several cases at high SNR. Let $\text{SNR}_r\sim\frac{P_s}{\sigma_r^2}$ and $\text{SNR}_s\sim\frac{P_r}{\sigma_s^2}$.

- If $\text{SNR}_s<\text{SNR}_r$, (48) is dominated by $\frac{1}{2}e^{-\frac{h_2^2P_r}{2\sigma_s^2}}$, while (47) is dominated by $e^{-\frac{h_1^2h_2^2P_r}{2(h_1^2+h_2^2)\sigma_r^2}}$. Therefore, the average error probability for ADF is at most $1/2$ of the one for DF.
- If $\text{SNR}_s>\text{SNR}_r$ and $1+\frac{h_2^2}{h_1^2}>\frac{P_r\sigma_r^2}{P_s\sigma_s^2}$ and $h_1>2h_2$, (48) is dominated by $e^{-\frac{h_2^2P_s}{2\sigma_r^2}}$, and (47) is dominated by $\frac{1}{2}e^{-\frac{h_2^2P_s}{2\sigma_r^2}}$. In this case, the average error probability for DF is $1/2$ of the one for ADF.
- If $\text{SNR}_s>\text{SNR}_r$ and $1+\frac{h_2^2}{h_1^2}<\frac{P_r\sigma_r^2}{P_s\sigma_s^2}$ and $\frac{h_1}{h_2}<\sqrt{\frac{5P_s\sigma_s^2}{P_r\sigma_r^2}}$, (48) is dominated by $e^{-\frac{h_2^2P_s}{2\sigma_r^2}}$, and (47) by $\frac{3}{4}e^{-\frac{h_2^2P_s}{2\sigma_r^2}}$. Hence, the average error probability for DF is $3/4$ of the one for ADF.
- If $\text{SNR}_s>\text{SNR}_r$ and $1+\frac{h_2^2}{h_1^2}<\frac{P_r\sigma_r^2}{P_s\sigma_s^2}$ and $2>\frac{h_1}{h_2}>\sqrt{\frac{5P_s\sigma_s^2}{P_r\sigma_r^2}}$, (48) is dominated by $e^{-\frac{h_2^2P_s}{2\sigma_r^2}}$, and (47) is dominated by $\frac{5}{8}e^{-\frac{h_2^2P_s}{2\sigma_r^2}}$. This leads to an average error probability for DF which is $5/8$ of the one for ADF.

These results suggest that when the channel is very asymmetric or the relay has greater power than the terminals we should use DF. When relay has almost the same power as the terminals we prefer ADF where the power savings by using the abs-based operation has a big impact on the overall performance. Note that from Section III-A2 we know that if h_1/h_2 is close to one DF with (18) performs better than DF with (14) or (16). Therefore, when the channel is symmetric and the relay has greater power than the terminals we should use DF with (18).

V. Higher Order Constellations

In industry standards such as the IEEE 802.11 series, usually higher order QAM constellations are employed to achieve high spectral efficiency. In the following, we assume $h_1=h_2=1$ for simplicity. We first define a mapping function $h(u)$ at the relay such that in the noise free case, each terminal can detect the other terminal's signal given its transmitted signal. This is equivalent to

$$\begin{aligned}&h(u_1+u_2)\neq h(u_1'+u_2), \quad \forall u_1\neq u_1' \text{ and}\\ &h(u_1+u_2)\neq h(u_1+u_2'), \quad \forall u_2\neq u_2',\ u_i,u_i'\in\mathcal{Q},\end{aligned} \tag{49}$$

$i=1,2$, where $\mathcal{Q}$ is the constellation set used by the two terminals. The classification of BPSK strategies into absolute and non-absolute value strategies can be generalized to a classification based on underlying relay mappings $h(u)$ satisfying the above condition. Condition (49) defines an undirected graph $\mathcal{G}$, where each node corresponds to a different value of u_1+u_2 and there is an edge between the node corresponding to u_1+u_2 and the node corresponding to $u_1'+u_2$, $u_1'\neq u_1$. Therefore, the relay function $h(u)$ corresponds to a valid vertex coloring of $\mathcal{G}$ such that any pair of adjacent nodes does not have the same color. To find the optimal relay function, we need to consider all possible colorings of graph $\mathcal{G}$. For each coloring, the strategies discussed for BPSK in Section III-A and Section III-B4 can be generalized using the underlying mapping $h(u)$ as described below, and the one achieving the minimum error rate is chosen. The minimum

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possible constellation size of the relay function is equal to the chromatic number of $\mathcal{G}$.

Another way of finding a feasible relay mapping $h(u)$ is, as above, to consider the sum $u_1 + u_2 = c_i$, $i = 1, \ldots, 2|\mathcal{V}| - 1$, for all $u_1, u_2 \in \mathcal{V}$, where $\mathcal{V}$ denotes the constellation set at the two terminals[3]. The quantity c_i takes elements from the set $\mathcal{W}$, where $|\mathcal{W}| = 2|\mathcal{V}| - 1$. The underlying (noise free) relay mapping $h(u)$ which maps the set $\mathcal{W}$ to a set $\mathcal{V}'$ of size $M \geq |\mathcal{V}|$ containing the constellation set to be received at the terminals, can now be found for every i by assigning the $k = (i \mod M)$-th element of $\mathcal{V}'$ to the values c_i. In principle, the M elements can be picked from $\mathcal{V}'$ in arbitrary order.

Note that rectangular QAM constellations can be easily transmitted as two PAM signals on quadrature carriers. In the following, we only consider PAM constellations, and we take 4-PAM as an example. The approach can be generalized to higher PAM constellations. For simplicity, we assume that the transmit signal by the terminals is chosen from the constellation set $\mathcal{V} = \{-3, -1, 1, 3\}$. In the absence of noise, the received signal at the relay is from the set $\mathcal{W} = \{-6, -4, -2, 0, 2, 4, 6\}$. We first consider the class of mapping functions such that they map $\mathcal{W}$ to $\mathcal{V}' = \mathcal{V}$. For example, we can choose

$$\begin{aligned} &h(-6) = -3,\ h(-4) = -1,\ h(-2) = 3, \\ &h(0) = 1,\ h(2) = -3,\ h(4) = -1,\ h(6) = 3, \end{aligned} \tag{50}$$

or

$$\begin{aligned} &h(-6) = -3,\ h(-4) = -1,\ h(-2) = 1, \\ &h(0) = 3,\ h(2) = -3,\ h(4) = -1,\ h(6) = 1. \end{aligned} \tag{51}$$

It is easy to verify that both (50) and (51) satisfy the condition in (49). Note that (51) is the physical network coding operation given in [8] using DF.

AAF can be readily generalized by setting the relay function to be a piecewise linear function based on $h(u)$ such as

$$f(u) = \begin{cases} \beta(u+3), & \text{if } u < -3, \\ \beta(u+5), & \text{if } -2 > u \geq -3, \\ \beta(1-u), & \text{if } 1 > u \geq -2, \\ \beta(-1-u), & \text{if } 2 > u \geq 1, \\ \beta(u-5), & \text{if } 5 > u \geq 2, \\ \beta(u-3), & \text{if } u \geq 5, \end{cases} \tag{52}$$

where β is a coefficient to maintain the average power constraint at the relay. The detection at each terminal is similar to the traditional 4-PAM demodulation by comparing with some thresholds. ADF can be adapted similarly. The relay defines hard decision regions for u, and sends a scaled/shifted version of $h(u)$. At high SNR, the ADF relay function based on (50) can be obtained as

$$f(u) = \begin{cases} -3\beta, & \text{if } u < -5, \\ -\beta, & \text{if } -3 > u \geq -5, \\ 3\beta, & \text{if } -1 > u \geq -3, \\ \beta, & \text{if } 1 > u \geq -1, \\ -3\beta, & \text{if } 3 > u \geq 1, \\ -\beta, & \text{if } 5 > u \geq 3, \\ 3\beta, & \text{if } u \geq 5. \end{cases} \tag{53}$$

[3]For the sake of simplicity we assume that the two terminals employ the same constellation set.

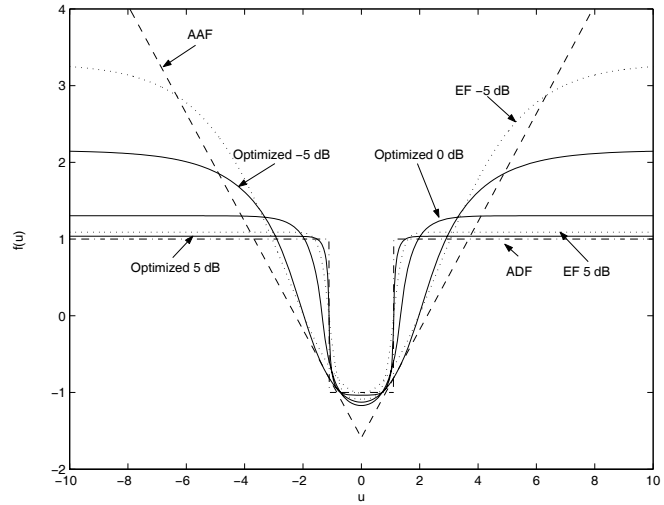

Fig. 3. Comparison of function $f(u)$ in different abs-based schemes with $\sigma_r^2 = \sigma_s^2$, $h_1 = h_2 = 1$ and $P_r = P_s = 1$.

For EF, we first consider the function $g(u)$ such that

$$g(u) = \arg\min_{g'(u)} E\left\{ \left| h(x_1 + x_2) - g'(u) \right|^2 \middle| u \right\}. \tag{54}$$

$f(u)$ is then a scaled version of $g(u)$, i.e., $f(u) = \beta\, g(u)$, where $\beta \geq 0$ is a scalar to satisfy the average power constraint. At the two terminals, there also exists an optimal decision threshold v. We can optimize v using the same approach as in AAF or just choose the conventional 4-PAM detection threshold. In all strategies, we can also apply a maximum likelihood detector at each terminal, i.e.,

$$\hat{x}_2 = \arg\min_{\tilde{x}_2 \in \mathcal{Q}} \left| y_1 - f\left(x_1 + \tilde{x}_2\right) \right|^2. \tag{55}$$

The relay mapping function can also perform a redundant mapping such that $\mathcal{W} = \{-6, -4, -2, 0, 2, 4, 6\}$ is mapped to a set $\mathcal{V}'$ with 5, 6, or 7 elements. For example, when $\mathcal{V}' = \{-4, -2, 0, 2, 4\}$, we can choose

$$\begin{aligned} &h(-6) = -4,\ h(-4) = -2,\ h(-2) = 0, \\ &h(0) = 2,\ h(2) = 4,\ h(4) = -2,\ h(6) = -4, \end{aligned} \tag{56}$$

or, when $\mathcal{V}' = \{-5, -3, -1, 1, 3, 5\}$, we can choose

$$\begin{aligned} &h(-6) = -5,\ h(-4) = -3,\ h(-2) = -1, \\ &h(0) = 1,\ h(2) = 3,\ h(4) = 5,\ h(6) = -5. \end{aligned} \tag{57}$$

When $\mathcal{V}' = \mathcal{W}$, we can simply choose $h(u) = u$. It is easy to verify that (56) and (57) satisfy the condition in (49).

VI. Simulation Results

In this section, we compare the performance of different strategies with $\sigma_r^2 = \sigma_s^2$ and $P_r = P_s = 1$ in all cases.

Fig. 2 shows the optimized non-abs-based relay function for different SNRs and different values of h_1 and h_2. At low SNR, the relay operation behaves like the AF strategy, while it looks like the DF strategy (14) at high SNR. Different abs-based relay functions $f(u)$ are compared in Fig. 3, where for AAF we choose $C = \sqrt{P_s} + \sigma_r/\sqrt{2}$. Unlike ADF with a hard limiter, the optimized relay adapts its transmit power according to the signal strength it receives which is the benefit of the average power constraint. If only a peak power constraint is

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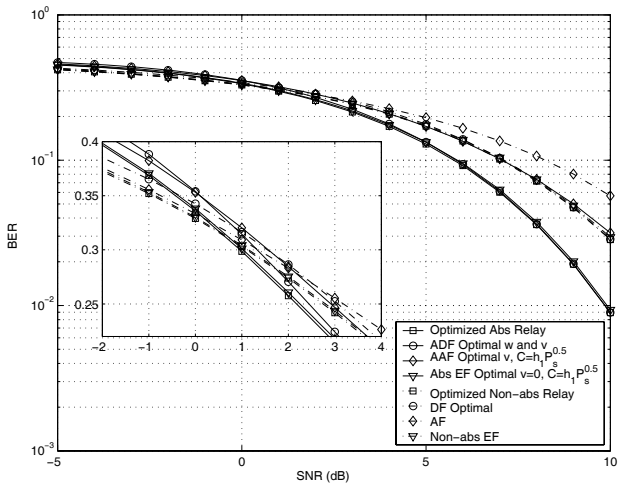

Fig. 4. Performance comparison between different abs-based and non-abs-based strategies when $h_1 = 1$ and $h_2 = 0.8$, $P_r = P_s = 1$. The subfigure shows the crossover between the abs-based and non-abs-based strategies.

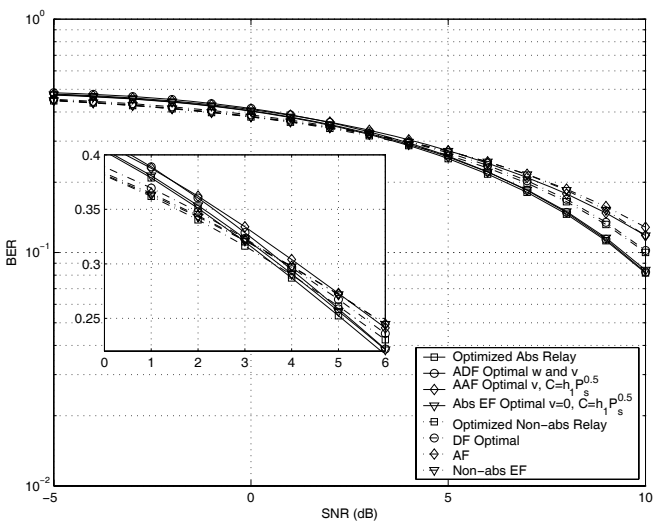

Fig. 5. Performance comparison between different abs-based and non-abs-based strategies when $h_1 = 1$ and $h_2 = 0.5$, $P_r = P_s = 1$. The subfigure shows the crossover between the abs-based and non-abs-based strategies.

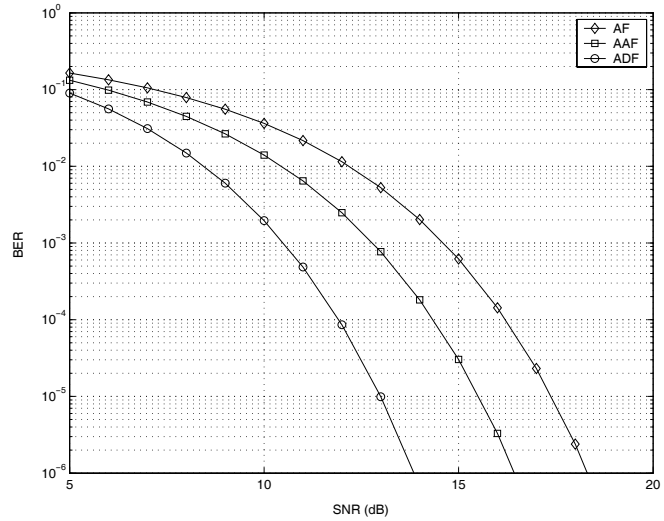

Fig. 6. Performance comparison of AF, AAF and ADF with the AF scheme for one-way relay channel when $\sigma_r^2 = \sigma_s^2$, $h_1 = h_2$ and $P_r = P_s = 1$.

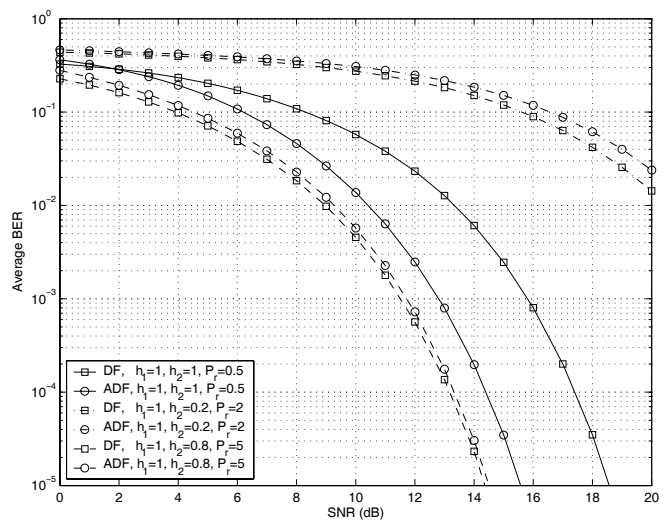

Fig. 7. Performance comparison of ADF with DF under different scenarios.

imposed at the relay, the optimal ADF achieves the minimum average probability of error. From Fig. 3, we can also see that when the SNR is small, the optimized relay function has a similar "V" shape-like behavior as the AAF strategy. As the SNR increases, the behavior of the optimized relay function is more related to the one for the ADF strategy. This suggests that ADF performs well at high SNR while AAF is effective at low SNR. Interestingly, the relay function of EF has almost the same shape as the optimized relay function in all SNRs.

Fig. 4 compares the bit error rate (BER) performance of different abs-based and non-abs-based strategies for BPSK when $h_1 = 1$ and $h_2 = 0.8$. We observe that at low SNR, the optimized non-abs-based (abs-based) relay performs according to the AF (AAF) strategy, while it behaves like the DF (ADF) strategy at high SNR. Also, EF performs close to the optimized strategy for all SNR values. It can also be seen that non-abs-based strategies perform better than abs-based strategies at low SNR in this scenario while the former performs worse than the latter at high SNR. The reason for this is that non-abs-based strategies do not exploit the fact that *a priori* information about the signal it has just transmitted is available at each terminal providing extra redundancy which is useful particularly at low SNR. A similar behavior is observed in Fig. 5 where the case $h_1 = 1$ and $h_2 = 0.5$ is considered. Compared to the results for $h_1 = 1$ and $h_2 = 0.8$ in Fig. 4 the threshold SNR below which non-abs-based strategies perform better than abs-based strategies is increased. Thus, non-abs-based strategies are beneficial for asymmetric channels.

In Fig. 6 we compare the BER for the AF, AAF, and ADF strategies on the two-way relay channel in the high SNR regime, where we assume that $\sigma_r^2 = \sigma_s^2$, $h_1 = h_2$ and $P_r = P_s = 1$. For the AAF strategy we set $C = 1$. Also, we do not include the optimized relay and EF strategies as their performances are very close to ADF at high SNR. We observe from Fig. 6 that AAF has a 2 dB gain over AF at a BER of 10^{-8}. Finally, we can see from Fig. 6 that ADF

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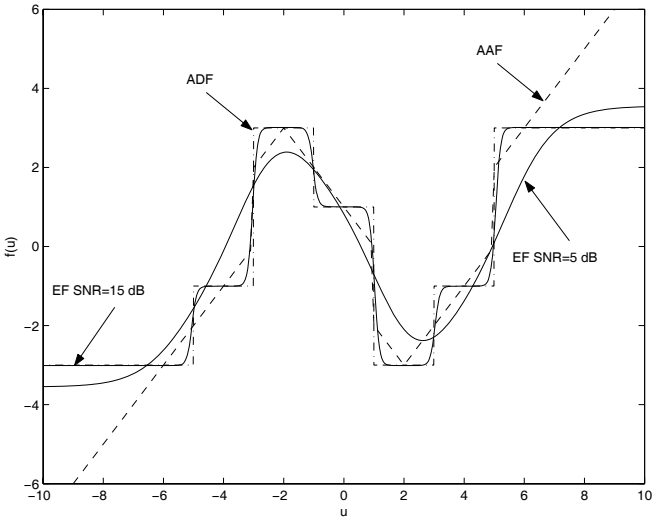

Fig. 8. Comparison of relay functions for AAF, ADF, and EF with $\sigma_r^2 = \sigma_s^2$ where both terminals use 4-PAM.

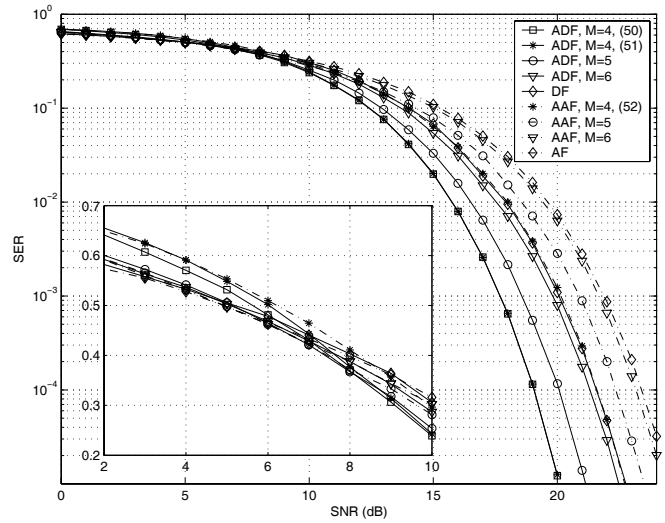

Fig. 9. SER comparison of ADF and AAF relay functions for 4-PAM with $M = |\mathcal{V}'| = 4, 5, 6, 7$ and $\sigma_r^2 = \sigma_s^2$. The subfigure shows the crossover between different strategies.

performs best, where a $2.7\,\text{dB}$ gain over AAF at a BER of 10^{-8} can be observed.

In Fig. 7 the average error probability of ADF and DF is compared for three different cases, which agrees with our analysis in Section IV very well. Fig. 8 compares the behavior of $f(u)$ for AAF, ADF, and EF strategies for $\sigma_r^2 = \sigma_s^2$, where both terminals use 4-PAM, i.e., $M = |\mathcal{V}'| = 4, 5, 6, 7$, and the SNR is chosen to be $5/\sigma_r^2$. The behavior of the relay function in Fig. 8 resembles the one in Fig. 3 for different strategies. In particular, when the SNR is small, EF has a similar behavior as AAF. As the SNR increases, we can observe that the EF relay function resembles the behavior of the ADF relay function.

In Fig. 9 the symbol error rate (SER) of different relay functions using ADF and AAF is compared, where the same parameters as in Fig. 8 are used. We can see that the performance degrades as M increases. Also, we can observe from Fig. 9 that a comparison between the mappings in (50) and (51) shows almost identical performance. There are two factors that affect the performance of relay functions with different M. First, a small M indicates a higher compression at the relay, which results in power savings. Second, when M is small, a detection error at the relay may affect the overall performance. At high SNR, it is clear that the power savings dominate the performance of ADF. At low SNR, we find that the performance degrades as M decreases, which means that $M = 7$ achieves the best performance. For example, at SNR$= 0$ dB, the SERs for $M = 4,\ 5,\ 6,\ 7$ are 0.6904, 0.6472, 0.6428, and 0.6146, respectively. This observation generalizes the one for the BPSK case, where the reason for this behavior is again that the redundancy in the constellation set increases for larger M.

VII. Conclusion

We have analyzed and optimized relaying strategies for memoryless TWRCs. In particular, we propose abs-based strategies where the relay processes the absolute value of the received signal. These techniques generally outperform non-abs-based strategies in the moderate to high SNR regime since they take into account that side information is available at the terminals which allows for additional power savings. Specifically, we have considered abs- and non-abs-based AF, DF and EF schemes, and also the optimization of the nonlinear processing function at the relay. We found that the non-abs-based DF performs better than the abs-based DF when the two-way channel is very asymmetric or the relay has greater power than the two terminals, while ADF performs better than DF when the relay has roughly the same power as the terminals. Although this work does not consider channel coding, the obtained expressions for the error probability allow for a rough determination of the required rate for an end-to-end channel code. Extensions of these results to higher order constellations such as QAM and PAM have also been presented, where similar observations can be made.

Appendix

In this appendix, we prove Theorem 1. We first give the following lemma.

Lemma 2: Let Z be a normal random variable with mean 0, and let $p_U(\mu)$, $p_V(\mu)$ denote two arbitrary probability density functions associated with the random variables U and V, respectively. If $p_U(\mu) - p_V(\mu)$ is nonnegative for $\mu \geq t$ and negative otherwise for some threshold t, then $p_{U+Z}(\nu) - p_{V+Z}(\nu)$ is nonnegative for $\nu \geq t'$ and negative otherwise for some threshold t'.

Proof: Denote by σ^2 the variance of Z. The result follows since (58) at the top of next page, where $\frac{1}{\sigma\sqrt{2\pi}} \exp\left\{+\frac{-\nu^2+2t\nu}{2\sigma^2}\right\} > 0$, and both integral terms are nondecreasing functions of ν. ■

Proof of Theorem 1: For brevity let $a \triangleq h_1\sqrt{P_s} + h_2\sqrt{P_s}$ and $b \triangleq h_1\sqrt{P_s} - h_2\sqrt{P_s}$.

Case 1: Non-abs strategies. When $x_1 = \sqrt{P_s}$, terminal 1's error-minimizing detection rule is to decide $x_2 = \sqrt{P_s}$ if $p_{f(a+N)+Z_1}(y_1) - p_{f(b+N)+Z_1}(y_1) \geq 0$ and $x_2 = -\sqrt{P_s}$ otherwise. Since $f(U)$ is an increasing function of U, we can apply Lemma 2 with $U = f(a+N)$, $V = f(b+N)$ and $Z = Z_1$ to give the result.

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$$\begin{aligned} p_{U+Z}(\nu) - p_{V+Z}(\nu) &= \int_{-\infty}^{\infty} p_Z(\nu-\mu)p_U(\mu)d\mu - \int_{-\infty}^{\infty} p_Z(\nu-\mu)p_V(\mu)d\mu \\ &= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\nu-\mu)^2}{2\sigma^2}\right\}(p_U(\mu)-p_V(\mu))\,d\mu \\ &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{+\frac{-\nu^2+2t\nu}{2\sigma^2}\right\}\left(\int_{-\infty}^{t} \exp\left\{\frac{2\nu(\mu-t)-\mu^2}{2\sigma^2}\right\}(p_U(\mu)-p_V(\mu))\,d\mu\right. \\ &\quad \left. + \int_{t}^{\infty} \exp\left\{\frac{2\nu(\mu-t)-\mu^2}{2\sigma^2}\right\}(p_U(\mu)-p_V(\mu))\,d\mu\right) \end{aligned} \tag{58}$$

Case 2: Abs strategies. When $x_1 = \sqrt{P_s}$, terminal 1's error-minimizing detection rule is to decide $x_2 = \sqrt{P_s}$ if $p_{f(|a+N|)+Z_1}(y_1) - p_{f(|b+N|)+Z_1}(y_1) \geq 0$ and $x_2 = -\sqrt{P_s}$ otherwise. Note that

$$\begin{aligned} &p_{|a+N|}(\mu) - p_{|b+N|}(\mu) \\ =&p_{a+N}(\mu) + p_{a+N}(-\mu) - p_{b+N}(\mu) - p_{b+N}(-\mu) \\ =&C(\mu)\,(D(\mu)-1) \end{aligned}$$

where $C(\mu) = \left(\exp\{-(\mu-b)^2/2\sigma^2\} + \exp\{-(-\mu-b)^2/2\sigma^2\}\right) / \sigma\sqrt{2\pi} > 0$ and

$$\begin{aligned} D(\mu) &= \frac{\exp\{-(\mu-a)^2/2\sigma^2\} + \exp\{-(-\mu-a)^2/2\sigma^2\}}{\exp\{-(\mu-b)^2/2\sigma^2\} + \exp\{-(-\mu-b)^2/2\sigma^2\}} \\ &= \exp\left\{\frac{-a^2+b^2}{2\sigma^2}\right\} \frac{\exp\{\mu a/\sigma^2\} + \exp\{-\mu a/\sigma^2\}}{\exp\{\mu b/\sigma^2\} + \exp\{-\mu b/\sigma^2\}} \end{aligned}$$

is an increasing function for $\mu \geq 0$. Thus, $p_{|a+N|}(\mu) - p_{|b+N|}(\mu)$ is nonnegative for $\mu \geq t$ and negative otherwise for some threshold t. Since $f(|U|)$ is a non-decreasing function of $|U|$, we can apply Lemma 2 with $U = f(|a+N|)$, $V = f(|b+N|)$ and $Z = Z_1$ to give the result.

In both cases, by symmetry, threshold detection is also optimal when $x_1 = -\sqrt{P_s}$. The same proof with all subscripts $_1$ and $_2$ interchanged applies for terminal 2. ■

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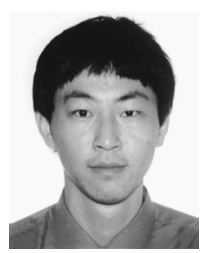

Tao Cui (S'04) received the M.Sc. degree in the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada, in 2005, and the M.S. degree from the Department of Electrical Engineering, California Institute of Technology, Pasadena, USA, in 2006. He is currently working toward the Ph.D. degree at the Department of Electrical Engineering, California Institute of Technology, Pasadena. His research interests are in the interactions between networking theory, communication theory, and information theory.

Tracey Ho (M'06) is an Assistant Professor in Electrical Engineering and Computer Science at the California Institute of Technology. She received a Ph.D. (2004) and B.S. and M.Eng degrees (1999) in Electrical Engineering and Computer Science (EECS) from the Massachusetts Institute of Technology (MIT). Her primary research interests are in information theory, network coding and communication networks.

Jörg Kliewer (S'97–M'99–SM'04) received the Dipl.-Ing. (M.Sc.) degree in electrical engineering from Hamburg University of Technology, Hamburg, Germany, in 1993 and the Dr.-Ing. degree (Ph.D.) in electrical engineering from the University of Kiel, Kiel, Germany, in 1999, respectively.

From 1993 to 1998, he was a Research Assistant at the University of Kiel, and from 1999 to 2004, he was a Senior Researcher and Lecturer with the same institution. In 2004, he visited the University of Southampton, Southampton, U.K., for one year, and from 2005 until 2007, he was with the University of Notre Dame, Notre Dame, IN, as a Visiting Assistant Professor. In August 2007, he joined New Mexico State University, Las Cruces, NM, as an Assistant Professor. His research interests include network coding, error-correcting codes, wireless communications, and communication networks.

Dr. Kliewer was the recipient of a Leverhulme Trust Award and a German Research Foundation Fellowship Award in 2003 and 2004, respectively. He is a Member of the Editorial Board of the EURASIP JOURNAL ON ADVANCES IN SIGNAL PROCESSING and Associate Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS.

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