Maximum Evaporation Rates for Nonisothermal Droplets

F. W. HARTWIG

Daniel and Florence Guggenheim Jet Propulsion Center, California Institute of Technology, Pasadena, Calif.

Calculations have been carried out in order to determine the rate of evaporation of a liquid droplet surrounded by hot gases. The present study represents an extension of earlier work by Penner on evaporation rates for isothermal droplets. In particular, allowance was made for temperature gradients within the droplet by considering a droplet composed of an isothermal core and an isothermal shell. Results obtained in the present investigation were found to be in satisfactory agreement with the known data for evaporation of isothermal droplets, thus justifying calculations for isothermal droplets as a reasonable first approximation.

Introduction

The purpose of this study is to extend earlier work on evaporation rates of isothermal droplets in rocket engines. Penner (1) treated the evaporation rate of a liquid droplet in a gas at constant temperature assuming Stokes flow. The limitations involved in this approximation, as well as references to the original literature, have been stated in (1) and need not be repeated here. Although the droplet temperature changed with time, it was assumed that the droplet remained isothermal during evaporation, i.e., that the thermal conduction coefficients of evaporating liquid droplets were, for all practical purposes, infinite.

We have considered a shell model of the evaporating droplet in which the droplet is divided into two parts, an inner core and an outer spherical shell. The inner sphere was assumed to be isothermal and to remain at the original temperature of the evaporating liquid; the outer shell was also assumed to be isothermal, but its temperature was determined by making an appropriate heat balance equation. The thickness of the spherical shell was treated as a variable parameter, and it was found that results substantially equivalent to those obtained for isothermal droplets are derived, independently of the thickness of the spherical shell.

Of the evaporating droplets treated previously (1) by assuming isothermal conditions during evaporation, we have chosen for study an aniline droplet with initial radius equal to \(5 \times 10^{-2}\) cm. The initial temperature was set equal to 300° K. Radiant heat transfer to the droplet was neglected since it is probably unimportant in liquid-fuel rocket motors (1).

The Shell Model of the Evaporating Droplet

The concept of an isothermal droplet during evaporation in combustion chambers represents a first approximation. In practice one would expect that temperature gradients are set up during the life of the droplet. If the thermal conductivity is sufficiently small, then it is clear that steep tempera-

Received July 26, 1952.

† Captain, USAF. This paper is abstracted from a thesis submitted by F. W. Hartwig to the graduate school of the California Institute of Technology in partial fulfillment of requirements for the degree of Aeronautical Engineer, June 1952. Present address: U. S. Air Force Institute of Technology, Wright-Patterson Air Force Base, Dayton, Ohio.

* Number in parentheses refers to the Reference on page 243.
The model of the evaporating droplet adopted in the present discussion requires allowance for another heat sink. Thus the assumption that $\epsilon$ is constant means that the inner surface of the isothermal shell must travel in the direction of the center of the sphere sufficiently rapidly to maintain $\epsilon$ constant. This travel of the inner surface of the outer shell means that some mass will be introduced from the colder core of the droplet into the outer shell. Since the shell must remain isothermal, heat must be added to this new mass to bring it up to the temperature of the shell. The surface area $4\pi(r - \epsilon)^2$ moves inward with a velocity $-dr/dt$. Hence the energy absorbed per unit time in order to keep $\epsilon$ constant and to keep the shell isothermal is

$$4\pi(r - \epsilon)^2(-dr/dt)\rho c(T_1 - T_0).$$

In order to correct for this heat sink, we may say that of the total heat $Q$ transferred to the droplet in unit time, only the amount

$$Q' = 4\pi\rho h\Delta T - 4\pi(r - \epsilon)^2(-dr/dt)\rho c(T_1 - T_0)$$

is effective in producing evaporation and heating the outer isothermal shell.

If Equation [1a] is used in place of Equation [1], Equation 3] becomes

$$(dT/dt)_b = 3\hbar\Delta T/r[1 - (1 - \epsilon/r)^2] \rho c - 3(1 - \epsilon/r)^2(-dr/dt)(T_1 - T_0)/r [1 - (1 - \epsilon/r)^2]$$

Neglecting heat transfer between the isothermal regions, it is apparent that

$$(dT/dt) = (dT/dt)_b + (dT/dt)_a$$

represents the rate of change of temperature in the isothermal shell with time. Using Equations [3a] and [6] we obtain the result

$$dT/dt = \{3[r/(1 - (1 - \epsilon/r)^2)]\{h\Delta T/\rho c\} - (1 - \epsilon/r)^2(-dr/dt)(T_1 - T_0) = (1 - (1 - \epsilon/r)^2)[7]$$

Similarly, for $\epsilon/r < 1$

$$dT/dt = h\Delta T/\rho c - (-dr/dt)(1/\epsilon c) + (T_1 - T_0)/\epsilon [7a]$$

and for $\epsilon/r = 1$

$$(dT/dt) = 3h\Delta T/\rho c - (3\pi/\epsilon)(-dr/dt) \ldots \ldots \ldots [7b]$$

Equation [7b] represents the known result for isothermal droplets (1).

In order to carry out approximate calculations for the rate of change of temperature $T_1$ and droplet radius $r$ with time, we may assume that $-dr/dt$ is given by the Knudsen equation with an evaporation coefficient $a$ set equal to unity. Thus

$$-dr/dt = (p/\rho)(M/2\pi RT)^{1/2} \ldots \ldots \ldots [8]$$

where $p$ is the saturated vapor pressure of the liquid at the temperature $T_1$, $M$ is the molecular weight, and $R$ is the molar gas constant. The pair of Equations [7], [7a], or [7b] together with Equation [8] can be solved by a simple iterative procedure (1).

Representative numerical values for aniline are the following: $\epsilon = 0.63$ cal/gm-K; $r^2 = 5 \times 10^{-5}$ cm; $h = k/\rho = 1.30 \times 10^{-4}$ cal/cm$^2$ sec K; $\Delta T = (3900 - T_1)$ K; $\rho = 1.03$ gm/cc; $c = 1 = 120$ cal gm$^{-1}$ sec$^{-1}$; $-dr/dt = \text{antilog}_{10}(7.09 - 2000/T_0)$; $T_0 = 300$ K; $T_1 = 1650$ K. The results of the present calculations for $\epsilon/r = 0.25$, $10^{-3}$, and $10^{-4}$ are plotted in Fig. 1 together with the known results for $\epsilon/r = 1$ (isothermal droplet).

Reference


ARS 1953 Junior Award Competition

Papers are now being invited for consideration for the 1953 Junior Award, to be presented at the ARS 8th Annual Convention to be held in December 1953.

The winning paper will be judged mainly on the basis of content which should reflect original thought and effort. The age of authors of papers should not exceed 25 years.

Please refer to first page, this issue of the JOURNAL, for acceptable scope of subject matter. Manuscripts are to be prepared in accordance with style instructions on second page of this issue.

Papers must be received no later than October 1, 1953, and should be clearly marked, "Submitted for Junior Award Competition."

Send papers to: The Secretary, American Rocket Society, 29 West 39th Street, New York 18, N. Y.