Lifetime of the Yukawa Particle

Recent investigations by various authors[1] have made it very probable that the hard rays of the cosmic radiation (mesotrons), now identified with the particle of Yukawa[2] of mass $\mu \sim 200m$ (m = mass of the electron), are unstable and will decay spontaneously into electrons and neutrinos. The lifetime for a mesotron at rest has been estimated from experience to be of the order $2\text{–}4\times10^{-6}$ sec.

Yukawa himself calculated the lifetime on the basis of his ideas to be of the order 0.25×10^{-6} sec., a result not far from the observed value. However, the present author[3] obtained on the same assumptions a much smaller value. The importance of this question may justify a restatement of the theoretical result and an explanation of this difference.

The final formulae for the lifetime obtained by both authors is the same, apart from differences in notation. It can be written in the form

$$\tau = \frac{G^2}{\hbar c} 4\pi^2 \left(\frac{m}{\mu}\right)^4 \frac{\hbar}{\mu c^2} \frac{1}{G_F^2}. \qquad (1)$$

In this formula $\hbar$, m, c have the usual meaning, and μ is the rest mass of the mesotron. G is the constant of dimension of a charge in the potential between nuclear particles $V(r) = (G^2/r)e^{-r\mu c/\hbar}$ following from Yukawa's theory. $G^2/\hbar c$ is of the order[3, 4] μ/M (M = mass of the proton) but probably somewhat larger than this quotient. The lifetime τ is therefore essentially proportional to μ^{-4}. G_F finally is the constant in Fermi's theory of β-decay, normalized to be a pure number. The form of interaction assumed for the coupling between proton, neutron and the electron neutrino field is

$$G_F mc^2(\hbar/mc)^3(\psi_N{}^*\beta\psi_P)(\varphi_\nu{}^*\beta\varphi_e) + \text{c.c.}$$

(ψ_N, ψ_P, φ_ν, φ_e being the wave functions of neutron, proton, neutrino and electron, respectively). This leads to the probability for emission of an electron of energy ϵ

$$w(\epsilon)d\epsilon = |M|^2 \frac{G_F^2}{(2\pi)^3} \frac{mc^2}{\hbar} \frac{(\epsilon_0-\epsilon)^2(\epsilon^2-mc^2)^{\frac{1}{2}}\epsilon d\epsilon}{(mc^2)^5},$$

where ϵ_0 is the maximum energy of the emitted electrons and M a matrix element from the motion of the heavy particles inside the nucleus.

The discrepancy in the calculated lifetimes comes from the different values used for the constant G_F. As discussed by Bethe and Bacher[5] and by Nordheim and Yost,[6] the experimental value of G_F depends quite appreciably on the group of elements which are taken for comparison, the difference being due in all probability to the matrix element M, which is smaller than unity for heavy elements but can be expected to be unity for light positron emitters. The value for G_F used by Yukawa (0.87×10^{-12} in our units) corresponds to the heavy natural radioactive elements, while the value deduced for the light positron emitters[6] is $G_F = 5.5\times10^{-12}$. It seems beyond doubt that this later value has to be taken for our purpose.

With the present most probable values $G^2/\hbar c = 0.3$; $\mu = 200m$; $G_F = 5.5\times10^{-12}$, we obtain from (1) $\tau = 1.6\times10^{-9}$ sec., i.e., a value about 10^{-3} times too small. A decrease in the assumed value for μ to $150m$ would increase τ only by a factor of order 3.

In view of this definite discrepancy the question arises whether any modifications of the theory could give a better result. It is to be noted firstly that the introduction of the Konopinski-Uhlenbeck form of the β-decay theory would only make matters worse as it would introduce roughly another factor $(m/\mu)^2$. A real improvement can only be expected by a complete reformulation of the theory. One possible suggestion would be to assume that the disintegration of a free mesotron is in first order approximation a forbidden transition, while in nuclei it is made allowed by the influence of the other nuclear particles.

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[5] H. Bethe and R. Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[6] L. W. Nordheim and F. Yost, Phys. Rev. **51**, 942 (1937). It has to be noted that the formula for τ_0 on p. 943 should be $\tau_0{}^{-1} = (G_F{}^2/(2\pi)^3) \times (mc^2/\hbar)$. The value of G_F is then determined from the empirical value $\tau_0{}^{-1} \cong 10^{-4}$.

The Scattering of Cosmic Rays by the Stars of a Galaxy

The problem dealt with in this note may be formulated in the following way: imagine a galaxy of N stars, each carrying a magnetic dipole of moment μ_n ($n = 1, 2, \cdots N$) and assume that the density, defined as the number of stars per unit volume, varies according to any given law, while the dipoles are oriented at random because of their very weak coupling. Under this condition the resultant field of the whole galaxy almost vanishes. Let there be an isotropic distribution of charged cosmic particles entering the galaxy from outside. Our problem is to find the intensity distribution in all directions around a point within the galaxy. Its importance arises from the fact that if the distribution should prove to be anisotropic a means would be available for determining whether cosmic rays come from beyond the galaxy, independent of the galactic rotation effect already considered by Compton and Getting.[1]

Suppose we consider a particle sent into an element of volume dV of scattering matter in a direction given by the vector R. Let the probability of emerging in the direction R' be given by a scattering function $f(R, R')$ per unit solid angle. Conversely a particle entering in the direction R' will have a probability $f(R', R)$ of emerging in the direction R. Let us assume that the scatterer (magnetic field of the star) has the reciprocal property so that $f(R, R') = f(R', R)$. In our case this property is satisfied provided the particle's sign is reversed at the same time as its direction of motion. That is, the probability of electron's going by any route is equal to the probability of positrons going by the reverse route. If it has the reciprocal property for each element of volume it will also have it for any extended distribution of matter, that is, $F(R, P; R', P') = F(R', P'; R, P)$, where $F(R, P; R', P')$ is the probability that a particle going in the direction R at the point P will emerge in the direction R' at the point P'. This is because the probability of following any route is equal to the probability of following the reverse route, through the same elements of volume. Thus the probability of a certain end result from a number of possible routes will equal the probability of the reversal of the result occurring through the reverse routes.

In our case the scatterer (star) is to a large extent nonabsorbing and noncapturing. The former is true except for particles colliding with the star, which can only happen when their energy is sufficiently great, and the latter is true except for particles which follow asymptotic or periodic orbits in the magnetic field of any one of the N stars. These orbits, however, almost certainly form a set of zero measure in the manifold of all possible orbits,[2] that is, they occur only exceptionally. Thus, while a dipole magnetic field can imprison charged particles starting from a point within it and can also keep them away if starting from infinity, depending on their energy and angular momentum, it can only exceptionally capture such particles starting from infinity. In our case, therefore, all particles starting in a direction R at a point P, sufficiently far from all neighboring stars, have only a small chance of being either absorbed or captured in a periodic orbit (of finite or infinite period), so that the great majority of them will emerge at infinity. For almost all particles, therefore, the probability of emerging at infinity must be unity, or

$$\int_{R'} F(R, P; R', \infty)dR' = 1 \tag{1}$$

almost always.

Now consider a beam of particles at infinity whose intensity in a direction R' is $I_\infty(R')$. The intensity at P observed in the direction R will be

$$I_p(R) = \int_{R'} F(R', \infty; R, P) I_\infty(R')dR'. \tag{2}$$

Using (1) and assuming an isotropic distribution at infinity such that $I_\infty(R')$ is a constant (independent of R'), we find that Eq. (2) becomes

$$I_p(R) = I_\infty \int_{R'} F(R, P; R', \infty)dR' = I_\infty \tag{3}$$

by (1). Therefore the intensity in any direction at P is the same and the distribution is isotropic at P if it is isotropic at infinity.

From the remark made previously, it is clear that if the distribution of positive and negative particles at infinity is isotropic, it will also be isotropic at any point P, except for small irregularities due to absorption by collision and by capture into periodic orbits. We conclude that particle scattering by magnetic fields of the stars is unable to contribute anything to the solution of the problem whether or not cosmic particles come from beyond our galaxy. The considerations developed in this note clearly hold so long as the scattering centers satisfy the conditions of being nonabsorbing and noncapturing, irrespective of the law of force which is responsible for the scattering. It need hardly be emphasized that they apply only to the case in which there is no resultant magnetic field for the whole galaxy, such as would exist if the dipoles were oriented along preferential directions. In this case particles would either be imprisoned if born within the galaxy, or kept out, if coming from outside, depending on their energy and angular momentum. The reciprocal property of paths would then break down in general, but would still hold for any allowed direction at any point within the galaxy.

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Nuclear Excitation of Indium by X-Rays

It has been shown recently[1, 2] that the stable nucleus In^{115}, when excited by fast neutrons or protons, may be left in a metastable excited state, designated by In^{115*}, from which it decays, emitting negative electrons, with a half-life time of 4.1 hours.

We have now observed that the same metastable state can be excited when indium is irradiated by x-rays. The x-rays were produced by bombarding a 2-mm thick lead target with electrons from an electrostatic generator. A thick indium foil, 1 inch in diameter, was placed directly behind the lead target. After 30 minutes irradiation at an electron energy of 1.73 Mev and a current of 10 μa, the indium foil showed an initial activity, recorded on a Geiger-Müller counter, of 45 counts per minute. The activity decayed with a period of approximately 4 hours. The walls of the counter reduced the intensity of the rather soft β-rays to about one-half. Until more is known about the effective x-rays, no well-defined cross section can be deduced from these data.

By varying the bombarding voltage it was established that the effect has a threshold at 1.35 ± 0.1 Mev. This result might be interpreted by assuming that In^{115} has at that energy an excited state which combines both with the ground state and the metastable excited state.

In a note which has just become known to us Pontecorvo and Lazar[3] also report the excitation of indium by x-rays. In their experiments the x-rays were produced with an impulse generator working at a peak voltage of 1850 kv.

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