ML CFO and PO Estimation in DCT OFDM Systems under Non-Circular Transmissions

Tao Cui
Department of Electrical Engineering
California Institute of Technology
Pasadena, CA 91125, USA
Email: taocui@caltech.edu

Feifei Gao and A. Nallanathan
Department of Electrical and Computer Engineering
National University of Singapore
Singapore 119260
Email: {feifeigao, elena}@nus.edu.sg

Chinthha Tellambura
Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB T6G 2V4, Canada
Email: chintha@ece.ualberta.ca

Abstract— Frequency synchronization is one of the most important components in orthogonal frequency-division multiplexing (OFDM) systems. Recently, the discrete cosine transform (DCT) based OFDM system has received wide attentions due to several advantages. Hence, the study of frequency synchronization issue for this newly raised system is well on its time. To provide a thorough study, we consider the non-circular transmissions, and the results can be easily generated to circular transmissions if the elliptic variance is set to zero. We present three joint maximum likelihood (ML) carrier frequency offset (CFO) and phase offset (PO) estimators. From both the theoretical analysis and the numerical comparisons, we found new advantages of the DCT-OFDM over the traditional discrete Fourier transform (DFT) based OFDM. These advantages, as well as those already studied in the early works on DCT-OFDM, support the belief that the DCT-OFDM is a new promising multi-carrier modulation (MCM) scheme.

I. INTRODUCTION

The traditional DFT based OFDM has already been enjoying its success when it is applied to European digital audio/video broadcasting (DAB, DVB) [1], [2], high performance local area network (HIPERLAN) [3] and IEEE 802.11a wireless LAN standards [4]. Another promising MCM, known as the DCT based OFDM has been proposed recently in [5], [6], where it is shown that the DCT-OFDM can, some times, offer additional benefits over the DFT-OFDM.

In MCM systems, it is well known that a CFO, caused by oscillators mismatch or Doppler effects, destroys the subcarriers orthogonality, and results in a substantial bit error rate (BER) degradation [7]. In the pioneer work [8], ML CFO estimation was developed for DFT-OFDM systems. The algorithm exploited the existence of the cyclic prefix (CP) and was mainly proposed for additive white Gaussian noise (AWGN) channels. However, this method fails to provide the PO estimation which is very important when the channel is (AWGN) channels. However, this method fails to provide the PO estimation which is very important when the channel is

II. NON-CIRCULAR MULTI-CARRIER TRANSMISSIONS

The $m$th OFDM block in the frequency domain is represented as

$$s_m = [s_{m,0}, s_{m,1}, \ldots, s_{m,N-1}]^T, \quad m = 0, 1, 2, \ldots$$

(1)

where $N$ denotes the number of the subcarriers adopted. The complex random variable $s_{m,i}$ is said to be non-circular at the order 2, if the elliptic variance $E\{s_{m,i}s_{m,i}\}$ is not zero. Therefore, we assume each $s_{m,i}$ has its variance and elliptic variance as

$$E\{s_{m,i}s_{m,i}\} = E_s \delta_{i1,i2},$$

$$E\{s_{m,i}s_{m,i}\} = E_c \delta_{i1,i2},$$

(2)

(3)

where $\delta_{ab}$ is the Kronecker delta function, $E_s$ denotes the average signal power, and $E_c$ may be a complex number.
with \(|E_c| \leq E_s\). Here, symbols transmitted on different subcarriers are reasonably assumed independent from each other. However, the symbol on the same subcarrier may be dependent from block to block. Clearly, lots of practical signal modulations contain non-circularity.

**Example 1: BPSK modulation.** Let \(s_{m,i} \in \{\sqrt{E_s}, -\sqrt{E_s}\}\).

Then, we have

\[
E_s = E\{s_{m,i}s_{m,i}^*\} = E\{s_{m,i}s_{m,i}\} = E_c. \quad (4)
\]

**Example 2: MSK modulation.** OFDM-MSK modulation has recently attracted lots of interests, whose original form was proposed in [11]. In this new modulation scheme, different branches of MSK symbols are transmitted through different subcarriers. The MSK symbol on the \(i\)th subcarrier is

\[
s_{m,i} = \overline{E_s} \exp \{ j\pi h \} a_{m,i} + a_{q,i}, \quad (5)
\]

where \(a_{m,i} \in \{\pm 1\}\) is independently generated and \(h\) is the modulation indices. Obviously,

\[
E\{s_{m,i}s_{m,i}^*\} = E_s, \quad (6)
\]

whereas,

\[
E_c = E\{s_{m,i}s_{m,i}\} = E_s E_c^{(m+1)} = \frac{1}{2} e^{j2\pi h} + e^{-j2\pi h} = E_s (\cos (2\pi h))^{m+1}. \quad (7)
\]

When \(h = 1/2\), as usually the case for MSK, \(E_c = (-1)^{m+1} E_s\). Clearly, OFDM-MSK also includes non-circularity.

### III. DISCRETE COSINE TRANSFORM BASED OFDM SYSTEM

**A. System Model**

In a DCT-OFDM system, instead of performing the inverse DFT (IDFT), we apply inverse DCT (IDCT) for the \(m\)th time domain OFDM block, resulting in

\[
u_m = [u_{m,0}, u_{m,1}, ..., u_{m,N-1}]^T = D^T s_m, \quad (8)
\]

where \(D\) is the DCT matrix with the \((a,b)\)th entry

\[
[D]_{ab} = \beta_a \frac{2}{N} \cos \left( \frac{(a-1)(2b-1)}{2N} \right), \quad a, b = 1, ..., N, \quad (9)
\]

and

\[
\beta_a = \begin{cases} \sqrt{2}, & a = 1 \\ 1/\sqrt{2}, & otherwise \end{cases} \quad (10)
\]

If the number of the subcarriers is sufficiently large, \(u_{m,i} \sim 0, ..., N-1\) can be well approximated as having a Gaussian distribution [12]. It can be readily verified that

\[
E\{u_m u_m^H\} = DE\{s_m s_m^H\} D^H = E_s I_N, \quad (11)
\]

\[
E\{u_m u_m^T\} = DE\{s_m s_m^T\} D^T = E_s I_N. \quad (12)
\]

Note that, \(u_{m,i}\), the time domain symbol after the IDCT, are independent from each other.

Suppose that the maximum channel delay is no longer than \(LT_s\), where \(T_s\) is the sampling period. From [5], we know that in order to diagonalize the channel matrix for the DCT-OFDM, we need to insert both the prefix and the suffix of length \(\mu = \frac{L}{2}\) into \(u_m\). Therefore, the effective OFDM block length is \((N + 2\mu)T_s\) and the symbols transmitted are represented by

\[
x_m = [x_{m,0}, x_{m,1}, ..., x_{m,N+2\mu-1}]^T = T_{ps} u_m, \quad (13)
\]

where

\[
T_{ps} = \begin{bmatrix} V_{\mu} & 0_{\mu \times (N-\mu)} \\ 0_{\mu \times (N-\mu)} & I_N \end{bmatrix}, \quad (14)
\]

and \(V_{\mu}\) is the \(\mu \times \mu\) anti-identity matrix:

\[
V_{\mu} = \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{bmatrix}. \quad (15)
\]

As in [8], we will only consider the AWGN channel for the time being but still keep both the prefix and the suffix for the estimation purpose. The baseband received signal could be expressed as

\[
r_m = [r_{m,0}, r_{m,1}, ..., r_{m,N+2\mu-1}]^T = e^{j2\pi m(N+2\mu-1)\epsilon_0} \Gamma(\epsilon_0) x_m + n_m, \quad (16)
\]

where \(\epsilon_0 = f_0 T_s\) is the normalized CFO by the sampling rate, and \(\phi_0\) is the PO. Each elements in \(n_m\) represents independent AWGN with variance \(E_n\), and \(\Gamma(\epsilon_0)\) is the diagonal CFO matrix of the form

\[
\Gamma(\epsilon_0) = \text{diag} \{ 1, e^{j2\pi \epsilon_0}, ..., e^{j2\pi (N+2\mu-1)\epsilon_0} \}. \quad (17)
\]

**B. The Joint CFO and PO Estimation**

The frequency synchronization should be carried on after the first OFDM block is received. Therefore, we consider the OFDM block with index number \(m = 0\), and the index \(m\) will be dropped for notation simplicity in the remaining of this paper unless otherwise mentioned. Define

\[
A = E_r T_{ps}^H T_{ps}^H + E_n I_{N+2\mu}, \quad (18)
\]

\[
B = E_e T_{ps}^H T_{ps}^H. \quad (19)
\]

Clearly, we have \(A^* = A\) while \(B^* \neq B\) unless \(E_c\) is a real number. Nevertheless, \(B^*\) can be represented as \(B^H\). Consequently, the covariance and the elliptic covariance of \(r\) are \(\Gamma(\epsilon_0) A \Gamma(\epsilon_0)^H\) and \(e^{j2\beta q_0} \Gamma(\epsilon_0) B \Gamma(\epsilon_0)\), respectively. Therefore, \(r\) is a non-circular Gaussian random vector and should be characterized by the joint probability density function (PDF) [13]:

\[
f(r, r^*|\epsilon_0, \phi_0) = \frac{1}{\pi^{N+L}(\det(R))^{1/2}} \times \exp -\frac{1}{2} [r^H, r^T] R^{-1} [r, r^*], \quad (20)
\]
where $R$ is the covariance matrix of $[r^T, r^H]^T$, defined as

$$ R = E[r^* r^T] = e^{j\phi_0} \Gamma(\varepsilon_0) \begin{pmatrix} 0 & A \end{pmatrix} \begin{pmatrix} B^* & A^* \end{pmatrix} R_s \begin{pmatrix} 0 & e^{j\phi_0} \Gamma(\varepsilon_0) \end{pmatrix} = e^{j\phi_0} \Gamma(\varepsilon_0)^* \begin{pmatrix} 0 & A \end{pmatrix} \begin{pmatrix} B^* & A^* \end{pmatrix} R_s. $$

From the inversion of the partitioned matrix lemma, we obtain

$$ R_s^{-1} = \begin{pmatrix} P & -Q \end{pmatrix} \begin{pmatrix} P^* & \end{pmatrix}, \quad (22) $$

where

$$ P = (A - B^H A^{-1} B)^{-1}, \quad Q = A^{-1} B P^*. \quad (23) $$

and $(\cdot)^*$ denotes $(\cdot)^{-1}$. Then $R^{-1} = \tilde{\Gamma} R_s^{-1} \tilde{\Gamma}^H$. From (21), we know that

$$ \det(R) = \det(\tilde{\Gamma}) \det(R_s) \det(\tilde{\Gamma}^H) = \det(R_s). \quad (25) $$

Therefore, $\det(R)$ is only a constant, and it is sufficient to consider the log-likelihood function only. Hereafter, we will provide three different types of the ML joint CFO and PO estimators.

1) MLE1: When $\varepsilon_0, \phi_0$ are both considered as deterministic values, their ML estimates can be found from

$$ \{\hat{\varepsilon}_0, \hat{\phi}_0\} = \arg \max_{\varepsilon, \phi} \log f(r, r^* | \varepsilon, \phi) = \arg \max_{\varepsilon, \phi} -\|r^H r^*\| R^{-1} r^* \quad (26) $$

where $\varepsilon, \phi$ are the trial parameters. After some manipulations, the ML estimates $\hat{\varepsilon}_0$ and $\hat{\phi}_0$ can be obtained by

$$ \{\hat{\varepsilon}_0, \hat{\phi}_0\} = \arg \max_{\varepsilon, \phi} -r^H Y r + \Re\{e^{-j2\phi} r^T U r\}, \quad (27) $$

where

$$ Y = \Gamma(\varepsilon) P \Gamma(\varepsilon)^*, \quad (28) $$

$$ U = \Gamma(\varepsilon)^* Q \Gamma(\varepsilon)^*. \quad (29) $$

Due to the special structure of $A$ and $B$, when $2\mu < N$, $-r^H Y r$ can be rewritten as (30)$^1$, where $\eta$ is some unimportant parameters unspecified for the concise of the paper, and $r^T U r$ can be expressed as (31). Note that only the second term in (27) is related with $\phi$. Then PO can be estimated via

$$ \hat{\phi}_0 = \arg \max_{\phi} \Re\{e^{-j2\phi} r^T U r\} = \frac{1}{2} \angle r^T U r, \quad (32) $$

where $\angle$ is used for extracting the angle. Clearly, the estimation range of PO is $[-\pi/2, \pi/2]$. Substituting (32) back into (27), $\hat{\varepsilon}_0$ can be found from

$$ \hat{\varepsilon}_0 = \arg \max_{\varepsilon} -r^H Y r + \Re\{e^{-j2\phi} r^T U r\}. \quad (33) $$

It is noted that the noise power $E_n$ is required by the estimator (27). This is not unexpected as is seen from [8], where the noise power is used to estimate the TO. The study on the noise power estimation can be found from [14] and references therein.

2) MLE2: Strictly speaking, MLE1 is not an ML estimator but a generalized likelihood ratio testing estimator [15]. To realize the true ML detection, we have to pursue a two dimensional search for both $\varepsilon$ and $\phi$ from equation (27). Due to its high complexity by performing two dimensional search, the MLE2 is only of theoretical interest and is proposed here as a benchmark for comparison with other estimators.

3) MLE3: When the statistics of either $\phi_0$ or $\hat{\varepsilon}_0$ is known, the ML estimation can be modified by using the marginal likelihood function. It is natural to assume that $\phi_0$ is a uniformly distributed random variable over $[-\pi, \pi]$. From (32), we know that $f(r, r^* | \varepsilon, \phi)$ is a periodic function of $\phi$ with the period $\pi$. Therefore we will assume that $\phi_0$ is uniformly distributed over the region $[-\pi/2, \pi/2]^2$. The average of $f(r, r^* | \varepsilon, \phi)$ over $\phi$ gives the marginal likelihood

$^1$Since in practice, $2\mu < N$ is always satisfied, we will only provide the explicit expression of the estimators for this case.

$^2$Normally the PO should be uniformly distributed over the region $[-\pi, \pi]$. However, if $|\phi_0| > \pi/2$, then only $\phi_0 \mod \pi - \pi/2$ can be found by the proposed estimator. Therefore the effective PO can reasonably be assumed to be uniformly distributed over the region $[-\pi/2, \pi/2]$. 

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function $f(r, r^*|\epsilon)$, which removes the likelihood dependence on \( \phi \). This marginal likelihood function can be expressed as

$$f(r, r^*|\epsilon) = E_\phi \{ f(r, r^*|\epsilon, \phi) \}$$  \hspace{1cm} (34)

$$= K \exp -r^H Yr \ E_\phi \exp(\Re\{e^{-j2\phi}r^T U_r\})$$  

$$= K \frac{\pi}{\exp -rf^H Yr} \ \frac{\pi/2}{\exp(\Re\{e^{-j2\phi}r^T U_r\})} \ d\phi,$$

where

$$K = \frac{1}{\pi^{N+L}(\text{det}(R))^{1/2}}.$$  \hspace{1cm} (35)

For notational simplicity, we define $w = |r^T U_r|$ and $\psi = \text{arg} r^T U_r$. Then

$$\frac{\pi/2}{\exp -rf^H Yr} \ \frac{\pi/2}{\exp(\Re\{e^{-j2\phi}r^T U_r\})} \ d\phi = \frac{\pi}{\exp(w \cos(\psi - 2\phi))} \ d\phi \pi - \frac{\pi}{\exp(w \cos(\psi))}$$

$$= \frac{1}{2} \exp(w \cos t) dt = \pi \exp(w \cos t) dt = \pi I_0(w),$$  \hspace{1cm} (36)

where $I_0(w)$ is the zeroth order modified Bessel function of the first kind. Finally, the expression for $f(r, r^*|\epsilon)$ is

$$f(r, r^*|\epsilon) = K \exp -r^H Yr \ I_0 |r^T U_r|.$$  \hspace{1cm} (37)

and \( \epsilon_0 \) should be estimated from

$$\hat{\epsilon}_0 = \arg \max_{\epsilon} f(r, r^*|\epsilon)$$

$$= \arg \max_{\epsilon} -r^H Yr \ I_0 |r^T U_r|.$$  \hspace{1cm} (38)

For high SNR, $I_0(w)$ can be approximated by

$$I_0(w) \approx \frac{e^w}{\sqrt{2\pi w}},$$  \hspace{1cm} (39)

and $f(r, r^*|\epsilon)$ is rewritten as

$$f(r, r^*|\epsilon) \approx K \frac{e^{\phi}}{\sqrt{2\pi |r^T U_r|^{1/2}}} = -K \frac{e^{\phi}}{\sqrt{2\pi |r^T U_r|^{1/2}}}.$$  \hspace{1cm} (40)

The ML estimation of \( \epsilon \) can be found from

$$\hat{\epsilon}_0 = \arg \max_{\epsilon} e^{\phi} |r^T U_r|^{1/2}.$$  \hspace{1cm} (41)

Unfortunately, a reasonable assumption on the distribution of \( \epsilon_0 \) is not available in the literature. Therefore, we are unable to average $f(r, r^*|\epsilon, \phi)$ over \( \epsilon \) to obtain $f(r, r^*|\phi)$. However, since the PDF of \( \phi \) is known, we may consider the maximum a posteriori (MAP) approach to detect \( \phi_0 \). From the Bayesian’s rule,

$$f(\phi|r, r^*, \hat{\epsilon}_0) = \frac{f(r, r^*|\epsilon, \phi)}{f(r, r^*|\epsilon, \hat{\epsilon}_0)} \cdot \frac{f(\epsilon_0|\phi)}{f(\epsilon_0|\hat{\epsilon}_0)}$$

$$= \frac{f(r, r^*|\epsilon_0, \phi)}{f(r, r^*|\epsilon_0)} \cdot \frac{f(\epsilon_0|\phi)}{f(\epsilon_0|\hat{\epsilon}_0)} \cdot \frac{f(\epsilon_0|\phi)}{f(\epsilon_0|\hat{\epsilon}_0)}.$$  \hspace{1cm} (42)

Note that the denominator is independent from \( \phi \). Then \( \phi_0 \) is estimated from

$$\hat{\phi}_0 = \arg \max_{\phi} f(r, r^*|\epsilon_0, \phi),$$  \hspace{1cm} (43)

This gives exactly the same result as (32).

C. Discussions

Since MLE3 includes the complicated Bessel function, we will mainly provide the discussions on MLE1. As will be seen in the later simulations, all these estimators perform identically within the regular SNR region.

1) Circular Transmissions: The ML CFO estimation for circular transmissions can be obtained by simply setting the elliptic variance of \( s_i \) as zero. In this case, \( B \) and \( U \) become zero matrices. Hence, the PO cannot be estimated. However, the CFO could still be estimated from

$$\hat{\epsilon}_0 = \arg \max_{\epsilon} -r^H Yr \ \mu - 1$$

$$= \arg \max_{\epsilon} \Re(r^*_i r_{2i-1} + r^*_i + N^r N_{2i-1}) \times e^{2\pi(2i-2\mu+\epsilon)}.$$  \hspace{1cm} (44)

Note that, \( \mu > 0 \) is required, otherwise $-r^H Yr$ is independent of \( \epsilon \), as explained previously. The estimator (44) can be easily understood in the noise free environment, in which case each term in (44) achieves its maximum at \( \epsilon = \epsilon_0 \). For example, \( r_{\text{new}} = r_{\text{old}} e^{j2\pi \epsilon_0} \) holds if there is no noise, and clearly, the term \( \Re(r_{\text{new}} e^{j2\pi \epsilon_0}) \) in (44) achieves its maximum at \( \epsilon = \epsilon_0 \). Estimator (44) could never provide the ML estimation for non-circular transmissions, and there is considerable performance loss if one still uses (44) for non-circular transmissions, as will be shown in the simulations later.

2) CFO Tracking: Tracking the residue CFO and PO is also possible by considering the block with index \( m > 0 \). Namely, for each new received block with \( m > 0 \), we can estimate new CFO and PO corresponding to this specific block. If the \( m \)th OFDM block is considered, we may regard the block with index \( m > 0 \) + \( \phi_0 \) as the equivalent PO and carry on the similar steps to estimate CFO and PO. The PO estimator (33) could then be rewritten as

$$\hat{\phi}_0 = \arg \max_{\phi} \Re\{e^{-j2\pi m(N+2\mu+\epsilon)|r^T U_r|}\}$$

$$= \frac{1}{2} \angle e^{-j2\pi m(N+2\mu)|r^T U_r|}.$$  \hspace{1cm} (45)

Nevertheless, the ML CFO estimator (33) remains unchanged.

IV. Simulation Results

In this section, we examine the proposed estimators under various scenarios. We consider OFDM systems with \( N = 64 \) subcarriers (consistent with IEEE 802.11a standard [4]). In each example, \( M_q = 500 \) Monte Carlo runs are conducted to average the simulation results. All symbols are assumed to be obtained from a BPSK constellation, namely, \( E_s = E_c \). The normalized mean square errors is defined as

$$\text{NMSE}(\epsilon) = \frac{1}{M_q} \sum_{i=1}^{M_q} \frac{|\epsilon_0,i - \epsilon_0|}{\omega^2},$$  \hspace{1cm} (46)

$$\text{NMSE}(\phi) = \frac{1}{M_q} \sum_{i=1}^{M_q} \frac{|\phi_0,i - \phi_0|}{\pi^2},$$  \hspace{1cm} (47)
where $\omega = 1/N$ if $\varepsilon_0 < 1/N$ and $\omega = \varepsilon_0$ otherwise. The parameters $\hat{\varepsilon}_{0,i}$ and $\hat{\phi}_{0,i}$ are the estimates of $\varepsilon_0$ and $\phi_0$ in the $i$th Monte Carlo run.

A. DCT-OFDM

The normalized CFO is chosen as large as $\varepsilon_0 = 0.2$ (more than 12 subcarrier spacings) and PO is set as $\pi/3$. We first compare our three estimators when both the prefix and suffix has length 8. The CFO and PO NMSEs versus SNR performances are shown in Fig. 1. The Cramer-Rao bounds (CRBs) [16] for CFO and PO estimations are displayed as well. From the figure, it can be seen that all three estimators provide the same performance. Moreover, they agree with the CRB very well, which is consistent with the general belief that the ML estimation can reach the CRBs asymptotically.

Next we examine how the performance differs by using different length of the prefix and the suffix. From the previous example, we know that all three proposed estimators perform identically. Hence, we will only present the result from MLE1 for the conciseness of the figure. The CFO and PO NMSEs versus SNR performances, for $\mu = 0, 2, 4, 8$ are drawn in Fig. 2 with their corresponding CRBs. From the figure, we can find that the CFO estimation accuracy is improved when $\mu$ is increased, say 4dB when $\mu$ is increased from 0 to 8. However, the PO estimation accuracy does not change much by varying the value of $\mu$.

We then illustrate the performance of the estimator (44) under the considered BPSK transmissions. Since $\mu$ for (44) must be greater than zero, we will select three values as $\mu = 2, 4, 8$. Fig. 3 shows CFO NMSEs versus SNR performance for both the estimator (44) and the ML estimator (33). It can be seen that the performance loss is significant if one ignore the non-circularity of the transmitted symbols and apply the circular estimator (44) directly. Especially when $\mu$ is small, the performance loss is amazingly large. Therefore, DCT-OFDM is sensitive to whether the transmission is circular or non-circular, and ignoring the non-circularity, if any, will greatly drop the performance of DCT OFDM systems.

B. Comparison Between DCT-OFDM and DFT-OFDM

One may be curious that how the performance of these two OFDM systems differs for the non-circular transmission considered. We will study this issue in this example. Since DFT-OFDM is also considered, the parameters change to $\varepsilon_0 = 0.15/N$, and $\phi_0 = \pi/3$. For a fair comparison, the total amount of redundancy added to one OFDM block is made the same, namely, $L = 2\mu$. The CFO and PO NMSEs versus SNR performances are shown in Fig. 4 and Fig. 5, respectively.
In this paper, we considered the non-circular transmissions and proposed three ML joint CFO and PO estimators for the newly raised DCT-OFDM systems. In addition to those mentioned in [5], [6], we found new advantages of the DCT-OFDM over the DFT-OFDM. First of all, if DCT based OFDM is applied, the CFO estimation range could be increased from only one subcarrier spacing to its maximum. Secondly, the CFO and PO estimation in DCT-OFDM is more accurate than those in DFT-OFDM. Lastly, the CFO estimation in DCT OFDM is robust to the amount of the redundancy inserted than in DFT-OFDM. Simulation results clearly support our analysis and discussions.

V. CONCLUSIONS

each figure, the solid lines from up to down represent the DCT-OFDM with $\mu = 2, 4, 8$, whereas the dashed lines from up to down represent the DFT-OFDM with $L = 4, 8, 16$. Two phenomena are observed. 1) The DCT OFDM could provide better performance than the DFT OFDM, especially when the amount of the redundancy added to one OFDM block is small. 2) The performance of the DCT-OFDM vary slightly when we change the value of the $\mu$. However, the performance of the DFT-OFDM is affected greatly when we change $L$. As a result, the estimators in DCT-OFDM is more robust to the amount of the redundancy than those in DFT-OFDM. Clearly, both these two phenomena suggest extra advantages of the DCT-OFDM over the DFT OFDM.

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