Search for $B^+ \rightarrow a_1^+ (1260) K^+ x$
We present a search for the decay $B^+ \rightarrow a_1^{(*)}(1260)K^-\bar{K}^0(892)$, the data, collected with the $B$-fact detector at the SLAC National Accelerator Laboratory, represent $465 \times 10^6 B\bar{B}$ pairs produced in $e^+e^-$ annihilation at the energy of the $Y(4S)$. We find no significant signal and set an upper limit at 90%
Measurements of the branching fractions and polarizations of charmless hadronic B decays are useful tests of the standard model and a means to search for new physics effects. In decays of B mesons to a pair of spin-one mesons, the longitudinal polarization, $f_L$, is particularly interesting. Simple helicity arguments favor $f_L$ to be close to 1, but several vector-vector (VV) decay modes such as $B \to \phi K^*$ [1] and $B^+ \to \rho^+ K^{*0}$ [2,3] are observed to favor $f_L \sim 0.5$. Possible explanations for this discrepancy have been proposed within the standard model [4] as well as in new physics scenarios [5].

New ways to explore the size of contributing amplitudes in charmless B meson decays and their helicity structure may come from measurements of the branching fractions and polarization of charmless decays of B mesons to an axial-vector meson and a vector meson (AV) or to an axial-vector meson and a pseudoscalar meson (AP) [6]. Theoretical decay rates have been predicted with the naive factorization (NF) [7] and QCD factorization (QCDF) [8] approaches. The NF calculations find the decay rates of $B \to AV$ modes to be smaller than the corresponding $B \to AP$ modes. The more complex QCDF calculations find the reverse. For example, QCDF predicts a branching fraction of $(1.1 \pm 0.4 \pm 0.9) \times 10^{-6}$ for $B^+ \to a_1^+ K^{*0}$ and $(32 \pm 16.5 \pm 12.0) \times 10^{-6}$ for $B^0 \to b_1^- \rho^+$, while NF predicts a branching fraction of $0.51 \times 10^{-6}$ and $1.6 \times 10^{-6}$, respectively. The first uncertainty on the QCDF prediction corresponds to the uncertainties due to the variation of Gegenbauer moments, decay constants, quark masses, form factors, and a B meson wave function parameter and the second uncertainty corresponds to the uncertainties due to the variation of penguin annihilation parameters. The NF prediction does not give an uncertainty on their value.

B meson decays to charmless AV final states are sensitive to penguin annihilation contributions, which enhance some decay modes while suppressing others. Thus, investigating decays to many final states will help determine the size of the contributing amplitudes.

A number of searches for AV decays to the final states $a_1^+ \rho^-$, $b_1 \rho^+$, and $b_1 K^*$ are presented in Refs. [9,10], with upper limits on the branching fractions of $30 \times 10^{-6}$ at 90% confidence level (C.L.) for $a_1^+ \rho^-$ and from 1.4 to $8.0 \times 10^{-6}$ at 90% C.L. for the $b_1 \rho$ and $b_1 K^*$ final states. In this paper, we present a search for the decay $B^+ \to a_1^+ K^{*0}$.

The data for this measurement were collected with the BABAR detector [11] at the PEP-II asymmetric-energy $e^+ e^-$ storage ring located at the SLAC National Accelerator Laboratory. An integrated luminosity of 424 fb$^{-1}$, corresponding to $(465 \pm 5) \times 10^6 B\bar{B}$ pairs, was produced in $e^+ e^-$ annihilation at the Y(4S) resonance (center-of-mass energy $\sqrt{s} = 10.58$ GeV).

A detailed Monte Carlo program (MC) is used to simulate the B meson production and decay sequences, and the detector response [12]. Dedicated samples of MC events for the decay $B^{+} \to a_1^{+} K^{*0}$ with $a_1^{+} \to \rho^{0} \pi^{+}$ and $K^{*0} \to K^{+} \pi^{-}$ were produced. For the $a_1^{+}$ meson parameters, we use the values given in Ref. [13] for studies with MC while for fits to the data we use a mass of 1229 MeV/c$^2$ and a width of 393 MeV/c$^2$, which were extracted from $B^{0} \to a_1^{+} \pi^{-}$ decays [14]. We account for the uncertainties of these resonance parameters in the determination of systematic uncertainties. The $a_1^{+} \to \pi^{+} \pi^{-} \pi^{0}$ decay proceeds mainly through the intermediate states $\rho^{0} \pi^{+}$ and $\sigma \pi^{+}$ [13]. No attempt is made to separate contributions of the dominant $P$ wave $\rho^{0}$ from the $S$ wave $\sigma$ in the channel $\pi^{+} \pi^{-}$. The difference in efficiency for the $S$ wave and $P$ wave cases is accounted for as a systematic uncertainty.

We reconstruct $a_1^{+}$ candidates through the decay sequence $a_1^{+} \to \rho^{0} \pi^{+}$ and $\rho^{0} \to \pi^{+} \pi^{-}$. The other primary daughter of the B meson is reconstructed as $K^{*0} \to K^{+} \pi^{-}$. Candidates for the charged kaons must have particle identification signatures consistent with those of kaons. Candidates for the charged pions must not be classified as protons, kaons, or electrons. We constrain the range of mass of reconstructed final-state candidates: between 0.55 and 1.0 GeV/c$^2$ for the $\rho^{0}$, between 0.9 and 1.8 GeV/c$^2$ for the $a_1^{+}$, and between 0.8 and 1.0 GeV/c$^2$ for the $K^{*0}$.

$B^{+}$ candidates are formed by combining $a_1^{+}$ and $K^{*0}$ candidates. The five final decay tracks in a candidate are fit to a common vertex. Candidates which have a $\chi^{2}$ probability for the fit greater than 0.01 are retained. For these candidates, we calculate the energy substituted mass, $m_{ES} = \sqrt{\Delta E} = \Delta E - \frac{1}{2} \sqrt{s}$, where $(E_B, p_B)$ is the B meson energy-momentum four-vector, all values being expressed in the $Y(4S)$ rest frame. We keep candidates with $5.25$ GeV/c$^2 < m_{ES} < 5.29$ GeV/c$^2$ and $|\Delta E| < 100$ MeV.

We also impose restrictions on the helicity-frame decay angle $\theta_{K^{*0}}$ of the $K^{*0}$ mesons. The helicity frame of a meson is defined as the rest frame of that meson, where the $z$ axis is the direction along which the boost is performed from the parent’s frame to this frame. For the decay $K^{*0} \to K^{+} \pi^{-}$, $\theta_{K^{*0}}$ is the polar angle of the daughter kaon, and for $a_1^{+} \to \rho^{0} \pi^{+}$, $\theta_{a_1^{+}}$ is the polar angle of the normal to the $a_1^{+} \to 3\pi$ decay plane. We define $H_i = \cos(\theta_i)$, where $i = (K^{*0}, a_1^{+})$. Since many background candidates accumulate near $|H_{K^{*0}}| = 1$, we require $-0.98 \leq H_{K^{*0}} \leq 0.8$.

Backgrounds arise primarily from random combinations of particles in continuum $e^+ e^- \to qq$ events ($q = u, d, s, c$). We reduce this background source with a requirement on the angle $\theta_T$ between the thrust axis [15] of the $B^+$
candidate in the \(Y(4S)\) frame and that of the charged tracks and neutral calorimeter clusters of the rest of the event.

The distribution is sharply peaked near \(|\cos\theta_1| = 1\) for jetlike continuum events, and nearly uniform for \(B^+\) meson decays. Optimizing the ratio of the signal yield to its (background dominated) uncertainty, we require \(|\cos\theta_1| < 0.8\).

A secondary source of background arises from \(b \to c\) transitions. We reduce this background by eliminating events in which one of the pions in the \(B^+\) candidate is also part of a \(D\) candidate.

Such \(D\) candidates, reconstructed from \(K^-\pi^+\) and \(K^-\pi^+\pi^+\), are required to have an invariant mass within 0.02 GeV/c\(^2\) of the nominal \(D\) meson mass.

The number of events which pass the selection is 15802. The average number of candidates found per event in the selected data sample is 1.5 (2.0 to 2.4 in signal MC depending on the polarization).

We define a neural network for use in selecting the best \(B^+\) candidate. The \(\chi^2\) probability of the vertex fit and the \(\rho\) meson mass were the input variables to the neural network. Thereby we find 13\% to 22\%, depending on the polarization, of the candidates were incorrectly reconstructed from particles in events that contain a true signal candidate.

To further discriminate against \(q\bar{q}\) background we construct a Fisher discriminant \(F\) \([16]\) which is a function of four variables: the polar angles of the \(B^+\) candidate momentum and of the \(B^+\) thrust axis with respect to the beam axis in the \(Y(4S)\) rest frame; and the zeroth (second) angular moment \(L_0\) (\(L_2\)) of the energy flow, excluding the \(B\) candidate, with respect to the \(B^+\) thrust axis. The moments are defined by \(L_j = \sum p_i \times |\cos \theta_i|^j\), where \(\theta_i\) is the angle with respect to the \(B^+\) thrust axis of a track or neutral cluster \(i\), and \(p_i\) is its momentum.

We obtain yields and the longitudinal polarization \(f_L\) from an extended maximum likelihood (ML) fit with the seven input observables \(\Delta E\), \(m_{ES}\), \(F\), the resonance masses \(m_{a_1}\) and \(m_{K^{*0}}\), and the helicity variables \(H_{K^{*0}}\) and \(H_{a_1}\).

Since the correlation between the observables in the selected data and in MC signal events is small, we take the probability density function (PDF) for each event to be a product of the PDFs for the individual observables. Corrections for the effects of possible correlations, referred to as fit bias yield, are made on the basis of MC studies described below. The components in the ML fit used are signal, \(q\bar{q}\) background, charm \(BB\) background, charmless \(BB\) background, and \(B^+ \to a_1^+\) \(K^{*0}\) background. The signal component includes true signal events where decay products of intermediate resonances are incorrectly assigned, or particles from the rest of the event are included.

We determine the PDFs for the signal and \(BB\) background components from fits to MC samples. We develop PDF parametrizations for the \(q\bar{q}\) background with fits to the data from which the signal region \((5.26 \text{ GeV}/c^2 < m_{ES} < 5.29 \text{ GeV}/c^2\) and \(|\Delta E| < 60\text{ MeV}\)) has been excluded.

For the signal, the \(m_{ES}\) and \(\Delta E\) distributions are parametrized as a sum of a crystal-ball function \([17]\) and a Gaussian function. In the case of \(m_{ES}\) for \(q\bar{q}\) and \(BB\) backgrounds we use the threshold function \(x\sqrt{1-x^2} \exp[-\xi(1-x^2)]\), where the argument \(x = 2m_{ES}/\sqrt{s}\) and \(\xi\) is a shape parameter. This function is discussed in more detail in Ref. \([18]\). In the case of \(\Delta E\) for \(q\bar{q}\) and \(BB\) backgrounds we use a polynomial function. The PDFs for the Fisher discriminant \(P_j(F)\) are parametrized as a single Gaussian function or a sum of two such functions. The PDFs for the invariant masses of the \(a_1^+\) and \(K^{*0}\) mesons for all components are constructed as sums of a relativistic Breit-Wigner function and a polynomial function. We use a joint PDF \(P_j(H_{K^{*0}}, H_{a_1})\) for the helicity distributions.

The ideal angular distribution from Ref. \([19]\) where \(\phi\), the angle between the decay planes of the \(a_1^+\) meson candidate and the \(K^{*0}\) meson candidate, is integrated out are

\[
P_{\text{ideal}}(H_{K^{*0}}, H_{a_1}) = f_L \times (1 - H_{K^{*0}}^2)H_{a_1}^2 + \frac{1}{2}(1 - f_L) \times (1 + H_{K^{*0}}^2)(1 - H_{a_1}^2)
\]

for signal component and

\[
P_{\text{ideal}}(H_{K^{*0}}, H_{a_1}) = f_L \times H_{K^{*0}}^2(1 - H_{K^{*0}})H_{a_1}^2 + \frac{1}{2}(1 - f_L) \times \frac{4}{3}H_{K^{*0}}^4 - 3H_{K^{*0}}^2 + 1 - H_{a_1}^2
\]

for the \(B^+ \to a_1^+\) \(K^{*0}\) background component. The helicity PDF for the \(q\bar{q}\) and \(BB\) background components is simply the product of the helicity PDFs for \(H_{K^{*0}}\) and \(H_{a_1}\). The \(H_j\) distributions for these components are based on Gaussian and polynomial functions.

The likelihood function is

\[
L = e^{-\left(\sum_i Y_i\right)/N} \prod_i \sum_j Y_j \times P_j(m_{ES})P_j(F)P_j(\Delta E)
\]

\[
\times P_j(m_{a_1})P_j(m_{K^{*0}})P_j(H_{K^{*0}})P_j(H_{a_1})
\]

where \(N\) is the number of events in the sample, and for each component \(j\) (signal, \(q\bar{q}\) background, \(b \to c\) transition \(BB\) background, charmless \(BB\) background, or \(B^+ \to a_1^+\) \(K^{*0}\) background), \(Y_j\) is the yield of component \(j\), and \(P_j(x)\) is the probability for variable \(x\) of event \(i\) to belong to component \(j\). We allow the most important parameters (first coefficient of the polynomial function for \(\Delta E\), the invariant masses of the \(a_1^+\) and \(K^{*0}\), and the width of the Breit-Wigner for the invariant mass of the \(K^{*0}\)) for the determination of the combinatorial background PDFs to vary in the fit, along with the yields for the signal, \(q\bar{q}\) background, and \(b \to c\) transition \(BB\) background.
We validate the fitting procedure by applying it to ensembles of simulated experiments with the \( q\bar{q} \) component drawn from the PDF, and with embedded known numbers of signal and \( B\bar{B} \) background events randomly extracted from the fully simulated MC samples. By tuning the number of embedded events until the fit reproduces the yields found in the data, we find a positive bias yield \( Y_b \), to be subtracted from the observed signal yield \( Y \). The fit bias yield arises from the neglected correlations in signal and \( B\bar{B} \) background events.

The corresponding numbers are reported in Table I. We do not find a significant signal thus we do not report a measurement on the quantity \( f_L \). In order to obtain the most conservative upper limit, we assume \( f_L = 1 \) in estimating the branching fraction.

![Image of Table I](image)

**TABLE I.** Summary of results for \( B^+ \to a_1^+ K^{0\ast} \). Signal yield \( Y \), fit bias yield \( Y_b \), the branching fraction \( \mathcal{B} = \mathcal{B}(B^+ \to a_1^+ K^{0\ast}) \times \mathcal{B}(a_1^+ \to \pi^+\pi^-\pi^0) \), significance \( S \) (see text), and upper limit (UL). The given uncertainties on fit yields are statistical only, while the uncertainties on the fit bias yield include the corresponding systematic uncertainties. The branching fraction of \( K^{0\ast} \to K^+\pi^- \) is assumed to be \( \frac{2}{3} \).

<table>
<thead>
<tr>
<th>( Y )</th>
<th>( Y_b )</th>
<th>( \mathcal{B}(10^{-6}) )</th>
<th>( S )</th>
<th>UL ( (10^{-6}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>61^{+23}_{-21}</td>
<td>34 \pm 17</td>
<td>0.7^{+0.5+0.6}_{-0.5-1.3}</td>
<td>0.5</td>
<td>1.8</td>
</tr>
</tbody>
</table>

We compute the branching fraction by subtracting the fit bias yield from the measured yield and dividing the result by the number of produced \( B\bar{B} \) pairs and by the product of the selection efficiency and the branching ratio for the \( B(K^{0\ast} \to K^+\pi^-) \) decay. We assume that the branching fractions of the \( Y(4S) \) to \( B^+B^- \) and \( B^0\bar{B}^0 \) are equal, consistent with measurements [13]. The efficiency for longitudinally and transversely polarized signal events, obtained from the MC signal model, is 12.9\% and 18.6\%, respectively. The results are given in Table I, along with the significance, \( S \), computed as the square root of the difference between the value of \( -2\ln L \) (with additive systematic uncertainties included) for zero signal and the value at its minimum. In Fig. 1 we show the projections of data with PDFs overlaid. The data plotted are subsamples enriched in signal with the requirement of a minimum value of the ratio of signal to total likelihood, computed without the plotted variable. We used 0.9 as the requirement on the ratio in Fig. 1 for each variable. The efficiency of these requirements for signal is between 57\% and 70\% depending on the variable.

Systematic uncertainties on the branching fraction arise from the imperfect knowledge of the PDFs, \( B\bar{B} \) backgrounds, fit bias yield, and efficiency. PDF uncertainties not already accounted for by free parameters in the fit are estimated from varying the signal-PDF parameters within their uncertainties. For \( K^{0\ast} \) resonance parameters we use the uncertainties from Ref. [13] and for the \( a_1^+ \) resonance parameters from Ref. [14]. The uncertainty from fit bias yield (Table I) includes its statistical uncertainty from the simulated experiments, and half of the correction itself, added in quadrature.

To determine the systematic uncertainty arising from our imperfect knowledge of the branching fractions of charmless \( B \) decays, we vary the charmless \( B\bar{B} \) background component yield by 100\%. We conservatively assume that the branching ratio of \( B^+ \to a_1^+ K^{0\ast} \) could be as large as that of \( B^+ \to a_2^+ K^{0\ast} \) and vary the \( B^+ \to a_2^+ K^{0\ast} \) from 0 to 18 events around a fixed yield of 9 events used for the \( B^+ \to a_2^+ K^{0\ast} \) component in the likelihood function.

The uncertainty associated with \( f_L \) is estimated by taking the difference in the measured branching fraction between the nominal fit \( (f_L = 1) \) and the maximum and minimum values found in the scan along the range [0, 1]. We divide these values by \( \sqrt{3} \), motivated by our assumption of a flat prior for \( f_L \) in its physical range.

Uncertainties in our knowledge of the tracking efficiency are 0.4\% per track in the \( B^+ \) candidate. This is estimated within the tracking efficiency determination, which is based on \( \tau \) lepton decays. The uncertainties in
The efficiency from the event selection are below 0.6%. The systematic uncertainty on the measurement of the integrated luminosity is 1.1%. All systematic uncertainties on the branching fraction are summarized in Table II.

We obtain a central value for the product of branching fractions:

\[
\mathcal{B}(B^+ \rightarrow a_1^+ K^{*0}) \times \mathcal{B}(a_1^+ \rightarrow \pi^+ \pi^- \pi^+) = (0.7^{+0.5+0.6}_{-0.5-1.3}) \times 10^{-6},
\]

where the first uncertainty quoted is statistical, the second systematic. Including systematic uncertainties, this result corresponds to an upper limit at 90% confidence level of \(3.6 \times 10^{-6}\).

This upper limit is in agreement with the prediction from naive factorization and lower than, but not inconsistent with that of QCD factorization.

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[2] Charge-conjugate reactions are implied.

