Simulating merging binary black holes with nearly extremal spins

Geoffrey Lovelace,1 Mark A. Scheel,2 and Béla Szilágyi2

1Center for Radiophysics and Space Research, Cornell University, Ithaca, New York, 14853, USA
2Theoretical Astrophysics 350-17, California Institute of Technology, Pasadena, California 91125, USA

(Received 13 October 2010; published 10 January 2011)

Astrophysically realistic black holes may have spins that are nearly extremal (i.e., close to 1 in dimensionless units). Numerical simulations of binary black holes are important tools both for calibrating analytical templates for gravitational-wave detection and for exploring the nonlinear dynamics of curved spacetime. However, all previous simulations of binary-black-hole inspiral, merger, and ringdown have been limited by an apparently insurmountable barrier: the merging holes’ spins could not exceed 0.93, which is still a long way from the maximum possible value in terms of the physical effects of the spin. In this paper, we surpass this limit for the first time, opening the way to explore numerically the behavior of merging, nearly extremal black holes. Specifically, using an improved initial-data method suitable for binary black holes with nearly extremal spins, we simulate the inspiral (through 12.5 orbits), merger and ringdown of two equal-mass black holes with equal spins of magnitude 0.95 antialigned with the orbital angular momentum.

DOI: 10.1103/PhysRevD.83.024010
PACS numbers: 04.25.dg, 04.30.–w

I. INTRODUCTION

Although there is considerable uncertainty, it is possible that astrophysical black holes exist with nearly extremal spins (i.e., in dimensionless units spins close to 1, the theoretical upper limit for a stationary black hole). Binary-black-hole (BBH) mergers in vacuum typically lead to remnant holes with dimensionless spins \( \chi \sim 0.7-0.8 \) [1–3], although if the merging holes are surrounded by matter the remnant’s spin typically could be higher than \( \chi \sim 0.9 \) [1–3]. Black holes can reach higher spins via prolonged accretion [4,5]: thin accretion disks (with magnetohydrodynamic effects neglected) lead to spins as large as \( \chi \sim 0.998 \) [6], while thick-disk accretion with magnetohydrodynamic effects included can lead to spins as large as \( \chi \sim 0.95 \) [7,8]. Even without accretion, at very high mass ratios with spins aligned with the orbital angular momentum, binary-black-hole mergers can also lead to holes with nearly extremal spins [9–11]. There is observational evidence suggesting the existence of black holes with nearly extremal spins in quasars [12], and some efforts to infer the spin of the black hole in microquasar GRS 1915 + 105 from its x-ray spectra suggest a spin larger than 0.98, though other analyses suggest the spin may be much lower [13–15].

Merging BBHs—possibly with nearly extremal spins—are among the most promising sources of gravitational waves for current and future detectors. Numerical simulations of BBHs are important tools both for predicting the gravitational waves that detectors will observe and for exploring the behavior of nonlinear, highly dynamical spacetimes. Following Pretorius’ breakthrough in 2005 [16], several groups have successfully simulated the inspiral, merger, and ringdown of two coalescing black holes in a variety of initial configurations; however, all prior BBH simulations have been limited to spins of 0.93 or less, which is quite far from extremal. The parameter \( \chi \) is a poor measure of how close a black hole is to extremality in terms of physical effects: a black hole with spin 0.93 has less than 60% of the rotational energy of an extremal hole with the same mass (Fig. 1).

Previous simulations have been unable to reach higher spins because of the way they construct their initial data. Just as initial data for Maxwell’s equations must satisfy

![Rotational energy / rotational energy if extremal](image)

FIG. 1 (color online). The rotational energy of a Kerr black hole as a function of the hole’s dimensionless spin parameter \( \chi \equiv \text{Spin}/(\text{Mass})^2 \). The thick red line indicates the Bowen-York limit: standard Bowen-York puncture initial data—used in almost all numerical binary-black-hole calculations to date—cannot yield rotational energies more than 60% of the way to extremality. By using instead initial data based on two superposed Kerr-Schild holes (“SKS initial data”), in this paper we surpass the Bowen-York limit (green circle), opening the way for numerical studies of merging, nearly extremal black holes.
constraints (the electric and magnetic fields must have vanishing divergence in vacuum), initial data for the Einstein equations must satisfy constraint equations. Most BBH simulations begin with puncture initial data [17], which assumes that the initial spatial metric is conformally flat (i.e., proportional to the metric of flat space). With this assumption, 3 of the 4 constraint equations can be solved analytically using the solutions of Bowen and York [18,19]; however, conformally flat initial data cannot describe single or binary black holes that are both in equilibrium and possess linear [20] or angular [21,22] momentum. Bowen-York puncture data can yield solutions of binary black holes with spins as large as \( \chi = 0.984 \) initially, but when such initial data are evolved, the holes quickly relax to spins of about \( \chi = 0.93 \) or less [23–25]. Several groups have evolved BBH puncture data with spins near but below this Bowen-York limit [9,26,27], with the simulation by Dain, Lousto, and Zlochower [28] coming the closest with spins of 0.967 at time \( t = 0 \) quickly falling to 0.924.

To reach spins beyond the Bowen-York limit, one must begin with initial data that is conformally curved. Recently, Liu and collaborators [29] have constructed and evolved conformally curved initial data based on that of Brandt and Seidel [30,31] for a single black hole with spins as high as \( \chi = 0.99 \). Hannam and collaborators [32] have constructed and evolved conformally curved BBH initial data [33,34] for head-on mergers of black holes with spins as large as \( \chi = 0.9 \). In Ref. [35], conformally curved BBH data with spins of \( \chi = 0.93 \) were constructed and evolved through the first 1.9 orbits of an inspiral, but no attempt was made to simulate the complete inspiral, merger, and ringdown.

In this paper, we demonstrate that conformally curved initial data is suitable for simulations with nearly extremal spins by using it to compute the first inspiral, merger, and ringdown of two black holes with spins larger than the Bowen-York limit. By surpassing this limit, our results open the way for numerical exploration of the gravitational waveforms and nonlinear dynamics of black holes that are nearly extremal.

## II. INITIAL DATA

We evolve a low-eccentricity initial data set: a BBH where the holes have equal masses and equal spins of magnitude 0.95 antialigned with the orbital angular momentum. Some properties of the initial data used in this paper are listed in Table I.

Following Ref. [35] and the references therein, we construct constraint-satisfying initial data by solving the extended conformal thin sandwich equations with quasiequilibrium boundary conditions [36–41] using a spectral elliptic solver [42]. The initial spatial metric is proportional to a weighted superposition of the metrics of two boosted, spinning Kerr-Schild black holes.

We measure the quasilocal spin \( S_{\text{AVK}} \) of each hole in the initial data using the approximate-Killing-vector method summarized in Appendix A of Ref. [35], which is very similar to the prescription previously published by Cook and Whiting [43]. The dimensionless spin of each hole \( \chi \) is then related to \( S_{\text{AVK}} \) by the formula

\[
\chi = S_{\text{AVK}}/M_{\text{chr}}
\]

where \( M_{\text{chr}} := \sqrt{M_{\text{irr}}^2 + S^2}/4M_{\text{irr}} \) is the Christodoulou mass, \( M_{\text{irr}} := \sqrt{A/16\pi} \) is the irreducible mass, and \( A \) is the area of the horizon. (For a single Kerr black hole, \( M_{\text{chr}} \) reduces to the usual Kerr mass parameter.)

To reduce eccentricity, we follow the iterative method of Ref. [44], which is an improvement of the earlier method of Ref. [45]. For each iteration, we construct an initial data set and evolve it for approximately 3 orbits. Then, the initial angular and radial motion of the holes are adjusted to minimize oscillations in the orbital frequency. Using this method, we reduce the orbital eccentricity to approximately \( 10^{-3} \).

## III. EVOLUTION

We evolve our initial data using the Spectral Einstein Code spec [46]. Building on the methods of Ref. [47] and the references therein, we have made several technical improvements to our code which both enable us to evolve our \( \chi = 0.95 \) initial data through merger and make our code more robust in general. Here we briefly summarize some of the most important improvements; full details of these techniques will be described in a future paper.

We use a computational domain with the singularities inside the horizons excised, and we use a time-dependent coordinate mapping to keep the excision boundaries inside the individual apparent horizons as the horizons orbit and slowly approach each other [48]. Our coordinate mapping also ensures that the excision surfaces’ shapes conform to those of the horizons which enclose them. One important ingredient of our improved binary-black-hole evolutions is that the coordinate mapping is adjusted adaptively throughout the evolution, which is helpful because the horizons’ dynamics change from slow to fast during the simulation.

Because we apply no boundary condition on the excision surfaces, these surfaces must be pure-outflow boundaries (i.e., must have no incoming characteristic fields) in order to reduce the equation of motion of characteristic fields to zero.

| \(|\chi_i|\) | \(J_{\text{ADM}}^2/M^2\) | \(\Omega_{0}\) | \(M^2/M\) | \(d_0/M\) | \(\alpha\) |
|---|---|---|---|---|---|
| 0.9498 | 0.6845 | 0.014508 | 0.5000 | 0.9933 | 15.566 |
| \(-0.99\) | \(0.713\) | 0.007139 |
for the evolution to be well posed. A second improvement to our code is that we now can adjust the velocity of each excision surface to keep the characteristic fields outgoing there. For the $\chi = 0.95$ simulation considered here, this characteristic speed control is necessary only during the last orbit before merger; earlier, it is sufficient to control the size of the excision surface using the method of Ref. [47].

A third element which we have recently added to SPEC is spectral adaptive mesh refinement. During the evolution, we monitor the truncation error of each evolved field, the resolution requirements of the apparent horizons, and the local magnitude of constraint violation; to maintain a desired accuracy, we then add or remove spectral basis functions as needed. In the simulation presented in this paper, we use spectral adaptive mesh refinement only during the final quarter orbit before merger. Throughout the entire simulation, we also adaptively adjust the resolution of the apparent horizon finder as the horizon becomes more distorted.

**IV. RESULTS**

In Fig. 2, we plot the dimensionless quasilocal spin $\chi$ measured on one individual horizon and also on the common horizon. From $t = 0$ to $t = 50M$, there is a sharp, numerically resolved drop in the magnitude of the dimensionless spin $\chi$ from 0.9498 to 0.9492. During the remainder of the inspiral, the spin drifts, with the amount of drift decreasing as resolution increases; at the highest resolution (N5), the spin remains $\chi = 0.949$ throughout the next 11.8 orbits, until just before merger, when the magnitude of the spin of each hole drops sharply. This result demonstrates that it is now possible to simulate BBHs where the holes retain spins beyond the Bowen-York limit throughout the inspiral; it also opens the way for future explorations of the strong-field dynamics of merging, nearly extremal holes—dynamics that can only be explored using numerical simulations.

During the ringdown, the spin $\chi$ of the common horizon quickly relaxes to its final value of $\chi = 0.3757 \pm 0.0002$ (where the uncertainty is estimated as the difference between the highest and second-highest resolutions). This is approximately consistent with but slightly larger than the predictions obtained by extrapolating fitting formulae from simulations with lower initial spins of Ref. [49] ($\chi_{\text{fit}} = 0.371$), Ref. [1] ($\chi_{\text{fit}} = 0.372 \pm 0.057$), Ref. [50] ($\chi_{\text{fit}} = 0.369 \pm 0.012$), and Ref. [9] ($\chi_{\text{fit}} = 0.366 \pm 0.002$). This result is a first step toward a better understanding of the relation between the properties of the remnant hole and those of the merging holes when the latter are nearly extremal. Numerical simulations that directly measure this relation (instead of extrapolating from lower-spin results) will yield greater understanding of the properties of black holes produced from merging extremal holes.

Figure 3 shows the individual horizon trajectories and the real part of the $(\ell, m) = (2, 2)$ spherical harmonic mode of the emitted gravitational waveform. We extract waves on a series of concentric spherical shells; the waveform shown was extracted on the outermost spherical shell.
M (which holds within initial relaxation and where the uncertainties of each hole has a constant mass and second-highest resolution. Under the assumption that holes have relaxed), the quantity 

\[ \frac{1}{C_0} \Delta \approx \frac{1}{C_0} \] 

remnant hole formed by a nearly extremal BBH merger, and to provide high-spin gravitational waveforms for data analysis.

**ACKNOWLEDGMENTS**

We are pleased to thank N. Taylor for a gauge modification that allows us to use the nonsmooth maps of Ref. [47] throughout our evolutions and L. Kidder, R. Owen, H. Pfeiffer, S. Teukolsky, and K. Thorne for helpful discussions. This work was supported in part by grants from the Sherman Fairchild Foundation to Caltech and Cornell and from the Brinson Foundation to Caltech; by NSF Grants No. PHY-0601459 and No. PHY-1005655 at Caltech; by NASA Grant No. NNX09AF97G at Caltech; by NSF Grants No. PHY-0969111 and No. PHY-1005426 at Cornell; and by NASA Grant No. NNX09AF96G at Cornell. The numerical computations presented in this paper were performed primarily on the Caltech compute cluster ZWICKY, which was cofunded by the Sherman Fairchild Foundation. Some computations were also performed on the GPC supercomputer at the SciNet HPC Consortium; SciNet is funded by the Canada Foundation for Innovation under the auspices of Compute Canada; the Government of Ontario; Ontario Research Fund—Research Excellence; and the University of Toronto. Some computations were performed in part using TeraGrid resources provided by NCSA’s RANGER cluster under Grant No. TG-PHY990007N.


