Charged particle multiplicities in pp interactions at \( \sqrt{s} = 0.9, 2.36, \) and 7 TeV

The CMS collaboration

Abstract: Measurements of primary charged hadron multiplicity distributions are presented for non-single-diffractive events in proton-proton collisions at centre-of-mass energies of \( \sqrt{s} = 0.9, 2.36, \) and 7 TeV, in five pseudorapidity ranges from \(|\eta| < 0.5\) to \(|\eta| < 2.4\). The data were collected with the minimum-bias trigger of the CMS experiment during the LHC commissioning runs in 2009 and the 7 TeV run in 2010. The multiplicity distribution at \( \sqrt{s} = 0.9 \text{ TeV} \) is in agreement with previous measurements. At higher energies the increase of the mean multiplicity with \( \sqrt{s} \) is underestimated by most event generators. The average transverse momentum as a function of the multiplicity is also presented. The measurement of higher-order moments of the multiplicity distribution confirms the violation of Koba-Nielsen-Olesen scaling that has been observed at lower energies.

Keywords: Hadron-Hadron Scattering
1 Introduction

The charged hadron multiplicity, or number of primary charged hadrons, $n$, is a basic global observable characterising final states in high-energy-collision processes. The multiplicity distribution, $P_n$, is the probability to produce $n$ charged hadrons in an event, either in full phase space or in restricted phase space domains. In this paper we report measurements of $P_n$ in non-single-diffractive [1] proton-proton collisions, at centre-of-mass energies $\sqrt{s} = 0.9$, 2.36, and 7 TeV at the Large Hadron Collider (LHC) [2]. The measurements are based on events recorded by the Compact Muon Solenoid (CMS) [3] experiment, using a minimum-bias trigger.

Energy-momentum and charge conservation significantly influence the multiplicity distribution for the full phase space. The distribution in restricted phase space, which is less affected by such constraints, is expected to be a more sensitive probe of the underlying dynamics and can be used to better constrain phenomenological models. Comprehensive reviews on the subject can be found in [1, 4, 5]. The measurements described in this paper are performed for intervals of increasing extent in pseudorapidity from $|\eta| < 0.5$ up to $|\eta| < 2.4$, where $\eta$ is defined as $-\ln[\tan(\theta/2)]$, and $\theta$ is the polar angle of the particle with respect to
the counterclockwise beam direction. In these measurements primary charged hadrons are defined as all charged hadrons produced in the interaction, including the products of the decays of objects with life-time less than $10^{-10}$ seconds; products of longer-lived particles, such as $K_S^0$ and $\Lambda$, and hadrons originating from secondary interactions are excluded.

Independent emission of single particles yields a Poissonian $P_n$. Deviations from this shape, therefore, reveal correlations. These correlations are predominantly short range in rapidity, attributed to cluster decays, and reflect local conservation of quantum numbers in the hadronisation process. In hadron-hadron interactions, additional large long-range rapidity correlations are observed, whose magnitude increases with $\sqrt{s}$ [6]. In contrast, in $e^+e^-$ annihilations such long-range correlations are much weaker and practically absent in two- and three-jet event samples [7].

The mean of the multiplicity distribution, $\langle n \rangle$, is equal to the integral of the inclusive single-particle density in the considered phase-space domain. Higher-order moments of $P_n$ measure event-to-event multiplicity fluctuations. They are related to the two-particle and higher-order inclusive density correlations and provide more detailed dynamical information than that contained in single-particle inclusive spectra [8–12]. The average transverse momentum of the charged particles, $\langle p_T \rangle$, exhibits a positive correlation with the event multiplicity in hadron-hadron collisions ([13–19] and references therein).

Traditionally, the $s$ dependence of $P_n$ and its moments has been much discussed [1, 4, 5] in relation to Koba-Nielsen-Olesen (KNO) scaling [20, 21]. In this framework, one studies the KNO function $\Psi(z) = \langle n \rangle P_n$, where $z = n/\langle n \rangle$. If KNO scaling holds, $\Psi(z)$ and the normalised moments $C_q = \langle n^q \rangle/\langle n \rangle^q$ are independent of $s$.

Throughout this paper, we compare the data with existing measurements at similar or lower centre-of-mass energies and with predictions of the multiplicity distribution and its mean value from analytical and event generator Monte Carlo (MC) models. The models are based on the assumption that hadrons are produced via the fragmentation of colour strings. This comparison should allow for a better tuning of the existing MC models to accurately simulate minimum-bias events and underlying-event effects.

The next section gives a short description of the CMS detector. Section 3 describes the MC models used in the analysis, while section 4 presents the data samples. The track reconstruction and acceptance are explained in section 5. Section 6 describes the corrections applied to the data. Section 7 lists all relevant systematic uncertainties. The results are discussed in section 8.

## 2 The CMS detector

A complete description of the CMS detector can be found in [3]. The CMS experiment uses a right-handed coordinate system, with the origin at the nominal interaction point (IP), the $x$ axis pointing to the centre of the LHC ring, the $y$ axis pointing up, and the $z$ axis along the counterclockwise beam direction.

The central feature of the CMS detector is a superconducting solenoid of 6 m internal diameter, providing an axial magnetic field with nominal strength of 3.8 T. Immersed in the magnetic field are the pixel tracker, the silicon-strip tracker (SST), the lead tungstate...
electromagnetic calorimeter, the brass/scintillator hadron calorimeter and the muon detection system. In addition to the barrel and endcap calorimeters, the steel/quartz-fibre forward calorimeter (HF) covers the region $2.9 < |\eta| < 5.2$.

Two of the CMS subdetectors acting as LHC beam monitors, the Beam Scintillation Counters (BSC) and the Beam Pick-up Timing for the eXperiments (BPTX) devices, were used to trigger the detector readout. The BSCs are located along the beam line on each side of the IP at a distance of 10.86 m and are sensitive in the range $3.23 < |\eta| < 4.65$. The two BPTX devices, which are located inside the beam pipe at distances of 175 m from the IP, are designed to provide precise information on the bunch structure and timing of the incoming beams, with a time resolution better than 0.2 ns.

The tracking detector consists of 1440 silicon-pixel and 15148 silicon-strip detector modules. The barrel part consists of 3 (10) layers of pixel (SST) modules around the IP at distances ranging from 4.4 cm to 1.1 m. Five out of the 10 strip layers are double sided and provide additional $z$ coordinate measurements. The two endcaps consist of 2 (12) disks of pixel (SST) modules that extend the pseudorapidity acceptance to $|\eta| < 2.5$. The tracker provides an impact parameter resolution of about 100 $\mu$m and a $p_T$ resolution of about 0.7% for 1 GeV/c charged particles at normal incidence [22, 23].

3 Models

The PYTHIA 6 [24] generator and its fragmentation model tuned to CDF data [25, 26], hereafter called PYTHIA D6T, is used as a baseline model to simulate inelastic pp collisions. However, at 7 TeV a dedicated PYTHIA tune [27] describing better the high multiplicities is used for correcting the data. Alternative tunings that differ mainly in the modelling of multiple parton interactions have also been considered [26, 28, 29]. PHOJET [30, 31] is used as an alternative event generator that differs mainly in the underlying dynamical model for particle production. While PYTHIA contains at least one hard scatter per event, particle production in PHOJET, which is based on the dual-parton model [32], is predominantly soft and contains in general multiple-string configurations derived from the dual-parton model with multi-Pomeron exchanges [32]. Each Pomeron exchange gives rise to two strings stretched between either valence or sea partons. At low energies the dominant process is single-Pomeron exchange, which leads to two strings stretched between valence quarks and diquarks. With increasing energy, additional Pomeron exchanges occur, forming strings stretched between sea partons. Because the sea partons carry on average only a small fraction of the momentum of the incident hadrons, these strings are concentrated in the central rapidity region. These extra strings [33, 34] are needed to explain the KNO scaling violations observed at high energies [35, 36], the increase of the central particle density with increasing energy [35–37], the $\langle p_T \rangle$ versus $n$ dependence [38, 39], and long-range rapidity correlations [40]. The distribution of the number of exchanged Pomerons can be obtained from perturbative Reggeon calculus and unitarity, by means of fits to the measured total, elastic, and diffractive cross sections as described in [41–43]. Other alternatives based on the Colour Glass Condensate (CGC) picture with saturated gluons [44] also make predictions for the multiplicity dependence of $\langle p_T \rangle$ and for the long-range rapidity correlations.
We also compare our measurements with a new fragmentation model implemented in Pythia 8 [45]. In addition to having $p_T$-ordered parton showers, rather than an ordering by virtuality, it differs from its predecessor in the modelling of multiple-parton interactions and the treatment of beam remnants and diffraction. The detailed MC simulation of the CMS detector response is based on Geant4 [46]. Simulated events were processed and reconstructed in the same manner as collision data.

4 Data sample

Near the end of 2009, the CMS experiment collected two datasets of proton-proton collisions at centre-of-mass energies of 0.9 and 2.36 TeV. In March 2010 a new running period at $\sqrt{s} = 7$ TeV started, of which data collected in the first days have been analysed for this paper. The corresponding inelastic proton-proton interaction rates were about 11, 3, and 50 Hz, respectively, for these datasets. At these rates, the fraction of bunch crossings in which two or more minimum-bias collisions occurred is negligible [47, 48].

Diffraction is commonly characterised by one (single-diffraction) or two (double-diffraction) colourless exchanges resulting in the observation of a large rapidity interval devoid of any hadron activity (rapidity gap). All results presented in this paper refer to inelastic non-single-diffractive (NSD) interactions and are based on an event selection that retains a large fraction of the non-diffractive (ND) and double-diffractive (DD) events, while disfavouring single-diffractive (SD) events.

The trigger and offline event selection are nearly identical to those used in [47, 48]; about three times more events were used in this analysis at $\sqrt{s} = 0.9$ and 7 TeV. The trigger required a signal in any of the BSC scintillator counters, in coincidence with either of the two BPTX devices indicating the presence of at least one proton bunch crossing the IP. Beam halo backgrounds are reduced by using the timing information from the BSC counters at opposite ends of the CMS detector. Additional beam-induced backgrounds are removed by requiring the cluster sizes in the pixel detector to be consistent with a single primary vertex, as described in [47]. NSD events are selected by requiring at least one HF calorimeter tower with more than 3 GeV of total energy in each of the positive-$z$ and negative-$z$ HF calorimeters. Finally, a reconstructed primary vertex is required with the $z$ coordinate within $\pm 15$ cm of the centre of the beam collision region.

For the final analysis, totals of about 132, 12, and 442 thousand events were retained at $\sqrt{s} = 0.9$, 2.36, and 7 TeV, respectively. Event yields at various stages of the selection are shown in table 1.

5 Track reconstruction and acceptance definition

The barrel and endcap pixel and SST detectors are used in the reconstruction of tracks within an acceptance of $|\eta| < 2.5$. Due to a large drop in reconstruction efficiency near the limits of this range, we restrict the computation of the multiplicity spectra to the region $|\eta| < 2.4$. 

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<table>
<thead>
<tr>
<th>Selection</th>
<th>√s (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam background rejection + L1 trigger</td>
<td>254666</td>
</tr>
<tr>
<td>all preceding + forward calorimeters</td>
<td>146658</td>
</tr>
<tr>
<td>all preceding + primary vertex</td>
<td>132294</td>
</tr>
</tbody>
</table>

Table 1. Event yields in each data sample after sequential trigger and event selection.

In proton-proton collisions at the LHC, the events selected by minimum-bias triggers involve predominantly soft interactions and contain mostly particles with small transverse momenta. These are reconstructed by extending the standard tracking algorithms of the CMS experiment, which are based on a combinatorial track finder [22] that performs multiple iterations. Hits that can be assigned unambiguously to tracks in one iteration are removed from the collection of tracker hits to create a smaller collection that can be used in the subsequent iteration. This iterative procedure was further optimised for primary track reconstruction with $p_T \geq 100\text{ MeV}/c$ in minimum-bias events [47]. The reconstruction efficiency, estimated by means of the detector simulation, exceeds 90% for tracks with $p_T > 500\text{ MeV}/c$ and drops below 70% for tracks with $p_T < 100\text{ MeV}/c$. Contamination from mis-reconstructed tracks is below 5% for $p_T < 500\text{ MeV}/c$.

After three iterations of the combinatorial track finder, the position of the primary vertex is recomputed and then used as an additional constraint in a refit of all previously reconstructed tracks, thus improving the overall resolution in $\eta$ and $p_T$. An agglomerative clustering algorithm followed by a Gaussian mixture model [49] were applied in order to optimise the vertex-finding efficiency.

The contamination due to decays of long-lived particles (denoted as $V^0$ decays) is dominated by charged pions and protons originating from $K^0_S$ and $\Lambda$ decays. The $K^0_S$ production rate is roughly 5% of that for all charged particles [50], and the resulting charged secondaries amount to 7% of all $\pi^\pm$. The $\Lambda$ production is measured to be 43% of $K^0_S$ [51, 52] in this kinematic domain, yielding another 3% of charged secondaries consisting of protons and pions. In order to reduce the contamination of these $V^0$ decay products and secondaries produced in interactions of charged particles with the material of the detector, we require all reconstructed tracks to be associated with the primary vertex. This is done by selecting tracks with a small impact parameter with respect to the primary vertex position both in the transverse plane and along the $z$ axis [47]. Only 1.5% of $K^0_S$ and $\Lambda$ decay products pass these selections, resulting in a final contamination of 0.2%.

The $K^\pm/\pi^\pm$ ratio is known to be fairly constant over a wide range of centre-of-mass energies and is between 8 and 12% [50]. The $K^\pm$ have a lower reconstruction efficiency at low $p_T$ than pions and mismodeling of the $K^\pm/\pi^\pm$ ratio could result in a change in the multiplicity distribution. However a doubling of the $K^\pm/\pi^\pm$ ratio yields a negligible shift of 0.25% in the multiplicity average. This is expected because the $\langle p_T \rangle$ of $K^\pm$ is substantially higher than $\pi^\pm$, and $K^\pm$ therefore contribute very little in the $p_T$ range where the reconstruction efficiencies differ. Finally, only tracks with a relative uncertainty
Table 2. The percentage of single-diffractive events (SD/inelastic) at each centre-of-mass energy as predicted by PYTHIA D6T and PHOJET.

<table>
<thead>
<tr>
<th>Generator</th>
<th>√s (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PYTHIA D6T</td>
<td>22.5</td>
</tr>
<tr>
<td>PHOJET</td>
<td>18.9</td>
</tr>
</tbody>
</table>

on their measured transverse momentum smaller than 10% are selected; this requirement rejects mainly low-quality and badly reconstructed tracks.

6 Corrections

Due to the requirement of significant activity in both ends of the HF, the event selection acceptance for SD events is small: 5% at 0.9 TeV and 7% at 7 TeV. This contribution is therefore subtracted based on simulated PYTHIA events. The PYTHIA and PHOJET predictions of the initial fractions of SD events differ substantially, as seen from table 2. The difference of the two predictions is taken as the systematic uncertainty related to the SD subtraction. It is customary to normalise the charged hadron multiplicity distribution \( P_n = \sigma_n/\sigma \), where \( \sigma_n \) denotes the cross section for a fixed multiplicity \( n \), to either the total inelastic cross section or the NSD cross section. For the results presented in this paper the normalisation factor \( \sigma \) corresponds to the latter.

The minimum-bias trigger and NSD selection unavoidably introduce a bias in the measured charged hadron multiplicity. Furthermore, a fraction of the events are removed by the requirement of a good quality primary vertex. These effects result in an accepted multiplicity given by

\[
T_n = \epsilon_n \cdot P_n, \tag{6.1}
\]

where \( \epsilon_n \) is the trigger and event reconstruction efficiency for multiplicity \( n \). This efficiency is close to 100% for multiplicities larger than \( n = 20 \) and drops gradually to 40% for \( n = 1 \).

Due to inefficiencies in track reconstruction and acceptance, the creation of secondary particles by the interaction of primaries with the beam pipe and the detector material, and the presence of decay products of long-lived hadrons, one will in general not measure the true multiplicity \( n \) but a statistically related quantity \( m \). The statistical distribution \( O_m \) of this observed multiplicity is related to the true accepted multiplicity distribution by the linear relation

\[
O_m = \sum_n R_{m,n} \cdot T_n. \tag{6.2}
\]

The problem of inverting the response matrix \( R_{m,n} \), here taken from MC, is well known and extensive literature on the topic exists [53, 54]. When dealing with limited event samples, an algebraic inversion of \( R_{m,n} \) turns out to be impossible and leads to unstable results. We therefore use a Bayesian unfolding method, as described in [55]. The unfolding procedure introduces large statistical correlations between adjacent bins of the multiplicity spectrum.
The full covariance matrix of the unfolded multiplicity spectrum was calculated using a resampling technique [54].

The average transverse momentum $\langle p_T \rangle$ as a function of $n$ was measured as well. For each bin in raw multiplicity, a Monte Carlo based correction factor $\langle p_{T\text{gen}}/p_{T\text{rec}} \rangle$ was applied to convert the measured $\langle p_T \rangle$ to the corresponding value for primary charged hadrons. Subsequently, the response matrix $R_{m,n}$, weighted by the corrected data, was applied to correct the multiplicities.

The correctness of the unfolding procedure was verified using MC events generated with our baseline PYTHIA D6T tune where we compared the multiplicity distribution of generated primary hadrons with that of reconstructed tracks after unfolding. At all energies considered, an excellent agreement with the generated hadrons is achieved, proving the stability of the procedure.

The track reconstruction efficiency of the minimum-bias tracking drops drastically for $p_T < 100 \text{ MeV}/c$, while the mis-reconstructed track rate increases. Rather than correcting for these effects using MC simulations, we reconstruct the $p_T$ spectrum in data themselves and calculate the fraction of charged hadrons with $p_T < 100 \text{ MeV}/c$ by extrapolating the measured spectrum in data, using a parametrisation based on an exponential of a third-degree polynomial in $p_T$. The fraction of charged hadrons added by this correction ranges between 5% and 7% depending on the centre-of-mass energy and the pseudorapidity interval under study. The functional form used to extrapolate to the lowest transverse momenta introduces an uncertainty of 1% on this fraction and is taken into account in the systematic uncertainties. The effect of all the correction procedures on the measured raw multiplicity distribution is illustrated in figure 1 for a centre-of-mass energy of 7 TeV.

7 Systematic uncertainties

Four main sources of systematic uncertainty contribute to the total uncertainty on the multiplicity distribution $P_n$ and $\langle p_T \rangle$ versus $n$: the uncertainty on the trigger and event selection efficiency, the uncertainty on the tracking efficiency and acceptance, the model dependence of the response matrix, and the model dependent SD subtraction. All four are discussed below. The effects of misalignment of the tracking detector, dead sensors, and the uncertainty on the vertex position are much smaller and are contained within the overall tracking systematic uncertainty. For $\langle p_T \rangle$ versus $n$, the effect of the event selection and SD subtraction was observed to be negligible.

Trigger and event selection efficiency. The corrections for the trigger and event selection efficiency are based entirely on MC simulation. The largest impact on the overall efficiency is that of the HF coincidence requirement. A cross-check of the multiplicity dependent efficiency with zero-bias events, which are by definition not biased at all (random trigger on collisions), shows good agreement within statistical errors between data and MC. A relative shift of the efficiency correction factors by $\pm 5\%$ for low multiplicities, decreasing to $\pm 1\%$ for $n \sim 20$, covers the trigger efficiency measured in zero-bias data. This leads to a maximum 5% systematic uncertainty.
Tracking efficiency and acceptance. A correct description of the tracking efficiency in the MC simulation of the detector is essential for obtaining a correct response matrix. At low transverse momenta, the efficiency drops due to a loss of hits on the tracks that are stopped within the tracking volume. As in [47], we assign a 2% uncertainty on this efficiency. The remaining contamination is mostly due to secondary tracks originating from interactions with the material of the LHC beam pipe around the interaction point. This is estimated in [47] to be no more than 1%, and is confirmed based on [50]. These uncertainties, together with smaller contributions from misalignment of the tracking detector, beam-halo background, multiple counting of tracks, and mis-reconstructed tracks are added in quadrature to produce a total tracking uncertainty of 2.5%. All correction factors related to track reconstruction are summarised in table 3. As was discussed in section 5, the contamination from $V^0$ decays after associating tracks with the primary vertex is small (0.2%) and already included in the systematic uncertainty for secondary tracks. The difference in reconstruction efficiency for charged kaons and pions also has a negligible impact on the measured multiplicity distribution. Finally, the extrapolation uncertainty from $p_T = 100 \text{MeV}/c$ to zero is 1% (section 6). For $\langle p_T \rangle$ versus $n$, this was taken into account by changing the mean transverse momentum by the average $p_T$ of tracks lost during the event reconstruction folded with the tracking efficiency uncertainty. The overall effect of this systematic uncertainty is less than 10% for small multiplicities, but can reach up to 30% for the high multiplicities, where it is the main source of systematic uncertainty.

Model dependence. The baseline MC model that is used to unfold the multiplicity distribution underestimates the single-particle densities at zero rapidity by 10% relatively...
Table 3. Summary of systematic uncertainties on the track reconstruction.

<table>
<thead>
<tr>
<th>Source</th>
<th>tracking uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking efficiency</td>
<td>2.0</td>
</tr>
<tr>
<td>Acceptance</td>
<td>1.0</td>
</tr>
<tr>
<td>Pixel hit efficiency</td>
<td>0.3</td>
</tr>
<tr>
<td>Pixel cluster splitting</td>
<td>0.2</td>
</tr>
<tr>
<td>Correction for secondaries</td>
<td>1.0</td>
</tr>
<tr>
<td>Misalignment</td>
<td>0.1</td>
</tr>
<tr>
<td>Beam halo</td>
<td>0.1</td>
</tr>
<tr>
<td>Multiple track counting</td>
<td>0.1</td>
</tr>
<tr>
<td>Mis-reconstructed tracks</td>
<td>0.5</td>
</tr>
<tr>
<td>Total</td>
<td>2.5</td>
</tr>
</tbody>
</table>

8 Results

8.1 Charged hadron multiplicity distributions

The NSD charged hadron multiplicity distributions are measured in increasing ranges of pseudorapidity from $|\eta| < 0.5$ to $|\eta| < 2.4$. The fully corrected results at $\sqrt{s} = 0.9$, 2.36, and 7 TeV are compared in figure 2 with earlier measurements in the same pseudorapidity ranges performed by the UA5 [35, 36] and ALICE [57, 58] collaborations. Our measurements were also compared with results obtained with a CMS cross-check analysis (not shown) on data at $\sqrt{s} = 0.9$ and 7 TeV, using a tracklet-based tracking algorithm as in ref. [47].
reconstruction efficiency exceeding 90% for $p_T > 50$ MeV/c, the latter provided a cross-check of the extrapolation for tracks below $p_T < 100$ MeV/c, including the use of data without magnetic field at 7 TeV. All measurements agree well within their total uncertainties.

In the largest pseudorapidity interval of $|\eta| < 2.4$, there is a change of slope in $P_n$ for $n > 20$, indicating a multicomponent structure as was discussed in [59] and [60] in terms of multiple-soft-Pomeron exchanges. This feature becomes more pronounced with increasing centre-of-mass energies, notably at $\sqrt{s} = 7$ TeV.

An extensive range of tunes [26–29] based on the PYTHIA 6 fragmentation model have been developed. They differ mainly in their parametrisation of the multiple-parton-interaction model. Some reproduce the charged hadron multiplicities better than others, but none is able to give a good description simultaneously at all the centre-of-mass en-

![Figure 2](image-url)

**Figure 2.** The fully corrected charged hadron multiplicity spectrum for $|\eta| < 0.5, 1.0, 1.5, 2.0, \text{and} 2.4$, (a) at $\sqrt{s} = 0.9$ TeV, (b) 2.36 TeV, and (c) 7 TeV, compared with other measurements in the same $\eta$ interval and at the same centre-of-mass energy [35, 36, 57, 58]. For clarity, results in different pseudorapidity intervals are scaled by powers of 10 as given in the plots. The error bars are the statistical and systematic uncertainties added in quadrature.
energies and in all pseudorapidity ranges. For clarity, only the baseline tune D6T is shown in comparison with other models having a different physical description of soft-particle production such as PHOJET [30, 31] and the new fragmentation model of PYTHIA 8 [45].

A comparison of our measurements with these three classes of models is shown in figure 3 for all charged hadrons and for those with $p_T > 500$ MeV/$c$. PYTHIA D6T underestimates drastically the multiplicity at all measured energies but improves when $p_T > 500$ MeV/$c$ is required. PYTHIA 8 is the only model that gives a reasonable description of the multiplicity distribution at all energies, but tends to overestimate the multiplicity at 7 TeV when $p_T > 500$ MeV/$c$ is required. PHOJET produces too few charged hadrons overall but gives a good description of the average transverse momentum $\langle p_T \rangle$ at fixed multiplicity $n$, as illustrated in figure 4. Among the three classes of models, PYTHIA 8 gives the best overall description of the multiplicity distribution and the dependence of the average transverse momentum on $n$. Inspired by [61] we fit a first-degree polynomial in $\sqrt{n}$ to the multiplicity dependence of $\langle p_T \rangle(n)$ for $n > 15$ at each energy, yielding a good description which is valid at all three energies (figure 4). The ratios of the data obtained at 7 and 2.36 TeV with respect to the data at 0.9 TeV show that the rise of the average transverse momentum with the multiplicity is weakly depending on energy.

All previous observations seem to indicate that the Monte Carlo models produce too few particles with low transverse momenta, especially at 7 TeV. The PYTHIA models tend to compensate for this by producing too many particles with high transverse momentum, which is related to the modelling of semi-hard multiple-parton interactions.

### 8.2 Violation of KNO scaling

The multiplicity distributions are shown in KNO form in figure 5 for a large pseudorapidity interval of $|\eta| < 2.4$, where we observe a strong violation of KNO scaling between
Figure 4. (a) A comparison of $\langle p_T \rangle$ versus $n$ for $|\eta| < 2.4$ with two different PYTHIA models and the PHOJET model at $\sqrt{s} = 0.9, 2.36,$ and $7$ TeV. For clarity, results for different energies are shifted by the values of $a$ shown in the plots. Fits to the high-multiplicity part ($n > 15$) with a linear form in $\sqrt{n}$ are superimposed. (b) The ratios of the higher-energy data to the fit at $0.9$ TeV indicate the approximate energy independence of $\langle p_T \rangle$ at fixed $n$.

Figure 5. The charged hadron multiplicity distributions in KNO form at $\sqrt{s} = 0.9$ and $7$ TeV in two pseudorapidity intervals, (a) $|\eta| < 2.4$ and (b) $|\eta| < 0.5$.

$\sqrt{s} = 0.9$ TeV and $7$ TeV, and for a small pseudorapidity interval of $|\eta| < 0.5$, where KNO scaling holds. Scaling is a characteristic property of the multiplicity distribution in cascade processes of a single jet with self-similar branchings and fixed coupling constant $[62–69]$.

The validity of KNO scaling is shown more quantitatively in figure 6 by the normalised
order-$q$ moments $C_q$ of the multiplicity distribution, complemented with measurements at lower energies [70–72]. For $|\eta| < 2.4$ the values of $C_q$ increase linearly with log $s$, while for $|\eta| < 0.5$ they remain constant up to $q = 4$ over the full centre-of-mass energy range, as illustrated by the fits in figure 6.

Multiplicity distributions for $e^+e^-$ annihilations up to the highest LEP energies show clear evidence for multiplicity scaling, both in small ranges ($\Delta \eta < 0.5$), in single hemispheres, and in full phase-space. However, at LEP energies, scaling is broken for intermediate-size ranges where, besides two-jet events, multi-jet events contribute most prominently [73–77].

For hadron-hadron collisions, approximate KNO scaling holds up to ISR energies [78, 79], but clear scaling violations become manifest above $\sqrt{s} \approx 200$ GeV both for the multiplicity distributions in full phase space and in central pseudorapidity ranges [59, 70, 80, 81]. In p$\bar{p}$ collisions, and for large rapidity ranges, the UA5 experiment was the first to observe a larger than expected high-multiplicity tail and a change of slope [59, 72], which was interpreted as evidence for a multi-component structure of the final states [34, 60, 82]. Our observation of strong KNO scaling violations at $\sqrt{s} = 7$ TeV, as well as a change of slope in $P_n$, confirm these earlier measurements.

All these observations, together with the sizable growth with energy of the non-diffractive inelastic cross section, point to the increasing importance of multiple hard, semi-hard, and soft partonic subprocesses in high energy hadron-hadron inelastic collisions [6, 32, 34, 59, 83, 84].

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**Figure 6.** Fits of the log $s$ dependence of the normalised moments $C_q$ of the multiplicity distribution for (a) $|\eta| < 2.4$ (assuming linear dependence) and (b) $|\eta| < 0.5$ (assuming no dependence), including data from lower energy experiments [70–72]. For $\sqrt{s} = 0.9$ TeV, data from experiments other than CMS were drawn shifted to lower $\sqrt{s}$ for clarity.
The mean multiplicity $\langle n \rangle$ for data, PYTHIA D6T, PYTHIA 8, and PHOJET for $|\eta| < 2.4$ at each centre-of-mass energy. For data, the quoted uncertainties are first statistical, then upward and downward systematic.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>Data</th>
<th>PYTHIA D6T</th>
<th>PYTHIA 8</th>
<th>PHOJET</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>$17.9 \pm 0.1^{+1.1}_{-1.1}$</td>
<td>14.7</td>
<td>14.9</td>
<td>17.1</td>
</tr>
<tr>
<td>2.36</td>
<td>$22.9 \pm 0.5^{+1.6}_{-1.5}$</td>
<td>16.7</td>
<td>17.8</td>
<td>18.7</td>
</tr>
<tr>
<td>7</td>
<td>$30.4 \pm 0.2^{+2.2}_{-2.0}$</td>
<td>21.2</td>
<td>25.8</td>
<td>23.2</td>
</tr>
</tbody>
</table>

Table 4. Mean multiplicity for data, PYTHIA D6T, PYTHIA 8, and PHOJET for $|\eta| < 2.4$ at each centre-of-mass energy.

8.3 Energy dependence of the mean multiplicity

The mean multiplicity $\langle n \rangle$ is the first moment of the multiplicity distribution and is equal to the integral of the corresponding single-particle inclusive density in the $\eta$ interval considered. The mean multiplicity $\langle n \rangle$ is observed to rise with increasing centre-of-mass energy in hadron-hadron collisions [35–37, 70, 78, 79, 85, 86]. The same behaviour is also observed in $e^+e^-$ collisions, in deep-inelastic scattering [87], and in heavy ion collisions [1].

Our measured mean multiplicity is compared with experimental data obtained at lower energies in figure 7. Recent Regge-inspired models [41–43] predict a power-like behaviour among which only ref. [42] describes the highest energy data very well. Parton saturation models (such as [44]) predict a strong rise of the central rapidity plateau as well. Table 4 gives an overview of $\langle n \rangle$ for the data and for the PYTHIA D6T, PYTHIA 8, and PHOJET event generators. As in section 8.1, PYTHIA D6T produces on average too few particles per event at all three energies. PHOJET is consistent with the data within uncertainties for $\sqrt{s} = 0.9$ TeV, but is not able to predict properly the mean multiplicity at higher energies. PYTHIA 8 describes best the 7 TeV data, but underestimates $\langle n \rangle$ systematically at all energies.

9 Conclusions

The charged hadron multiplicity distributions of non-single-diffractive events were measured from an analysis of the minimum-bias datasets collected by CMS at three centre-of-mass energies: $\sqrt{s} = 0.9$, 2.36, and 7 TeV. The excellent tracking capabilities of the silicon pixel and strip detectors of CMS, combined with an optimised tracking and vertexing algorithm, allow the reconstruction of charged tracks down to $p_T = 100$ MeV/c with high efficiency and low background contamination. A full correction for detector resolution and acceptance effects and an extrapolation to zero transverse momentum yield measurements of the charged hadron multiplicity distribution for increasing central pseudorapidity ranges from $|\eta| < 0.5$ to $|\eta| < 2.4$, which can be compared with models of soft-particle production and with experimental data at lower energies.

Although some event generators provide an adequate description of Tevatron and LEP data, none is able to describe simultaneously the multiplicity distributions and the $p_T$ spectrum at $\sqrt{s} = 7$ TeV. In general, models predict too few low-momentum particles,
Figure 7. The evolution of the mean charge multiplicity with the centre-of-mass energy for $|\eta| < 2.4$, including data from lower-energy experiments for $|\eta| < 2.5$ [37, 70–72]. The data are compared with predictions from three analytical Regge-inspired models [41–43] and from a saturation model [44].

indicating that by increasing the amount of multiple-parton interactions one effectively introduces too many hard scatters in the event.

The change of slope in $P_n$ in the widest central pseudorapidity intervals observed at $\sqrt{s} = 7$ TeV, combined with the strong linear increase of the $C_q$ moments, indicates a clear violation of KNO scaling with respect to lower energies. This observation merits further studies.

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