Cosmological Signature of New Parity-Violating Interactions

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Are there any manifestations of parity violation other than those observed in weak interactions? A map of the cosmic microwave background (CMB) temperature and polarization will provide a new signature of $P$ violation. We examine two classes of $P$-violating interactions that would give rise to such a signature. The first interaction leads to a cosmological birefringence, possibly driven by quintessence. The other interaction leads to an asymmetry in the amplitude of right- versus left-handed gravitational waves produced during inflation. The Planck Surveyor should improve upon the current sensitivity to birefringence. While the primordial effect would most likely elude detection by MAP and Planck, it may be detectable with a future dedicated CMB polarization experiment.

The discovery of parity ($P$) violation [1] was central to the development of what has now become the standard model. Nevertheless, this symmetry violation occurs strictly within the weak interaction sector. Presumably, its ultimate origin lies in the grand-unified and/or Planck-scale physics that yields the standard model as its low-energy limit. If so, might there be some remnant of $P$ violation in gravitational interactions or in some other, still undiscovered, sector?

Some tantalizing clues do exist. The baryon asymmetry of the Universe requires charge conjugation ($C$) violation as well as $CP$ violation [2], likely in new physics beyond the standard model. Moreover, extensions of the standard model, including grand unified theories and supersymmetry, naturally suggest nonstandard $P$ and $CP$ violating interactions. Carroll has argued that a certain class of quintessence models should generically produce such $P$ asymmetric physics (“cosmological birefringence”) [3], and other cosmological physics may also give rise to parity breaking [4].

In the next few years, high-precision temperature and polarization maps of the cosmic microwave background (CMB) will become available [5,6]. These maps will provide a wealth of data concerning the physics of the early Universe. Although the primary purpose of these observations is not to explore $P$ violation, certain temperature/polarization cross-correlation functions can provide a probe of $P$ violation. Their relevance has heretofore been disregarded since they vanish if the underlying physics—in particular gravity and inflation—is $P$ symmetric, as has been assumed until now.

In this Letter, we explore the possibility of probing exotic $P$ violating physics using the CMB. We first lay out the details of the CMB correlation functions needed to detect $P$ violation. We then explain the features of fundamental interactions and early-Universe mechanisms required to produce such a preferred macroscopic orientation. We then provide two examples of interactions and mechanisms that can produce this $P$ violating signature and discuss their detectability.

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The existence of interactions that yield $P$ asymmetric physics alone is insufficient to produce a preferred cosmological orientation from a $P$ symmetric initial state. Since the CMB signature is charge blind, then the CPT theorem suggests that the required interaction must violate time-reversal $(T)$ invariance as well as $P$ invariance in a fashion that preserves $PT$. If we have an interaction that is $P$ and $T$ violating, then any mechanism that defines an arrow of time could conceivably drive the Universe to a preferred orientation. Such a $T$ asymmetric process might be the expansion of the Universe or maybe some entropy-producing process. Another possibility, and that which we focus on here, is that the $T$ symmetry is broken by the rolling of some scalar field.

If there is some $P$ and $T$ violating physics that appears at some large energy scale $\mu$ that involves a new scalar field $\chi$, then at lower energies we would expect terms in our effective Lagrangian like

$$L_{\text{int}} = g(\chi) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (1)$$

where $g(\chi)$ is a dimensionless function of a scalar field and $F_{\mu\nu}$ is the electromagnetic field-strength tensor. The scalar field $\chi$ has been identified, e.g., with that in scalar-tensor theories of gravity [4] or with a quintessence field [3]. If $\chi$ is constant in space and time, then the term has no effect on electrodynamics, since the term can be written as a total derivative. However, if $\chi$ is spatially homogeneous but changing with time, then the polarization vector of a photon is rotated by an angle $\Delta \alpha \propto \Delta g(\chi)$, where $\Delta g(\chi)$ is the change in the function $g(\chi)$ as the photon propagates from source to observer [4]; this effect has been referred to as “cosmological birefringence.”

The effect of such a rotation is to alter a $P$-symmetric CMB as it propagates from the surface of last scatter to the observer. Each $Y_{lm}^G$ tensor field is orthogonal to the $Y_{lm}^C$ of the same $l$ and $m$ at each point on the sky, so rotating the polarization of each photon everywhere by the same amount simply mixes the $G$ and $C$ modes. Any mechanism that produces temperature anisotropies also produces a polarization pattern with a gradient component, and it also produces a nonzero $TG$ cross correlation. If the CMB has some nonzero $C_{fG}^{TG}$ moments at the surface of last scatter, and the polarization vector of each photon is rotated by an angle $\Delta \alpha$, then it induces $TC$ moments, $C_{fT}^{TC} = C_{fG}^{TG} \sin 2\Delta \alpha$. Furthermore, the shape of the $C_{fT}^{TC}$ power spectrum (as a function of $l$) is the same as that of the $C_{fG}^{TG}$ power spectrum. The dashed curve in Fig. 1 shows an example of such a $C_{fT}^{TC}$ power spectrum. This curve was generated assuming a flat model with a matter density $\Omega_m = 0.3$, a cosmological constant $\Lambda = 0.7$, a baryon density $\Omega_b h^2 = 0.02$, and Hubble parameter $h = 0.65$, with a nearly scale-invariant spectrum of primordial adiabatic perturbations and no gravitational waves.

Let us now consider the consequences of another class of terms that generically appears in our effective action

$$L_{\text{int}} = f(\Phi) R_{\sigma\mu\nu}^A \tilde{R}^{\sigma\mu\nu}_A. \quad (2)$$

In contrast to our earlier discussion, here we identify scalar field $\Phi$ with the inflaton field. These terms arise in exact analogy from whatever physics that produces terms like Eq. (1). For example, let $\Phi$ be an axion- or pionlike field axially coupled to heavy fermions. Then, radiative fermion loops generate both Eqs. (1) and (2). Another class of examples appears in [11].

So long as the scalar field is homogeneous and constant in time, Eq. (2) becomes a pure surface term, and thus does not contribute at all to classical gravity dynamics. Thus we expect that after inflation, when the inflaton has come to rest, $P$ asymmetric gravity dynamics is not present, suggesting no current observed constraint on Eq. (2). Nevertheless, the term has relevant effects during inflation which may be observed through the CMB.

The homogeneous dynamics of the inflaton is identical to that without Eq. (2). We may take any conventional slow-roll inflation scenario where $\Phi \neq 0$. Moreover, the conventional flat Robertson-Walker metric is still a solution to the metric equations of motion with the new interactions, implying the overall cosmology is not affected by the new term. However, metric perturbations are affected by these terms. For simplicity, take the metric $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ in a flat-space background, $\eta_{\alpha\beta}$. Linearizing the metric equations of motion in the harmonic gauge ($\partial^\rho h_{\mu\nu} = \frac{1}{2} \partial_\mu h^\nu_\rho$), we find

$$M^2 \Box h_{\alpha\beta} = 2 f'' \Phi^2 e^{i k \cdot x} \eta_{\alpha\beta} (\partial_\gamma \partial_\delta h_{0k} + \partial_0 \partial_k h_{\gamma\delta}) + 2 f' \Phi e^{i k \cdot x} \eta_{\gamma\delta} \partial_\gamma \partial_\delta h_{0k} + (\alpha \leftrightarrow \beta). \quad (3)$$

assuming the acceleration of the inflaton is negligible. Here, the prime on $f$ denotes differentiation with respect to $\Phi$, and the Latin indices indicate spatial indices only. Let us look at plane-wave perturbations of the form $h_{\alpha\beta} = e_{\alpha\beta} e^{-i k \cdot x}$ where $e_{\alpha\beta}$ is a constant polarization.
matrix. Assuming the effects of the new terms are small, we find the following plane-wave solutions
\[e^R_{\mu \nu} = e^{2i/\lambda \phi_x k/m_0^2} e^{-ik + ikz}, \quad e^L_{\mu \nu} = e^{-2i/\lambda \phi_x k/m_0^2} e^{-ik + ikz},\]
where \(e^R, e^L\) are the polarization tensors for right- and left-handed polarized waves, respectively. Thus, right-handed gravitational waves (GWs) are amplified as they propagate while left-handed GWs are attenuated. These solutions preserve PT although they violate P and T individually.

Let us apply this result to our scenario where the Universe inflates. While their wavelength is much smaller than the horizon size, right-handed GWs amplify while left-handed GWs attenuate. Eventually, the fluctuations expand past the horizon and freeze out. To estimate the discrepancy between left-handed and right-handed tensor fluctuations in the early Universe, we assume that the fluctuations of both handednesses are equal in amplitude and behave classically as they expand beyond a wavelength \(1/\mu\) and then freeze as the wavelength becomes comparable to the horizon scale. When the waves exit the horizon scale, we can estimate the fraction of accumulated discrepancy through the index \(\epsilon\):
\[\epsilon \sim (M_p/\mu)(H/M_p)^3(\dot{\Phi}/H^2)^2,\]
where \(H\) is the Hubble scale and \(f_H^2\) is characterized by the scale \(1/\mu^2\). The factor \(H^2/\dot{\Phi}\) is associated with the amplitude of scalar density perturbations \(\sim 10^{-5}\), while the factor \(H/M_p\) is associated with the amplitude of tensor perturbations \(\sim 3 \times 10^{-6}\). Given fixed cosmological parameters, one may think of a limit on \(\epsilon\) as a lower bound on \(\mu/M_p\).

Let us describe how this physics is reflected in the CMB. Long-wavelength GWs produce temperature anisotropies and also a curl component of the polarization [9,10]. An excess of right over left (or vice versa) circularly polarized GWs produces a nonzero \(C_1^{TC}\). Consider a single right-handed circularly polarized GW with wave number \(k\) propagating in the \(+\hat{z}\) direction. This can be written as an out-of-phase combination of a linearly polarized GW with + polarization and another of equal amplitude with \(\times\) polarization. We can always choose the \(x\) and \(y\) axes so that the amplitude of the + component has a crest at the origin and the \(\times\) component has a zero at the origin. Doing so, the multipole coefficients \(a_{(lm)}^T\) for the temperature pattern induced on the sky by this particular circularly polarized wave is [12]
\[a_{(lm)}^T = \begin{cases} \left(\delta_{m,2} + \delta_{m,-2}\right)A_l^T(k), & \text{even } l (\times), \\ -i(\delta_{m,2} - \delta_{m,-2})A_l^T(k), & \text{odd } l (\times), \end{cases}\]
where, as indicated, the even-\(l\) contribution is from the + mode and the odd-\(l\) contribution is from the \(\times\) mode. For a left-handed circularly polarized wave, the sign of the odd-\(l\) moments (from the \(\times\) contribution) are reversed. The \(A_l^T(k)\) are temperature brightness functions (see [7]).

The multipole coefficients for the \(G\) component of the CMB polarization are similar, except that the \(A_l^T(k)\) are replaced by some polarization functions \(A_l^G(k)\). The multipole coefficients for the \(C\) component of the CMB polarization are similar,
\[a_{(lm)}^C = \begin{cases} (\delta_{m,2} + \delta_{m,-2})A_l^C(k), & \text{even } l (\times), \\ -i(\delta_{m,2} - \delta_{m,-2})A_l^C(k), & \text{odd } l (\times), \end{cases}\]
except note that the even-\(l\) moments now come from the \(\times\) mode and the odd-\(l\) moments come from the + mode. For a left-handed wave the sign of the even-\(l\) moments is reversed.

Equations (6) and (7) indicate why \(C_1^{TC} = 0\) (and why \(C_0^{TC} = 0\)) for linearly polarized waves. For example, if we have only a + polarized wave, then the \(T\) pattern induces only even-\(l\) modes and the \(C\) pattern induces only odd-\(l\) modes. But these equations also show that a circularly polarized wave induces a nonzero \(C_1^{TC}\). Recall that we measure a given \(C_1^{TC}\) by averaging the quantity \(a_{(lm)}^C a_{(lm)}^C \delta_{(lm)}\) over all \(2l+1\) values of \(m\). Doing so, we find that this right-handed GW induces a nonvanishing \(C_1^{TC} = 2(2l + 1)^{-1} A_l^T(k)A_l^T(k)\), and a left-handed GW induces the same quantity but with the opposite sign. Since \(C_1^{TC}\) is rotationally invariant, the result is independent of the direction of propagation of the GW.

The solid curve in Fig. 1 shows \(C_1^{TC,R}\), the \(TC\) power spectrum expected for a GW background made of only right-handed GWs. This curve was generated assuming the same classical cosmological parameters as were used for the dashed curve, but here we have assumed the presence of a nearly scale-invariant spectrum of GWs with a tensor-to-scalar ratio of \(T/S = 0.7\). For a more general mixture of right- and left-handed GWs, \(C_1^{TC} = \epsilon C_1^{TC,R}\). The solid curve in Fig. 2 shows the smallest \(\epsilon\) that could be distinguished from a null result from the \(C_1^{TC}\) moments at the 1\(\sigma\) level as a function of detector sensitivity \(s\) for a one-year experiment that maps the temperature and polarization of the entire sky. The calculation was done using the same cosmological parameters as were used in Fig. 1.

The sensitivity to \(\epsilon\) remains finite even as \(s \to 0\), since the measurement is ultimately cosmic-variance limited. Figure 1 is only meant to be illustrative; the precise sensitivity differs for different cosmological parameters. The sensitivity to \(\epsilon\) will of course be degraded if the tensor amplitude is smaller. Because \(\epsilon \ll 1\) is a strict constraint, neither MAP (\(s = 150 \mu K\sqrt{\sec}\)) [5] nor Planck (\(s = 35 \mu K\sqrt{\sec}\)) [6] will be able to detect any such left-right asymmetry, but a post-Planck experiment might conceivably be able to discriminate a value as small as \(\epsilon \sim 0.08\). With the cosmological parameters used, this value of \(\epsilon\) corresponds to having the \(P\) and \(T\) violating physics occur at the scale \(\mu \sim 4 \times 10^{-3} M_p\). This discussion is in some sense conservative since we have not considered the additional information provided.
by the $C_l^{TC}$ moments or the improved sensitivity possible with a deeper map of a smaller region of sky.

Similarly, the dashed curve in Fig. 2 shows the smallest rotation angle $\Delta \alpha$ induced by the term in Eq. (1) that could be distinguished from a null result at the $1\sigma$ level as a function of $s$. Again, the underlying cosmological parameters are taken to be those used in Fig. 1. Note that with no tensor perturbations, there is no cosmic-variance limit to the detectability of $\Delta \alpha$. 

The correlation between the elongation axes and polarization vectors of distant radio galaxies and quasars can put constraints on $\Delta \alpha$ at the order of $1^\circ$ [3]. Figure 2 shows that the Planck Surveyor is slightly more sensitive, while a future high-precision CMB polarization could provide much better sensitivity, e.g., $\Delta \alpha \lesssim 0.01^\circ$ for $s \lesssim 1 \mu K/\text{sec}$. Moreover, radio sources only probe the motion of the scalar field between now and redshifts of a few, whereas the CMB probes the motion of the scalar field out to redshifts $z \approx 1100$. Thus, the CMB should provide a better probe of models such as quintessence models with a tracking solution [13], in which the scalar field is expected to do most of its rolling at early times.

Nonzero $C_l^{TC}$ can similarly be induced by Faraday rotation due to intervening magnetic fields [14]. However, Faraday rotation depends on the CMB photon frequency [15], whereas the effects we are considering are frequency independent. Furthermore, Faraday rotation is an anisotropic effect, so it affects the $l$ dependence of the $C_l^{TC}$ (unless the magnetic field is very homogeneous, in which case only the very lowest-$l$ modes would be affected).

Should inflationary or quintessence physics be $P$ and $T$ violating, these effects should in general be present, and if detected, would provide a valuable window to cosmological physics. There may be other sources of parity breaking in addition to those we discuss that would engender the CMB signature considered. A dedicated CMB polarization experiment would be poised to yield a wealth of new information about the early Universe.

We have shown here that such observations would also be capable of providing unique tests of exotic $P$ violation.

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FIG. 2. The smallest $\epsilon$ for the (GW model) and $\Delta \alpha$ (for the polarization rotation model) detectable at the $1\sigma$ level with a one-year CMB temperature/polarization experiment with detector sensitivity $s$. For this calculation, a beamwidth of $0.1^\circ$ is assumed (although results for $\epsilon$ are roughly the same for a beamwidth as large as $0.5^\circ$).