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ON THE QUANTITATIVE THEORY OF AN ELECTROSTATIC VOLTAGE MULTIPLIER

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Abstract

The detailed quantitative theory of a symmetrical, four carrier, electrostatic voltage multiplier is developed. It is shown that a machine of this type steps up a given applied d.c. potential in a definite ratio, and a formula for this (transformation) ratio is deduced in terms of the coefficients of capacity and the coefficients of induction of the elements of the machine in a definite configuration. A formula which gives the potential of the receivers every geometrical cycle (quarter turn) is deduced. While the current furnished by a device of this sort is extremely small, it may be used separately to excite an ordinary static machine, so that this machine will give a constant current at a constant high potential.

It has been pointed out in a previous paper[2] that the method of attack used in developing the quantitative theory of the static machine can be applied also to an electrostatic voltage multiplier.[3] The present work is concerned with the detailed analysis of a simple machine of this type.

The machine studied is represented diagramatically in Fig. 1. It consists of two inductors 7, 8, on which are impressed the constant voltages $+E$ and $-E$ respectively; two receivers 5, 6; and four revolving carriers 1, 2, 3, 4. The neutralizing rod 9 and the midpoint of the battery are connected together and grounded.

The action of the machine is seen readily from the figure. When the carriers are in the positions 1, 2, a charge of sign opposite to that of the inductor near which it happens to be is induced on each carrier, these charges are then carried forward and shared with the receivers, and so forth.

In practice such a device will charge the receivers to a difference of potential which is much greater than that applied to the inductors, and so is a sort of direct current transformer, although actually the increased potential comes from the work done in separating charges of opposite signs, i.e. is the result of a generating action rather than of a transformer action.

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[2] A. W. Simon, Phys. Rev. **26**, 111 (1925).
[3] A. W. Simon, J. Opt. Soc. Am. **10**, 669 (1925).

Our problem is to follow the action of this machine quantitatively. If we use the same notation as that employed in previous papers,[4] with the one modification that instead of denoting the geometrical cycle by a subscript we write it as the argument of the functions Q and V, we have as the fundamental equations for the charges for two successive geometrical cycles:

$$Q_5(n+1)+Q_3(n+1)=Q_5(n)+Q_1(n) \tag{1}$$

$$Q_6(n+1)+Q_4(n+1)=Q_6(n)+Q_2(n) \ . \tag{2}$$

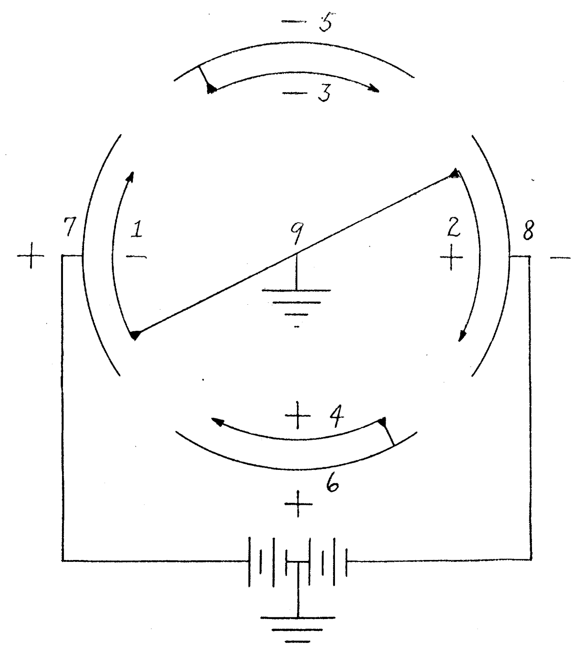

Fig. 1. A four-carrier, direct electrostatic voltage multiplier of the symmetrical type.

If now we substitute for each of the Q's its value in terms of the corresponding potentials and electric coefficients, and note that we have two-fold electrical symmetry, we obtain:

$$aV_3(n+1)+bV_5(n+1)+cE=a'V_3(n)+b'V_4(n)+c'E \tag{3}$$

$$bV_3(n+1)+aV_5(n+1)-cE=b'V_3(n)+a'V_4(n)-c'E \tag{4}$$

[4] For details both as to notation and general method employed in this paper, the reader must be referred to the author's previous work on the theory of the static machine, in particular to Phys. Rev. **24,** 690 (1924); **25,** 368 (1925); **26,** 111 (1925).

where a, b, c, a', b', c', are given by:

$$a = a_{35,35} = a_{64,64} \qquad a' = a_{35,51} = a_{64,62}$$
$$b = a_{64,35} = a_{35,64} \qquad b' = a_{64,51} = a_{35,62}$$
$$c = a_{7,35} - a_{8,35} = a_{7,64} - a_{8,64} = 0 \qquad c' = a_{7,51} - a_{8,51} = a_{7,62} - a_{8,62} \;.$$

The solution of Eqs. (3) and (4) is:

$$V_3(n) = C_1 r_1^n + C_2 r_2^n + C_3 \tag{5}$$
$$V_4(n) = C_1' r_1^n + C_2' r_2^n + C_3' \;, \tag{6}$$

where the C's and r's are given by the following:

$$C_1 = C_1' = \tfrac{1}{2}(V_3(0) + V_4(0)) \tag{7}$$
$$C_2 = -C_2' = \tfrac{1}{2}(V_3(0) - V_4(0)) - \rho E \tag{8}$$
$$C_3 = -C_3' = -\rho E = -(a_{71} - a_{81})E/(a_{35,3} - a_{35,4}) \;. \tag{9}$$
$$r_1 = (a_{35,51} + a_{35,62})/(a_{35,35} + a_{35,64}) \tag{10}$$
$$r_2 = (a_{35,51} - a_{35,62})/(a_{35,35} - a_{35,64}) \;. \tag{11}$$

With regard to the magnitudes of r_1 and r_2, it is possible to prove the following relation from the general properties of electric coefficients:

$$0 < r_1 < r_2 < 1$$

Let us now turn to the physical interpretation of these results. Taking account of (5), (6), (9), and (12), we note that both $V_3(n)$ and $V_4(n)$ approach constant values as n approaches infinity. In particular, we have:

$$\begin{aligned} &\lim_{n\to\infty} V_3(n) = V_3(\infty) = -\rho E \\ &\lim_{n\to\infty} V_4(n) = V_4(\infty) = +\rho E \;. \end{aligned} \tag{13}$$

The quantity ρ represents the transformation ratio of the machine, and it is of interest to investigate on what this ratio depends and how it may be varied. For this purpose Eq. (9) is rewritten as follows:

$$\rho = -(a_{71} - a_{81})/(a_{33} + a_{53} - a_{43} - a_{63}) \;. \tag{14}$$

In order to make ρ large, we must surround the carrier as completely as possible in position 1 thus making a_{71} large and a_{81} very small; we must also surround the carrier as completely as possible in position 3, thus making a_{53} almost equal in absolute value to a_{33}, and both a_{43}

and a_{63} small.[5] In the limit, if the carrier in position 3 were completely surrounded, i.e. if $a_{33}=-a_{53}$, and $a_{43}=a_{63}=0$, the transformation ratio would become infinite. It is interesting to note that this would occur even though the carrier were not surrounded in position 1. In this respect our machine is an exceptional case, for ordinarily to produce an infinite ratio, the carriers must be surrounded completely in both positions. The explanation in our case is that since the potential of the receivers eventually become equal and opposite Eq. (13) their effect on the carrier when the latter receives its charge (position 1) is exactly zero, so that no matter how highly the receivers become charged, they do not counteract the effect of the inductors.

Another important point to note is that the capacity coefficient a_{55} does not appear in the expression for ρ Eq. (14). This means that the transformation ratio does not depend on the capacity of the receivers or any capacities connected to the receivers. In other words, the transformation ratio is independent of the load.

Let us next inquire how the machine builds up. For this purpose Eqs. (5), (6), (7), (8), are of value. We can assume of course that the receivers are originally charged to any potentials whatever. However, two cases are of special interest. (1) The receivers 5, 6, are originally charged to potentials $-\rho E$ and $+\rho E$ respectively. For this case we obtain $C_1=C_1'=C_2=C_2'=0$, i.e. no change takes place, as we should expect. (2) The receivers originally have equal but opposite potentials. For this case we obtain $C_1=C_1'=0$, i.e. only the r_2 terms are effective in building up. The r_1 terms therefore represent the tendency for the two receivers to attain equal and opposite potentials, and this occurs even before these potentials have reached their final values, for from inequality (12) it is seen that the r_1 terms represent transients of shorter duration than the r_2 terms.

We have so far spoken of the machine as building up. In particular, however, if the original potentials are higher than the equilibrium (final) potentials, it is also evident from the equations that the machine will build "down," again with an r_1 transient, if the original potentials are not equal and opposite, or none if they are, etc.

In conclusion a word must be said with regard to the currents furnished by devices of this sort. Strictly speaking, a device of the type just discussed can be used only for charging a condenser, not for furnishing steady currents through a resistance. But even for capacity

[5] It is of course always assumed that the geometrical symmetry of the machine is preserved, i.e., that the carrier in position 2 is treated exactly the same as that in position 1, the carrier in position 4 exactly the same as that in position 3, and so on.

loads, due to the fact that the capacities involved in the machine itself are small, the currents attained are usually of the order of microamperes, so that a device of this sort is inferior even to the ordinary static machine. However, a modified device can easily be constructed, which will not be inferior to the static machine in this respect, and, furthermore, furnish a constant and a controllable potential across a (high) resistance. For this purpose we have merely to impress the constant high potential furnished by an electrostatic voltage multiplier on the inductors of a static machine of the Holtz type (the inductor brushes being removed) and connect the resistance across the terminals. By this means not only more or less continuous currents can be produced, depending on the size of the sectors of the static machine and the resistance of the load, but also these currents are comparable in magnitude with those furnished by the ordinary static machine. Used in this way, the static machine becomes essentially a separated excited electrostatic generator, and as such has the advantages that it is a constant potential machine and that its output voltage can be controlled. So far no experiments along this line have been made, but they would be of interest.

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