A THREE INDUCTOR STATIC MACHINE

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Abstract
The quantitative theory of a three inductor static machine of the Holtz type is given. It is shown that such a machine is essentially a three phase electrostatic alternator, and that it can be star or mesh connected. A method for demonstrating the action of the machine visually by means of the glow in discharge tubes is described.

Multiple inductor static machines of the Holtz type have been studied very little before either theoretically or experimentally. Holtz constructed a machine of this type with twenty inductors, and found that the quantity of electricity was greatly increased but that such a machine was very liable to reverse its polarity. The latter point is significant in the light of our present considerations.

![Diagram](image)

Fig. 1. A three inductor Holtz machine, Y connected.

Since the ordinary Toepler Holtz static machine is a two inductor machine, it is logical in the study of multiple inductor machines to consider next a three inductor machine of this type, and this is the object of our present paper.

A three inductor static machine of the Holtz type is illustrated in Fig. 1. It consists of three fixed inductors 1, 4, 7; and six revolving carriers 2, 3, 5, 6, 8, 9; and operates in a manner readily seen from the figure. In order to simplify the theory we shall assume that the three neutralizing rods are connected together and grounded.

1 National Research Fellow.
2 V. E. Johnson, "High Speed Influence Machines," p. 115.
I. Theory of a three Inductor Static Machine

If we use the same notation and general method explained in detail in a previous paper, we have as the fundamental equations for the charges:

\[
\begin{align*}
Q_1(n+1) + Q_2(n+1) &= Q_1(n) + Q_3(n), \\
Q_2(n+1) + Q_3(n+1) &= Q_4(n) + Q_5(n), \\
Q_3(n+1) + Q_4(n+1) &= Q_5(n) + Q_6(n).
\end{align*}
\]

If next we eliminate the charges in the usual way by substituting for each charge its value in terms of the electric coefficients and the corresponding potentials, we have a set of three equations, in which the matrix of the coefficients of the left members (which involve as unknowns \(V_1(n+1), V_4(n+1), V_5(n+1)\), in order), is conveniently represented as follows:

\[
\begin{array}{ccc}
1,2 & 4,5 & 7,8 \\
1,2 & a & c \\
4,5 & b & a \\
7,8 & c & b \\
\end{array}
\]

and the matrix of the coefficients of the right members (which involve as unknowns \(V_2(n), V_4(n), V_5(n)\)) similarly by:

\[
\begin{array}{ccc}
1,2 & 4,5 & 7,8 \\
1,9 & a' & c' \\
4,3 & b' & a' \\
7,6 & c' & b' \\
\end{array}
\]

The definition of this representation is as follows:

Any cell of the square represents the coefficient occupying the corresponding place in the matrix of the coefficients of the potential equations; and this coefficient is obtained in terms of the fundamental electric coefficients (coefficients of capacity and of induction) by combining the numbers at the head of the column with the numbers at the head of the row in which the coefficient stands according to the notation:

\[
a_{p_{x}, v_{y}} = a_{px} + a_{py} + a_{qy} + a_{qs}
\]

the significance of which has already been given in a previous paper.

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4 A. W. Simon, loc.

In order to solve these equations we make use of another matrix

\[
\begin{array}{ccc}
\omega_0^0 & \omega_2^1 & \omega_2^2 \\
\omega_1^0 & \omega_1^1 & \omega_1^2 \\
\omega_0^1 & \omega_1^0 & \omega_2^1 \\
\end{array}
\]

where \(\omega_j\) is the cube root of unity corresponding to an amplitude of \(j2\pi/3\). The detailed solution is carried out in accordance with the method already outlined in the preceding paper for the case of an eight inductor electrostatic alternator. We have, provided the initial potentials satisfy the relations:

\[ V_1(0) = V_1(0) \cos(0), \quad V_2(0) = V_1(0) \cos(-2\pi/3), \quad V_3(0) = V_1(0) \cos(-4\pi/3), \]

as the solutions:

\[ V_3(n) = \rho^n V_1(0) \cos(n\theta - 0), \]
\[ V_2(n) = \rho^n V_1(0) \cos(n\theta - 2\pi/3), \]
\[ V_1(n) = \rho^n V_1(0) \cos(n\theta - 4\pi/3). \]

From these equations we see directly that a three inductor static machine of the Holtz type is an electrostatic alternator; in particular, it is essentially a three phase electrostatic alternator—a most remarkable result. This conclusion also shows that the Holtz machine of the ordinary two inductor type really belongs to a family of electrostatic alternators, and is a direct current machine only by accident, so to speak. The fundamental matrix of the Holtz machine is

\[
\begin{array}{cc}
\omega_0^0 & \omega_0^1 \\
\omega_1^0 & \omega_1^1 \\
\end{array}
\]

where \(\omega_0, \omega_1\) are square roots of unity with an amplitude of 0 and \(\pi\) respectively, and are therefore both real instead of complex, being equal to 1 and \(-1\), respectively.

It is interesting to inquire whether the analogy of a three phase alternator can be carried farther to include star and mesh (\(Y\) and \(\delta\)) connections. The answer is that it can (if resistances are inserted in the brush circuits) the case illustrated in Fig. 1 being a star connection, while the case given in Fig. 2 would represent a mesh connection.

II. EXPERIMENTAL VERIFICATION OF THE THEORY

Experimentally these two connections can be demonstrated very strikingly in the following way: Between each brush and the neutral point in the case of the star connection, or between each pair of ad-
jacent brushes in the case of the mesh connection, straight discharge
tubes, (neon, helium, or nitrogen) are inserted, and shortcircuited
for a moment until the machine has built up. The fact that the polarity
reverses, as well as the fact that a wave travels around the machine,
can then be detected readily by the glow in the tubes. Particularly
striking is the delta connection, in which the tubes light up in suc-
cession, and a wave can be seen to travel around the circuit formed by
the delta.

Fig. 2. A three inductor Holtz machine, delta connected.

The machine just discussed corresponds, of course, to a Holtz
machine with collector brushes and Leyden jars removed, and a dis-
charge tube inserted in the neutralizing rod. Vice versa, we can add
neutralizing rods, and Leyden jars to the three inductor machine,
and perform a variety of new experiments.

The theory and experiments just given can be extended to ma-
chines with more than three inductors and various types of con-
nections.

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