

1

INTRODUCTION TO MULTIPHASE FLOW

1.1 INTRODUCTION

1.1.1 *Scope*

In the context of this book, the term *multiphase flow* is used to refer to any fluid flow consisting of more than one phase or component. For brevity and because they are covered in other texts, we exclude those circumstances in which the components are well mixed above the molecular level. Consequently, the flows considered here have some level of phase or component separation at a scale well above the molecular level. This still leaves an enormous spectrum of different multiphase flows. One could classify them according to the state of the different phases or components and therefore refer to gas/solids flows, or liquid/solids flows or gas/particle flows or bubbly flows and so on; many texts exist that limit their attention in this way. Some treatises are defined in terms of a specific type of fluid flow and deal with low Reynolds number suspension flows, dusty gas dynamics and so on. Others focus attention on a specific application such as slurry flows, cavitating flows, aerosols, debris flows, fluidized beds and so on; again there are many such texts. In this book we attempt to identify the basic fluid mechanical phenomena and to illustrate those phenomena with examples from a broad range of applications and types of flow.

Parenthetically, it is valuable to reflect on the diverse and ubiquitous challenges of multiphase flow. Virtually every processing technology must deal with multiphase flow, from cavitating pumps and turbines to electrophotographic processes to papermaking to the pellet form of almost all raw plastics. The amount of granular material, coal, grain, ore, etc. that is transported every year is enormous and, at many stages, that material is required to flow. Clearly the ability to predict the fluid flow behavior of these processes is central to the efficiency and effectiveness of those processes. For

example, the effective flow of toner is a major factor in the quality and speed of electrophotographic printers. Multiphase flows are also a ubiquitous feature of our environment whether one considers rain, snow, fog, avalanches, mud slides, sediment transport, debris flows, and countless other natural phenomena to say nothing of what happens beyond our planet. Very critical biological and medical flows are also multiphase, from blood flow to semen to *the bends* to lithotripsy to laser surgery cavitation and so on. No single list can adequately illustrate the diversity and ubiquity; consequently any attempt at a comprehensive treatment of multiphase flows is flawed unless it focuses on common phenomenological themes and avoids the temptation to digress into lists of observations.

Two general topologies of multiphase flow can be usefully identified at the outset, namely *disperse flows* and *separated flows*. By *disperse flows* we mean those consisting of finite particles, drops or bubbles (the disperse phase) distributed in a connected volume of the continuous phase. On the other hand *separated flows* consist of two or more continuous streams of different fluids separated by interfaces.

1.1.2 Multiphase flow models

A persistent theme throughout the study of multiphase flows is the need to model and predict the detailed behavior of those flows and the phenomena that they manifest. There are three ways in which such models are explored: (1) experimentally, through laboratory-sized models equipped with appropriate instrumentation, (2) theoretically, using mathematical equations and models for the flow, and (3) computationally, using the power and size of modern computers to address the complexity of the flow. Clearly there are some applications in which full-scale laboratory models are possible. But, in many instances, the laboratory model must have a very different scale than the prototype and then a reliable theoretical or computational model is essential for confident extrapolation to the scale of the prototype. There are also cases in which a laboratory model is impossible for a wide variety of reasons.

Consequently, the predictive capability and physical understanding must rely heavily on theoretical and/or computational models and here the complexity of most multiphase flows presents a major hurdle. It may be possible at some distant time in the future to code the Navier-Stokes equations for each of the phases or components and to compute every detail of a multiphase flow, the motion of all the fluid around and inside every particle or drop, the position of every interface. But the computer power and speed

required to do this is far beyond present capability for most of the flows that are commonly experienced. When one or both of the phases becomes turbulent (as often happens) the magnitude of the challenge becomes truly astronomical. Therefore, simplifications are essential in realistic models of most multiphase flows.

In disperse flows two types of models are prevalent, *trajectory models* and *two-fluid models*. In trajectory models, the motion of the disperse phase is assessed by following either the motion of the actual particles or the motion of larger, representative *particles*. The details of the flow around each of the particles are subsumed into assumed drag, lift and moment forces acting on and altering the trajectory of those particles. The thermal history of the particles can also be tracked if it is appropriate to do so. Trajectory models have been very useful in studies of the rheology of granular flows (see chapter 13) primarily because the effects of the interstitial fluid are small. In the alternative approach, *two-fluid models*, the disperse phase is treated as a second continuous phase intermingled and interacting with the continuous phase. Effective conservation equations (of mass, momentum and energy) are developed for the two fluid flows; these included interaction terms modeling the exchange of mass, momentum and energy between the two flows. These equations are then solved either theoretically or computationally. Thus, the two-fluid models neglect the discrete nature of the disperse phase and approximate its effects upon the continuous phase. Inherent in this approach, are averaging processes necessary to characterize the properties of the disperse phase; these involve significant difficulties. The boundary conditions appropriate in two-fluid models also pose difficult modeling issues.

In contrast, separated flows present many fewer issues. In theory one must solve the single phase fluid flow equations in the two streams, coupling them through appropriate kinematic and dynamic conditions at the interface. Free streamline theory (see, for example, Birkhoff and Zarantonello 1957, Tulin 1964, Woods 1961, Wu 1972) is an example of a successful implementation of such a strategy though the interface conditions used in that context are particularly simple.

In the first part of this book, the basic tools for both trajectory and two-fluid models are developed and discussed. In the remainder of this first chapter, a basic notation for multiphase flow is developed and this leads naturally into a description of the mass, momentum and energy equations applicable to multiphase flows, and, in particular, in two-fluid models. In chapters 2, 3 and 4, we examine the dynamics of individual particles, drops and bubbles. In chapter 7 we address the different topologies of multiphase

flows and, in the subsequent chapters, we examine phenomena in which *particle* interactions and the particle-fluid interactions modify the flow.

1.1.3 Multiphase flow notation

The notation that will be used is close to the standard described by Wallis (1969). It has however been slightly modified to permit more ready adoption to the Cartesian tensor form. In particular the subscripts that can be attached to a property will consist of a group of uppercase subscripts followed by lowercase subscripts. The lower case subscripts (i , ij , etc.) are used in the conventional manner to denote vector or tensor components. A single uppercase subscript (N) will refer to the property of a specific phase or component. In some contexts generic subscripts $N = A, B$ will be used for generality. However, other letters such as $N = C$ (continuous phase), $N = D$ (disperse phase), $N = L$ (liquid), $N = G$ (gas), $N = V$ (vapor) or $N = S$ (solid) will be used for clarity in other contexts. Finally two uppercase subscripts will imply the difference between the two properties for the two single uppercase subscripts.

Specific properties frequently used are as follows. *Volumetric fluxes* (volume flow per unit area) of individual components will be denoted by j_{Ai} , j_{Bi} ($i = 1, 2$ or 3 in three dimensional flow). These are sometimes referred to as superficial component velocities. The *total volumetric flux*, j_i is then given by

$$j_i = j_{Ai} + j_{Bi} + \dots = \sum_N j_{Ni} \quad (1.1)$$

Mass fluxes are similarly denoted by G_{Ai} , G_{Bi} or G_i . Thus if the densities of individual components are denoted by ρ_A, ρ_B it follows that

$$G_{Ai} = \rho_A j_{Ai} ; G_{Bi} = \rho_B j_{Bi} ; G_i = \sum_N \rho_N j_{Ni} \quad (1.2)$$

Velocities of the specific phases are denoted by u_{Ai} , u_{Bi} or, in general, by u_{Ni} . The relative velocity between the two phases A and B will be denoted by u_{ABi} such that

$$u_{Ai} - u_{Bi} = u_{ABi} \quad (1.3)$$

The volume fraction of a component or phase is denoted by α_N and, in the case of two components or phases, A and B , it follows that $\alpha_B = 1 - \alpha_A$. Though this is clearly a well defined property for any finite volume in the flow, there are some substantial problems associated with assigning a

value to an infinitesimal volume or point in the flow. Provided these can be resolved, it follows that the volumetric flux of a component, N , and its velocity are related by

$$j_{Ni} = \alpha_N u_{Ni} \quad (1.4)$$

and that

$$j_i = \alpha_A u_{Ai} + \alpha_B u_{Bi} + \dots = \sum_N \alpha_N u_{Ni} \quad (1.5)$$

Two other fractional properties are only relevant in the context of one-dimensional flows. The *volumetric quality*, β_N , is the ratio of the volumetric flux of the component, N , to the total volumetric flux, i.e.

$$\beta_N = j_N / j \quad (1.6)$$

where the index i has been dropped from j_N and j because β is only used in the context of one-dimensional flows and the j_N , j refer to cross-sectionally averaged quantities.

The *mass fraction*, x_A , of a phase or component, A , is simply given by $\rho_A \alpha_A / \rho$ (see equation 1.8 for ρ). On the other hand the *mass quality*, \mathcal{X}_A , is often referred to simply as *the quality* and is the ratio of the mass flux of component, A , to the total mass flux, or

$$\mathcal{X}_A = \frac{G_A}{G} = \frac{\rho_A j_A}{\sum_N \rho_N j_N} \quad (1.7)$$

Furthermore, when only two components or phases are present it is often redundant to use subscripts on the volume fraction and the qualities since $\alpha_A = 1 - \alpha_B$, $\beta_A = 1 - \beta_B$ and $\mathcal{X}_A = 1 - \mathcal{X}_B$. Thus unsubscripted quantities α , β and \mathcal{X} will often be used in these circumstances.

It is clear that a multiphase mixture has certain *mixture* properties of which the most readily evaluated is the *mixture* density denoted by ρ and given by

$$\rho = \sum_N \alpha_N \rho_N \quad (1.8)$$

On the other hand the specific enthalpy, h , and specific entropy, s , being defined as per unit mass rather than per unit volume are weighted according to

$$\rho h = \sum_N \rho_N \alpha_N h_N ; \quad \rho s = \sum_N \rho_N \alpha_N s_N \quad (1.9)$$

Other properties such as the *mixture* viscosity or thermal conductivity cannot be reliably obtained from such simple weighted means.

Aside from the relative velocities between phases that were described earlier, there are two other measures of relative motion that are frequently used. The *drift velocity* of a component is defined as the velocity of that component in a frame of reference moving at a velocity equal to the total volumetric flux, j_i , and is therefore given by, u_{NJi} , where

$$u_{NJi} = u_{Ni} - j_i \quad (1.10)$$

Even more frequent use will be made of the *drift flux* of a component which is defined as the volumetric flux of a component in the frame of reference moving at j_i . Denoted by j_{NJi} this is given by

$$j_{NJi} = j_{Ni} - \alpha_N j_i = \alpha_N (u_{Ni} - j_i) = \alpha_N u_{NJi} \quad (1.11)$$

It is particularly important to notice that the sum of all the drift fluxes must be zero since from equation 1.11

$$\sum_N j_{NJi} = \sum_N j_{Ni} - j_i \sum_N \alpha_N = j_i - j_i = 0 \quad (1.12)$$

When only two phases or components, A and B , are present it follows that $j_{AJi} = -j_{BJi}$ and hence it is convenient to denote both of these drift fluxes by the vector j_{ABi} where

$$j_{ABi} = j_{AJi} = -j_{BJi} \quad (1.13)$$

Moreover it follows from 1.11 that

$$j_{ABi} = \alpha_A \alpha_B u_{ABi} = \alpha_A (1 - \alpha_A) u_{ABi} \quad (1.14)$$

and hence the drift flux, j_{ABi} and the relative velocity, u_{ABi} , are simply related.

Finally, it is clear that certain basic relations follow from the above definitions and it is convenient to identify these here for later use. First the relations between the volume and mass qualities that follow from equations 1.6 and 1.7 only involve ratios of the densities of the components:

$$\mathcal{X}_A = \beta_A / \sum_N \left(\frac{\rho_N}{\rho_A} \right) \beta_N \quad ; \quad \beta_A = \mathcal{X}_A / \sum_N \left(\frac{\rho_A}{\rho_N} \right) \mathcal{X}_N \quad (1.15)$$

On the other hand the relation between the volume fraction and the volume quality necessarily involves some measure of the relative motion between the phases (or components). The following useful results for two-phase (or

two-component) one-dimensional flows can readily be obtained from 1.11 and 1.6

$$\beta_N = \alpha_N + \frac{j_{NJ}}{j} ; \beta_A = \alpha_A + \frac{j_{AB}}{j} ; \beta_B = \alpha_B - \frac{j_{AB}}{j} \quad (1.16)$$

which demonstrate the importance of the drift flux as a measure of the relative motion.

1.1.4 Size distribution functions

In many multiphase flow contexts we shall make the simplifying assumption that all the disperse phase particles (bubbles, droplets or solid particles) have the same size. However in many natural and technological processes it is necessary to consider the distribution of particle size. One fundamental measure of this is the size distribution function, $N(v)$, defined such that the number of particles in a unit volume of the multiphase mixture with volume between v and $v + dv$ is $N(v)dv$. For convenience, it is often assumed that the particles size can be represented by a single linear dimension (for example, the diameter, D , or radius, R , in the case of spherical particles) so that alternative size distribution functions, $N'(D)$ or $N''(R)$, may be used. Examples of size distribution functions based on radius are shown in figures 1.1 and 1.2.

Often such information is presented in the form of cumulative number distributions. For example the cumulative distribution, $N^*(v^*)$, defined as

$$N^*(v^*) = \int_0^{v^*} N(v)dv \quad (1.17)$$

is the total number of particles of volume less than v^* . Examples of cumulative distributions (in this case for coal slurries) are shown in figure 1.3.

In these disperse flows, the evaluation of global quantities or characteristics of the disperse phase will clearly require integration over the full range of particle sizes using the size distribution function. For example, the volume fraction of the disperse phase, α_D , is given by

$$\alpha_D = \int_0^\infty v N(v)dv = \frac{\pi}{6} \int_0^\infty D^3 N'(D)dD \quad (1.18)$$

where the last expression clearly applies to spherical particles. Other properties of the disperse phase or of the interactions between the disperse and continuous phases can involve other moments of the size distribution function (see, for example, Friedlander 1977). This leads to a series of mean

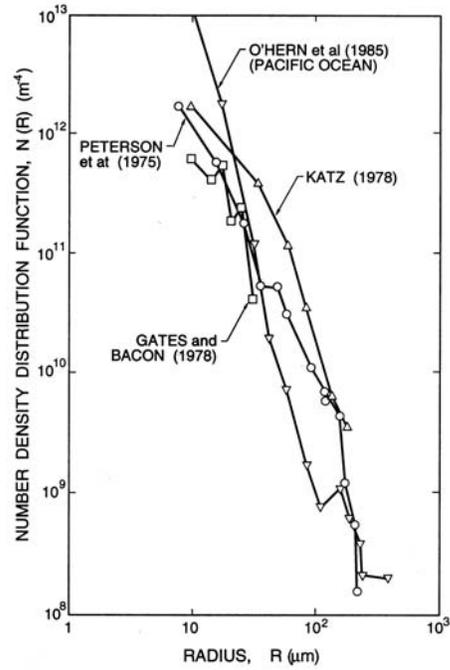


Figure 1.1. Measured size distribution functions for small bubbles in three different water tunnels (Peterson *et al.* 1975, Gates and Bacon 1978, Katz 1978) and in the ocean off Los Angeles, Calif. (O'Hern *et al.* 1985).

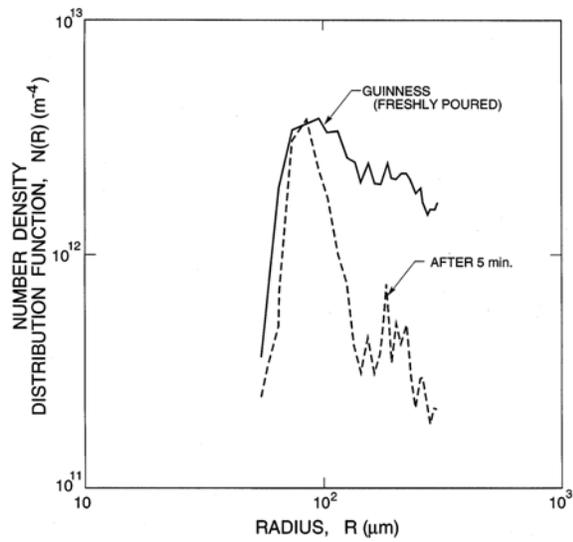


Figure 1.2. Size distribution functions for bubbles in freshly poured Guinness and after five minutes. Adapted from Kawaguchi and Maeda (2003).

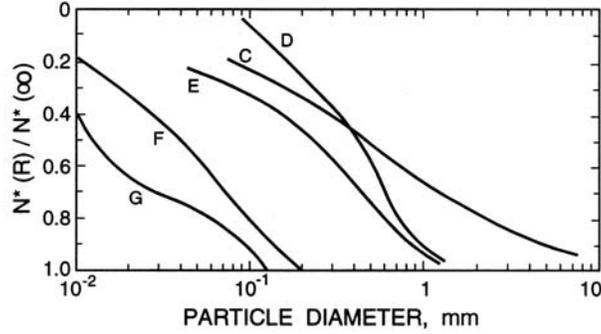


Figure 1.3. Cumulative size distributions for various coal slurries. Adapted from Shook and Roco (1991).

diameters (or sizes in the case of non-spherical particles) of the form, D_{jk} , where

$$D_{jk} = \left[\frac{\int_0^\infty D^j N'(D) dD}{\int_0^\infty D^k N'(D) dD} \right]^{\frac{1}{j-k}} \quad (1.19)$$

A commonly used example is the *mass mean* diameter, D_{30} . On the other hand processes that are controlled by particle surface area would be characterized by the *surface area mean* diameter, D_{20} . The surface area mean diameter would be important, for example, in determining the exchange of heat between the phases or the rates of chemical interaction at the disperse phase surface. Another measure of the average size that proves useful in characterizing many disperse particulates is the Sauter mean diameter, D_{32} . This is a measure of the ratio of the particle volume to the particle surface area and, as such, is often used in characterizing particulates (see, for example, chapter 14).

1.2 EQUATIONS OF MOTION

1.2.1 Averaging

In the section 1.1.3 it was implicitly assumed that there existed an *infinitesimal* volume of dimension, ϵ , such that ϵ was not only very much smaller than the typical distance over which the flow properties varied significantly but also very much larger than the size of the individual phase elements (the disperse phase particles, drops or bubbles). The first condition is necessary in order to define derivatives of the flow properties within the flow field. The second is necessary in order that each *averaging* volume (of volume ϵ^3)

contain representative samples of each of the components or phases. In the sections that follow (sections 1.2.2 to 1.2.9), we proceed to develop the effective differential equations of motion for multiphase flow assuming that these conditions hold.

However, one of the more difficult hurdles in treating multiphase flows, is that the above two conditions are rarely both satisfied. As a consequence the averaging volumes contain a finite number of finite-sized particles and therefore flow properties such as the continuous phase velocity vary significantly from point to point within these averaging volumes. These variations pose the challenge of how to define appropriate average quantities in the averaging volume. Moreover, the gradients of those averaged flow properties appear in the equations of motion that follow and the mean of the gradient is not necessarily equal to the gradient of the mean. These difficulties will be addressed in section 1.4 after we have explored the basic structure of the equations in the absence of such complications.

1.2.2 Continuum equations for conservation of mass

Consider now the construction of the effective differential equations of motion for a disperse multiphase flow (such as might be used in a two-fluid model) assuming that an appropriate elemental volume can be identified. For convenience this elemental volume is chosen to be a *unit* cube with edges parallel to the x_1, x_2, x_3 directions. The mass flow of component N through one of the faces perpendicular to the i direction is given by $\rho_N j_{Ni}$ and therefore the net outflow of mass of component N from the cube is given by the divergence of $\rho_N j_{Ni}$ or

$$\frac{\partial(\rho_N j_{Ni})}{\partial x_i} \quad (1.20)$$

The rate of increase of the mass of component N stored in the elemental volume is $\partial(\rho_N \alpha_N)/\partial t$ and hence conservation of mass of component N requires that

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{\partial(\rho_N j_{Ni})}{\partial x_i} = \mathcal{I}_N \quad (1.21)$$

where \mathcal{I}_N is the rate of transfer of mass to the phase N from the other phases per unit total volume. Such mass exchange would result from a phase change or chemical reaction. This is the first of several phase interaction terms that will be identified and, for ease of reference, the quantities \mathcal{I}_N will be termed the *mass interaction terms*.

Clearly there will be a continuity equation like 1.21 for each phase or component present in the flow. They will be referred to as the Individual Phase Continuity Equations (IPCE). However, since mass as a whole must be conserved whatever phase changes or chemical reactions are happening it follows that

$$\sum_N \mathcal{I}_N = 0 \quad (1.22)$$

and hence the sum of all the IPCEs results in a Combined Phase Continuity Equation (CPCE) that does not involve \mathcal{I}_N :

$$\frac{\partial}{\partial t} \left(\sum_N \rho_N \alpha_N \right) + \frac{\partial}{\partial x_i} \left(\sum_N \rho_N j_{Ni} \right) = 0 \quad (1.23)$$

or using equations 1.4 and 1.8:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} \left(\sum_N \rho_N \alpha_N u_{Ni} \right) = 0 \quad (1.24)$$

Notice that only under the conditions of *zero* relative velocity in which $u_{Ni} = u_i$ does this reduce to the Mixture Continuity Equation (MCE) which is identical to that for an equivalent single phase flow of density ρ :

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \quad (1.25)$$

We also record that for one-dimensional duct flow the individual phase continuity equation 1.21 becomes

$$\frac{\partial}{\partial t} (\rho_N \alpha_N) + \frac{1}{A} \frac{\partial}{\partial x} (A \rho_N \alpha_N u_N) = \mathcal{I}_N \quad (1.26)$$

where x is measured along the duct, $A(x)$ is the cross-sectional area, u_N, α_N are cross-sectionally averaged quantities and $A\mathcal{I}_N$ is the rate of transfer of mass to the phase N per unit length of the duct. The sum over the constituents yields the combined phase continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left(A \sum_N \rho_N \alpha_N u_n \right) = 0 \quad (1.27)$$

When all the phases travel at the same speed, $u_N = u$, this reduces to

$$\frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} (\rho A u) = 0 \quad (1.28)$$

Finally we should make note of the form of the equations when the two components or species are intermingled rather than separated since we will

analyze several situations with gases diffusing through one another. Then both components occupy the entire volume and the void fractions are effectively unity so that the continuity equation 1.21 becomes:

$$\frac{\partial \rho_N}{\partial t} + \frac{\partial(\rho_N u_{Ni})}{\partial x_i} = \mathcal{I}_N \quad (1.29)$$

1.2.3 Disperse phase number continuity

Complementary to the equations of conservation of mass are the equations governing the conservation of the number of bubbles, drops, particles, etc. that constitute a disperse phase. If no such *particles* are created or destroyed within the elemental volume and if the number of *particles* of the disperse component, D , per unit total volume is denoted by n_D , it follows that

$$\frac{\partial n_D}{\partial t} + \frac{\partial}{\partial x_i} (n_D u_{Di}) = 0 \quad (1.30)$$

This will be referred to as the Disperse Phase Number Equation (DPNE).

If the volume of the particles of component D is denoted by v_D it follows that

$$\alpha_D = n_D v_D \quad (1.31)$$

and substituting this into equation 1.21 one obtains

$$\frac{\partial}{\partial t} (n_D \rho_D v_D) + \frac{\partial}{\partial x_i} (n_D u_{Di} \rho_D v_D) = \mathcal{I}_D \quad (1.32)$$

Expanding this equation using equation 1.30 leads to the following relation for \mathcal{I}_D :

$$\mathcal{I}_D = n_D \left(\frac{\partial(\rho_D v_D)}{\partial t} + u_{Di} \frac{\partial(\rho_D v_D)}{\partial x_i} \right) = n_D \frac{D_D}{D_D t} (\rho_D v_D) \quad (1.33)$$

where $D_D/D_D t$ denotes the Lagrangian derivative following the disperse phase. This demonstrates a result that could, admittedly, be assumed, a priori. Namely that the rate of transfer of mass to the component D in each particle, \mathcal{I}_D/n_D , is equal to the Lagrangian rate of increase of mass, $\rho_D v_D$, of each particle.

It is sometimes convenient in the study of bubbly flows to write the bubble number conservation equation in terms of a population, η , of bubbles per unit *liquid* volume rather than the number per unit total volume, n_D . Note

that if the bubble volume is v and the volume fraction is α then

$$\eta = \frac{n_D}{(1 - \alpha)} ; n_D = \frac{\eta}{(1 + \eta v)} ; \alpha = \eta \frac{v}{(1 + \eta v)} \quad (1.34)$$

and the bubble number conservation equation can be written as

$$\frac{\partial u_{Di}}{\partial x_i} = -\frac{(1 + \eta v)}{\eta} \frac{D_D}{D_D t} \left(\frac{\eta}{1 + \eta v} \right) \quad (1.35)$$

If the number population, η , is assumed uniform and constant (which requires neglect of slip and the assumption of liquid incompressibility) then equation 1.35 can be written as

$$\frac{\partial u_{Di}}{\partial x_i} = \frac{\eta}{1 + \eta v} \frac{D_D v}{D_D t} \quad (1.36)$$

In other words the divergence of the velocity field is directly related to the Lagrangian rate of change in the volume of the bubbles.

1.2.4 Fick's law

We digress briefly to complete the kinematics of two interdiffusing gases. Equation 1.29 represented the conservation of mass for the two gases in these circumstances. The kinematics are then completed by a statement of Fick's Law which governs the interdiffusion. For the gas, A , this law is

$$u_{Ai} = u_i - \frac{\rho D}{\rho_A} \frac{\partial}{\partial x_i} \left(\frac{\rho_A}{\rho} \right) \quad (1.37)$$

where D is the diffusivity.

1.2.5 Continuum equations for conservation of momentum

Continuing with the development of the differential equations, the next step is to apply the momentum principle to the elemental volume. Prior to doing so we make some minor modifications to that control volume in order to avoid some potential difficulties. Specifically we deform the bounding surfaces so that they never cut through disperse phase particles but everywhere are within the continuous phase. Since it is already assumed that the dimensions of the particles are very small compared with the dimensions of the control volume, the required modification is correspondingly small. It is possible to proceed without this modification but several complications arise. For example, if the boundaries cut through particles, it would then be necessary to determine what fraction of the control volume surface is acted

upon by tractions within each of the phases and to face the difficulty of determining the tractions within the particles. Moreover, we shall later need to evaluate the interacting force between the phases within the control volume and this is complicated by the issue of dealing with the parts of particles intersected by the boundary.

Now proceeding to the application of the momentum theorem for either the disperse ($N = D$) or continuous phase ($N = C$), the flux of momentum of the N component in the k direction through a side perpendicular to the i direction is $\rho_N j_{Ni} u_{Nk}$ and hence the net flux of momentum (in the k direction) out of the elemental volume is $\partial(\rho_N \alpha_N u_{Nk})/\partial x_i$. The rate of increase of momentum of component N in the k direction within the elemental volume is $\partial(\rho_N \alpha_N u_{Nk})/\partial t$. Thus using the momentum conservation principle, the net force in the k direction acting on the component N in the control volume (of unit volume), \mathcal{F}_{Nk}^T , must be given by

$$\mathcal{F}_{Nk}^T = \frac{\partial}{\partial t} (\rho_N \alpha_N u_{Nk}) + \frac{\partial}{\partial x_i} (\rho_N \alpha_N u_{Ni} u_{Nk}) \quad (1.38)$$

It is more difficult to construct the forces, \mathcal{F}_{Nk}^T in order to complete the equations of motion. We must include body forces acting within the control volume, the force due to the pressure and viscous stresses on the exterior of the control volume, and, most particularly, the force that each component imposes on the other components within the control volume.

The first contribution is that due to an external force field on the component N within the control volume. In the case of gravitational forces, this is clearly given by

$$\alpha_N \rho_N g_k \quad (1.39)$$

where g_k is the component of the gravitational acceleration in the k direction (the direction of g is considered vertically downward).

The second contribution, namely that due to the tractions on the control volume, differs for the two phases because of the small deformation discussed above. It is zero for the disperse phase. For the continuous phase we define the stress tensor, σ_{Cki} , so that the contribution from the surface tractions to the force on that phase is

$$\frac{\partial \sigma_{Cki}}{\partial x_i} \quad (1.40)$$

For future purposes it is also convenient to decompose σ_{Cki} into a pressure, $p_C = p$, and a deviatoric stress, σ_{Cki}^D :

$$\sigma_{Cki} = -p \delta_{ki} + \sigma_{Cki}^D \quad (1.41)$$

where δ_{ki} is the Kronecker delta such that $\delta_{ki} = 1$ for $k = i$ and $\delta_{ij} = 0$ for $k \neq i$.

The third contribution to \mathcal{F}_{Nk}^T is the force (per unit total volume) imposed on the component N by the other components within the control volume. We write this as \mathcal{F}_{Nk} so that the Individual Phase Momentum Equation (IPME) becomes

$$\begin{aligned} \frac{\partial}{\partial t} (\rho_N \alpha_N u_{Nk}) + \frac{\partial}{\partial x_i} (\rho_N \alpha_N u_{Ni} u_{Nk}) \\ = \alpha_N \rho_N g_k + \mathcal{F}_{Nk} - \delta_N \left\{ \frac{\partial p}{\partial x_k} - \frac{\partial \sigma_{Cki}^D}{\partial x_i} \right\} \end{aligned} \quad (1.42)$$

where $\delta_D = 0$ for the disperse phase and $\delta_C = 1$ for the continuous phase.

Thus we identify the second of the interaction terms, namely the *force interaction*, \mathcal{F}_{Nk} . Note that, as in the case of the mass interaction \mathcal{I}_N , it must follow that

$$\sum_N \mathcal{F}_{Nk} = 0 \quad (1.43)$$

In disperse flows it is often useful to separate \mathcal{F}_{Nk} into two components, one due to the pressure gradient in the continuous phase, $-\alpha_D \partial p / \partial x_k$, and the remainder, \mathcal{F}'_{Dk} , due to other effects such as the relative motion between the phases. Then

$$\mathcal{F}_{Dk} = -\mathcal{F}_{Ck} = -\alpha_D \frac{\partial p}{\partial x_k} + \mathcal{F}'_{Dk} \quad (1.44)$$

The IPME 1.42 are frequently used in a form in which the terms on the left hand side are expanded and use is made of the continuity equation 1.21. In single phase flow this yields a Lagrangian time derivative of the velocity on the left hand side. In the present case the use of the continuity equation results in the appearance of the mass interaction, \mathcal{I}_N . Specifically, one obtains

$$\begin{aligned} \rho_N \alpha_N \left\{ \frac{\partial u_{Nk}}{\partial t} + u_{Ni} \frac{\partial u_{Nk}}{\partial x_i} \right\} \\ = \alpha_N \rho_N g_k + \mathcal{F}_{Nk} - \mathcal{I}_N u_{Nk} - \delta_N \left\{ \frac{\partial p}{\partial x_k} - \frac{\partial \sigma_{Cki}^D}{\partial x_i} \right\} \end{aligned} \quad (1.45)$$

Viewed from a Lagrangian perspective, the left hand side is the normal rate of increase of the momentum of the component N ; the term $\mathcal{I}_N u_{Nk}$ is the

rate of increase of the momentum in the component N due to the gain of mass by that phase.

If the momentum equations 1.42 for each of the components are added together the resulting Combined Phase Momentum Equation (CPME) becomes

$$\begin{aligned} \frac{\partial}{\partial t} \left(\sum_N \rho_N \alpha_N u_{Nk} \right) + \frac{\partial}{\partial x_i} \left(\sum_N \rho_N \alpha_N u_{Ni} u_{Nk} \right) \\ = \rho g_k - \frac{\partial p}{\partial x_k} + \frac{\partial \sigma_{Cki}^D}{\partial x_i} \end{aligned} \quad (1.46)$$

Note that this equation 1.46 will only reduce to the equation of motion for a single phase flow in the absence of relative motion, $u_{Ck} = u_{Dk}$. Note also that, in the absence of any motion (when the deviatoric stress is zero), equation 1.46 yields the appropriate hydrostatic pressure gradient $\partial p / \partial x_k = \rho g_k$ based on the mixture density, ρ .

Another useful limit is the case of uniform and constant sedimentation of the disperse component (volume fraction, $\alpha_D = \alpha = 1 - \alpha_C$) through the continuous phase under the influence of gravity. Then equation 1.42 yields

$$\begin{aligned} 0 &= \alpha \rho_D g_k + \mathcal{F}_{Dk} \\ 0 &= \frac{\partial \sigma_{Cki}}{\partial x_i} + (1 - \alpha) \rho_C g_k + \mathcal{F}_{Ck} \end{aligned} \quad (1.47)$$

But $\mathcal{F}_{Dk} = -\mathcal{F}_{Ck}$ and, in this case, the deviatoric part of the continuous phase stress should be zero (since the flow is a simple uniform stream) so that $\sigma_{Ckj} = -p$. It follows from equation 1.47 that

$$\mathcal{F}_{Dk} = -\mathcal{F}_{Ck} = -\alpha \rho_D g_k \quad \text{and} \quad \partial p / \partial x_k = \rho g_k \quad (1.48)$$

or, in words, the pressure gradient is hydrostatic.

Finally, note that the equivalent one-dimensional or duct flow form of the IPME is

$$\frac{\partial}{\partial t} (\rho_N \alpha_N u_N) + \frac{1}{A} \frac{\partial}{\partial x} (A \rho_N \alpha_N u_N^2) = -\delta_N \left\{ \frac{\partial p}{\partial x} + \frac{P \tau_w}{A} \right\} + \alpha_N \rho_N g_x + \mathcal{F}_{Nx} \quad (1.49)$$

where, in the usual pipe flow notation, $P(x)$ is the perimeter of the cross-section and τ_w is the wall shear stress. In this equation, $A \mathcal{F}_{Nx}$ is the force imposed on the component N in the x direction by the other components per unit length of the duct. A sum over the constituents yields the combined

phase momentum equation for duct flow, namely

$$\frac{\partial}{\partial t} \left(\sum_N \rho_N \alpha_N u_N \right) + \frac{1}{A} \frac{\partial}{\partial x} \left(A \sum_N \rho_N \alpha_N u_N^2 \right) = -\frac{\partial p}{\partial x} - \frac{P\tau_w}{A} + \rho g_x \quad (1.50)$$

and, when all phases travel at the same velocity, $u = u_N$, this reduces to

$$\frac{\partial}{\partial t} (\rho u) + \frac{1}{A} \frac{\partial}{\partial x} (A \rho u^2) = -\frac{\partial p}{\partial x} - \frac{P\tau_w}{A} + \rho g_x \quad (1.51)$$

1.2.6 Disperse phase momentum equation

At this point we should consider the relation between the equation of motion for an individual particle of the disperse phase and the Disperse Phase Momentum Equation (DPME) delineated in the last section. This relation is analogous to that between the number continuity equation and the Disperse Phase Continuity Equation (DPCE). The construction of the equation of motion for an individual particle in an infinite fluid medium will be discussed at some length in chapter 2. It is sufficient at this point to recognize that we may write Newton's equation of motion for an individual particle of volume v_D in the form

$$\frac{D_D}{D_D t} (\rho_D v_D u_{Dk}) = F_k + \rho_D v_D g_k \quad (1.52)$$

where $D_D/D_D t$ is the Lagrangian time derivative following the particle so that

$$\frac{D_D}{D_D t} \equiv \frac{\partial}{\partial t} + u_{Di} \frac{\partial}{\partial x_i} \quad (1.53)$$

and F_k is the force that the surrounding continuous phase imparts to the particle in the direction k . Note that F_k will include not only the force due to the velocity and acceleration of the particle relative to the fluid but also the *buoyancy* forces due to pressure gradients within the continuous phase. Expanding 1.52 and using the expression 1.33 for the mass interaction, \mathcal{I}_D , one obtains the following form of the DPME:

$$\rho_D v_D \left\{ \frac{\partial u_{Dk}}{\partial t} + u_{Di} \frac{\partial u_{Dk}}{\partial x_i} \right\} + u_{Dk} \frac{\mathcal{I}_D}{n_D} = F_k + \rho_D v_D g_k \quad (1.54)$$

Now examine the implication of this relation when considered alongside the IPME 1.45 for the disperse phase. Setting $\alpha_D = n_D v_D$ in equation 1.45, expanding and comparing the result with equation 1.54 (using the continuity

equation 1.21) one observes that

$$\mathcal{F}_{Dk} = n_D F_k \quad (1.55)$$

Hence the appropriate force interaction term in the disperse phase momentum equation is simply the sum of the fluid forces acting on the individual particles in a unit volume, namely $n_D F_k$. As an example note that the steady, uniform sedimentation interaction force \mathcal{F}_{Dk} given by equation 1.48, when substituted into equation 1.55, leads to the result $F_k = -\rho_D v_D g_k$ or, in words, a fluid force on an individual particle that precisely balances the weight of the particle.

1.2.7 Comments on disperse phase interaction

In the last section the relation between the force interaction term, \mathcal{F}_{Dk} , and the force, F_k , acting on an individual particle of the disperse phase was established. In chapter 2 we include extensive discussions of the forces acting on a single particle moving in a infinite fluid. Various forms of the fluid force, F_k , acting *on* the particle are presented (for example, equations 2.47, 2.49, 2.50, 2.67, 2.71, 3.20) in terms of (a) the particle velocity, $V_k = u_{Dk}$, (b) the fluid velocity $U_k = u_{Ck}$ that would have existed at the center of the particle in the latter's absence and (c) the relative velocity $W_k = V_k - U_k$.

Downstream of some disturbance that creates a relative velocity, W_k , the drag will tend to reduce that difference. It is useful to characterize the rate of equalization of the particle (mass, m_p , and radius, R) and fluid velocities by defining a velocity *relaxation* time, t_u . For example, it is common in dealing with gas flows laden with small droplets or particles to assume that the equation of motion can be approximated by just two terms, namely the particle inertia and a Stokes drag, which for a spherical particle is $6\pi\mu_C R W_k$ (see section 2.2.2). It follows that the relative velocity decays exponentially with a time constant, t_u , given by

$$t_u = m_p / 6\pi R \mu_C \quad (1.56)$$

This is known as the velocity relaxation time. A more complete treatment that includes other parametric cases and other fluid mechanical effects is contained in sections 2.4.1 and 2.4.2.

There are many issues with the equation of motion for the disperse phase that have yet to be addressed. Many of these are delayed until section 1.4 and others are addressed later in the book, for example in sections 2.3.2, 2.4.3 and 2.4.4.

1.2.8 Equations for conservation of energy

The third fundamental conservation principle that is utilized in developing the basic equations of fluid mechanics is the principle of conservation of energy. Even in single phase flow the general statement of this principle is complicated when energy transfer processes such as heat conduction and viscous dissipation are included in the analysis. Fortunately it is frequently possible to show that some of these complexities have a negligible effect on the results. For example, one almost always neglects viscous and heat conduction effects in preliminary analyses of gas dynamic flows. In the context of multiphase flows the complexities involved in a general statement of energy conservation are so numerous that it is of little value to attempt such generality. Thus we shall only present a simplified version that neglects, for example, viscous heating and the global conduction of heat (though not the heat transfer from one phase to another).

However these limitations are often minor compared with other difficulties that arise in constructing an energy equation for multiphase flows. In single-phase flows it is usually adequate to assume that the fluid is in an equilibrium thermodynamic state at all points in the flow and that an appropriate thermodynamic constraint (for example, *constant* and *locally uniform* entropy or temperature) may be used to relate the pressure, density, temperature, entropy, etc. In many multiphase flows the different phases and/or components are often *not* in equilibrium and consequently thermodynamic equilibrium arguments that might be appropriate for single phase flows are no longer valid. Under those circumstances it is important to evaluate the heat and mass transfer occurring between the phases and/or components; discussion on this is delayed until the next section 1.2.9.

In single phase flow application of the principle of energy conservation to the control volume (CV) uses the following statement of the first law of thermodynamics:

$$\begin{aligned} & \text{Rate of heat addition to the CV, } \mathcal{Q} \\ & + \text{Rate of work done on the CV, } \mathcal{W} \\ & = \\ & \text{Net flux of total internal energy out of CV} \\ & + \text{Rate of increase of total internal energy in CV} \end{aligned}$$

In chemically non-reacting flows the total internal energy per unit mass, e^* , is the sum of the internal energy, e , the kinetic energy $u_i u_i / 2$ (u_i are the velocity components) and the potential energy gz (where z is a coordinate

measured in the vertically upward direction):

$$e^* = e + \frac{1}{2}u_i u_i + gz \quad (1.57)$$

Consequently the energy equation in single phase flow becomes

$$\frac{\partial}{\partial t}(\rho e^*) + \frac{\partial}{\partial x_i}(\rho e^* u_i) = \mathcal{Q} + \mathcal{W} - \frac{\partial}{\partial x_j}(u_i \sigma_{ij}) \quad (1.58)$$

where σ_{ij} is the stress tensor. Then if there is no heat addition to ($\mathcal{Q} = 0$) or external work done on ($\mathcal{W} = 0$) the CV and if the flow is steady with no viscous effects (no deviatoric stresses), the energy equation for single phase flow becomes

$$\frac{\partial}{\partial x_i} \left\{ \rho u_i \left(e^* + \frac{p}{\rho} \right) \right\} = \frac{\partial}{\partial x_i} \{ \rho u_i h^* \} = 0 \quad (1.59)$$

where $h^* = e^* + p/\rho$ is the total enthalpy per unit mass. Thus, when the total enthalpy of the incoming flow is uniform, h^* is constant everywhere.

Now examine the task of constructing an energy equation for each of the components or phases in a multiphase flow. First, it is necessary to define a total internal energy density, e_N^* , for each component N such that

$$e_N^* = e_N + \frac{1}{2}u_{Ni}u_{Ni} + gz \quad (1.60)$$

Then an appropriate statement of the first law of thermodynamics for each phase (the individual phase energy equation, IPEE) is as follows:

$$\begin{aligned} & \text{Rate of heat addition to } N \text{ from outside CV, } \mathcal{Q}_N \\ & + \text{Rate of work done to } N \text{ by the exterior surroundings, } \mathcal{W}\mathcal{A}_N \\ & + \text{Rate of heat transfer to } N \text{ within the CV, } \mathcal{Q}\mathcal{I}_N \\ & + \text{Rate of work done to } N \text{ by other components in CV, } \mathcal{W}\mathcal{I}_N \\ & = \\ & \text{Rate of increase of total kinetic energy of } N \text{ in CV} \\ & + \text{Net flux of total internal energy of } N \text{ out of the CV} \end{aligned}$$

where each of the terms is conveniently evaluated for a unit total volume.

First note that the last two terms can be written as

$$\frac{\partial}{\partial t}(\rho_N \alpha_N e_N^*) + \frac{\partial}{\partial x_i}(\rho_N \alpha_N e_N^* u_{Ni}) \quad (1.61)$$

Turning then to the upper part of the equation, the first term due to external heating and to conduction of heat from the surroundings into the control volume is left as \mathcal{Q}_N . The second term contains two contributions: (i) minus

the rate of work done by the stresses acting on the component N on the surface of the control volume and (ii) the rate of external *shaft work*, \mathcal{W}_N , done on the component N . In evaluating the first of these, we make the same modification to the control volume as was discussed in the context of the momentum equation; specifically we make small deformations to the control volume so that its boundaries lie wholly within the continuous phase. Then using the continuous phase stress tensor, σ_{Cij} , as defined in equation 1.41 the expressions for $\mathcal{W}\mathcal{A}_N$ become:

$$\mathcal{W}\mathcal{A}_C = \mathcal{W}_C + \frac{\partial}{\partial x_j} (u_{Cj} \sigma_{Cij}) \quad \text{and} \quad \mathcal{W}\mathcal{A}_D = \mathcal{W}_D \quad (1.62)$$

The individual phase energy equation may then be written as

$$\begin{aligned} \frac{\partial}{\partial t} (\rho_N \alpha_N e_N^*) + \frac{\partial}{\partial x_i} (\rho_N \alpha_N e_N^* u_{Ni}) = \\ \mathcal{Q}_N + \mathcal{W}_N + \mathcal{Q}\mathcal{I}_N + \mathcal{W}\mathcal{I}_N + \delta_N \frac{\partial}{\partial x_j} (u_{Cj} \sigma_{Cij}) \end{aligned} \quad (1.63)$$

Note that the two terms involving internal exchange of energy between the phases may be combined into an *energy interaction* term given by $\mathcal{E}_N = \mathcal{Q}\mathcal{I}_N + \mathcal{W}\mathcal{I}_N$. It follows that

$$\sum_N \mathcal{Q}\mathcal{I}_N = O \quad \text{and} \quad \sum_N \mathcal{W}\mathcal{I}_N = O \quad \text{and} \quad \sum_N \mathcal{E}_N = O \quad (1.64)$$

Moreover, the work done terms, $\mathcal{W}\mathcal{I}_N$, may clearly be related to the interaction forces, \mathcal{F}_{Nk} . In a two-phase system with one disperse phase:

$$\mathcal{Q}\mathcal{I}_C = -\mathcal{Q}\mathcal{I}_D \quad \text{and} \quad \mathcal{W}\mathcal{I}_C = -\mathcal{W}\mathcal{I}_D = -u_{Di} \mathcal{F}_{Di} \quad \text{and} \quad \mathcal{E}_C = -\mathcal{E}_D \quad (1.65)$$

As with the continuity and momentum equations, the individual phase energy equations can be summed to obtain the combined phase energy equation (CPEE). Then, denoting the total rate of external heat added (per unit total volume) by \mathcal{Q} and the total rate of external shaft work done (per unit total volume) by \mathcal{W} where

$$\mathcal{Q} = \sum_N \mathcal{Q}_N \quad \text{and} \quad \mathcal{W} = \sum_N \mathcal{W}_N \quad (1.66)$$

the CPEE becomes

$$\frac{\partial}{\partial t} \left(\sum_N \rho_N \alpha_N e_N^* \right) + \frac{\partial}{\partial x_i} \left(-u_{Cj} \sigma_{Cij} + \sum_N \rho_N \alpha_N u_{Ni} e_N^* \right) = \mathcal{Q} + \mathcal{W} \quad (1.67)$$

When the left hand sides of the individual or combined phase equations, 1.63 and 1.67, are expanded and use is made of the continuity equation 1.21 and the momentum equation 1.42 (in the absence of deviatoric stresses), the results are known as the *thermodynamic* forms of the energy equations. Using the expressions 1.65 and the relation

$$e_N = c_{vN}T_N + \text{constant} \quad (1.68)$$

between the internal energy, e_N , the specific heat at constant volume, c_{vN} , and the temperature, T_N , of each phase, the thermodynamic form of the IPEE can be written as

$$\begin{aligned} \rho_N \alpha_N c_{vN} \left\{ \frac{\partial T_N}{\partial t} + u_{Ni} \frac{\partial T_N}{\partial x_i} \right\} = \\ \delta_N \sigma_{Cij} \frac{\partial u_{Ci}}{\partial x_j} + \mathcal{Q}_N + \mathcal{W}_N + \mathcal{Q}\mathcal{I}_N + \mathcal{F}_{Ni}(u_{Di} - u_{Ni}) - (e_N^* - u_{Ni}u_{Ni})\mathcal{I}_N \end{aligned} \quad (1.69)$$

and, summing these, the thermodynamic form of the CPEE is

$$\begin{aligned} \sum_N \left\{ \rho_N \alpha_N c_{vN} \left(\frac{\partial T_N}{\partial t} + u_{Ni} \frac{\partial T_N}{\partial x_i} \right) \right\} = \\ \sigma_{Cij} \frac{\partial u_{Ci}}{\partial x_j} - \mathcal{F}_{Di}(u_{Di} - u_{Ci}) - \mathcal{I}_D(e_D^* - e_C^*) + \sum_N u_{Ni}u_{Ni}\mathcal{I}_N \end{aligned} \quad (1.70)$$

In equations 1.69 and 1.70, it has been assumed that the specific heats, c_{vN} , can be assumed to be constant and uniform.

Finally we note that the one-dimensional duct flow version of the IPEE, equation 1.63, is

$$\frac{\partial}{\partial t} (\rho_N \alpha_N e_N^*) + \frac{1}{A} \frac{\partial}{\partial x} (A \rho_N \alpha_N e_N^* u_N) = \mathcal{Q}_N + \mathcal{W}_N + \mathcal{E}_N - \delta_N \frac{\partial}{\partial x} (p u_C) \quad (1.71)$$

where $A\mathcal{Q}_N$ is the rate of external heat addition to the component N per unit length of the duct, $A\mathcal{W}_N$ is the rate of external work done on component N per unit length of the duct, $A\mathcal{E}_N$ is the rate of energy transferred to the component N from the other phases per unit length of the duct and p is the pressure in the continuous phase neglecting deviatoric stresses. The CPEE, equation 1.67, becomes

$$\frac{\partial}{\partial t} \left(\sum_N \rho_N \alpha_N e_N^* \right) + \frac{1}{A} \frac{\partial}{\partial x} \left(\sum_N A \rho_N \alpha_N e_N^* u_N \right) = \mathcal{Q} + \mathcal{W} - \frac{\partial}{\partial x} (p u_C) \quad (1.72)$$

where AQ is the total rate of external heat addition to the flow per unit length of the duct and AW is the total rate of external work done on the flow per unit length of the duct.

1.2.9 Heat transfer between separated phases

In the preceding section, the rate of heat transfer, QI_N , to each phase, N , from the other phases was left undefined. Now we address the functional form of this rate of heat transfer in the illustrative case of a two-phase flow consisting of a disperse solid particle or liquid droplet phase and a gaseous continuous phase.

In section 1.2.7, we defined a relaxation time that typifies the natural attenuation of velocity differences between the phases. In an analogous manner, the temperatures of the phases might be different downstream of a flow disturbance and consequently there would be a second *relaxation* time associated with the equilibration of temperatures through the process of heat transfer between the phases. This temperature relaxation time is denoted by t_T and can be obtained by equating the rate of heat transfer from the continuous phase to the particle with the rate of increase of heat stored in the particle. The heat transfer to the particle can occur as a result of conduction, convection or radiation and there are practical flows in which each of these mechanisms are important. For simplicity, we shall neglect the radiation component. Then, if the relative motion between the particle and the gas is sufficiently small, the only contributing mechanism is conduction and it will be limited by the thermal conductivity, k_C , of the gas (since the thermal conductivity of the particle is usually much greater). Then the rate of heat transfer to a particle (radius R) will be given approximately by $2\pi Rk_C(T_C - T_D)$ where T_C and T_D are representative temperatures of the gas and particle respectively.

Now we add in the component of heat transfer by the convection caused by relative motion. To do so we define the Nusselt number, Nu , as twice the ratio of the rate of heat transfer with convection to that without convection. Then the rate of heat transfer becomes Nu times the above result for conduction. Typically, the Nusselt number is a function of both the Reynolds number of the relative motion, $Re = 2WR/\nu_C$ (where W is the typical magnitude of $(u_{Di} - u_{Ci})$), and the Prandtl number, $Pr = \rho_C \nu_C c_{pC}/k_C$. One frequently used expression for Nu (see Ranz and Marshall 1952) is

$$Nu = 2 + 0.6Re^{\frac{1}{2}}Pr^{\frac{1}{3}} \quad (1.73)$$

and, of course, this reduces to the pure conduction result, $Nu = 2$, when the second term on the right hand side is small.

Assuming that the particle temperature has a roughly uniform value of T_D , it follows that

$$\mathcal{QI}_D = 2\pi Rk_C Nu(T_C - T_D)n_D = \rho_D \alpha_D c_{sD} \frac{DT_D}{Dt} \quad (1.74)$$

where the material derivative, D/Dt , follows the particle. This provides the equation that must be solved for T_D namely

$$\frac{DT_D}{Dt} = \frac{Nu(T_C - T_D)}{2 t_T} \quad (1.75)$$

where

$$t_T = c_{sD} \rho_D R^2 / 3k_C \quad (1.76)$$

Clearly t_T represents a typical time for equilibration of the temperatures in the two phases, and is referred to as the *temperature relaxation time*.

The above construction of the temperature relaxation time and the equation for the particle temperature represents perhaps the simplest formulation that retains the essential ingredients. Many other effects may become important and require modification of the equations. Examples are the rarefied gas effects and turbulence effects. Moreover, the above was based on a uniform particle temperature and steady state heat transfer correlations; in many flows heat transfer to the particles is highly transient and a more accurate heat transfer model is required. For a discussion of these effects the reader is referred to Rudinger (1969) and Crowe *et al.* (1998).

1.3 INTERACTION WITH TURBULENCE

1.3.1 Particles and turbulence

Turbulent flows of a single Newtonian fluid, even those of quite simple external geometry such as a fully-developed pipe flow, are very complex and their solution at high Reynolds numbers requires the use of empirical models to represent the unsteady motions. It is self-evident that the addition of particles to such a flow will result in;

1. complex unsteady motions of the particles that may result in non-uniform spatial distribution of the particles and, perhaps, particle segregation. It can also result in particle agglomeration or in particle fission, especially if the particles are bubbles or droplets.
2. modifications of the turbulence itself caused by the presence and motions of the

particles. One can visualize that the turbulence could be damped by the presence of particles, or it could be enhanced by the wakes and other flow disturbances that the motion of the particles may introduce.

In the last twenty five years, a start has been made in the understanding of these complicated issues, though many aspects remain to be understood. The advent of laser Doppler velocimetry resulted in the first measurements of these effects; and the development of direct numerical simulation allowed the first calculations of these complex flows, albeit at rather low Reynolds numbers. Here we will be confined to a brief summary of these complex issues. The reader is referred to the early review of Hetsroni (1989) and the text by Crowe *et al.* (1998) for a summary of the current understanding.

To set the stage, recall that turbulence is conveniently characterized at any point in the flow by the Kolmogorov length and time scales, λ and τ , given by

$$\lambda = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}} \quad \text{and} \quad \tau = \left(\frac{\nu}{\epsilon}\right)^{\frac{1}{2}} \quad (1.77)$$

where ν is the kinematic viscosity and ϵ is the mean rate of dissipation per unit mass of fluid. Since ϵ is proportional to U^3/ℓ where U and ℓ are the typical velocity and dimension of the flow, it follows that

$$\lambda/\ell \propto Re^{-\frac{3}{4}} \quad \text{and} \quad U\tau/\ell \propto Re^{-\frac{1}{2}} \quad (1.78)$$

and the difficulties in resolving the flow either by measurement or by computation increase as Re increases.

Gore and Crowe (1989) collected data from a wide range of turbulent pipe and jet flows (all combinations of gas, liquid and solid flows, volume fractions from 2.5×10^{-6} to 0.2, density ratios from 0.001 to 7500, Reynolds numbers from 8000 to 100,000) and constructed figure 1.4 which plots the fractional change in the turbulence intensity (defined as the rms fluctuating velocity) as a result of the introduction of the disperse phase against the ratio of the particle size to the turbulent length scale, D/ℓ_t . They judge that the most appropriate turbulent length scale, ℓ_t , is the size of the most energetic eddy. Single phase experiments indicate that ℓ_t is about 0.2 times the radius in a pipe flow and 0.039 times the distance from the exit in a jet flow. To explain figure 1.4 Gore and Crowe argue that when the particles are small compared with the turbulent length scale, they tend to follow the turbulent fluid motions and in doing so absorb energy from them thus reducing the turbulent energy. It appears that the turbulence reduction is a strong function of Stokes number, $St = m_p/6\pi R\mu\tau$, the ratio of the particle

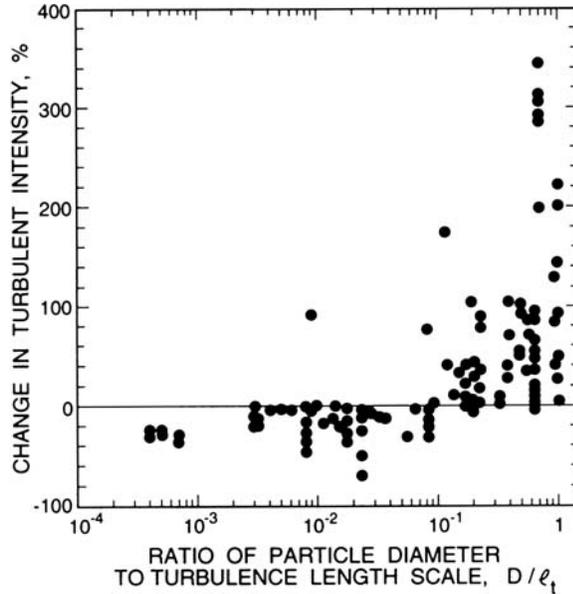


Figure 1.4. The percentage change in the turbulence intensity as a function of the ratio of particle size to turbulence length scale, D/ℓ_t , from a wide range of experiments. Adapted from Gore and Crowe (1989).

relaxation time, $m_p/6\pi R\mu$, to the Kolmogorov time scale, τ . A few experiments (Eaton 1994, Kulick *et al.* 1994) suggest that the maximum reduction occurs at St values of the order of unity though other features of the flow may also influence the effect. Of course, the change in the turbulence intensity also depends on the particle concentration. Figure 1.5 from Paris and Eaton (2001) shows one example of how the turbulent kinetic energy and the rate of viscous dissipation depend on the mass fraction of particles for a case in which D/ℓ_t is small.

On the other hand large particles do not follow the turbulent motions and the relative motion produces wakes that tend to add to the turbulence (see, for example, Parthasarathy and Faeth 1990). Under these circumstances, when the response times of the particles are comparable with or greater than the typical times associated with the fluid motion, the turbulent flow with particles is more complex due to the effects of relative motion. Particles in a gas tend to be centrifuged out of the more intense vortices and accumulate in the shear zones in between. Figure 1.6 is a photograph of a turbulent flow of a gas loaded with particles showing the accumulation of particles in shear zones between strong vortices. On the other hand, bubbles in a liquid flow tend to accumulate in the center of the vortices.

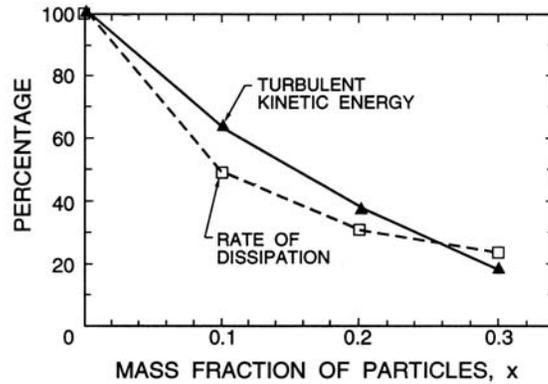


Figure 1.5. The percentage change in the turbulent kinetic energy and the rate of viscous dissipation with mass fraction for a channel flow of $150\mu m$ glass spheres suspended in air (from Paris and Eaton 2001).



Figure 1.6. Image of the centerplane of a fully developed, turbulent channel flow of air loaded with $28\mu m$ particles. The area is $50mm$ by $30mm$. Reproduced from Fessler *et al.*(1994) with the authors' permission.

Analyses of turbulent flows with particles or bubbles are currently the subject of active research and many issues remain. The literature includes a number of heuristic and approximate quantitative analyses of the enhancement of turbulence due to particle relative motion. Examples are the work of Yuan and Michaelides (1992) and of Kenning and Crowe (1997). The latter relate the percentage change in the turbulence intensity due to the particle wakes; this yields a percentage change that is a function not only of

D/ℓ_t but also of the mean relative motion and the density ratio. They show qualitative agreement with some of the data included in figure 1.4.

An alternative to these heuristic methodologies is the use of direct numerical simulations (DNS) to examine the details of the interaction between the turbulence and the particles or bubbles. Such simulations have been carried out both for solid particles (for example, Squires and Eaton 1990, Elghobashi and Truesdell 1993) and for bubbles (for example, Pan and Banerjee 1997). Because each individual simulation is so time consuming and leads to complex consequences, it is not possible, as yet, to draw general conclusions over a wide parameter range. However, the kinds of particle segregation mentioned above are readily apparent in the simulations.

1.3.2 Effect on turbulence stability

The issue of whether particles promote or delay transition to turbulence is somewhat distinct from their effect on developed turbulent flows. Saffman (1962) investigated the effect of dust particles on the stability of parallel flows and showed theoretically that if the relaxation time of the particles, t_u , is small compared with ℓ/U , the characteristic time of the flow, then the dust destabilizes the flow. Conversely if $t_u \gg \ell/U$ the dust stabilizes the flow.

In a somewhat similar investigation of the effect of bubbles on the stability of parallel liquid flows, d'Agostino *et al.* (1997) found that the effect depends on the relative magnitude of the most unstable frequency, ω_m , and the natural frequency of the bubbles, ω_n (see section 4.4.1). When the ratio, $\omega_m/\omega_n \ll 1$, the primary effect of the bubbles is to increase the effective compressibility of the fluid and since increased compressibility causes increased stability, the bubbles are stabilizing. On the other hand, at or near resonance when ω_m/ω_n is of order unity, there are usually bands of frequencies in which the flow is less stable and the bubbles are therefore destabilizing.

In summary, when the response times of the particles or bubbles (both the relaxation time and the natural period of volume oscillation) are short compared with the typical times associated with the fluid motion, the particles simply alter the effective properties of the fluid, its effective density, viscosity and compressibility. It follows that under these circumstances the stability is governed by the effective Reynolds number and effective Mach number. Saffman considered dusty gases at low volume concentrations, α , and low Mach numbers; under those conditions the net effect of the dust is to change the density by $(1 + \alpha\rho_S/\rho_G)$ and the viscosity by $(1 + 2.5\alpha)$. The ef-

fective Reynolds number therefore varies like $(1 + \alpha\rho_S/\rho_G)/(1 + 2.5\alpha)$. Since $\rho_S \gg \rho_G$ the effective Reynolds number is increased and the dust is therefore destabilizing. In the case of d'Agostino *et al.* the primary effect of the bubbles (when $\omega_m \gg \omega_n$) is to change the compressibility of the mixture. Since such a change is stabilizing in single phase flow, the result is that the bubbles tend to stabilize the flow.

On the other hand when the response times are comparable with or greater than the typical times associated with the fluid motion, the particles will not follow the motions of the continuous phase. The disturbances caused by this relative motion will tend to generate unsteady motions and promote instability in the continuous phase.

1.4 COMMENTS ON THE EQUATIONS OF MOTION

In sections 1.2.2 through 1.2.8 we assembled the basic form for the equations of motion for a multiphase flow that would be used in a two-fluid model. However, these only provide the initial framework for there are many additional complications that must be addressed. The relative importance of these complications vary greatly from one type of multiphase flow to another. Consequently the level of detail with which they must be addressed varies enormously. In this general introduction we can only indicate the various types of complications that can arise.

1.4.1 Averaging

As discussed in section 1.2.1, when the ratio of the particle size, D , to the typical dimension of the averaging volume (estimated as the typical length, ϵ , over which there is significant change in the averaged flow properties) becomes significant, several issues arise (see Hinze 1959, Vernier and Delhay 1968, Nigmatulin 1979, Reeks 1992). The reader is referred to Slattery (1972) or Crowe *et al.* (1997) for a systematic treatment of these issues; only a summary is presented here. Clearly an appropriate volume average of a property, Q_C , of the continuous phase is given by $\langle Q_C \rangle$ where

$$\langle Q_C \rangle = \frac{1}{V_C} \int_{V_C} Q_C dV \quad (1.79)$$

where V_C denotes the volume of the continuous phase within the control volume, V . For present purposes, it is also convenient to define an average

$$\overline{Q_C} = \frac{1}{V} \int_{V_C} Q_C dV = \alpha_C \langle Q_C \rangle \quad (1.80)$$

over the whole of the control volume.

Since the conservation equations discussed in the preceding sections contain derivatives in space and time and since the leading order set of equations we seek are versions in which all the terms are averaged over some local volume, the equations contain averages of spatial gradients and time derivatives. For these terms to be evaluated they must be converted to derivatives of the volume averaged properties. Those relations take the form (Crowe *et al.* 1997):

$$\frac{\partial \overline{Q_C}}{\partial x_i} = \frac{\partial \overline{Q_C}}{\partial x_i} - \frac{1}{V} \int_{S_D} Q_C n_i dS \quad (1.81)$$

where S_D is the total surface area of the particles within the averaging volume. With regard to the time derivatives, if the volume of the particles is not changing with time then

$$\frac{\partial \overline{Q_C}}{\partial t} = \frac{\partial \overline{Q_C}}{\partial t} \quad (1.82)$$

but if the location of a point on the surface of a particle relative to its center is given by r_i and if r_i is changing with time (for example, growing bubbles) then

$$\frac{\partial \overline{Q_C}}{\partial t} = \frac{\partial \overline{Q_C}}{\partial t} + \frac{1}{V} \int_{S_D} Q_C \frac{Dr_i}{Dt} dS \quad (1.83)$$

When the definitions 1.81 and 1.83 are employed in the development of appropriate averaged conservation equations, the integrals over the surface of the disperse phase introduce additional terms that might not have been anticipated (see Crowe *et al.* 1997 for specific forms of those equations). Here it is of value to observe that the magnitude of the additional surface integral term in equation 1.81 is of order $(D/\epsilon)^2$. Consequently these additional terms are small as long as D/ϵ is sufficiently small.

1.4.2 Averaging contributions to the mean motion

Thus far we have discussed only those additional terms introduced as a result of the fact that the gradient of the average may differ from the average of the

gradient. Inspection of the form of the basic equations (for example the continuity equation, 1.21 or the momentum equation 1.42) readily demonstrates that additional averaging terms will be introduced because the average of a product is different from the product of averages. In single phase flows, the *Reynolds stress terms* in the averaged equations of motion for turbulent flows are a prime example of this phenomenon. We will use the name *quadratic rectification terms* to refer to the appearance in the averaged equations of motion of the mean of two fluctuating components of velocity and/or volume fraction. Multiphase flows will, of course, also exhibit conventional Reynolds stress terms when they become turbulent (see section 1.3 for more on the complicated subject of turbulence in multiphase flows). But even multiphase flows that are not turbulent in the strictest sense will exhibit variations in the velocities due the flows around particles and these variations will yield quadratic rectification terms. These must be recognized and modeled when considering the effects of locally non-uniform and unsteady velocities on the equations of motion. Much more has to be learned of both the laminar and turbulent quadratic rectification terms before these can be confidently incorporated in model equations for multiphase flow. Both experiments and computer simulation will be valuable in this regard.

One simpler example in which the fluctuations in velocity have been measured and considered is the case of concentrated granular flows in which direct particle-particle interactions create particle velocity fluctuations. These particle velocity fluctuations and the energy associated with them (the so-called granular temperature) have been studied both experimentally and computationally (see chapter 13) and their role in the effective continuum equations of motion is better understood than in more complex multiphase flows.

With two interacting phases or components, the additional terms that emerge from an averaging process can become extremely complex. In recent decades a number of valiant efforts have been made to codify these issues and establish at least the forms of the important terms that result from these interactions. For example, Wallis (1991) has devoted considerable effort to identify the inertial coupling of spheres in inviscid, locally irrotational flow. Arnold, Drew and Lahey (1989) and Drew (1991) have focused on the application of cell methods (see section 2.4.3) to interacting multiphase flows. Both these authors as well as Sangani and Didwania (1993) and Zhang and Prosperetti (1994) have attempted to include the fluctuating motions of the particles (as in granular flows) in the construction of equations of motion for the multiphase flow; Zhang and Prosperetti also provide a useful compar-

ative summary of these various averaging efforts. However, it is also clear that these studies have some distance to go before they can be incorporated into any real multiphase flow prediction methodology.

1.4.3 Averaging in pipe flows

One specific example of a quadratic rectification term (in this case a discrepancy between the product of an average and the average of a product) is that recognized by Zuber and Findlay (1965). In order to account for the variations in velocity and volume fraction over the cross-section of a pipe in constructing the one-dimensional equations of pipe flow, they found it necessary to introduce a distribution parameter, C_0 , defined by

$$C_0 = \frac{\overline{\alpha j}}{\overline{\alpha} \overline{j}} \quad (1.84)$$

where the overbar now represents an average over the cross-section of the pipe. The importance of C_0 is best demonstrated by observing that it follows from equations 1.16 that the cross-sectionally averaged volume fraction, $\overline{\alpha_A}$, is now related to the volume fluxes, $\overline{j_A}$ and $\overline{j_B}$, by

$$\overline{\alpha_A} = \frac{1}{C_0} \frac{\overline{j_A}}{(\overline{j_A} + \overline{j_B})} \quad (1.85)$$

Values of C_0 of the order of 1.13 (Zuber and Findlay 1965) or 1.25 (Wallis 1969) appear necessary to match the experimental observations.

1.4.4 Modeling with the combined phase equations

One of the simpler approaches is to begin by modeling the combined phase equations 1.24, 1.46 and 1.67 and hence avoid having to codify the mass, force and energy interaction terms. By defining mixture properties such as the density, ρ , and the total volumetric flux, j_i , one can begin to construct equations of motion in terms of those properties. But none of the summation terms (equivalent to various weighted averages) in the combined phase equations can be written accurately in terms of these mixture properties. For example, the summations,

$$\sum_N \rho_N \alpha_N u_{Ni} \quad \text{and} \quad \sum_N \rho_N \alpha_N u_{Ni} u_{Nk} \quad (1.86)$$

are not necessarily given with any accuracy by ρj_i and $\rho j_i j_k$. Indeed, the discrepancies are additional rectification terms that would require modeling

in such an approach. Thus any effort to avoid addressing the mass, force and energy interaction terms by focusing exclusively on the mixture equations of motion immediately faces difficult modeling questions.

1.4.5 Mass, force and energy interaction terms

Most multiphase flow modeling efforts concentrate on the individual phase equations of motion and must therefore face the issues associated with construction of \mathcal{I}_N , the mass interaction term, \mathcal{F}_{Nk} , the force interaction term, and \mathcal{E}_N , the energy interaction term. These represent the core of the problem in modeling multiphase flows and there exist no universally applicable methodologies that are independent of the topology of the flow, the flow pattern. Indeed, efforts to find systems of model equations that would be applicable to a range of flow patterns would seem fruitless. Therein lies the main problem for the user who may not be able to predict the flow pattern and therefore has little hope of finding an accurate and reliable method to predict flow rates, pressure drops, temperatures and other flow properties.

The best that can be achieved with the present state of knowledge is to attempt to construct heuristic models for \mathcal{I}_N , \mathcal{F}_{Nk} , and \mathcal{E}_N given a particular flow pattern. Substantial efforts have been made in this direction particularly for dispersed flows; the reader is directed to the excellent reviews by Hinze (1961), Drew (1983), Gidaspow (1994) and Crowe *et al.* (1998) among others. Both direct experimentation and computer simulation have been used to create data from which heuristic expressions for the interaction terms could be generated. Computer simulations are particularly useful not only because high fidelity instrumentation for the desired experiments is often very difficult to develop but also because one can selectively incorporate a range of different effects and thereby evaluate the importance of each.

It is important to recognize that there are several constraints to which any mathematical model must adhere. Any violation of those constraints is likely to produce strange and physically inappropriate results (see Garabedian 1964). Thus, the system of equations must have appropriate frame-indifference properties (see, for example, Ryskin and Rallison 1980). It must also have real characteristics; Prosperetti and Jones (1987) show that some models appearing in the literature do have real characteristics while others do not.

In this book chapters 2, 3 and 4 review what is known of the behavior of individual particles, bubbles and drops, with a view to using this information to construct \mathcal{I}_N , \mathcal{F}_{Nk} , and \mathcal{E}_N and therefore the equations of motion for particular forms of multiphase flow.