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## Undergraduate Texts in Mathematics

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- James:** Topological and Uniform Spaces.
- Jänich:** Linear Algebra.
- Jänich:** Topology.
- Kemeny/Snell:** Finite Markov Chains.
- Kinsey:** Topology of Surfaces.
- Klambauer:** Aspects of Calculus.
- Lang:** A First Course in Calculus. Fifth edition.
- Lang:** Calculus of Several Variables. Third edition.
- Lang:** Introduction to Linear Algebra. Second edition.
- Lang:** Linear Algebra. Third edition.
- Lang:** Undergraduate Algebra. Second edition.
- Lang:** Undergraduate Analysis.
- Lax/Burstein/Lax:** Calculus with Applications and Computing. Volume 1.
- LeCuyer:** College Mathematics with APL.
- Lidl/Pilz:** Applied Abstract Algebra. Second edition.
- Macki-Strauss:** Introduction to Optimal Control Theory.
- Malitz:** Introduction to Mathematical Logic.
- Marsden/Weinstein:** Calculus I, II, III. Second edition.
- Martin:** The Foundations of Geometry and the Non-Euclidean Plane.
- Martin:** Geometric Constructions.
- Martin:** Transformation Geometry: An Introduction to Symmetry.
- Millman/Parker:** Geometry: A Metric Approach with Models. Second edition.
- Moschovakis:** Notes on Set Theory.
- Owen:** A First Course in the Mathematical Foundations of Thermodynamics.
- Palka:** An Introduction to Complex Function Theory.
- Pedrick:** A First Course in Analysis.
- Peressini/Sullivan/Uhl:** The Mathematics of Nonlinear Programming.
- Prenowitz/Jantosciak:** Join Geometries.
- Priestley:** Calculus: An Historical Approach.
- Protter/Morrey:** A First Course in Real Analysis. Second edition.
- Protter/Morrey:** Intermediate Calculus. Second edition.
- Roman:** An Introduction to Coding and Information Theory.
- Ross:** Elementary Analysis: The Theory of Calculus.
- Samuel:** Projective Geometry. *Readings in Mathematics.*
- Scharlau/Opolka:** From Fermat to Minkowski.
- Sethuraman:** Rings, Fields, and Vector Spaces: An Approach to Geometric Constructability.
- Sigler:** Algebra.
- Silverman/Tate:** Rational Points on Elliptic Curves.
- Simmonds:** A Brief on Tensor Analysis. Second edition.
- Singer:** Geometry: Plane and Fancy.
- Singer/Thorpe:** Lecture Notes on Elementary Topology and Geometry.
- Smith:** Linear Algebra. Second edition.
- Smith:** Primer of Modern Analysis. Second edition.
- Stanton/White:** Constructive Combinatorics.
- Stillwell:** Elements of Algebra: Geometry, Numbers, Equations.
- Stillwell:** Mathematics and Its History.
- Stillwell:** Numbers and Geometry. *Readings in Mathematics.*
- Strayer:** Linear Programming and Its Applications.
- Thorpe:** Elementary Topics in Differential Geometry.
- Toth:** Glimpses of Algebra and Geometry.



## **Undergraduate Texts in Mathematics**

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**Troutman:** Variational Calculus and  
Optimal Control. Second edition.

**Valenza:** Linear Algebra: An

Introduction to Abstract Mathematics.

**Whyburn/Duda:** Dynamic Topology.

**Wilson:** Much Ado About Calculus.

29.  $\int \operatorname{csch} x \, dx = \ln \left| \tanh \frac{x}{2} \right| = -\frac{1}{2} \ln \frac{\cosh x + 1}{\cosh x - 1}$
30.  $\int \sinh^2 x \, dx = \frac{1}{4} \sinh 2x - \frac{1}{2} x$
31.  $\int \cosh^2 x \, dx = \frac{1}{4} \sinh 2x + \frac{1}{2} x$
32.  $\int \operatorname{sech}^2 x \, dx = \tanh x$
33.  $\int \sinh^{-1} \frac{x}{a} \, dx = x \sinh^{-1} \frac{x}{a} - \sqrt{x^2 + a^2} \quad (a > 0)$
34.  $\int \cosh^{-1} \frac{x}{a} \, dx = \begin{cases} x \cosh^{-1} \frac{x}{a} - \sqrt{x^2 - a^2} & \left[ \cosh^{-1} \left( \frac{x}{a} \right) > 0, a > 0 \right] \\ x \cosh^{-1} \frac{x}{a} + \sqrt{x^2 - a^2} & \left[ \cosh^{-1} \left( \frac{x}{a} \right) < 0, a > 0 \right] \end{cases}$
35.  $\int \tanh^{-1} \frac{x}{a} \, dx = x \tanh^{-1} \frac{x}{a} + \frac{a}{2} \ln |a^2 - x^2|$
36.  $\int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \ln(x + \sqrt{a^2 + x^2}) = \sinh^{-1} \frac{x}{a} \quad (a > 0)$
37.  $\int \frac{1}{a^2 + x^2} \, dx + \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (a > 0)$
38.  $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \quad (a > 0)$
39.  $\int (a^2 - x^2)^{3/2} \, dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} \quad (a > 0)$
40.  $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} \quad (a > 0)$
41.  $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$
42.  $\int \frac{1}{(a^2 - x^2)^{3/2}} \, dx = \frac{x}{a^2 \sqrt{a^2 - x^2}}$
43.  $\int \sqrt{x^2 \pm a^2} \, dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln |x + \sqrt{x^2 \pm a^2}|$
44.  $\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln |x + \sqrt{x^2 - a^2}| = \cosh^{-1} \frac{x}{a} \quad (a > 0)$
45.  $\int \frac{1}{x(a+bx)} \, dx = \frac{1}{a} \ln \left| \frac{x}{a+bx} \right|$
46.  $\int x\sqrt{a+bx} \, dx = \frac{2(3bx-2a)(a+bx)^{3/2}}{15b^2}$
47.  $\int \frac{\sqrt{a+bx}}{x} \, dx = 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} \, dx$
48.  $\int \frac{x}{\sqrt{a+bx}} \, dx = \frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$
49.  $\int \frac{1}{x\sqrt{a+bx}} \, dx = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right| \quad (a > 0)$   
 $= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bx}{-a}} \quad (a < 0)$
50.  $\int \frac{\sqrt{a^2 - x^2}}{x} \, dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
51.  $\int x\sqrt{a^2 - x^2} \, dx = -\frac{1}{3} (a^2 - x^2)^{3/2}$
52.  $\int x^2 \sqrt{a^2 - x^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} \quad (a > 0)$

Continued on overleaf.

53.  $\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
54.  $\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$
55.  $\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \quad (a > 0)$
56.  $\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$
57.  $\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \cos^{-1} \frac{a}{|x|}$   
 $= \sqrt{x^2 - a^2} - a \sec^{-1} \left( \frac{x}{a} \right) \quad (a > 0)$
58.  $\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$
59.  $\int \frac{1}{x\sqrt{x^2 + a^2}} dx = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{x^2 + a^2}} \right|$
60.  $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \cos^{-1} \frac{a}{|x|} \quad (a > 0)$
61.  $\int \frac{1}{x^2\sqrt{x^2 \pm a^2}} dx = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$
62.  $\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$
63.  $\int \frac{1}{ax^2 + bx + c} dx = \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| \quad (b^2 > 4ac)$   
 $= \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} \quad (b^2 < 4ac)$
64.  $\int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{1}{ax^2 + bx + c} dx$
65.  $\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln |2ax + b + 2\sqrt{a} \sqrt{ax^2 + bx + c}| \quad (a > 0)$   
 $= \frac{1}{\sqrt{-a}} \sin^{-1} \frac{-2ax - b}{\sqrt{b^2 - 4ac}} \quad (a < 0)$
66.  $\int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$
67.  $\int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$
68.  $\int \frac{1}{x\sqrt{ax^2 + bx + c}} dx = \frac{-1}{\sqrt{c}} \ln \left| \frac{2\sqrt{c} \sqrt{ax^2 + bx + c} + bx + 2c}{x} \right| \quad (c > 0)$   
 $= \frac{1}{\sqrt{-c}} \sin^{-1} \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}} \quad (c < 0)$
69.  $\int x^3 \sqrt{x^2 + a^2} dx = \left( \frac{1}{5} x^2 - \frac{2}{15} a^2 \right) \sqrt{(a^2 + x^2)^3}$
70.  $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \frac{\mp \sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}$
71.  $\int \sin ax \sin bx dx = \frac{\sin(a - b)x}{2(a - b)} - \frac{\sin(a + b)x}{2(a + b)} \quad (a^2 \neq b^2)$

Continued on inside back cover.

$$72. \int \sin ax \cos bx \, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$73. \int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$74. \int \sec x \tan x \, dx = \sec x$$

$$75. \int \csc x \cot x \, dx = -\csc x$$

$$76. \int \cos^m x \sin^n x \, dx = \frac{\cos^{m-1} x \sin^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x \, dx \\ = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \sin^{n-2} x \, dx$$

$$77. \int x^n \sin ax \, dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$78. \int x^n \cos ax \, dx = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

$$79. \int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$$

$$80. \int x^n \ln ax \, dx = x^{n+1} \left[ \frac{\ln ax}{n+1} - \frac{1}{(n+1)^2} \right]$$

$$81. \int x^n (\ln ax)^m \, dx = \frac{x^{n+1}}{n+1} (\ln ax)^m - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx$$

$$82. \int e^{ax} \sin bx \, dx = \frac{e^{ax} (a \sin bx - b \cos bx)}{a^2 + b^2}$$

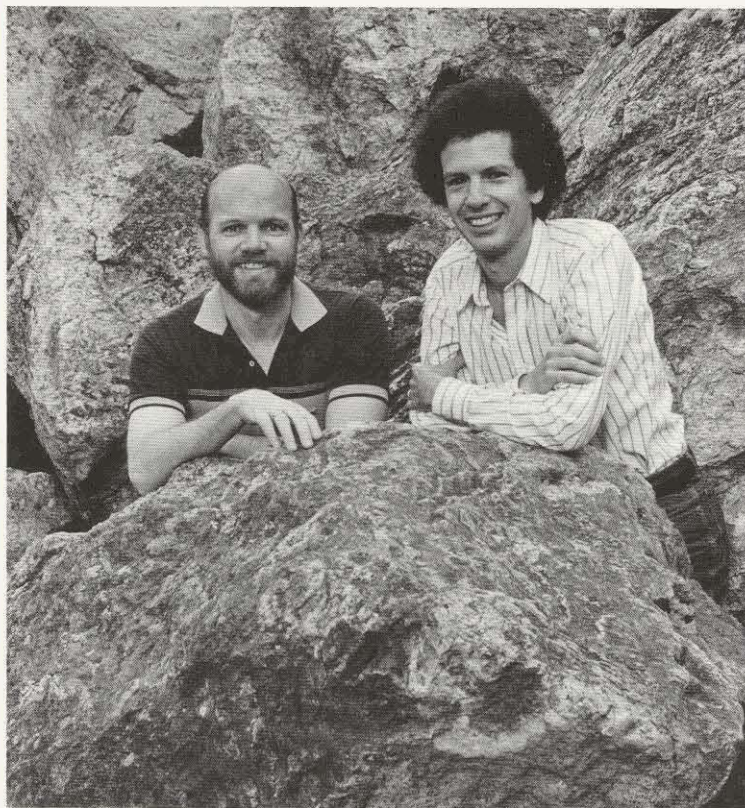
$$83. \int e^{ax} \cos bx \, dx = \frac{e^{ax} (b \sin bx + a \cos bx)}{a^2 + b^2}$$

$$84. \int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x$$

$$85. \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x$$

### Greek Alphabet

$\alpha$	alpha	$\iota$	iota	$\rho$	rho
$\beta$	beta	$\kappa$	kappa	$\sigma$	sigma
$\gamma$	gamma	$\lambda$	lambda	$\tau$	tau
$\delta$	delta	$\mu$	mu	$\upsilon$	upsilon
$\epsilon$	epsilon	$\nu$	nu	$\phi$	phi
$\zeta$	zeta	$\xi$	xi	$\chi$	chi
$\eta$	eta	$\omicron$	omicron	$\psi$	psi
$\theta$	theta	$\pi$	pi	$\omega$	omega



*Photograph by Margo Weinstein*

This book, the first of a three-volume work, is the outgrowth of the authors' experiences teaching calculus at Berkeley. It covers all of the basic concepts and results of differential calculus, and provides an introduction to integral calculus. Throughout the book, the authors motivate the study of calculus using its applications, including many solved problems. Extensive exercises are given at the end of each section; in addition, a separate student study guide has been prepared.

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