Dissipation

As when the massy substance of the Earth quivers. Marlowe

Real materials are not perfectly elastic. Solids creep when a high stress is applied, and the strain is a function of time. These phenomena are manifestations of anelasticity. The attenuation of seismic waves with distance and of normal modes with time are other examples of anelastic behavior. Generally, the response of a solid to a stress can be split into an elastic or instantaneous part and an anelastic or time-dependent part. The anelastic part contains information about temperature, stress and the defect nature of the solid. In principle, the attenuation of seismic waves can tell us about such things as small-scale heterogeneity, melt content, dislocation density and defect mobility. These, in turn, are controlled by temperature, pressure, stress, history and the nature of the defects. If these parameters can be estimated from seismology, they can be used to estimate other anelastic properties such as viscosity.

For example, the dislocation density of a crystalline solid is a function of the non-hydrostatic stress. These dislocations respond to an applied oscillatory stress, such as a seismic wave, but they are out of phase because of the finite diffusion time of the atoms around the dislocation. The dependence of attenuation on frequency can yield information about the dislocations. The longer-term motions of these same dislocations in response to a higher tectonic stress gives rise to a solid-state viscosity. Seismic waves also attenuate due to macroscopic phenomena, such as scattering and interactions between fluid, or molten, parts of the interior and the solid matrix. Anelasticity causes the elastic moduli to vary with frequency; elastic constants are not constant. The equations in this section can be used to correct seismic velocities for temperature effects due to anelasticity. These are different from the anharmonic effects discussed in other chapters.

Seismic-wave attenuation

The travel time or velocity of a seismic wave provides an incomplete description of the material it has propagated through. The amplitude and frequency of the wave provide some more information. Seismic waves attenuate or decay as they propagate. The rate of attenuation contains information about the anelastic properties of the propagation medium.

A propagating wave can be written

$$A = A_0 \exp(\omega t - \kappa x)$$

where $A$ is the amplitude, $\omega$ the frequency, $\kappa$ the wave number, $t$ the travel time, $x$ the distance and $c = \omega / \kappa$ the phase velocity. If spatial attenuation occurs, then $\kappa$ is complex. The imaginary part of $\kappa$, $\kappa^*$ is called the spatial attenuation coefficient.

The elastic moduli, $M$, are now also complex:

$$M = M + iM^*$$
The specific quality factor, a convenient dimensionless measure of dissipation, is

\[ Q^{-1} = M^*/M \]

This is related to the energy dissipated per cycle. Since the phase velocity

\[ c = \frac{\omega}{k} = \sqrt{M/\rho} \]

it follows that

\[ Q^{-1} = 2 \frac{k^*}{k} = \frac{M^*}{M} \quad \text{for} \quad Q \gg 1 \]

In general, all the elastic moduli are complex, and each wave type has its own \( Q \) and velocity, both frequency dependent. For an isotropic solid the imaginary parts of the bulk modulus and rigidity are denoted as \( K^* \) and \( G^* \). Most mechanisms of seismic-wave absorption affect the rigidity more than the bulk modulus, and shear waves more than compressional waves.

### Frequency dependence of attenuation

In a perfectly elastic homogenous body, the elastic wave velocities are independent of frequency. Variations with temperature, and pressure, or volume, are controlled by anharmonicity. In an imperfectly elastic, or anelastic, body the velocities are dispersive; they depend on frequency, and this introduces another mechanism for changing moduli, and seismic velocities with temperature. This is important when comparing seismic data taken at different frequencies or when comparing seismic and laboratory data. When long-period seismic waves started to be used in seismology, it was noted that the free oscillation, or normal mode, models, differed from the classical body wave-models, which were based on short-period seismic waves. This is the body-wave-normal-mode discrepancy. The discrepancy was resolved when it was realized that in a real solid, as opposed to an ideally elastic one, the elastic moduli were functions of frequency. One has to allow for this when using body waves, surface waves and normal modes in the inversion for velocity vs. depth. The absorption, or dissipation, of energy, and the frequency dependence of seismic velocity, can be due to intrinsic anelasticity, or due to scattering.

A variety of physical processes contribute to attenuation in a crystalline material: motions of point defects, dislocations, grain boundaries and so on. These processes all involve a high-frequency, or unrelaxed, modulus and a low-frequency, or relaxed, modulus. At sufficiently high frequencies, or low temperatures, the defects, which are characterized by a time constant, do not have time to contribute, and the body behaves as a perfectly elastic body. Attenuation is low and \( Q \), the seismic quality factor, is high in the high-frequency limit. At very low frequencies, or high temperature, the defects have plenty of time to respond to the applied force and they contribute an additional strain. Because the stress cycle time is long compared with the response time of the defect, stress and strain are in phase and again \( Q \) is high. Because of the additional relaxed strain, however, the modulus is low and the relaxed seismic velocity is low. When the frequency is comparable to the characteristic time of the defect, attenuation reaches a maximum, and the wave velocity changes rapidly with frequency. Similar effects are seen in porous or partially molten solids; the elastic moduli depend on frequency.

These characteristics are embodied in the standard linear solid, which is composed of an elastic spring and a dashpot (or viscous element) arranged in a parallel circuit, which is then attached to another spring. At high frequencies the second, or series, spring responds to the load, and this spring constant is the effective modulus that controls the total extension. At low frequencies the other spring and dashpot both extend, with a time constant characteristic of the dashpot, the total extension is greater, and the effective modulus is therefore lower. This system is sometimes described as a viscoelastic solid. The temperature dependence of the spring constant, or modulus, represents the anharmonic contribution to the temperature dependence of the overall modulus of the system. The temperature dependence of the viscosity of the dashpot introduces another term – the anelastic term – in
the temperature dependence of moduli or seismic velocities.

The $Q^{-1}$ of such a system is

$$Q^{-1}(\omega) = \frac{k_2}{k_1} \frac{\omega \tau}{1 + (\omega \tau)^2}$$

where $k_2$ and $k_1$ are, respectively, the spring constants (or moduli) of the series spring and the parallel spring and $\tau$ is the relaxation time.

Clearly, $Q^{-1}$, the dimensionless attenuation, is a maximum at $\omega \tau = 1$, and

$$Q^{-1}(\omega) = 2Q_{\text{max}}^{-1} \frac{\omega \tau}{1 + (\omega \tau)^2}$$

(1)

The resulting absorption peak is shown in Figure 19.1. This can be considered a plot of attenuation and velocity vs. either frequency, or temperature, since $\tau$ is a function of temperature, an exponential function for thermally activated processes.

The phase velocity is approximately given by

$$c(\omega) = c_0 \left(1 + Q_{\text{max}}^{-1} \frac{(\omega \tau)^2}{1 + (\omega \tau)^2}\right)$$

(2)

where $c_0$ is the zero-frequency velocity. The high-frequency or elastic velocity is

$$c_\infty = c_0 \left(1 + Q_{\text{max}}^{-1}\right)$$

(3)

Far away from the absorption peak, the velocity can be written

$$c(\omega) \approx c_0 \left(1 + \frac{k_1}{2k_2} Q^{-2}\right) \text{ for } \omega \tau \ll 1$$

and

$$= c_\infty \left(1 - \frac{k_1^2}{(2k_1 + k_2)k_2} Q^{-2}\right) \text{ for } \omega \tau \gg 1$$

and the $Q$ effect is only second order. In these limits, velocity is nearly independent of frequency, but $Q$ is not; $Q$ and $c$ cannot both be independent of frequency. Velocity depends on the attenuation. When $Q$ is constant, or nearly so, the fractional change in phase velocity becomes a first-order effect.

For activated processes,

$$\tau = \tau_0 \exp \frac{E^*/RT}{4}$$

(4)

where $E^*$ is an activation energy. This is where the temperature dependence of seismic velocities comes in, in anelastic processes. Velocity is not a simple linear function of temperature.

For activated processes, then,

$$Q^{-1}(\omega) = 2Q_{\text{max}}^{-1} \frac{\omega \tau_0 \exp E^*/RT}{1 + (\omega \tau_0)^2} \exp 2E^*/RT$$

(5)

The relaxation peak can be defined either by changing $\omega$ or changing $T$.

At high temperatures, or low frequencies,

$$Q^{-1}(\omega) = 2Q_{\text{max}}^{-1} \omega \tau_0 \exp E^*/RT$$

(6)

This is contrary to the general intuition that attenuation must increase with temperature. However, if $\tau$ differs greatly from seismic periods, it is possible that we may be on the low-temperature or high-frequency portion of the absorption peak, and

$$Q^{-1}(\omega) = 2Q_{\text{max}}^{-1} / (\omega \tau_0 \exp E^*/RT), \omega \tau \gg 1$$

(7)
In that case \( Q \) does decrease with an increase in \( T \), and in that regime \( Q \) increases with frequency. This appears to be the case for short-period waves in the mantle. It is also generally observed that low-\( Q \) and low-velocity regions of the upper mantle are in tectonically active and high heat-flow areas. Thus, seismic frequencies appear to be near the high-frequency, low-temperature side of the absorption peak in the Earth's upper mantle. This may not be true in the lower mantle; the absorption band shifts with frequency, as in the absorption band model for mantle attenuation.

The characteristic relaxation time also changes with pressure,

\[
\tau = \tau_0 \exp\left(\frac{E^* + PV^*}{RT}\right)
\]

(8)

where \( V^* \), the activation volume, controls the effect of pressure on \( \tau \) and \( Q \), and seismic velocity.

Most mechanisms of attenuation at seismic frequencies and mantle temperatures can be described as activated relaxation effects. Increasing temperature drives the absorption peak to higher frequencies (characteristic frequencies increase with temperature). Increasing pressure drives the peak to lower frequencies.

Absorption in a medium with a single characteristic frequency gives rise to a bell-shaped Debye peak centered at a frequency \( \omega \tau = 1 \), as shown in Figure 19.1. The specific dissipation function and phase velocity satisfy the differential equation for the standard linear solid and can be written

\[
c^2(\omega) = \frac{c_0^2}{\left(1 + \omega^2 \tau^2 c_\infty^2/c_0^2\right)^2} \times \left[1 + \omega^2 \tau^2 + 2\omega^2 \tau^2 Q_{\text{max}}^{-1}\right]^{1/2}
\]

The high-frequency \( (c) \) and low-frequency velocities are related by

\[
\frac{c_\infty^2 - c_0^2}{c_0 c_\infty} = 2Q_{\text{max}}^{-1}
\]

so that the total dispersion depends on the magnitude of the peak dissipation. For a \( Q \) of 200, a typical value for the upper mantle, the total velocity dispersion is 2%.

Solids in general, and mantle silicates in particular, are not characterized by a single relaxation time and a single Debye peak. A distribution of relaxation times broadens the peak and gives rise to an absorption band (Figure 19.2). \( Q \) can be weakly dependent on frequency in such a band. Seismic \( Q \) values are nearly constant with frequency over much of the seismic band. A nearly constant \( Q \) can be explained by involving a spectrum of relaxation times and a superposition of elementary relaxation peaks, giving

\[
Q^{-1}(\omega) = \left(2Q_{\text{max}}^{-1}/\pi\right) \tan^{-1}\left[\omega(\tau_1 - \tau_2)/(1 + \omega^2 \tau_1 \tau_2)\right]
\]

and

\[
c(\omega) = c_0 \left(1 + (Q_{\text{max}}^{-1}/2\pi) \ln \left[(1 + \omega^2 \tau_1^2)/(1 + \omega^2 \tau_2^2)\right]\right)
\]

For \( \tau_1 \ll \omega^{-1} \ll \tau_2 \) the value of \( Q^{-1} \) is constant and equal to \( Q_{\text{max}}^{-1} \). The total dispersion in this case is

\[
\frac{c_\infty - c_0}{c_0} = Q_{\text{max}}^{-1}/\pi \ln(\tau_1/\tau_2)
\]

Which depends on the ratio \( \tau_1/\tau_2 \), which is the width of the absorption band. The spread in \( \tau \) can be due to a distribution of \( \tau_0 \) or of \( E^* \).
Attenuation mechanisms
The actual physical mechanism of attenuation in the mantle is uncertain, but it is likely to be a relaxation process involving a distribution of relaxation times, or a scattering process involving a distribution of scatterers. Many of the attenuation mechanisms that have been identified in solids occur at relatively low temperatures and high frequencies and can therefore be eliminated from consideration. These include point-defect and dislocation resonance mechanisms, which typically give absorption peaks at kilohertz and megahertz frequencies at temperatures below about half the melting point. The so-called grain-boundary and cold-work peak and the 'high-temperature background' occur at lower frequencies and higher temperatures. These mechanisms involve the stress-induced diffusion of dislocations. The Bordoni peak occurs at relatively low temperature in cold-worked metals but may be a higher-temperature peak in silicates. It is apparently due to the motion of dislocations since it disappears upon annealing. Because of the large wavelengths of seismic waves, it is not required that the dissipation mechanism be microscopic, or grain-scale.

Spectrum of relaxation times
Relaxation mechanisms lead to an internal friction peak of the form

\[ Q^{-1}(\omega) = \Delta \int_{-\infty}^{\infty} D(\tau) [\omega \tau/(1 + \omega^2 \tau^2)] d\tau \]

where \( D(\tau) \) is called the retardation spectrum and \( \Delta \) is the modulus defect, the relative difference between the high-frequency, unrelaxed shear modulus and the low-frequency, relaxed modulus. The modulus defect is also a measure of the total reduction in modulus that is obtained in going from low temperature to high temperature.

Convenient forms of the retardation spectrum are given in Minster and Anderson (1981) and discussed at greater length in Theory of the Earth. These equations are now widely used in correcting seismic velocities in the mantle to standard temperatures and frequencies, the so-called anelastic correction. For simple dislocation and grain boundary networks the difference between the relaxed and unrelaxed moduli is about 8%. Anharmonic and anelastic corrections to seismic velocities are now routinely applied in comparisons with laboratory values. These are more complex, but more physically based, than the simple linear scalings between velocity and temperature or density that were used in the past.

The relationship between retardation spectra and transient creep and the Jeffreys-Lomnitz creep law are given in Theory of the Earth, Chapter 14. Jeffreys used this law as an argument against continental drift.

Partial melting
Seismic studies indicate that increased absorption, particularly of S-waves, occurs below volcanic zones and is therefore presumably related to partial melting. Regional variations in seismic absorption are a powerful tool in mapping the thermal state of the crust and upper mantle. It has also been suggested that partial melting is the most probable cause of the low-velocity layer in the upper mantle of the Earth. Thus the role of partial melting in the attenuation of seismic waves may be a critical one, at least in certain regions of the Earth. Studies of the melting of polycrystalline solids have shown that melting begins at grain boundaries, often at temperatures far below the melting point of the main constituents of the grains. This effect is caused by impurities that have collected at the grain boundaries during the initial solidification.

In principle, a large anelastic contribution can cause a large decrease in seismic velocity without partial melting but many of the low-Q regions are volcanic. In high heat-flow areas it is difficult to design a geotherm that does not imply upper mantle melting. Partial melting is a possible cause of seismic attenuation, and low velocity, particularly at very low frequencies.

Figure 19.3 shows the Q in an ice-brine-NaCl system for a concentration of 2% NaCl. Note the abrupt drop in Q as partial melting is
initiated at the eutectic temperature. There is a corresponding, but much less pronounced, drop in velocity at the same temperature.

**Bulk attenuation**

Most of the mechanisms of seismic-wave attenuation operate in shear. Shear losses, generally, are much larger than compressional losses, and therefore shear waves attenuate more rapidly than compressional waves. Bulk or volume attenuation can become important in certain circumstances. One class of such mechanisms is due to thermoelastic relaxation. An applied stress changes the temperature of a sample relative to its surrounding, or of one part of a specimen relative to another. The heat flow that then occurs in order to equalize the temperature difference gives rise to energy dissipation and to anelastic behavior. The difference between the unrelaxed and relaxed moduli is the difference between the adiabatic and isothermal moduli. Under laboratory conditions this is typically 1% for oxides and silicates.

**Absorption-band Q model for the Earth**

Attenuation in solids and liquids, as measured by the quality factor, or specific dissipation function, $Q$, is frequency dependent. In seismology, however, $Q$ is often assumed to be independent of frequency. The success of this assumption is a reflection of the limited precision, resolving power and bandwidth of seismic data and the trade-off between frequency and depth effects rather than a statement about the physics of the Earth's interior.

Frequency-independent $Q$ models provide an adequate fit to most seismic data including the normal-mode data. There is evidence, however, that short-period body waves may require higher $Q$ values. Some geophysical applications require estimates of the elastic properties of the Earth outside the seismic band. These include calculations of tidal Love numbers, Chandler periods and high-frequency moduli for comparison with ultrasonic data. The anelastic temperature derivatives of the seismic velocities, required for the interpretation of tomographic images, requires a theoretically sound dissipation function. The constant-$Q$ models cannot be used for these purposes. For these reasons it is important to have a good attenuation model for the Earth.

The theory of seismic attenuation has been worked out in some detail and discussed more fully than here in *Theory of the Earth*, Chapter 14.

For a solid characterized by a single relaxation time $Q^{-1}$ is a Debye function with a single narrow peak. For a solid with a spectrum of relaxation times, the band is broadened and the maximum attenuation is reduced. For a polycrystalline solid with a variety of grain sizes, orientations and activation energies, the absorption band can be appreciably more than several decades wide. If, as seems likely, the attenuation mechanism in the mantle is an activated process, the relaxation times should be a strong function of temperature and pressure. The location of the absorption band, therefore, changes with depth. $Q$ can be a weak function of frequency only over a limited bandwidth. If the material has a finite elastic modulus at high frequency and a nonzero modulus at low frequency, there must be high- and low-frequency cutoffs in the relaxation spectrum. Physically this means that relaxation times cannot take on arbitrarily high and low values. The anelastic temperature correction, likewise, depends on temperature and frequency. In some published studies the anelastic correction for temperature implies an infinite or semi-infinite absorption band.

The relationship between $Q$ and bandwidth indicates that a finite $Q$ requires a finite bandwidth of relaxation times and therefore an absorption band of finite width. $Q$ can be a weak function of frequency, and velocity can be a strong function of temperature, only in this band.

We can approximate the seismic absorption band in the mantle and core in the following...
DISSIPATION

Q of ice containing 2% NaCl. At low temperatures this is a solid solution. At temperatures higher than the eutectic the system is an ice–brine mixture (Spetzler and Anderson, 1968).

A fundamental

First Overtone

Second Overtone

O

J

\log_{10} f

Q = Q_{\text{min}}(f \tau_2)^{-1}, \quad f < 1/\tau_2

Q = Q_{\text{min}}(f \tau_2)^{\nu}, \quad 1/\tau_2 < f < 1/\tau_1

Q = Q_{\text{min}}(\tau_2 / \tau_1)^{\nu}(f \tau_1), \quad f > 1/\tau_1

Fig. 19.4 Schematic illustration of an absorption band.

The relaxation time for an activated process depends exponentially on temperature and pressure. Characteristic lengths, such as dislocation or grain size, are a function of tectonic stress, which is a function of depth. The location of the band, therefore, depends on tectonic stress, temperature and pressure, at least for microscopic mechanisms. The width of the band is controlled by the distribution of relaxation times, which in turn depends on the distribution of grain sizes, dislocation lengths and so on.

The effect of pressure dominates over temperature for most of the upper mantle, and tectonic stress decreases with depth and away from shear boundary layers. The absorption band is expected to move to longer periods with increasing depth. A reversal of this trend may be caused by steep stress or temperature gradients across boundary layers, or by enhanced diffusion due to the presence of fluids, changes in crystal structure or in the nature of the point defects. If we assume that the parameters of the absorption band are constant throughout the mantle, we can use the seismic data to determine the location of the band as a function of depth. This assumption is equivalent to assuming that the activation energy, \( E^a \), and activation volume, \( V^a \),
are fixed. By assuming that the characteristics of the absorption band are invariant with depth, we are assuming that the width of the band is controlled by a distribution of characteristic relaxation times rather than a distribution of activation energies. Although this assumption can be defended, to some extent it has been introduced to reduce the number of model parameters. If a range of activation energies is assumed, the shape of the band (its width and height) varies with temperature and pressure. The parameters of the absorption bands are given in Table 19.1. The locations of the bands as a function of depth are shown in Figures 19.5 through 19.7. I refer to the absorption-band model as ABM.

The variation of the characteristic times with depth in the mantle is shown in Figure 19.5. Note that both decrease with depth in the uppermost mantle. This is expected in regions of steep thermal gradient. They increase slightly below 250 km and abruptly at 400 km. No abrupt change occurs at 670 km. Apparently, a steep
### Table 19.2

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>( Q_s )</th>
<th>( Q_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1s</td>
<td>10s</td>
</tr>
<tr>
<td>5142</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td>4044</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2887</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2843</td>
<td>184</td>
<td>130</td>
</tr>
<tr>
<td>2400</td>
<td>184</td>
<td>130</td>
</tr>
<tr>
<td>2200</td>
<td>11350</td>
<td>1135</td>
</tr>
<tr>
<td>671</td>
<td>8919</td>
<td>892</td>
</tr>
<tr>
<td>421</td>
<td>5691</td>
<td>569</td>
</tr>
<tr>
<td>421</td>
<td>190</td>
<td>134</td>
</tr>
<tr>
<td>200</td>
<td>157</td>
<td>111</td>
</tr>
<tr>
<td>11</td>
<td>200</td>
<td>141</td>
</tr>
<tr>
<td>11</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

### Table 19.3

<table>
<thead>
<tr>
<th>Region</th>
<th>( Q_s )</th>
<th>( Q_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1s</td>
<td>1s</td>
</tr>
<tr>
<td>Upper mantle</td>
<td>379</td>
<td>267</td>
</tr>
<tr>
<td>Lower mantle</td>
<td>1068</td>
<td>721</td>
</tr>
<tr>
<td>Whole mantle</td>
<td>691</td>
<td>477</td>
</tr>
</tbody>
</table>

### Table 19.4

<table>
<thead>
<tr>
<th>Region</th>
<th>( \tau_1 (s) )</th>
<th>( \tau_2 (s) )</th>
<th>( Q_s )</th>
<th>( Q_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1s</td>
<td>10s</td>
</tr>
<tr>
<td>Upper mantle</td>
<td>0</td>
<td>3.33</td>
<td>479</td>
<td>1200</td>
</tr>
<tr>
<td>Lower mantle</td>
<td>0</td>
<td>0.20</td>
<td>2000</td>
<td>( 2 \times 10^4 )</td>
</tr>
<tr>
<td>Outer core</td>
<td>15.1</td>
<td>66.7</td>
<td>7530</td>
<td>753</td>
</tr>
<tr>
<td>Inner core</td>
<td>3.01</td>
<td>13.3</td>
<td>1506</td>
<td>418</td>
</tr>
</tbody>
</table>
temperature gradient and high tectonic stresses can keep the absorption band at high frequencies throughout most of the upper mantle, but these effects are overridden below 400 km where most mantle minerals are in the cubic structure. A phase change, along with high pressure and low stress, may contribute to the lengthening of the relaxation times. Relaxation times change only slightly through most of the lower mantle.

Most of the lower mantle therefore has high $Q_\omega$ for body waves and low $Q_\omega$ for free oscillations (Tables 19.2 and 19.3). The relationship between the $P$, $S$ and Rayleigh wave $Q$ is:

$$Q_p^{-1} = L Q_s^{-1} + (1 - L) Q_k^{-1}$$

where $L = (4/3)(\beta/\alpha)^2$ and $\beta$ and $\alpha$ are the shear and compressional velocities.

For a Poisson solid with $Q_k^{-1} = 0$,

$$Q_p = (9/4)Q_s$$