Frederick H. Soon

Student's Guide to CALCULUS

by J. Marsden and A. Weinstein
Volume III

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Dedicated to:

Henry, Ora, Dennis, and Debbie

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FOREWORD

This Student Guide is exceptional, maybe even unique, among such guides in that its author, Fred Soon, was actually a student user of the textbook during one of the years we were writing and debugging the book. (He was one of the best students that year, by the way.) Because of his background, Fred has taken, in the Guide, the point of view of an experienced student tutor helping you to learn calculus. While we do not always think Fred's jokes are as funny as he does, we appreciate his enthusiasm and his desire to enter into communication with his readers; since we nearly always agree with the mathematical judgements he has made in explaining the material, we believe that this Guide can serve you as a valuable supplement to our text.

To get maximum benefit from this Guide, you should begin by spending a few moments to acquaint yourself with its structure. Once you get started in the course, take advantage of the many opportunities which the text and Student Guide together provide for learning calculus in the only way that any mathematical subject can truly be mastered — through attempting to solve problems on your own. As you read the text, try doing each example and exercise yourself before reading the solution; do the same with the quiz problems provided by Fred.

Fred Soon knows our textbook better than anyone with the (possible) exception of ourselves, having spent hundreds of hours over the past ten years assisting us with its creation and proofreading. We have enjoyed our association with him over this period, and we hope now that you, too, will benefit from his efforts.

Jerry Marsden

Alan Weinstein

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HOW TO USE THIS BOOK

As the title implies, this book is intended to guide the student's study of calculus. Realizing that calculus is not the only class on the college student's curriculum, my objective in writing this book is to maximize understanding with a minimum of time and effort.

For each new section of the text, this student guide contains sections entitled Prerequisites, Prerequisite Quiz, Goals, Study Hints, Solutions to Every Other Odd Exercise, Section Quiz, Answers to Prerequisite Quiz, and Answers to Section Quiz. For each review section, I have included the solutions to every other odd exercise and a chapter test with solutions.

A list of prerequisites, if any, is followed by a short quiz to help you decide if you're ready to continue. If some prerequisite seems vague to you, review material can be found in the section or chapter of the text listed after each prerequisite. If you have any difficulty with the simple prerequisite quizzes, you may wish to review.

As you study, keep the goals in mind. They may be used as guidelines and should help you to grasp the most important points.

The study hints are provided to help you use your time efficiently. Comments have been offered to topics in the order in which they appear in the text. I have tried to point out what is worth memorizing and what isn't. If time permits, it is advisable to learn the derivations of formulas rather than just memorizing them. You will find that the course will be more meaningful

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to you and that critical parts of a formula can be recalled even under the stress of an exam. Other aspects of the study hints include clarification of text material and "tricks" which will aid you in solving the exercises. Finally, please be aware that your instructor may choose to emphasize topics which I have considered less important.

Detailed solutions to every other odd exercise, i.e., 1,5,9, etc. are provided as a study aid. Some students may find it profitable to try the exercises first and then compare the method employed in this book. Since the authors of the text wrote most of the exercises in pairs, the answers in this book may also be used as a guide to solving the corresponding even exercises. In order to save space, fractions have been written on one line, so be careful about your interpretations. Thus, $1/x + y$ means y plus $1/x$, whereas $1/(x + y)$ means the reciprocal of $x + y$. Transcendental functions such as cos, sin, ln, etc. take precedence over division, so $\cos ax/b$ means take the cosine of ax and then divide by b, whereas $\cos (ax/b)$ has an unambiguous meaning. $\ln a/2$ means half of $\ln a$, not the natural logarithm of $a/2$. Also, everything in the term after the slash is in the denominator, so $1/2\int xdx + 1$ means add 1 to the reciprocal of $2\int xdx$. It does not mean add 1 to half of the integral. The latter would be denoted $(1/2)\int xdx + 1$.

Section quizzes are included for you to evaluate your mastery of the material. Some of the questions are intended to be tricky, so do not be discouraged if you miss a few of them. The answers to these "hard" questions should add to your knowledge and prepare you for your exams. Since most students seem to fear word problems, each quiz contains at least one word problem to help you gain familiarity with this type of question.

Finally, answers have been provided to both the prerequisite and section quizzes. If you don't understand how to arrive at any of the answers, be sure

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to ask your instructor.

In the review sections, I have written more questions and answers which may appear on a typical test. These may be used along with the section quizzes to help you study for your tests.

Since Calculus was intended for a three semester course, I have also included three-hour comprehensive exams at the end of Chapters 3, 6, 9, 12, 15, and 18. These should help you prepare for your midterms and final examinations. Best of luck with all of your studies.

ACKNOWLEDGEMENTS

Several individuals need to be thanked for helping to produce this book. I am most grateful to Jerrold Marsden and Alan Weinstein for providing the first edition of Calculus from which I, as a student, learned about derivatives and integrals. Also, I am deeply appreciative for their advice and expertise which they offered during the prepartion of this book. Invaluable aid and knowledgeable reviewing were provided by my primary assistants: Stephen Hook, Frederick Daniels, and Karen Pao. Teresa Ling should be recognized for laying the groundwork with the first edition of the student guide. Finally, my gratitude goes to my father, Henry, who did the artwork; to Charles Olver and Betty Hsi, my proofreaders; to Sean Bates for his contributions; and to Ruth Edmonds and Esther Zack, whose typing made this publication a reality.

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Berkeley, California

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