Removal of Particles from Gas Streams

Particulate removal devices operate basically on the principle that a gas stream containing particles is passed through a region where the particles are acted on by external forces or caused to intercept obstacles, thereby separating them from the gas stream. When acted upon by external forces, the particles acquire a velocity component in a direction different from that of the gas stream. In order to design a separation device based on particulate separation by external forces, one must be able to compute the motion of a particle under such circumstances.

A preliminary selection of suitable particulate emission control systems is generally based on knowledge of four items: particulate concentration in the stream to be cleaned, the size distribution of the particles to be removed, the gas flow rate, and the final allowable particulate emission rate. Once the systems that are capable of providing the required efficiencies at the given flow rates have been chosen, the ultimate selection is generally made on the basis of the total cost of construction and operation. The size of a collector, and therefore its cost, is directly proportional to the volumetric flow rate of gas that must be cleaned. The operating factors that influence the cost of a device are the pressure drop through the unit, the power required, and the quantity of liquid needed (if a wet scrubbing system). In this chapter we concentrate on the design equations that are generally used for calculating efficiencies of various types of particulate emission control equipment. We shall not consider the estimation of capital or operating costs.

Devices that remove particles from gas streams rely on one or more of the following physical mechanisms:

1. Sedimentation. The particle-containing gas stream is introduced into a device or chamber where the particles settle under gravity to the floor of the chamber. Devices of this type are called settling chambers.
2. *Migration of charged particle in an electric field.* The particle-containing gas stream is introduced into a device in which the particles are charged and then subjected to an electric field. The resulting electrostatic force on the particles causes them to migrate to one of the surfaces of the device, where they are held and collected. Devices of this type are called electrostatic precipitators.

3. *Inertial deposition.* When a gas stream changes direction as it flows around an object in its path, suspended particles tend to keep moving in their original direction due to their inertia. Particulate collection devices based on this principle include cyclones, scrubbers, and filters.

4. *Brownian diffusion.* Particles suspended in a gas are always in Brownian motion. When the gas stream flows around obstacles, the natural random motion of the particles will bring them into contact with the obstacles, where they adhere and are collected. Because we know that Brownian motion is more pronounced the smaller the particle, we expect that devices based on diffusion as the separation mechanism will be most effective for small particles.

The key parameter that influences the choice of which device to employ in a particular case is the particle diameter $D_p$. As we will see, the physical mechanisms above vary greatly in their effectiveness depending on the size of the particle. Thus one of our major objectives in this chapter is to understand the effectiveness of particulate removal devices as a function of particle size.

There are several different classes of particulate control equipment that we consider in this chapter. The simplest particulate control device is a *settling chamber*, a large chamber in which the gas velocity is slowed, allowing the particles to settle out by gravity. A *cyclone* operates by causing the entire gas stream to flow in a spiral pattern inside a tapered tube. Because of the centrifugal force, particles migrate outward and collect on the wall of the tube. The particles slide down the wall and fall to the bottom, where they are removed. The clean gas generally reverses its flow and exits out of the top of the cyclone. An *electrostatic precipitator* utilizes the electrostatic force on charged particles in an electric field to separate particles from the gas stream. A high voltage drop is established between two electrodes, and particles passing through the resulting electric field acquire charge. The charged particles migrate to and are collected on an oppositely charged plate while the clean gas flows out through the device. Periodically, the plates are cleaned by rapping to shake off the layer of dust that has accumulated. A variety of *filters* operate on the principle that the particulate-laden gas is forced through an assemblage of collecting elements, such as a fiber or a filter mat. As the gas passes through the assemblage, particles accumulate on the collectors. Wet collection devices called *scrubbers* operate on the basis of the collision of particles with droplets of water that can easily be separated from the gas because of their large size.

Some general statements can be made about the nature of the various types of particulate gas-cleaning equipment. Mechanical collectors such as settling chambers or cyclones are typically much less expensive than the others but are generally only moderately efficient in particle removal. Since they are much better for large particles than for small ones, they often are used as precleaners for the more efficient final control
devices, especially at high particulate loadings. Electrostatic precipitators can treat large volumetric flow rates of gas at relatively low pressure drops with very high removal efficiencies. However, electrostatic precipitators are expensive and are relatively inflexible to changes in process operating conditions. Fabric filters tend to have very high efficiencies but are expensive and are generally limited to dry, low-temperature conditions. Scrubbing can also achieve high efficiencies and offers the auxiliary advantage that gaseous pollutants can be removed simultaneously with particles. However, scrubbers can be expensive to operate, owing to their high pressure drop and to the fact that they produce a wet sludge that must be treated or disposed of.

We begin the chapter with a discussion of how the collection or removal efficiency of a device may be defined.

### 7.1 COLLECTION EFFICIENCY

We define the collection efficiency $\eta(D_p)$ of a device for particles of diameter $D_p$ as

$$\eta(D_p) = 1 - \frac{\text{number of particles of diameter } D_p \text{ per m}^3 \text{ of gas out}}{\text{number of particles of diameter } D_p \text{ per m}^3 \text{ of gas in}} \quad (7.1)$$

The overall efficiency of the device based on particle number is

$$\eta = 1 - \frac{\text{number of particles per m}^3 \text{ of gas out}}{\text{number of particles per m}^3 \text{ of gas in}} \quad (7.2)$$

These efficiencies can be expressed in terms of the particle size distribution functions at the inlet and outlet sides of the device,

$$\eta(D_p) = \frac{n_{\text{in}}(D_p) \, dD_p - n_{\text{out}}(D_p) \, dD_p}{n_{\text{in}}(D_p) \, dD_p}$$

$$= 1 - \frac{n_{\text{out}}(D_p)}{n_{\text{in}}(D_p)} \quad (7.3)$$

and

$$\eta = \frac{\int_0^\infty \left[ n_{\text{in}}(D_p) - n_{\text{out}}(D_p) \right] \, dD_p}{\int_0^\infty n_{\text{in}}(D_p) \, dD_p}$$

$$= \frac{\int_0^\infty \eta(D_p) \, n_{\text{in}}(D_p) \, dD_p}{\int_0^\infty n_{\text{in}}(D_p) \, dD_p} \quad (7.4)$$
The definition of overall efficiency above is based on particle number. We can also define overall efficiencies based on other particle properties, such as surface area and volume (or mass). For example, the collection efficiency based on particle mass \( \eta_m \) is defined as

\[
\eta_m(D_p) = 1 - \frac{\text{mass of particles of diameter } D_p \text{ per m}^3 \text{ of gas out}}{\text{mass of particles of diameter } D_p \text{ per m}^3 \text{ of gas in}}
\]

and the overall efficiency is

\[
\eta_m = \frac{\int_{D_p}^{\infty} \left[ \frac{\pi}{6} \rho_p D_p^3 n_{\text{in}}(D_p) - \frac{\pi}{6} \rho_p D_p^3 n_{\text{out}}(D_p) \right] dD_p}{\int_{D_p}^{\infty} \left( \frac{\pi}{6} \rho_p D_p^3 n_{\text{in}}(D_p) \right) dD_p}
\]

The overall collection efficiency by mass is usually the easiest to measure experimentally. The inlet and outlet streams may be sampled by a collection device, such as a filter, that collects virtually all of the particles.

A term that is sometimes used to express collection efficiency is the penetration. The penetration is based on the amount emitted rather than captured; penetration based on particle mass is just \( P_m = 1 - \eta_m \). Alternatively, the penetration can be defined on the basis of particle number, \( P = 1 - \eta \).

We have called the relationship between collection efficiency and particle size simply the collection efficiency. Other terms that are used for this quantity are the grade efficiency or the fractional efficiency. An important point on the collection efficiency curve is the size for which \( \eta = 0.5 \). The particle size at this point is called the cut size or the cut diameter.

### 7.2 SETTLING CHAMBERS

Gravitational settling is perhaps the most obvious means of separating particles from a flowing gas stream. A settling chamber is, in principle, simply a large box through which the effluent gas stream flows and in which particles in the stream settle to the floor by gravity. Gas velocities through a settling chamber must be kept low enough so that settling particles are not reentrained. The gas velocity is usually reduced by expanding the ducting into a chamber large enough so that sufficiently low velocities result. Although in principle settling chambers could be used to remove even the smallest particles, practical limitations in the length of such chambers restrict their applicability to the removal of particles larger than about 50 \( \mu \text{m} \). Thus settling chambers are normally
used as precleaners to remove large and possibly abrasive particles, prior to passing the gas stream through other collection devices. Settling chambers offer the advantages of (1) simple construction and low cost, (2) small pressure drops, and (3) collection of particles without need for water. The main disadvantage of settling chambers is the large space that they require.

A settling chamber is, as noted above, simply a horizontal chamber through which the particle-laden gas flows and to the floor of which the particles settle. Figure 7.1 shows a simple gravity settling chamber design. Actually, the chamber may contain a number of relatively closely spaced horizontal plates so that the distance that a particle must settle to be collected is considerably smaller than the height of the overall device.

In analyzing the performance of a settling chamber, the key feature is the nature of the gas flow through the device. We can distinguish three basic idealized flow situations: (1) laminar flow, (2) plug flow (velocity uniform across the cross section) with no vertical mixing of particles, (3) plug flow with complete vertical mixing of particles. Laminar flow is characterized by a parabolic-type velocity profile; such a flow would only be realized for Reynolds numbers below that for transition to turbulent flow. In a laminar flow, the time required for a particle at height $y$ above the floor of the chamber to settle is $y/v_t$, where $v_t$ is the particle's settling velocity, and vertical mixing of particles is absent in laminar flow. (The effect of Brownian motion is generally neglected relative to the steady downward movement due to settling.) The second flow category above, plug flow with no vertical mixing of particles, is, in a sense, an approximation to laminar flow in that vertical mixing of particles is still ignored, but a flat velocity profile is assumed and the particles all settle at their settling velocities. The third category, plug flow with thorough vertical mixing, is the model for turbulent flow. In a turbulent flow settling chamber the gas velocity is assumed to be uniform across the chamber due to the turbulent mixing. Moreover, the turbulent mixing in the core of the...
chamber overwhelms the tendency of the particles to settle and maintains a uniform particle concentration vertically across the chamber. Removal by settling can be assumed to occur in a thin layer at the bottom of the chamber.

### 7.2.1 Laminar Flow Settling Chamber

In the laminar flow settling chamber the gas velocity profile is parabolic, as shown in Figure 7.2, and as a particle below the center streamline settles, it encounters fluid moving more slowly, and thus its residence time in the chamber increases over what it would have been on the higher streamline. Conversely, particles initially above the center streamline encounter faster moving streamlines as they fall until they pass the center streamline.

Consider the laminar flow settling chamber shown in Figure 7.2. The gas velocity profile for laminar flow between two parallel plates separated by a distance $H$ with the centerline of the chamber taken as $y = 0$ is

$$u_x = \frac{3}{2} \bar{u} \left[ 1 - \left( \frac{2y}{H} \right)^2 \right]$$  \hspace{1cm} (7.7)

where $\bar{u}$ is the mean velocity across the plates. We assume that particles are introduced uniformly across the entrance to the channel at concentration $N_0$.

There will be a critical height $y^*$ such that a particle of diameter $D_p$ initially at $x = 0$, $y = y^*$ will be at $y = -H/2$ at $x = L$. This particle will be the “last” particle of diameter $D_p$ collected in the device. Particles of diameter $D_p$ that entered the chamber above $y = y^*$ will not be collected; clearly, the value of $y^*$ depends on the particular $D_p$ of interest. This “last” particle collected takes time $t_f$ to fall a vertical distance $y^* + H/2$. Since $v_t$ is a constant,

$$t_f = \frac{y^* + H/2}{v_t}$$  \hspace{1cm} (7.8)

The vertical position of the particle at any time after entering the chamber is given by $dy/dt = v_y = -v_t$, which can be integrated to give

$$y = y^* - v_t t$$  \hspace{1cm} (7.9)

The horizontal position is given by $dx/dt = v_x$, or

$$\frac{dx}{dt} = \frac{3}{2} \bar{u} \left[ 1 - \frac{4}{H^2} \left( y^* - v_t t \right)^2 \right]$$  \hspace{1cm} (7.10)

---

Figure 7.2 Laminar flow settling chamber.
where the local horizontal velocity of the particle is that of the gas (7.7). Integrating (7.10) from the entrance to the exit of the chamber, we obtain

\[
\frac{LH^2}{6u} = t_f \left( \frac{H^2}{4} - y^* y^* \right) + v_i y^* t_f - \frac{v_i^2}{3} t_f^3
\]  

(7.11)

Using (7.8) and (7.11) gives

\[
\beta = \left[ 1 - 4 \left( \frac{y^*}{H} \right)^2 \right] \left( \frac{\alpha}{2} + \alpha \frac{y^*}{H} \right) + 4\alpha \frac{y^*}{H} \left( \frac{1}{2} + \frac{y^*}{H} \right)^2 - \frac{4}{3} \alpha \left( \frac{1}{2} + \frac{y^*}{H} \right)^3
\]

(7.12)

where \( \beta = 2v_i/3u \) and \( \alpha = H/L \).

To determine the expression for the collection efficiency, we need to compute the fraction of particles of a size \( D_p \) that is collected over a length \( L \). The flow of particles into the chamber, in number of particles per unit time, for a chamber of width \( W \), is

\[
\int_{-H/2}^{H/2} N_0 u_x(y) W dy = N_0 \bar{u} WH
\]

The number of particles collected per unit time is that portion of the inlet flow of particles between \( y = -H/2 \) and \( y = y^* \),

\[
\int_{-H/2}^{y^*} N_0 u_x(y) W dy = N_0 W \int_{-H/2}^{y^*} u_x(y) dy
\]

Therefore, the collection efficiency is just the ratio of the flow of particles collected to the total inlet flow,

\[
\eta(D_p) = \frac{N_0 W \int_{-H/2}^{y^*} u_x(y) dy}{N_0 \bar{u} WH} = \frac{1}{H \bar{u}} \int_{-H/2}^{y^*} u_x(y) dy
\]

(7.13)

Using (7.7), (7.13) becomes

\[
\eta(D_p) = \frac{1}{2} + \frac{3}{2} \frac{y^*}{H} - 2 \left( \frac{y^*}{H} \right)^3
\]

(7.14)

We now have two equations, (7.12) and (7.14), for the two unknowns \((y^*/H)\) and \( \eta \). We can simplify these further by letting \( z = \frac{1}{2} + (y^*/H) \). In doing so, (7.12) becomes

\[
\frac{\beta}{\alpha} = 2z^2 - \frac{4}{3} z^3
\]

(7.15)

Similarly, (7.14) can be expressed as

\[
\eta(D_p) = 3z^2 - 2z^3
\]

(7.16)
Combining (7.15) and (7.16), we see immediately that

$$\eta(D_p) = \frac{3\beta}{2\alpha} = \frac{\nu_t L}{\bar{u} H}$$  \hspace{1cm} (7.17)

This is the equation governing the collection efficiency of a laminar flow settling chamber that consists of two parallel plates of length $L$ separated by a distance $H$, with a mean gas velocity of $\bar{u}$.

To evaluate the efficiency of the laminar flow settling chamber, we need only to determine the settling velocity $\nu_t$. If the particle is sufficiently small to be in the Stokes law regime, then $\nu_t = \rho_p g D_p^2 / 18 \mu$, as derived in (5.30). Because of the large particle sizes of interest, we need not include the slip correction factor. For particles that are too large for Stokes’ law to apply, the terminal settling velocity can be determined using the drag coefficient, as outlined in Section 5.3.4.

**Example 7.1 Efficiency of a Laminar Flow Settling Chamber in the Stokes Law Regime**

Consider a settling chamber for which $H = 0.1$ m, $L = 10$ m, $\bar{u} = 0.1$ m s$^{-1}$, and $\rho_p = 1$ g cm$^{-3}$. At 298 K, $\nu_{air} = 0.15$ cm$^2$ s$^{-1}$ and $\mu = 1.8 \times 10^{-4}$ g cm$^{-1}$ s$^{-1}$. Under these conditions the Reynolds number for the channel flow is 667, so laminar flow conditions exist. From (7.17) and (5.30) we find that $\eta = 0.03024 D_p^3$, with $D_p$ in $\mu$m. Thus, for these particular conditions, the collection efficiency depends on particle diameter as follows:

<table>
<thead>
<tr>
<th>$D_p$ ($\mu$m)</th>
<th>$\eta(D_p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.03</td>
</tr>
<tr>
<td>2.0</td>
<td>0.12</td>
</tr>
<tr>
<td>3.0</td>
<td>0.27</td>
</tr>
<tr>
<td>4.0</td>
<td>0.48</td>
</tr>
<tr>
<td>5.0</td>
<td>0.76</td>
</tr>
<tr>
<td>5.75</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Thus all particles with diameter exceeding 5.75 $\mu$m are totally collected in this chamber.

### 7.2.2 Plug Flow Settling Chamber

The second type of flow situation we consider is that of plug flow with no vertical mixing of particles. We assume that the particles are distributed uniformly across the entrance to the chamber. Whether a particle is collected is determined solely by the height $y$ at its entrance above the collecting surface. A critical height $y^*$ can be defined such that all particles entering with $y \leq y^*$ are collected and those for which $y > y^*$ escape collection. The collection efficiency is then just

$$\eta(D_p) = \frac{\nu_t L}{\bar{u} H}$$
which is precisely the expression (7.17) obtained for the laminar flow settling chamber. Thus, in the parabolic velocity profile case, even though the particle falls across streamlines with different velocities, the overall effect is as if the particle were simply falling across streamlines all having a velocity equal to the mean velocity of the flow.

### 7.2.3 Turbulent Flow Settling Chamber

The flow in a rectangular channel can be assumed to be turbulent if the Reynolds number $Re_{e} > 4000$ (McCabe and Smith, 1976, p. 52). For a duct the Reynolds number can be defined as $Re_{e} = \frac{2Q}{\nu(H + NW)}$, where $r_{H}$ is the hydraulic radius, defined as the ratio of the cross-sectional area to the perimeter. Thus, for a duct of height $H$ and width $W$, $r_{H} = HW/[2(H + W)]$. The average velocity $\bar{u}$ is just the volumetric flow rate $Q$ divided by the cross-sectional area $HW$. If the duct contains $N$ horizontal plates, each space receives a volumetric flow of $Q/N$ and has a height $H/N$ (neglecting the effect of plate thickness). The Reynolds number for the flow in each space is then

$$Re_{e} = \frac{2Q}{\nu(H + NW)}$$

The turbulent flow settling chamber is shown schematically in Figure 7.3. In the laminar flow settling chamber just considered, particles settle at all heights above the floor of the chamber, the key to the analysis being to calculate the overall residence time of the particles as they fall across streamlines. The mechanism of collection in a turbulent flow settling chamber is, although ultimately based on the settling of particles under gravity, rather different from that in the laminar flow chamber. The difference is due to the turbulent flow in the chamber. In the bulk flow in the chamber, turbulent mixing is vigorous enough so that particles are overwhelmed by the flow and do not settle. We shall assume that the turbulent mixing maintains a uniform particle concentration over the height of the chamber. Very near the floor of the chamber a thin layer can be assumed to exist across which particles settle the short distance to the floor. Thus, once a particle, vigorously mixed in the core of the flow, enters this layer, it settles to the floor.

Consider a particle close to the wall. In time $dt$ the particle travels forward a distance $dx = \bar{u} dt$, where $\bar{u}$ is the mean velocity of the flow in the chamber. (Thus, we assume that the mean velocity $\bar{u}$ extends into the layer in spite of the absence of turbulent mixing in the layer.) During the time interval $dt$ the particle settles a distance $dy = v_{t} dt$. Therefore, the distances $dx$ and $dy$ are related by $dy = v_{t} dx/\bar{u}$.

---

Figure 7.3 Turbulent flow settling chamber.
In order to develop an overall design equation for the turbulent flow settling chamber, let us form a particle balance over the vertical section \( dx \) in Figure 7.3. At the entrance to the section \( dx \) there is a uniform distribution of particles across the entire chamber. The fraction of particles in the thin layer of thickness \( dy \) is just \( dy/H \). Since \( dy \) was defined in terms of \( dx \) such that \( dx \) is just the distance a particle moves in the horizontal direction while it falls the distance \( dy \), all particles in \( dy \) are collected over the distance \( dx \). Thus the fraction of particles collected in \( dx \) is \( dy/H = v_t \, dx/\bar{u}H \).

If the cross-sectional area of the device is \( A_c \), a particle number balance over the section \( dx \) is

\[
\bar{u}A_c(N\big|_x - N\big|_{x+dx}) = \left( \frac{v_t \, dx}{\bar{u}H} \right) \bar{u}A_c N\big|_x
\]

The left-hand side of (7.18) is the difference in flows in particles \( s^{-1} \) into and out of the volume \( A_c \, dx \), and the right-hand side is the number of particles \( s^{-1} \) removed in that volume. Dividing by \( dx \) and taking the limit as \( dx \to 0 \) yields

\[
\frac{dN}{dx} = - \frac{v_t}{\bar{u}H} N
\]

If the particle number concentration at the entrance to the chamber is \( N_0 \), then

\[
N(x) = N_0 \exp \left( -\frac{v_t x}{\bar{u}H} \right)
\]

Note that this equation holds for particles of each diameter since the particles are assumed not to interact with each other. Particle size dependence enters through the settling velocity \( v_t \). Thus, if desired, we can indicate the particle size dependence of \( N \) explicitly by \( N(x; D_p) \), where \( N \) is strictly the number of particles in the diameter range \( (D_p, D_p + dD_p) \).

The collection efficiency of a settling chamber of length \( L \) is

\[
\eta(D_p) = 1 - \frac{N(L)}{N_0} = 1 - \exp \left( -\frac{v_t L}{\bar{u}H} \right)
\]

We can express the collection efficiency explicitly in terms of particle diameter for Stokes law settling as

\[
\eta(D_p) = 1 - \exp \left( -\frac{LW \rho_p g D_p^2}{18 \mu Q} \right)
\]

where \( Q = \bar{u} HW \), the volumetric flow rate of gas through the chamber, and \( W \) is the width of the chamber.

We note a rather fundamental difference between the collection efficiencies for the settling chamber for laminar (and plug) and turbulent flows. The laminar flow collection
efficiency (7.17) predicts that \( \eta(D_p) = 1.0 \) for all particles large enough that \( \nu_t \geq \bar{u}H/L \). If \( \nu_t \sim D_p^2 \), the Stokes law case, then \( \eta(D_p) \) versus \( D_p \) is a parabolic curve. On the other hand, in the case of turbulent flow (7.22), \( \eta(D_p) \) approaches 1.0 asymptotically as \( D_p \to \infty \). These features are illustrated schematically in Figure 7.4. The abscissa of Figure 7.4 is the group \((\nu_t L/\bar{u} H)^{1/2}\), which for Stokes law settling is directly proportional to \( D_p \). Collection efficiency curves for actual chambers tend to have the S-shaped behavior of the turbulent flow curve in Figure 7.4 since any real unit will exhibit some degree of mixing in the flow.

**Example 7.2 Design of a Turbulent Flow Settling Chamber**

Determine the length of a settling chamber required to achieve 90% efficiency for 50-\(\mu\)m particles of density 2.0 g cm\(^{-3}\) from an airstream of 1 m\(^3\) s\(^{-1}\) at 298 K, 1 atm. The chamber is to be 1 m wide and 1 m high.

We first evaluate the Reynolds number for the chamber to determine if the flow will be laminar or turbulent.

\[
Re_c = \frac{H \bar{u} \rho}{\mu} = \frac{Q}{W \bar{u}}
\]

Using \( \nu = 0.15 \) cm\(^2\) s\(^{-1}\), \( Q = 10^6 \) cm\(^3\) s\(^{-1}\), \( W = 100 \) cm, we find \( Re_c = 6.67 \times 10^4 \). Thus the flow will be turbulent.

Now we need to determine the settling velocity of a 50-\(\mu\)m particle under the conditions of operation. We do not know ahead of time whether Stokes law will be valid for particles of this size, so to be safe we will determine the settling velocity using the drag coefficient. From (5.54), \( \text{Ga} = C_D \text{Re}^2 \), where \( \text{Re} \) is the particle Reynolds number, we can determine the value of Ga and then from Figure 5.6 we can determine the value of Re at
that value of $CD \cdot Re^2$. An alternative is to use (5.55) and (5.56). We will use Figure 5.6. The Galileo number in this problem is

$$Ga = \frac{4gD^3 \rho (\rho_p - \rho)}{3\mu^2} = CD \cdot Re^2 \cdot$$

$$= 12.1$$

From Figure 5.6, at this value of $CD \cdot Re^2$, $Re = 0.7$ and $v_1 = 21 \text{ cm s}^{-1}$. The length of the chamber can be determined from (7.21),

$$L = -\frac{\bar{u}H \ln (1 - \eta)}{v_1}$$

$$= 11 \text{ m}$$

If we had used Stokes law to calculate the settling velocity and (7.22) for the efficiency, the chamber length predicted would have been 15.2 m. Thus we see the effect of the fact that Stokes law is no longer strictly valid for 50-\mu m particles under the conditions of this example.

### 7.3 CYCLONE SEPARATORS

Cyclone separators are gas cleaning devices that utilize the centrifugal force created by a spinning gas stream to separate particles from a gas. A standard tangential inlet vertical reverse flow cyclone separator is shown in Figure 7.5. The gas flow is forced to follow the curved geometry of the cyclone while the inertia of particles in the flow causes them to move toward the outer wall, where they collide and are collected. A particle of mass $m_p$ moving in a circular path of radius $r$ with a tangential velocity $v_{\theta}$ is acted on by a centrifugal force of $m_p v_{\theta}^2 / r$. At a typical value of $v_{\theta} = 10 \text{ m s}^{-1}$, $r = 0.5 \text{ m}$, this force is 20.4 times that of gravity on the same particle. Thus we see the substantially enhanced force on the particle over that of settling alone that can be achieved in a cyclone geometry. In a cyclone the particles in the spinning gas stream move progressively closer to the outer wall as they flow through the device. As shown in Figure 7.5, the gas stream may execute several complete turns as it flows from one end of the device to the other. One way to pose the question of the design of a cyclone separator is: For a given gas flow rate and inner and outer radii, how long must the body of the cyclone be to ensure that a desired collection efficiency for particles of a given size be attained? Since the length of the body of a cyclone is related through the gas flow rate to the number of turns executed by the gas stream, the design problem is often posed in terms of computing the number of turns needed to achieve a specified collection efficiency.

There are a variety of designs of cyclone separators, differing in the manner in which the rotating motion is imparted to the gas stream. Conventional cyclones can be placed in the following categories:

1. **Reverse-flow cyclones** (tangential inlet and axial inlet)
2. Straight-through-flow cyclones
3. Impeller collectors

Figure 7.5 shows a conventional reverse-flow cyclone with a tangential inlet. The dirty gas enters at the top of the cyclone and is given a spinning motion because of its tangential entry. Particles are forced to the wall by centrifugal force and then fall down the wall due to gravity. At the bottom of the cyclone the gas flow reverses to form an inner core that leaves at the top of the unit. In a reverse-flow axial-inlet cyclone, the inlet gas is introduced down the axis of the cyclone, with centrifugal motion being imparted by permanent vanes at the top.

In straight-through-flow cyclones the inner vortex of air leaves at the bottom (rather than reversing direction), with initial centrifugal motion being imparted by vanes at the top. This type is used frequently as a precleaner to remove fly ash and large particles. The chief advantages of this unit are low pressure drop and high volumetric flow rates.

In the impeller collector, gases enter normal to a many-bladed impeller and are swept out by the impeller around its circumference while the particles are thrown into an annular slot around the periphery of the device. The principal advantage of this unit is its compactness; its chief disadvantage is a tendency toward plugging from solid buildup in the unit.

Cyclones can be constructed of any material, metal or ceramic, for example, that is capable of withstanding high temperatures, abrasive particles, or corrosive atmo-

![Figure 7.5](image)
spheres. It is necessary that the interior surface be smooth so that the collected particles may slide easily down the wall to the hopper. There are no moving parts to a cyclone, so operation is generally simple and relatively free of maintenance. Their low capital cost and maintenance-free operation make them ideal for use as precleaners for more efficient final control devices, such as electrostatic precipitators. Although cyclones have traditionally been regarded as relatively low efficiency collectors, some cyclones currently available from manufacturers can achieve efficiencies greater than 98% for particles larger than 5 μm. Generally, cyclones routinely achieve efficiencies of 90% for particles larger than 15 to 20 μm.

Consider a particle entering tangentially onto a horizontal plane of a spinning gas stream at $r_3$, as shown in Figure 7.6. Because of a centrifugal force of $m_p v_\theta^2/r$, the particle will follow a path outward across the flow streamlines. Its velocity vector will have a tangential component ($v_\theta$) and a radial component ($v_r$). Because the flow is actually into the page, there is an axial component ($v_z$) also. The velocity of the spinning gas is assumed to have only a tangential component, $u_\theta$, with $u_r = 0$. Tangential gas flows of this type usually are of the form $u_\theta r^n = \text{constant}$. As we will see shortly, for an ideal fluid in such a vortex flow $n = 1$, although in real flows the value of $n$ may range downward to 0.5. We begin our analysis of cyclone performance with the case of the ideal flow, which we will refer to as the laminar flow cyclone. Then we consider the turbulent flow cyclone in which, as in the case of the turbulent flow settling chamber, mixing in the flow maintains a uniform particle concentration at any tangential position in the cyclone. Since both of these represent idealized cases that are not attained in real cyclones, we turn finally to a semiempirical theory that has been widely used in practical cyclone design.

### 7.3.1 Laminar Flow Cyclone Separators

The so-called laminar flow cyclone does not have laminar flow in the sense of the laminar flow settling chamber, but rather a frictionless flow in which the streamlines follow the contours of the cyclone as shown in Figure 7.6. The velocity in the case of ideal flow is given in ($r, \theta$) coordinates as (Crawford, 1976, pp. 259–262).
where the entering flow is through a rectangular slot of area $W(r_2 - r_1)$.

To determine the collection efficiency consider a particle entering the cyclone at $r = r_3$ that strikes the wall at $\theta_f$. The particle’s velocity components at any point on its trajectory are $v_r$ and $v_\theta$. The radial velocity component is the terminal velocity of the particle when acted on by the centrifugal force $F_c = \frac{m_p v_\theta^2}{r}$, which, in the case in which the drag force can be given by Stokes law, is

$$v_r = \frac{F_c}{3\pi \mu D_p}$$

(7.24)

Since the $\theta$-component of the particle’s velocity is that of the fluid, $v_\theta = u_\theta$, and

$$F_c = \frac{m_p v_\theta^2}{r} = \frac{\pi}{6} \rho_p D_p^3 \frac{v_\theta^2}{r}$$

$$= \frac{\pi}{6} \rho_p D_p^3 \frac{Q^2}{W^2 r^3 (\ln r_2/r_1)^2}$$

(7.25)

Thus, combining (7.24) and (7.25), we obtain

$$v_r = \frac{\rho_p Q^2 D_p^2}{18\mu r^3 W^2 (\ln r_2/r_1)^2}$$

(7.26)

We now want to obtain an equation for the trajectory of a particle in the cyclone. The distance traveled in the $\theta$-direction in a time interval $dt$ is $v_\theta dt = r d\theta$. Also, the distance the particle moves in the $r$-direction in time $dt$ is $dr = v_r dt$. Then over a time interval $dt$, $r d\theta/v_\theta = dr/v_r$. From this relation we have

$$\frac{r}{dr} = \frac{v_\theta}{v_r}$$

(7.27)

and substituting the expressions for $v_\theta$ and $v_r$ gives us

$$\frac{d\theta}{dr} = \frac{18\mu W \ln (r_2/r_1) r}{\rho_p Q D_p^2}$$

(7.28)

a differential equation describing the particle’s trajectory. If the particle enters the device at $r = r_3$ and hits the outer wall at $\theta = \theta_f$, then integrating (7.28) gives

$$\theta_f = \frac{9\mu W \ln (r_2/r_1) r}{\rho_p Q D_p^2} (r_2^2 - r_3^2)$$

(7.29)

Conversely, we can solve (7.29) for $r_3$ to find the entrance position $r_3$ of a particle that hits the outer wall at $\theta = \theta_f$,

$$r_3 = \left[ r_2^2 - \frac{\rho_p Q D_p^2 \theta_f}{9\mu W \ln (r_2/r_1)} \right]^{1/2}$$

(7.30)
We can now determine an expression for the collection efficiency of a cyclone. Assume that the cyclone has an angle $\theta_f$. All particles that enter the cyclone at $r \geq r_3$ hit the wall over $0 \leq \theta \leq \theta_f$. If the entering particle concentration and gas velocity are uniform across the cross section, the collection efficiency is just that fraction of the particles in the entering flow that hits the outer wall before $\theta = \theta_f$.

$$\eta = \frac{r_2 - r_3}{r_2 - r_1}$$  \hspace{1cm} (7.31)

which is

$$\eta(D_p) = \frac{1 - \left[1 - \frac{\rho_p QD_p^2 \theta_f}{9 \mu W r_2^2 \ln (r_2/r_1)}\right]^{1/2}}{1 - r_1/r_2}$$ \hspace{1cm} (7.32)

The value of $\theta_f$ at which $\eta = 1$ is the value of $\theta_f$ when $r_3 = r_1$,

$$\theta_f = \frac{9 \mu W \ln (r_2/r_1)}{\rho_p QD_p^2} (r_2^2 - r_1^2)$$ \hspace{1cm} (7.33)

We had earlier noted a comparison of the centrifugal force acting on a particle to that due to gravity. Using $F_c$ from (7.25) and $F_g = (\pi/6) \rho_p D_p^3 g$, we obtain the ratio of the centrifugal to gravity force as

$$\frac{F_c}{F_g} = \frac{Q^2}{gr^3 W^2 \ln (r_2/r_1)^2}$$ \hspace{1cm} (7.34)

which, as can be shown, for typical cyclones, $F_c/F_g >> 1$.

### 7.3.2 Turbulent Flow Cyclone Separators

The model of the turbulent flow cyclone separator is shown in Figure 7.7. Because of turbulent mixing the particle concentration is assumed to be uniform across the cyclone, and, as in the case of the turbulent flow settling chamber, removal occurs across a thin layer at the outer wall. For lack of a better approximation, we continue to use the inviscid gas velocity components given by (7.23) to represent the fluid velocity field in the turbulent flow cyclone. Thus the key difference between the laminar and turbulent cyclones relates to the assumption made concerning particle behavior in the cyclone. The distance a particle travels in the $\theta$-direction in the laminar sublayer over a time interval $dt$ is $v_\theta dt = r_2 d\theta$, where we can evaluate $v_\theta$ at $r = r_2$. For the particle to be captured across the layer of thickness $dr$, $dr = v_r dt = v_r r_2 d\theta/v_\theta$, where $v_r$ is also evaluated at $r = r_2$.

To derive an expression for the change in particle number concentration with $\theta$, we perform a particle balance over the sector of angle $d\theta$. The fractional number of particles removed over $d\theta$ is just the fraction of particles that are in the boundary layer,
Sec. 7.3 Cyclone Separators

Figure 7.7 One-half complete turn of a turbulent flow cyclone.

\[ 2r_2 dr / (r_2^2 - r_1^2) \]. Thus

\[ N|_\theta - N|_{\theta + d\theta} = \frac{2r_2}{(r_2^2 - r_1^2)} \frac{dr}{d\theta} N|_\theta \]  

where we need not include the product of mean velocity and cross-sectional area since it appears on both sides of the equation. Using \( dr = \frac{v_{r_2} r_2}{\theta} \ d\theta / v_{\theta_2} \), dividing by \( d\theta \), and taking the limit as \( d\theta \) approaches zero gives us

\[ \frac{dN}{d\theta} = -\frac{v_{r_2}}{v_{\theta_2}} \frac{2r_2^2}{r_2^2 - r_1^2} N \]  

where \( v_{\theta_2} \) is \( v_\theta \) at \( r = r_2 \). This equation is to be integrated subject to \( N = N_0 \) at \( \theta = 0 \). The result is

\[ N(\theta) = N_0 \exp \left( -\frac{v_{r_2}}{v_{\theta_2}} \frac{2r_2^2}{r_2^2 - r_1^2} \theta \right) \]  

The collection efficiency of a cyclone that has an angle \( \theta_f \) is

\[ \eta(D_p) = 1 - \frac{N(\theta_f)}{N_0} \]

\[ = 1 - \exp \left[ -\frac{2v_{r_2} r^2 \theta_f}{v_{\theta_2} (r_2^2 - r_1^2)} \right] \]  

Using the explicit expressions for the two velocity components,

\[ v_{\theta_2} = \frac{Q}{Wr_2 \ln (r_2 / r_1)} \]  

\[ v_{r_2} = \frac{\rho_p Q^2 D_p^2}{18\mu r_2^3 W^2 \left( \ln r_2 / r_1 \right)^2} \]
we can express the collection efficiency in terms of the physical variables of the cyclone,

\[ \eta(D_p) = 1 - \exp \left( - \frac{\rho_p Q D_p^2 \theta_f}{9 \mu W (r_2^2 - r_1^2) \ln \left( \frac{r_2}{r_1} \right)} \right) \]  

(7.41)

This equation can be inverted to determine the angle of turn \( \theta_f \) needed to achieve a given collection efficiency for a given particle size.

7.3.3 Cyclone Dimensions

Cyclone collection efficiency increases with increasing (1) particle size, (2) particle density, (3) inlet gas velocity, (4) cyclone body length, (5) number of gas revolutions, and (6) smoothness of the cyclone wall. On the other hand, cyclone efficiency decreases with increasing (1) cyclone diameter, (2) gas outlet duct diameter, and (3) gas inlet area. For any specific cyclone whose ratio of dimensions is fixed, the collection efficiency increases as the cyclone diameter is decreased. The design of a cyclone separator represents a compromise among collection efficiency, pressure drop, and size. Higher efficiencies require higher pressure drops (i.e., inlet gas velocities) and larger sizes (i.e., body length).

The dimensions required to specify a tangential-entry, reverse-flow cyclone are shown in Figure 7.8. In classic work that still serves as the basis for cyclone design, Shepherd and Lapple determined "optimum" dimensions for cyclones. All dimensions were related to the body diameter \( D_c \). A common set of specifications is given on the right-hand side of Figure 7.8. Other standard cyclone dimensions are given by Licht (1984) and Cooper and Alley (1986). The number of revolutions that the gas makes in the outer vortex can be approximated by

\[ n_r = \frac{1}{H_c} \left( \frac{L_c + Z_c}{2} \right) \]

where the dimensions are shown in Figure 7.8.

Besides collection efficiency the other major consideration in cyclone specification is pressure drop. While higher efficiencies are obtained by forcing the gas through the cyclone at higher velocities, to do so results in an increased pressure drop. Since increased pressure drop requires increased energy input into the gas, there is ultimately an economic trade-off between collection efficiency and operating cost. A simple pressure-drop equation for cyclones is given by Cooper and Alley (1986). Cyclone pressure drops range from 250 to 4000 Pa.

7.3.4 Practical Equation for Cyclone Efficiency

We have analyzed the collection efficiency of a cyclone assuming that the particles behave as if they are in either a laminar or a turbulent flow. Actually, the flow pattern in a cyclone is a complex one, and the two models that we have presented represent extremes in cyclone performance. Although a Reynolds number for a cyclone can be de-
Figure 7.8 Geometric specifications for the design of a cyclone separator. The dimensions given on the right-hand side of the figure are those of the classic design of Shepherd and Lapple. This particular set of specifications appears in *Perry's Handbook* (Perry and Chilton, 1973; Figure 20-96, p. 20-82); reprinted by permission of McGraw-Hill Publishing Company.

fined as $\text{Re}_c = (\rho u / \mu) \left(4 A_c / \pi\right)^{1/2}$, where $A_c$ is the cross-sectional area so that $(4 A_c / \pi)^{1/2}$ is an equivalent diameter, and for the velocity it is sufficient to use $u = Q / W(r_2 - r_1)$, a characteristic velocity in the cyclone, a precise criterion for transition from laminar to turbulent flow in a cyclone does not exist. The laminar flow theory predicts a well-defined critical value for the smallest particle size that may be collected completely, whereas the turbulent flow result gives an asymptotic approach to complete collection as particle size increases. Experimentally determined collection efficiency curves generally approach 100% efficiency asymptotically and thus appear to conform more closely to turbulent than to laminar flow conditions. Since operating cyclones do not conform to either of these limiting cases, one must resort to semiempirical design equations to predict cyclone performance.

There has been a great deal of effort devoted to predicting the performance of
cyclones. Our primary goal in this section has been to present the general theoretical approaches to the problem so that the various analyses in the literature will be accessible to the reader. Surveys of design equations are available elsewhere (see, e.g., Bhatia and Cheremisinoff, 1977; Licht, 1980, 1984). We will present one such semiempirical design equation that has been applied successfully to cyclone design.

If the flow can be considered to be one of the two limiting cases analyzed above, the collection efficiency may be computed as shown earlier for a given geometry, flow rate, and number of turns. Practical design equations are generally derived by considering the particle trajectories under more realistic assumptions concerning the flow in the cyclone.

A theory developed by Leith and Licht (1972) has proved useful in practical cyclone design. In that theory, account is taken of the fact that the velocity profile in a cyclone usually does not adhere strictly to the ideal form (7.23). As we noted, a more general form of the velocity profile is \( u_0 r^n = \text{constant} \) [where (7.23) is \( n = 1 \)], where experimental observations indicate that in a cyclone \( n \) may range between 0.5 and 0.9, depending on the size of the unit and the temperature. It has been found experimentally that the exponent \( n \) may be estimated from (Licht, 1980, p. 239)

\[
n = 1 - (1 - 0.67D_c^{0.14}) \left( \frac{T}{283} \right)^{0.3}
\]

where \( D_c \) is the cyclone diameter in meters and \( T \) is the gas temperature in kelvin. The collection efficiency is given by

\[
\eta(D_p) = 1 - \exp \left( -MD_p^N \right) \tag{7.42}
\]

where \( N = 1/(n + 1) \) and

\[
M = 2 \left[ \frac{KQ \rho_p (n + 1)}{D_c^3} \right]^{N/2}
\]

where \( D_p \) is in cm, \( \rho_p \) is in g cm\(^{-3} \), \( Q \) is the gas volumetric flow rate in m\(^3\) s\(^{-1} \), \( \mu \) is in g cm\(^{-1} \) s\(^{-1} \) and \( K \) is a geometric configuration parameter that depends only on the relative dimensions of the unit. For the relative dimensions suggested in Figure 7.8, \( K = 402.9 \); for other dimensions the values of \( K \) are given by Licht (1980, 1984). The calculation of \( K \) is explained by Leith and Licht (1972) and Licht (1980).

**Example 7.3 Cyclone Collection Efficiency**

Three design equations for cyclone collection efficiency were presented in this section. We wish to compare the collection efficiencies predicted by each approach. To do so, consider a cyclone having \( W = 4 \) m and \( Q = 20 \) m\(^3\) s\(^{-1} \), inner and outer radii of 0.5 m and 1 m, respectively, and an angle of turn of 12\(\pi \). Assume that the particle size range of interest is from 1 to 30 \(\mu\)m and that the particles have a density of 2 g cm\(^{-3} \). The relative dimensions of the cyclone are those suggested in Figure 7.8. Assume \( T = 293 \) K.

Figure 7.9 shows the collection efficiencies for this cyclone predicted by the laminar flow theory (7.32), the turbulent flow theory (7.41), and the theory of Leith and Licht (7.42). We see that the laminar flow theory, which is based on computing particle trajec-
Electrostatic precipitators are one of the most widely used particulate control devices, ranging in size from those installed to clean the flue gases from the largest power plants to those used as small household air cleaners. The basic principle of operation of the electrostatic precipitator is that particles are charged, then an electric field is imposed on the region through which the particle-laden gas is flowing, exerting an attractive force on the particles and causing them to migrate to the oppositely charged electrode at right angles to the direction of gas flow. Electrostatic precipitation differs from mechanical methods of particle separation in that the external force is applied directly to the individual particles rather than indirectly through forces applied to the entire gas stream (e.g., in a cyclone separator). Particles collect on the electrode. If the particles collected are liquid, then the liquid flows down the electrode by gravity and is removed at the bottom of the device. If the particles are solid, the collected layer on the electrode is removed periodically by rapping the electrode. Particle charging is achieved by gener-

Figure 7.9  Collection efficiency curves for the conditions of Example 7.3 based on assuming laminar flow, turbulent flow and using the Leith and Licht equation.
ating ions by means of a corona established surrounding a highly charged electrode like a wire. The electric field is applied between that electrode and the collecting electrode. If the same pair of electrodes serves for particle charging and collecting, the device is called a single-stage electrostatic precipitator. Figure 7.10 shows a cylindrical single-stage electrostatic precipitator. A wire serving as the discharge electrode is suspended down the axis of a tube and held in place by a weight attached at the bottom. The sides of the cylinder form the collecting electrode. The collected particles which form a layer on the collecting electrode are removed to the dust hopper by rapping the collecting electrode. In a two-stage electrostatic precipitator, separate electrode pairs perform the charging and collecting functions.

Most industrially generated particles are charged during their formation by such means as flame ionization and friction, but usually only to a low or moderate degree. These natural charges are far too low for electrostatic precipitation (White, 1984). The

![Figure 7.10 Cylindrical single-stage electrostatic precipitator.](image-url)
high-voltage dc corona is the most effective means for particle charging and is universally used for electrostatic precipitation. The corona is formed between an active high-voltage electrode such as a fine wire and a passive ground electrode such as a plate or pipe. The corona surrounding the discharge electrode can lead to the formation of either positive or negative ions that migrate to the collecting electrode. The ions, in migrating from the discharging to the collecting electrode, collide with the particulate matter and charge the particles. Because the gas molecule ions are many orders of magnitude smaller than even the smallest particles and because of their great number, virtually all particles that flow through the device become charged. The charged particles are then transported to the collecting electrode, to which they are held by electrostatic attraction. The particles build a thickening layer on the collecting electrode. The charge slowly bleeds from the particles to the electrode. As the layer grows, the charges on the most recently collected particles must be conducted through the layer of previously collected particles. The resistance of the dust layer is called the dust resistivity.

As the particle layer grows in thickness, the particles closest to the plates lose most of their charge to the electrode. As a result, the electrical attraction between the electrode and these particles is weakened. However, the newly arrived particles on the outside layer have a full charge. Because of the insulating layer of particles, these new particles do not lose their charge immediately and thus serve to hold the entire layer against the electrode. Finally, the layer is removed by rapping, so that the layer breaks up and falls into a collecting hopper.

Of direct interest is the determination of the collection efficiency of a given precipitator as a function of precipitator geometry, gas flow rate, particle size, and gas properties. Flow in commercial electrostatic precipitators is turbulent. The prediction of the migration of particles therefore requires consideration of the motion of particles in turbulent flow subject to both electrostatic and inertial forces. Because one cannot describe exactly the motion of particles in turbulent flow, even in the absence of electric forces, there does not exist a rigorous general theory for the design of turbulent-flow electrostatic precipitators. In order to obtain design equations for collection efficiency, we resort, as we have been doing, to an idealized representation of the turbulent mixing process in the device and removal in a thin layer at the collector wall.

Electrostatic precipitators are commonly employed for gas cleaning when the volumetric throughput of gas is high. Such units are used routinely for fly ash removal from power plant flue gases. Electrostatic precipitators are also widely employed for the collection of particles and acid mists in the chemical and metallurgical process industries.

### 7.4.1 Overall Design Equation for the Electrostatic Precipitator

Figure 7.11 depicts the wall region of an electrostatic precipitator, a chamber of perimeter $P$ and cross-sectional area $A$, through which a gas containing charged particles is flowing. Turbulent flow will be assumed so that, as before, the particle number concentration is uniform at any point across the device. Again, we will suppose the existence of a thin layer adjacent to the walls of the device across which the particle migration
and collection occur. To reiterate, because the turbulent mixing in the core of the flow overwhelms the tendency of particles to migrate, the only migration occurs across a layer close to the wall. Thus, from the point of view of the overall design equation, the turbulent flow electrostatic precipitator is quite analogous to the turbulent flow settling chamber and cyclone; only the physical mechanism leading to particle migration differs.

Assume for the moment that the charge on a particle and the electric field between the electrodes are known. As we noted in Chapter 5, the electrostatic force on a particle with charge $q$ in an electric field of strength $E$ is $F_{el} = qE$. The electrical migration velocity of a particle of diameter $D_p$ in such a field is that given by (5.43)

$$v_e = \frac{qE C_c}{3\pi\mu D_p}$$

(7.43)

where we now retain the slip correction factor $C_c$ because we will be dealing with submicron-sized particles. The charge $q$ is equal to the product of the number of charges $z_p$ and the charge on an electron $e$.

As we have done before, we will define the wall layer thickness $dy$ such that all particles in $dy$ are captured over the distance $dx$, that is, $dy = v_e dt = v_e dx/\bar{u}$. The fraction of particles captured in distance $dx$ is just the ratio of the cross-sectional area of the wall layer to the overall cross-sectional area of the device, $P dy/A_c$. A balance on particle number over the section $dx$ gives

$$\bar{u} A_c (N|_x - N|_{x+dx}) = \left[ \left( \frac{P dy}{A_c} \right) N|_x \right] \bar{u} A_c$$

(7.44)

Using $dy = v_e dx/\bar{u}$ and then taking the limit as $dx \to 0$ give

$$\frac{dN}{dx} = -\frac{P v_e}{\bar{A_c} \bar{u}} N$$

(7.45)

where the electrical migration velocity $v_e$ is evaluated at conditions at the collector surface. Equation (7.45) is to be integrated subject to $N(0) = N_0$. 

Figure 7.11 Wall region of an electrostatic precipitator.
Sec. 7.4 Electrostatic Precipitation

The migration velocity \( v_e \) depends on the number of charges on the particle, which, as we will see, is a function of particle size as well as the electric field and ion density conditions in the precipitator, and on the local electric field strength. Both \( q \) and \( E \) are in general a function of distance \( x \) down the precipitator. If it can be assumed that \( v_e \) is independent of the number concentration \( N \), integration over a unit of length \( L \) yields\(^*\)

\[
N(L) = N_0 \exp \left( -\frac{P}{A_c \bar{u}} \int_0^L v_e \, dx \right)
\]

\[
= N_0 \exp \left( -\frac{A/L}{Q} \int_0^L v_e \, dx \right)
\]

(7.46)

where \( A_c \bar{u} = Q \), the volumetric flow rate of gas through the unit, and \( PL = A \), the collector surface area. Furthermore, if the electrical migration velocity can be assumed to be constant, then (7.46) gives

\[
N(L) = N_0 \exp \left( -\frac{A v_e}{Q} \right)
\]

and the collection efficiency is given by

\[
\eta = 1 - \exp \left( -\frac{A v_e}{Q} \right)
\]

(7.47)

Equation (7.47) is seen to be analogous to that derived for a turbulent flow settling chamber, with only the physical mechanism leading to particle migration differing. This equation was first used in an empirical form in 1919 by Evald Anderson and derived theoretically by W. Deutsch in 1922 (White, 1984). It has generally been referred to as the Deutsch equation and sometimes as the Deutsch-Anderson equation.

Although the Deutsch equation can be used to estimate the collection efficiency of an electrostatic precipitator, the assumption of constant \( v_e \) is overly restrictive. In the remainder of this section, we take into account the variation in migration velocity with position in the precipitator. Our development will focus on the cylinder and wire configuration, although it can be carried through in a similar fashion for other geometries, such as parallel plates. We should point out, however, that even though it is possible to derive theoretically the electric fields and migration velocities in devices with well-defined geometry, the idealized conditions corresponding to the theory seldom exist in actual practice. Factors such as particle reentrainment and gas channeling around the collecting zones cannot be accounted for theoretically. Because of these uncertainties, industrial precipitator design is often based on empirical migration velocities for use in the Deutsch equation (White, 1984). Nevertheless, it is important to understand the underlying fundamental relationships among the variables in an electrostatic precipitator, and we will develop these relationships subsequently.

### 7.4.2 Generation of the Corona

The mechanism for particle charging in an electrostatic precipitator is the generation of a supply of ions that attach themselves to the particles. The corona is the mechanism for

\*We will see subsequently that the migration velocity is, in fact, a function of the local number concentration.
forming ions. The corona can be either positive or negative. A gas usually has a few free electrons and an equal number of positive ions, a situation that is exploited in generating a corona. When a gas is placed between two electrodes, small amount of current results as the free electrons migrate to the positive electrode and the positive ions migrate to the negative electrode.

In the positive corona the discharge electrode, the wire in the cylindrical electrostatic precipitator, is at a positive potential. The few free electrons normally present in the gas migrate toward the wire. As the electrons approach the wire, their energy increases due to an increased attractive force. These free electrons collide with gas molecules, the collision leading in some cases to the ejection of an electron from the molecule, producing two free electrons and a positive ion. The two free electrons continue toward the positive electrode, gaining energy, until they collide with two more gas molecules, producing four free electrons and two positive ions. This process is referred to as an electron avalanche. The positive ions formed migrate to the negative electrode. It is these positive ions that must migrate across the entire device to the negative electrode that collide with and attach to the particles in the gas. The region immediately surrounding the wire in which the electron avalanche is established is the corona. Thus, with a positive corona the particles become positively charged. The term "corona" arises from the fact that the electron avalanche is often accompanied by the production of light.

In the negative corona the discharge electrode is maintained at a negative potential. The electron avalanche begins at the outer surface of the wire and proceeds radially outward. Close to the wire the electrons are sufficiently energetic to form positive ions upon collision with gas molecules, thus initiating the electron avalanche. The positive ions formed migrate the short distance to the wire. As the electrons migrate outward into a region of lower electric field strength, they are slowed down by collisions with gas molecules. These electrons eventually have lower energy than those that are accelerated toward the positive electrode in the positive corona. These relatively low energy electrons, rather than ejecting an electron from the gas molecule upon collision, are absorbed by the gas molecules to produce negative ions. The formation of negative ions, which begins to occur at the outer edge of the corona, essentially absorbs all the free electrons produced in the electron avalanche at the wire surface. These negative ions then migrate to the positive electrode, in the course of which attaching to gas molecules and forming negative ions. For a negative corona to be effective it is necessary that the gas molecules can absorb free electrons to form negative ions. Sulfur dioxide is one of the best electron-absorbing gases of those present in flue gases. Oxygen, CO₂ and H₂O are also effective electron absorbers. The negative corona is generally more stable than the positive corona, so it is preferred in most industrial applications. A by-product of the negative corona is the production of O₃, which is an undesirable feature of the household use of an electrostatic precipitator with a negative corona. Moreover, since the positive corona does not need an electron-absorbing gas, it is more suitable for domestic application.

A few comments are in order about the collecting, or passive, electrode. As the electrostatic precipitator is operated, a layer of the collected material builds up on the collecting electrode. Particle deposits on the precipitator collection surface must possess at least a small degree of electrical conductivity in order to conduct the ion currents from
the corona to ground. The minimum conductivity required is about $10^{-10}$ (Ω cm)$^{-1}$, which is the inverse of resistivity. A conductivity of $10^{-10}$ (Ω cm)$^{-1}$, or a resistivity of $10^{10}$ Ω cm, is small compared to that of ordinary metals but is much greater than that of good insulators such as silica and most plastics. The resistivity of a material is determined by establishing a current flow through a slab of known thickness of the material. As long as the resistivity of the collected dust layer is less than about $10^{10}$ Ω cm, the layer will surrender its charge to the electrode. A typical dust has a resistivity of about $10^8$ Ω cm at room temperature, due to a layer of water on the surface of the particles. As the temperature is increased beyond 373 K, the water is evaporated and the resistivity increases to a value characteristic of the collected solids. Fly ash resistivities can vary from $10^8$ to $10^{13}$ Ω cm. When the resistivity of the layer exceeds about $10^{10}$ Ω cm, the potential across the layer increases so that the voltage that can be maintained across the electrostatic precipitator decreases and the collection efficiency decreases. The electrical resistivity of collected particulate matter depends on its chemical composition, the constituents of the gas, and the temperature (Bickelhaupt, 1979; Han and Ziegler, 1984). The resistivity of fly ash is dependent on the content of SO$_3$, Na$_2$O, and to a lesser extent, hydrophilic compounds (Fe$_2$O$_3$, K$_2$, Li$_2$O) in the ash and on the water content in the flue gas. When sulfur-containing coal is burned, from 1 to 5% of the SO$_2$ is oxidized to SO$_3$ in the combustion process. The SO$_3$ condenses on the fly ash as H$_2$SO$_4$ and lowers its resistivity. Materials with very low resistivities, such as carbon black with a resistivity in the range $10^{-3}$ Ω cm, are difficult to collect because these materials assume the charge of the collecting electrode upon contact and are repelled toward the discharge electrode.

**7.4.3 Particle Charging**

Particle charging in an electrostatic precipitator occurs in the gas space between the electrodes where the gas ions generated by the corona bombard and become attached to the particles. The gas ions may reach concentrations as high as $10^{15}$ ions m$^{-3}$. The level of charge attained by a particle depends on the gas ion concentration, the electric field strength, the conductive properties of the particle, and the particle size. A 1-μm particle typically acquires the order of 300 electron charges, whereas a 10-μm particle can attain 30,000 electron charges. Predicting the level of charge acquired by a particle is necessary in order to predict the particle’s migration velocity, on the basis of which the collection efficiency can be calculated for a given set of operating conditions.

There are actually two mechanisms by which particles become charged in an electrostatic precipitator. In the first mechanism particle charging occurs when ions that are migrating toward the collecting electrode encounter particles to which they become attached. In migrating between the electrodes the ions follow the electric flux lines, which are curves everywhere tangent to the electric field vector. When the particle first enters the device and is uncharged, the electric flux lines deflect toward the particle, resulting in the capture of even a larger number of ions than would be captured if the ions followed their normal path between the electrodes. As the particle becomes charged, ions begin to be repelled by the particle, reducing the rate of charging. Eventually, the particle will
acquire a saturation charge and charging will cease. This mechanism is called ion bom-
bardment or field charging. The second mode of particle charging is diffusion charging,
in which the particle acquires a charge by virtue of the random thermal motion of ions
and their collision with and adherence to the particles.

The theories of both field and diffusion charging, in their full generality, are quite
complex and have received a great deal of attention. Strictly speaking, field and diffusion
charging occur simultaneously once a particle enters an electrostatic precipitator, and
hence to predict the overall charge acquired by a particle, one should consider the two
mechanisms together. However, because, as we shall see, diffusion charging is predom-
inant for particles smaller than about 1 μm in diameter and field charging is predominant
for particles larger than about 1 μm, the two mechanisms often are treated in electrostatic
precipitator design as if they occur independently. In doing so, one estimates the total
charge on a particle as the sum of the charges resulting from each of the two separate
mechanisms.

7.4.4 Field Charging

When a dielectric particle of radius \( R_p \) containing charge \( q \) is placed in a preexisting,
uniform electric field \( E_\infty \) with an initially unipolar ion density \( N_{i\infty} \), the electric potential
at point \((r, \theta, \phi)\) in the region outside the sphere is (Stratton, 1941)

$$V = \frac{q}{4\pi\varepsilon_0 r} + \left( \frac{r}{R_p} - \frac{\kappa - 1}{\kappa + 2} \frac{R_p^2}{r^2} \right) E_\infty R_p \cos \theta$$

where \( \kappa \) is the dielectric constant of the sphere and \( \varepsilon_0 \) is the permittivity of free space
\((8.85 \times 10^{-12} \text{ F m}^{-1})\). The range of values of the dielectric constant \( \kappa \) is \( \kappa = 1 \) for a
perfect insulator and \( \kappa = \infty \) for a perfect conductor. The dielectric constants of insulating
particles of mineral origin commonly are of order 2 to 10. The value of \( \kappa \) for air is
approximately 1.

The first term in \( V \) is the Coulombic potential, and the second term combines the
\( r \)-component external potential uniformly built by \( E_\infty \) and the \( r \)-component image po-
tential resulting from the sphere dielectric polarization in \( E_\infty \). The electric field around
the sphere is just the negative gradient of the potential \( V \). For the \( r \)-component of the
electric field,

$$E_r = -\frac{\partial V}{\partial r} = -E_\infty \cos \theta \left( 1 + 2 \frac{\kappa - 1}{\kappa + 2} \frac{R_p^3}{r^3} \right) + \frac{q}{4\pi\varepsilon_0 r^2}$$

At the surface of the sphere

$$E_r \bigg|_{r=R_p} = -E_\infty \cos \theta \frac{3\kappa}{\kappa + 2} + \frac{q}{4\pi\varepsilon_0 R_p^2}$$

Field charging occurs as the ions that are migrating in the field \( E_\infty \) become close
to a particle (the sphere) and follow the distorted electric field lines around the particle
and impinge on it. The electric field at the surface of the particle has a zero-potential
circle at $\theta = \theta_0$ such that for $\theta \leq \theta_0$ ions will impinge on the surface and for $\theta \geq \theta_0$ ions will drift past the particle. We find $\theta_0$ by setting $E_r |_{r = R_p} = 0$,

$$\theta_0 = \cos^{-1}\left(\frac{\kappa + 2}{3\kappa} \frac{q}{4\pi \varepsilon_0 R_p^2 E_\infty} \frac{1}{\varepsilon_0}\right)$$

To determine the impingement rate of ions (in number per second), we need to integrate the ion flux,

$$\mp B_i E_r |_{r = R_p} N_\infty$$

where $B_i$ is the ion mobility, as given in (5.44) (minus sign for positive charging, plus sign for negative charging) from $\theta = 0$ to $\theta = \theta_0$ and from $\phi = 0$ to $\phi = 2\pi$,

$$J_{fc} = \int_0^{\theta_0} \int_0^{2\pi} (\mp B_i E_r |_{r = R_p} N_\infty) R_p^2 \sin \theta \, d\theta \, d\phi$$

which gives

$$J_{fc} = \pm \frac{q_s B_i N_\infty}{4\varepsilon_0} \left(1 - \frac{q}{q_s}\right)^2$$  \hspace{1cm} (7.48)$$

where $q_s$ is the saturation charge,

$$q_s = 4\pi \varepsilon_0 \left(\frac{3\kappa}{\kappa + 2}\right) E_\infty R_p^2$$  \hspace{1cm} (7.49)$$

Since the rate of charging of the particle for singly charged ions equals the ion impingement rate $J_{fc}$ multiplied by the unit charge, $\pm e$, we obtain for the rate of charging of the particle,

$$\frac{dq}{dt} = \frac{q_s e B_i N_\infty}{4\varepsilon_0} \left(1 - \frac{q}{q_s}\right)^2 \, |q| < |q_s|$$  \hspace{1cm} (7.50)$$

which can be integrated subject to $q = 0$ at $t = 0$ to give the time-dependent field charge,

$$q = \frac{q_s e B_i N_\infty t}{e B_i N_\infty t + 4\varepsilon_0}$$  \hspace{1cm} (7.51)$$

Under usual operating conditions in an electrostatic precipitator, the saturation charge is attained soon after the particles enter the device (White, 1984). For our purposes, then, it suffices to assume that the field-charging contribution to total particle charge is given by (7.49)

$$q_{fc} = \left(\frac{3\kappa}{\kappa + 2}\right) \pi \varepsilon_0 E D_p^2$$  \hspace{1cm} (7.49)$$

and that this charge is attained by particles immediately upon entrance into the precipitator.
We can examine the validity of this approximation from (7.51). In order for \( q \) from (7.51) to be approximated by \( q_s \), it is necessary that \( t \gg 4\epsilon_0/eB_iN_{i\infty} \). Now, \( \epsilon_0 \) is the order of \( 10^{-11} \) C m\(^{-1}\) V\(^{-1}\), \( e \) is the order of \( 10^{-19} \) C, \( B_i \) is the order of \( 10^{-4} \) m\(^2\) V\(^{-1}\) s\(^{-1}\), and \( N_{i\infty} \) is the order of \( 10^{13} \) m\(^{-3}\). Thus we find that under usual conditions \( q = q_s \) for \( t > 0.1 \) s, and therefore approximating the field-charging contribution by (7.49) is valid for electrostatic precipitators since the residence time of the particles in the precipitator will generally exceed 1 s or so.

**Example 7.4 Field Charging**

The saturation charge on a particle attained by field charging in an electric field of strength \( E \) is given by (7.49). Charging electric fields in an electrostatic precipitator are typically in the range 300 to 600 kV m\(^{-1}\), but may exceed 1000 kV m\(^{-1}\) in special cases. Let us calculate the magnitude of this charge for the following conditions: \( D_p = 1 \) \( \mu \text{m} \), \( E = 500 \) kV m\(^{-1}\), \( K \gg 1 \) (conducting particle). Then from (7.49)

\[
q_{fc} = 4.17 \times 10^{-17} \text{ C}
\]

The number of electronic charges to which this charge corresponds is

\[
z_p = \frac{q_{fc}}{e} = \frac{4.17 \times 10^{-17}}{1.60 \times 10^{-19}} = 260 \text{ electronic charges}
\]

### 7.4.5 Diffusion Charging

Diffusion charging occurs as the ions in their random thermal motion collide with a particle and surrender their charge to it. In that sense the mechanism of diffusion charging is identical to that of the diffusion of uncharged vapor molecules to the surface of a particle (Section 5.5). However, because both the particle and the ions are charged, the random thermal motion of the ions in the vicinity of a particle is influenced by an electrostatic force. This force gives rise to a tendency of the ions to migrate away from the particle as the particle charge increases. The overall flux of ions to a particle thus must include both the random diffusive motion and the electrical migration. As in the case of diffusion of gas molecules to a particle, the particular flux expression depends on the ratio of the ion mean free path, \( \lambda_i \), to the particle radius, that is, the ion Knudsen number. We neglect the effect of the background electric field in the precipitator in analyzing the flux of ions to a particle.

In the free molecule regime a kinetic theory argument can be used to deduce the rate of diffusion charging. If the particle has a charge \( q \), the radial distribution of ions around the particle should be given by a Boltzmann expression (White, 1963)

\[
N_{i,s} = N_{i}\left|_{r=R_p} = N_{i\infty} \exp \left( \mp \frac{qe}{2\pi\epsilon_0 k T D_p} \right) \right.
\]

(minus sign for positive charging; plus sign for negative charging). The rate at which ions strike the surface of the particle per unit surface area is given by the effusion flux,
\[ dq \quad \frac{dt}{dt} = \pm \frac{\pi D_p^2}{4} N_{i\infty} \epsilon_i e \exp \left( \mp \frac{q e}{2 \epsilon_0 kT D_p} \right) \]

Thus the rate of accumulation of charge is

\[ q = \pm \frac{2 \epsilon_0 kT D_p}{e} \ln \left[ \exp \left( \pm \frac{q_0 e}{2 \epsilon_0 kT D_p} \right) + \frac{N_{i\infty} q^2 D_p \epsilon_i t}{8 \epsilon_0 kT} \right] \tag{7.52} \]

which can be integrated subject to \( q = q_0 \) at \( t = 0 \) to give

\[ q = \pm \frac{2 \epsilon_0 kT D_p}{e} \ln \left( 1 + \frac{N_{i\infty} e^2 D_p \epsilon_i t}{8 \epsilon_0 kT} \right) \tag{7.53} \]

where the plus sign is for positive charging and the minus sign for negative charging.

In the continuum regime the flux of ions toward the particle at any distance \( r \) is given by

\[ J_i = 4 \pi r^2 \left( D_i \frac{dN_i}{dr} \mp B_i E N_i \right) \tag{7.54} \]

The first term of the right-hand side is the diffusive contribution to the flux and the second is that due to the field-induced migration in the vicinity of the particle. The steady-state ion concentration profile cannot be prescribed to be Boltzmann equilibrium distributed since it is now influenced by the presence of the particle. The local electric field around the particle is, since we are neglecting the overall field in the precipitator, the Coulombic field

\[ E = \frac{q}{4 \pi \epsilon_0 r^2} \tag{7.55} \]

At steady state \( J_i \) is a constant independent of \( r \). We substitute (7.55) into (7.54), and solve the differential equation subject to \( N_i = N_{i\infty} \) as \( r \to \infty \) to get

\[ N_i = \mp J_i \epsilon_0 \frac{B_i q \epsilon_{i\infty}}{B_i q} + \left( \mp \frac{J_i \epsilon_0}{B_i q} + N_{i\infty} \right) \exp \left( \mp \frac{q e}{2 \epsilon_0 kT D_p} \right) \]

(upper sign for positive charging; lower sign for negative charging). Note that if \( J_i = 0 \), we recover the Boltzmann distribution.

To determine \( J_i \) we assume that \( N_i = 0 \) at \( r = R_p \),

\[ J_i = \pm \frac{B_i q N_{i\infty}}{\epsilon_0} \left[ \exp \left( \pm \frac{q e}{2 \epsilon_0 kT D_p} \right) - 1 \right]^{-1} \]

The rate of accumulation of charge is just

\[ dq \quad \frac{dt}{dt} = \frac{B_i q e N_{i\infty}}{\epsilon_0} \left[ \exp \left( \pm \frac{q e}{2 \epsilon_0 kT D_p} \right) - 1 \right]^{-1} \]
This equation can be integrated subject to \( q = q_0 \) at \( t = 0 \) to give the following implicit expression for \( q \) as a function of time:

\[
\sum_{m=1}^{\infty} \frac{1}{m!} \left( \frac{e}{2 \pi \varepsilon_0 k T D_p} \right)^m \left[ (\pm q)^m - (\pm q_0)^m \right] = \frac{B_i e N_{\infty} t}{\varepsilon_0} \tag{7.56}
\]

We have now developed diffusion charging results in the free molecule, (7.53), and continuum regimes, (7.56). Lassen (1961) obtained an expression that spans the two regimes,

\[
\frac{dq}{dt} = \frac{B_i q e N_{\infty}}{\varepsilon_0} \left[ \left( 1 \pm \frac{2qe Kn}{a \pi \varepsilon_0 k T D_p} \right) \exp \left( \pm \frac{qe}{2 \pi \varepsilon_0 k T D_p} \right) - 1 \right]^{-1} \tag{7.57}
\]

where the ion Knudsen number \( Kn = 2\lambda_i/D_p \). The ion mean free path is related to its molecular diffusivity by

\[
D_i = \frac{1}{a} \lambda_i c_i
\]

Lassen used the value of \( a = 3 \), which we recall is used in conjunction with the Fuchs-Sutugin interpolation formula [i.e., (5.95)].

In 1918 Enskog obtained the following expression for the binary diffusivity of species \( i \) in a background gas \( j \),

\[
D_{ij} = \frac{3\pi}{32} \left( \frac{30z^2 + 16z + 12}{30z^2 + 16z + 12} \right) (1 + z)\lambda_i c_i
\]

where \( z = m_i/m_j \), the ratio of the mass \((m_i)\) to that of the background gas \((m_j)\). This equation results from the second-order Chapman-Enskog solution to the Boltzmann equation for a hard sphere model. The term in the first parentheses is the correction factor to the first-order solution [recall (5.11)].

Equation (7.57) can be integrated subject to \( q = q_0 \) at \( t = 0 \) to give

\[
\sum_{m=1}^{\infty} \left( \frac{4 Kn}{a} + \frac{1}{m!} \right) \left( \frac{e}{2 \pi \varepsilon_0 k T D_p} \right)^m \left[ (\pm q)^m - (\pm q_0)^m \right] = \frac{B_i e N_{\infty} t}{\varepsilon_0} \tag{7.58}
\]

For \( Kn \gg 1 \) and \( D_i = kT \lambda_i / e \), (7.58) reduces to (7.52).

Many treatments of electrostatic precipitation confine their analysis of the diffusion charging contribution to particle charge to the free molecule result (7.53). One relevant question concerns the difference in charge predicted by that equation as compared with the more complete result (7.58). We will examine that difference in Example 7.5.

The classical diffusion charging equations derived above are based on the absence of an external electric field and the neglect of the electrostatic image force between the ions and the dielectric particles. Diffusion charging can be enhanced by an external electric field, so-called "field-enhanced diffusion." Results on combined field and diffusion charging have been obtained by Liu and Yeh (1968), Brock and Wu (1973), Smith and McDonald (1976), Liu and Kapadia (1978), and Withers and Melcher (1981). The
effect of the electrostatic image force has been considered by Marlow and Brock (1975), Liu and Pui (1977), and Davison and Gentry (1985). In the transition regime Fuchs obtained the flux for diffusion charging by flux matching. Fuchs' formula includes the electrostatic force on the trajectory of an ion in the vicinity of a particle and has shown good agreement with recent experimental results (Adachi et al., 1985).

Example 7.5 Particle Charging by Field and Diffusion Charging Mechanisms

Let us compute the charge acquired by particles as a function of $D_p$ by field and diffusion charging mechanisms separately. We consider the following conditions:

$$T = 293 \, \text{K} \quad E = 2 \times 10^5 \, \text{V m}^{-1}$$
$$N_{\infty} = 10^{13} \, \text{m}^{-3}$$
$$\kappa = 1$$

We need to select an ion mass. The ion masses are difficult to determine accurately, as the ions tend to form clusters that may change with time. Adachi et al. (1985) have considered the available data and have recommended:

$$m^+_i = 109 \text{ to } 130 \, \text{amu}$$
$$m^-_i = 50 \text{ to } 100 \, \text{amu}$$

$$B^+_i = 1.4 \times 10^{-4} \, \text{m}^2 \text{V}^{-1} \text{s}^{-1}$$
$$B^-_i = 1.9 \times 10^{-4} \, \text{m}^2 \text{V}^{-1} \text{s}^{-1}$$

Using these values, we obtain:

$$\bar{c}^+_i = 2.18 \times 10^2 \text{ to } 2.38 \times 10^2 \, \text{m s}^{-1}$$
$$\bar{c}^-_i = 2.48 \times 10^2 \text{ to } 3.52 \times 10^2 \, \text{m s}^{-1}$$
$$\lambda^+_i = 1.44 \times 10^{-8} \text{ to } 1.46 \times 10^{-8} \, \text{m}$$
$$\lambda^-_i = 1.79 \times 10^{-8} \text{ to } 1.94 \times 10^{-8} \, \text{m}$$
$$D^+_i = 3.54 \times 10^{-6} \, \text{m}^2 \text{s}^{-1}$$
$$D^-_i = 4.80 \times 10^{-6} \, \text{m}^2 \text{s}^{-1}$$

Figure 7.12 shows the number of elementary charges as a function of particle diameter for both field and diffusion charging. The field charging line is (7.49), which, since this is a log-log plot, is a straight line of slope 2. The saturation charge from field charging depends, in addition to size, only on the dielectric constant of the particle and the field strength. The diffusion charging contribution varies with time. That contribution as given by (7.58) assuming no initial charge is shown at $t = 1$ and 10 s. Also, we show by the dashed line the classic free molecule result (7.53). We note that the free molecule result is quite close to the more complete equation (7.58) for particle diameters less than about 1 \( \mu \text{m} \), the regime where diffusion charging dominates. For this reason and because the field charging contribution reaches saturation very quickly, it will suffice henceforth to use (7.52) as an approximation to the diffusion charging contribution in electrostatic precipitation with $q_0$ equal to the field charge.

Example 7.6 Migration Velocity

The charged particle migration velocity in an electric field was given by (7.43). Let us compute the migration velocities of the particles of Figure 7.12 at a charging time of $t = 1 \, \text{s}$ in air at 298 K. At this temperature, the mean free path of the air molecules is $\lambda_{m} = 0.065 \, \mu \text{m}$. The migration velocity is shown in Figure 7.13, and the individual contributions
Figure 7.12 Particle charging by field and diffusion charging mechanisms. The following conditions are assumed: \( T = 293 \, \text{K} \), \( m^* = 130 \, \text{amu} \), \( B^* = 1.4 \times 10^{-4} \, \text{m}^2 \, \text{V}^{-1} \, \text{s}^{-1} \), \( c_i^* = 218 \, \text{m} \, \text{s}^{-1} \), \( \lambda_i^* = 1.46 \times 10^{-8} \, \text{m} \), \( D_i^* = 3.54 \times 10^{-6} \, \text{m} \, \text{s}^{-2} \), \( a = 0.905 \), \( E = 2 \times 10^5 \, \text{V} \, \text{m}^{-1} \), \( N_{ioo} = 10^{13} \, \text{m}^{-3} \), \( \kappa = 1 \).

Figure 7.13 Particle migration velocity. Conditions are the same as Figure 7.12.
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Figure 7.14 Contributions to particle migration velocity from field and diffusion charging. Conditions are the same as in Figure 7.12.

are shown in Figure 7.14. We note that the migration velocity is relatively large for very small particles and very large particles. Even though the particle charge is lower for small particles, the mobility of such small particles is large enough to more than compensate for the relatively lower level of charge. Large particles are able to acquire such a substantial charge that even the increased Stokes drag cannot overcome the charge effect. This can be seen simply from the fact that the particle charge by field charging increases as $D_p$, whereas the Stokes drag increases only as $D_p$, leading to an overall increase in the migration velocity with $D_p$ as particle diameter increases.

7.4.6 The Electric Field

Our final step in developing the information needed to design an electrostatic precipitator is to calculate the electric field in the device. The chamber consists of two electrodes, the discharge and the collecting electrodes. Between the electrodes the gas contains free electrons, ions, and charged particles.

The electric field intensity, $E$, is defined in terms of the potential $V$ by

$$ E = - \nabla V $$

and in a medium with space charge density $q_v$, is governed by Poisson's equation

$$ \nabla \cdot E = \frac{q_v}{\varepsilon_0} $$

[Actually, the denominator on the right-hand side of (7.60) is the product of the dielectric constant of the medium and $\varepsilon_0$. Since $\kappa$ is essentially unity for gases under all realistic precipitation conditions, we simply use $\varepsilon_0$ in (7.60).]
If we consider, for example, two concentric cylinders, with inner radius \( r_0 \) and outer radius \( r_c \), the inner radius at voltage \( V_0 \) and the outer at \( V = 0 \), the solutions of (7.59) and (7.60) are

\[
V = \frac{q_v}{4\varepsilon_0} (r_c^2 - r^2) + \left[ V_0 - \frac{q_v}{4\varepsilon_0} (r_c^2 - r_0^2) \right] \frac{\ln (r_c/r)}{\ln (r_c/r_0)} \tag{7.61}
\]

\[
E = \frac{q_v r}{2\varepsilon_0} + \left[ \frac{V_0 - q_v (r_c^2 - r_0^2)/4\varepsilon_0}{\ln (r_c/r_0)} \right] \frac{1}{r} \tag{7.62}
\]

The electric field strength at \( r = r_c \) (the collector) is

\[
E_c = \frac{q_v r_c}{2\varepsilon_0} + \frac{V_0 - q_v (r_c^2 - r_0^2)/4\varepsilon_0}{r \ln (r_c/r_0)} \tag{7.63}
\]

As noted above, the species contributing to the space charge density are ions, electrons, and charged particles. In computing \( q_v \), it can be assumed that the gas molecules capture all the free electrons so that only the ions and charged particles contribute to \( q_v \). Actually, an ionic current flows in the direction of the electric field consisting of ions charged with the same polarity as the charging electrode and moving to the collecting electrode. The ions migrate to the collecting electrode with a velocity large enough to be unaffected by the turbulent flow in the chamber. The space charge density due to the ionic current depends on the local value of \( E \). Accounting for the dependence of \( q_v \) on \( E \) leads to a nonlinear equation for \( E \) that cannot be solved exactly. Under usual conditions of electrostatic precipitation operation, the effect of the ionic current on \( q_v \) can be neglected (Crawford, 1976).

The edge of the corona, at \( r = r_0 \) in the cylindrical case, is defined by the electric field strength \( E_0 \). An empirical expression for the electric field strength at the edge of the corona is given by White (1963) as

\[
E_0 = \pm 3 \times 10^6 f \left[ \frac{T_0 \rho_0}{T \rho_0} + 0.03 \left( \frac{T_0 \rho_0}{T \rho_0 \rho_0} \right)^{1/2} \right] \quad \text{V m}^{-1} \tag{7.64}
\]

where \( T_0 = 293 \) K, \( \rho_0 = 1 \) atm, \( r_0 \) is the radius at the edge of the corona (m), and \( f \) is a roughness factor that accounts for rough spaces on the wire surface. The effect of roughness is to reduce the field strength needed to form the corona. For a clean smooth wire, \( f = 1 \); for practical applications \( f = 0.6 \) is a reasonable value to use in the absence of other information. For a positive corona, the positive sign is used in (7.64); the negative sign for a negative corona.

We are now in a position to summarize the basic design equations for the electrostatic precipitator. To do so, we follow the treatment of Crawford (1976). The electric field strength at the edge of the corona is fixed and given by (7.64). The electric field strength (7.62) can be written in terms of \( E_0 \) as

\[
E = \frac{q_v}{2\varepsilon_0} \left( r - \frac{r_0^2}{r} \right) + \frac{r_0 E_0}{r} \quad \text{V m}^{-1} \tag{7.65}
\]
The voltage at the edge of the corona is

\[ V_0 = \frac{q_v}{4\epsilon_0}\left( r_c^2 - r_0^2 - 2r_0^2 \ln \frac{r_c}{r_0} \right) + r_0 E_0 \ln \frac{r_c}{r_0} \]  

(7.66)

Since it is usually the case that \( r_c \gg r_0 \), (7.66) may be approximated as

\[ V_0 = \frac{q_v r_c^2}{4\epsilon_0} + r_0 E_0 \ln \frac{r_c}{r_0} \]  

(7.67)

Even though we have been evaluating \( E \) as a function of radial position, keep in mind that if \( q_v \) varies down the length of the precipitator, \( E \) is also a function of axial position \( x \).

At this point we need to specify \( q_v \). As we noted above, we will not include the ion current as a contribution to the space charge density, only the charge on the particles. Field charging occurs rapidly after the particles enter the precipitator, so that if the particle concentration is substantially less than the ion concentration, it is reasonable to assume that every particle acquires the saturation charge given by (7.49) corresponding to the field strength at the entrance. For this purpose it is sufficient to use the mean electric field strength across the entrance,

\[ \hat{E}(0) = \frac{\int_{r_0}^{r_c} 2\pi r E(0, r) \, dr}{\pi (r_c^2 - r_0^2)} \]  

(7.68)

Thus the saturation charge from field changing is

\[ q_{fc} = \frac{3\kappa}{\kappa + 2} \pi \epsilon_0 D_p^2 \hat{E}(0) \]  

(7.69)

where in evaluating \( \hat{E}(0) \) we invoke the approximation that \( r_c \gg r_0 \),

\[ \hat{E}(0) = \frac{q_v(0) r_c}{3\epsilon_0} + \frac{2E_0 r_0 (r_c - r_0)}{r_c^2} \]  

(7.70)

Let us recapitulate. Particles at number concentration \( N_0 \) enter the precipitator and are assumed to be immediately charged to \( q_{fc} \), corresponding to the mean electric field strength \( \hat{E}(0) \) across the entrance to the precipitator. The space charge density at the entrance to the precipitator is then just the product of the charge on each particle and the number concentration of the particles,

\[ q_v(0) = q_{fc} N_0 \]  

(7.71)

where \( q_{fc} \) is given by (7.69). If we combine (7.69)–(7.71), we can eliminate \( \hat{E}(0) \) and obtain a relation for \( q_{fc} \) in terms of known quantities,

\[ q_{fc} = \frac{2E_0 r_0 (r_c - r_0) \epsilon_0}{r_c^2 \left[ (\kappa + 2)/3\kappa \pi D_p^2 - r_c N_0/3 \right]} \]  

(7.72)
This is the relation for the charge/particle, due to field charging, at the entrance to the precipitator. We noted above that we are assuming that each particle immediately acquires the saturation field charge upon entrance into the precipitator, and that for this assumption to hold, the number concentration of particles must be substantially less than the ion concentration. The mean electric field at the entrance $\hat{E}(0)$ is computed taking into account the space charge density due to the particles that are charged due to $\hat{E}(0)$. The contribution to $\hat{E}(0)$ from the space charge density [i.e., the first term on the right-hand side of (7.70)] cannot exceed $\hat{E}(0)$ itself. This restriction is reflected in the fact that for (7.72) to be valid, it is necessary that the denominator of (7.72) be positive. Given $D_p$, $\kappa$, and $r_c$, this condition places an upper limit on the value of $N_0$ for which the approach is valid. In fact, for the theory to be applicable, we require that

$$\frac{\kappa + 2}{3\kappa\pi D_p^2} \gg \frac{r_c N_0}{3}$$

For example, if $\kappa = 1$, $r_c = 0.3$ m, and $D_p = 1$ $\mu$m, we require that $N_0 \ll 3 \times 10^{12}$ m$^{-3}$. For $D_p = 0.1$ $\mu$m, $N_0 \ll 3 \times 10^{14}$ m$^{-3}$.

As the particles flow through the precipitator, each particle retains its charge $q_p$ and may gain additional charge due to diffusion charging. The space charge density at any point is the product of the charge $q$ on each particle and the number concentration of particles,

$$q_v(x) = q N(x) \quad (7.73)$$

If we neglect the charging contribution from diffusion charging, an assumption valid for particles larger than about 0.5 $\mu$m, the charge on each particle is just $q_p$, as given by (7.72). The space charge density $q_v(x)$ decreases down the precipitator as particles are deposited on the collecting electrode,

$$q_v(x) = \frac{2E_0 r_0 (r_c - r_0) e_0 N(x)}{r_c^2 \left[ (\kappa + 2)/3\kappa\pi D_p^2 - r_c N_0/3 \right]} \quad (7.74)$$

The electric field strength at any radial and axial position is found by combining (7.65) and (7.74),

$$E(x, r) = \frac{E_0 r_0 (r_c - r_0) (r - r_0^2/r) N(x)}{r_c^2 \left[ (\kappa + 2)/3\kappa\pi D_p^2 - r_c N_0/3 \right]} + \frac{r_0 E_0}{r} \quad (7.75)$$

The electric field strength, through its dependence on $q_v$, and therefore on $N$, also varies down the length of the precipitator.

The electric field strength at the collector is obtained from (7.75), using the approximation $r_c \gg r_0$, as

$$E_c(x) = \frac{E_0 r_0 (r_c - r_0) N(x)}{r_c \left[ (\kappa + 2)/3\kappa\pi D_p^2 - r_c N_0/3 \right]} + \frac{r_0 E_0}{r_c} \quad (7.76)$$

We are now ready to return to the basic design equation for the electrostatic precipitator. We recall that we performed a balance on the number of particles of a given
size across a differential section of the precipitator that led to (7.44). For the concentric

cylinder geometry we have been considering, the wall layer of thickness \( dr \) is defined

by \( dx/\bar{u} = dr/v_c \). The fraction of the cross-sectional area occupied by the wall layer is

\[ 2\pi r_c \, dr/(\pi (r_c^2 - r_0^2)) \]. If we assume that \( r_c \gg r_0 \), this fraction is just \( 2 \, dr/r_c \). Then

the form of (7.44) appropriate to this geometry is

\[
\bar{u} A_c \left( N\bigg|_x - N\bigg|_{x+dx} \right) = \bar{u} A_c \frac{2dr}{r_c} N\bigg|_x
\]

\[ = \bar{u} A_c \left( \frac{2v_c}{r_c} dx \right) N\bigg|_x \]  

(7.77)

Taking the limit of (7.77) as \( dx \to 0 \) and using (7.43) gives

\[
\frac{dN}{dx} = -\frac{2qE_cC_c}{3\pi\mu D_p\bar{u}r_c} N
\]

(7.78)

Using the volumetric flow rate \( Q = \bar{u}\pi (r_c^2 - r_0^2) \equiv \bar{u}\pi r_c^2 \), (7.78) can be written as

\[
\frac{dN}{dx} = -\frac{2qE_c r_c C_c}{3\mu D_p Q} N
\]

(7.79)

Now, substituting (7.76) for \( E_c(x) \) in (7.79) gives

\[
\frac{dN}{dx} = -a(bN + 1)N
\]

(7.80)

where

\[
a = \frac{2q r_0 E_0 C_c}{3\mu D_p Q}
\]

\[
b = \frac{3(r_c - r_0)}{(\kappa + 2)\kappa \pi D_p^2 - r_c N_0}
\]

Integrating (7.80) from \( N = N_0 \) at \( x = 0 \) to \( N \) at \( x = L \) gives

\[
L = \frac{1}{a} \ln \left( \frac{N_0/(bN_0 + 1)}{N/(bN + 1)} \right)
\]

(7.81)

Since the collection efficiency is \( \eta(D_p) = 1 - N/N_0 \), (7.81) can be written in

terms of the collection efficiency as

\[
L = \frac{1}{a} \ln \left( \frac{bN_0 + (1 - \eta)^{-1}}{bN_0 + 1} \right)
\]

(7.82)

or as the collection efficiency achieved by a given length,

\[
\eta = 1 - \left[ \left( bN_0 + 1 \right) e^{aL} - bN_0 \right]^{-1}
\]

(7.83)

In our derivation of the design equations for the electrostatic precipitator we assumed that the particle charge in the device is due solely to field charging at the
entrance to the precipitator. We know, however, that for particles smaller than about 0.5 \( \mu \text{m} \) diameter, diffusion charging dominates field charging. Let us see how the precipitator design equations are modified to include diffusion charging.

The space charge density at any point is given by (7.73), now written as

\[
q_r(x) = q(x)N(x) \tag{7.84}
\]

Since we assume that field charging occurs immediately at the entrance and that diffusion charging begins only for \( x > 0 \), \( q_r(0) = q_f N_0 \). The value of \( q_f \) is that given by (7.72). From (7.53) the net charge is

\[
q(x) = \pm \frac{2\pi \varepsilon_0 k TD_p}{e} \ln \left[ \exp \left( \pm \frac{q_0 e}{2\pi \varepsilon_0 k TD_p} \right) + \frac{N_{\infty} e^2 D_p}{2\varepsilon_0 k T} \left( \frac{kT}{2\pi m_f} \right)^{1/2} \frac{x}{u} \right] \tag{7.85}
\]

where time \( t \) is replaced by \( x/u \).

The electric field strength at any position in the unit is given by (7.65)

\[
E(x, r) = \frac{q_r(x)}{2\varepsilon_0} \left( r - \frac{r_0^2}{r} \right) + \frac{r_0 E_0}{r} \tag{7.86}
\]

and that at the collector surface, assuming that \( r_c \gg r_0 \), is

\[
E_c(x) = \frac{r_c q(x)N(x)}{2\varepsilon_0} + \frac{r_0 E_0}{r_c} \tag{7.87}
\]

Finally, (7.79) becomes

\[
\frac{dN}{dx} = -\frac{2r_c C_c}{3\mu D_y Q} q(x) \left( \frac{r_c}{2\varepsilon_0} q(x)N(x) + \frac{r_0 E_0}{r_c} \right) N(x) \tag{7.88}
\]

which must be solved to obtain the collection efficiency.

In this section we have focused on developing the basic equations for predicting electrostatic precipitator collection efficiency. In the design of an actual electrostatic precipitator one must specify the configuration (e.g., parallel plates or wire in tube), the plate area and spacing, the corona power and current, and the pressure drop. These will depend on the gas velocity, the particle loading, the required removal efficiency, and the resistivity of the particulate matter. White (1977) presents an analysis of all of these factors in the design of an electrostatic precipitator.

**Example 7.7 Electrostatic Precipitator Design**

An airstream flowing at 1.5 m s\(^{-1}\) at 573 K, 1 atm containing a particle mass concentration of \( 3 \times 10^{-3} \text{ g m}^{-3} \) with a particle density of 1.75 g cm\(^{-3}\) is to be treated by a cylindrical electrostatic precipitator. All particles can be assumed to have \( \kappa = 3 \). The electrostatic precipitator is to consist of a cylinder of dimensions \( r_0 = 0.005 \text{ m} \) and \( r_c = 0.1 \text{ m} \). A value of \( f = 0.7 \) can be assumed. Assume a negative corona. We want to determine the efficiency of the precipitator as a function of particle diameter and precipitator length.
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Figure 7.15 Overall efficiency of the electrostatic precipitator in Example 7.7 as a function of particle diameter and length.

Figure 7.16 Overall efficiency of the electrostatic precipitator in Example 7.7 as a function of particle diameter and length. Comparison of efficiencies calculated with field and diffusion charging and field charging only.
The volumetric flow rate of air through the precipitator is \( Q = 0.0471 \) m\(^3\) s\(^{-1}\). At 573 K the density and viscosity of air are \( \rho = 6.126 \times 10^{-4} \) g cm\(^{-3}\) and \( \mu = 2.85 \times 10^{-4} \) g cm\(^{-1}\) s\(^{-1}\). The Reynolds number is thus \( \text{Re} = \frac{u(2r_c)\rho}{\mu} = 6448 \), and the flow will be turbulent. The initial number concentration of particles depends on the particle diameter. If all the entering particles are of diameter \( D_p \), then, for a mass concentration of \( 3 \times 10^{-3} \) g cm\(^{-3}\), the feed number concentration is

\[
N_0 = 3.274 \times 10^9 D_p^{-3} \text{ m}^{-3}
\]

with \( D_p \) in \( \mu \)m.

From (7.64) the electric field strength at the edge of the corona is \( E_0 = -1.7109 \times 10^6 \) V m\(^{-1}\). The charge/particle due to field charging at the entrance to the precipitator \( q_{fc} \) is given by (7.72). The value of \( q_{fc} \) depends on the size of the feed particles. For \( D_p = 0.5 \) \( \mu \)m, for example, \( q_{fc} = -2.037 \times 10^{-18} \) C. The diffusion charging contribution to the particle charge is given by (7.85). The background ion concentration will be taken as \( N_{io0} = 10^{13} \) m\(^{-3}\), and the ion mass \( m_i \) will be estimated as in Example 7.5.

Equation (7.88) can be integrated numerically subject to \( N = N_0 \) at \( x = 0 \). The efficiency at any length \( x \) is then \( \eta = 1 - N(x)/N_0 \). Figure 7.15 shows the overall efficiency as a function of particle diameter for precipitator lengths of 1, 2, and 3 m. In this figure we also show the efficiency calculated assuming that the particle charge is the sum of independent field and diffusion charging contributions. This assumption is seen to lead to substantial errors especially in the region of minimum efficiency. Figure 7.16 gives the same result compared to that considering field charging only. We see that for particles of diameter smaller than 1 \( \mu \)m, diffusion charging cannot be neglected. Figure 7.17 shows the overall efficiency as a function of precipitator length at various particle diameters.

![Figure 7.17](image-url)

**Figure 7.17** Overall efficiency of the electrostatic precipitator in Example 7.7 as a function of precipitator length.
7.5 FILTRATION OF PARTICLES FROM GAS STREAMS

A major class of particulate air pollution control devices relies on the filtration of particles from gas streams. A variety of filter media is employed, including fibrous beds, packed beds, and fabrics. Fibrous beds used to collect airborne particles are typically quite sparsely packed, usually only about 10% of the bed volume being fibers. Packed-bed filters consist of solid packing, say, a tube and tend to have higher packing densities than do fibrous filters. Both fibrous and packed beds are widely used in ventilation systems. Fabric filters are frequently used to remove solid particles from industrial gases, whereby the dusty gas flows through fabric bags and the particles accumulate on the cloth.

The physical mechanisms by which the filtration is accomplished vary depending on the mode of filtration. Conventional sparsely packed fibrous beds can be viewed as assemblages of cylinders. In such a filter the characteristic spacing between fibers is much larger than the size of the particles being collected. Thus the mechanism of collection is not simply sieving, in which the particles are trapped in the void spaces between fibers; rather, the removal of particles occurs by the transport of particles from the gas to the surface of a single collecting element. Because the filtration mechanisms in a fibrous bed can be analyzed in terms of a single collector, it is possible to describe them in considerable theoretical detail. Packed-bed filters are sometimes viewed as assemblages of interacting, but essentially separate, spherical collectors, although the close proximity of individual packing elements casts doubt as to the validity of this approach. Because of the relatively closer packing in packed-bed filters, and the resulting difficulty of describing the particle collection process in clean theoretical terms, predicting collection in such systems is more empirically based than for fibrous filters. Fabric filter efficiencies must be predicted strictly empirically since the accumulated particle layer actually does the collecting. We will devote most of our attention in this section to filtration by fibrous filters wherein theoretical predictions may be made.

We begin with an analysis of the overall collection efficiency of a fibrous filter bed. Then we consider the mechanisms of collection by a single cylinder placed in a particulate-laden gas flow. Finally, we discuss briefly industrial fabric filters and packed-bed filters.

7.5.1 Collection Efficiency of a Fibrous Filter Bed

A fibrous filter bed is viewed as a loosely packed assemblage of single cylinders. Even though the fibers are oriented in all directions in the bed, from a theoretical point of view the bed is treated as if every fiber is normal to the gas flow through the bed. Since, as we have noted, the solid fraction of the filter, α, is generally the order of only 10%, we assume, in addition, that each fiber acts more or less independently as a collector. (As we will see later, there is assumed to be an effect of the other fibers on the flow field around an individual fiber.) Thus, to compute the particle removal by a filter bed, we basically need to determine the number of fibers per unit volume of the bed and then multiply that quantity by the efficiency of a single fiber.
Figure 7.18 shows a schematic of a filter bed. Let $D_f$ be the uniform diameter of each fiber comprising the bed. We will perform a balance on the number concentration of particles of diameter $D_p$ across the bed, and, as usual, to do so we consider the balance over a slice of thickness $dx$. Let the cross-sectional area of the bed be $A_c$, and let $L_f$ be the total length of fiber per unit volume of the bed. Then the solid fraction of the filter can be expressed in terms of $D_f$ and $L_f$ as

$$\alpha = \frac{\pi D_f^2 L_f}{4} \quad (7.89)$$

The gas velocity inside the filter is greater than that approaching the filter, $\bar{u}$, due to the volume of flow excluded by the fibers. The volumetric flow rate of air through the filter is $Q = A_c \bar{u}$, so the velocity inside the bed, $u_\infty$, is related to that upstream of the bed, $\bar{u}$, by

$$u_\infty = \frac{Q}{A_c (1 - \alpha)} = \frac{\bar{u}}{1 - \alpha} \quad (7.90)$$

The particle flows into and out of the element $dx$ are $QN \big|_x$ and $QN \big|_{x + dx}$, respectively. The number of particles removed per unit time in the element $dx$ is the product of the flow of particles into the element and the fractional removal of particles by fibers. Let the collection efficiency of a single fiber $\eta$ be defined as the ratio of the number of particles collected to the total number of particles in the projected upstream area ($D_f L_f$) of the fiber. Thus the particle balance over $dx$ is

$$A_c \bar{u} (N \big|_x - N \big|_{x + dx}) = (D_f L_f \eta) u_\infty N \big|_x (A_c \, dx)$$
Taking the limit as $dx \to 0$ and using (7.89) and (7.90), we obtain

$$\frac{dN}{dx} = -\frac{4\alpha \eta}{\pi (1 - \alpha) D_f} N$$  \hspace{1cm} (7.91)

which, when integrated over a bed of length $L$, subject to $N(0) = N_0$, gives

$$\frac{N(L)}{N_0} = \exp \left[ -\frac{4\alpha \eta L}{\pi (1 - \alpha) D_f} \right]$$  \hspace{1cm} (7.92)

The overall efficiency of the bed is

$$\eta_b = 1 - \frac{N(L)}{N_0} = 1 - \exp \left[ -\frac{4\alpha \eta L}{\pi (1 - \alpha) D_f} \right]$$  \hspace{1cm} (7.93)

The quantity $\pi (1 - \alpha) D_f / 4\alpha \eta$ can be viewed as a characteristic depth of penetration of suspended particles in the bed. Since experiments on collection by an isolated fiber are difficult, the isolated fiber collection efficiency $\eta$ is sometimes determined from (7.92) by measuring $N(L)$ and $N_0$ over a bed of length $L$ and known $\alpha$ and $D_f$.

### 7.5.2 Mechanics of Collection by a Single Fiber

As we have just seen, the basis of predicting the collection efficiency of a filter bed is the collection efficiency of a single filter element in the bed. That filter element is taken as an isolated cylinder normal to the gas flow. Three distinct mechanisms can be identified whereby particles in the gas reach the surface of the cylinder:

1. **Particles in a gas undergo Brownian diffusion** that will bring some particles in contact with the cylinder due to their random motion as they are carried past the cylinder by the flow. A concentration gradient is established after the collection of a few particles and acts as a driving force to increase the rate of deposition over that which would occur in the absence of Brownian motion. Because the Brownian diffusivity of particles increases as particle size decreases, we expect that this removal mechanism will be most important for very small particles. When analyzing collection by Brownian diffusion, we treat the particles as diffusing massless points.

2. **Interception** takes place when a particle, following the streamlines of flow around a cylinder, is of a size sufficiently large that its surface and that of the cylinder come into contact. Thus, if the streamline on which the particle center lies is within a distance $D_p/2$ of the cylinder, interception occurs.

3. **Inertial impaction** occurs when a particle is unable to follow the rapidly curving streamlines around an obstacle and, because of its inertia, continues to move toward the obstacle along a path of less curvature than the flow streamlines. Thus, collision occurs because of the particle's momentum. Note that the mechanism of inertial impaction is based on the premise that the particle has mass but no size, whereas interception is based on the premise that the particle has size but no mass.
Collection may also result from electrostatic attraction when either particles or fiber or both possess a static charge. These electrostatic forces may be either direct, when both particle and fiber are charged, or induced, when only one of them is charged. Such charges are usually not present unless deliberately introduced during the manufacture of the fiber. We will not discuss the mechanisms of electrostatic attraction here. Such a discussion is presented by Strauss (1966).

The size ranges in which the various mechanisms of collection are important are:

- Inertial impaction: $>1 \text{ \mu m}$
- Interception: $>1 \text{ \mu m}$
- Diffusion: $<0.5 \text{ \mu m}$
- Electrostatic attraction: 0.01 to 5 \text{ \mu m}

It is common to analyze the mechanisms of collection separately and then combine the individual efficiencies to give the overall collection efficiency for the cylinder or other obstacle. To see how to combine efficiencies, let us consider two independent mechanisms of collection: one with efficiency $\eta_1$, the other with efficiency $\eta_2$. The probability that a particle will escape collection by mechanism 1 is $(1 - \eta_1)$. If it escapes collection by mechanism 1, the probability that it will escape collection altogether is that probability times the probability of escaping collection by mechanism 2, $(1 - \eta_1)(1 - \eta_2)$. Thus the probability that it will be collected is $1 - (1 - \eta_1)(1 - \eta_2)$, or $\eta_1 + \eta_2 - \eta_1\eta_2$. With $n$ independent mechanisms, the probability of collection is $1 - (1 - \eta_1)(1 - \eta_2) \cdots (1 - \eta_n)$. For two independent mechanisms of collection, we see that the overall collection efficiency is

$$\eta = \eta_1 + \eta_2 - \eta_1\eta_2.$$  

(7.94)

Because collection efficiencies of two independent mechanisms, such as those listed above, are frequently such that one mechanism is dominant in a particular range of particle size, the overall efficiency is often calculated simply as $\eta = \eta_1 + \eta_2$. Later when we present collection efficiencies for impaction/interception (mechanism 1) and Brownian diffusion (mechanism 2), we will use this approximation.

Most developments of particle collection assume, for lack of better information, that particles transported to the surface of a fiber are retained by the fiber. Experiments have shown, however, that for a variety of substances and filter media, the fraction of particles striking the collector surface that adhere is generally less than unity and may in some cases be as low as 0.5. All of the results we will present can be modified by including an accommodation coefficient if one is known, although we will not discuss this factor further here.

### 7.5.3 Flow Field around a Cylinder

We begin our analysis of the collection of particles by a cylinder with a brief discussion of the velocity field around a cylinder placed normal to the flow. The Reynolds number,
based on the cylinder diameter, \( \text{Re} = \frac{D_f u_\infty \rho}{\mu} \), for the flows of interest to us is usually of order unity or smaller. It is customary to determine the flow field around the cylinder based on the assumption of creeping flow (i.e., \( \text{Re} \ll 1 \)). There exists no solution of the creeping flow equations of motion that satisfies simultaneously the condition of zero velocity at the cylinder surface and that of \( u_\infty \) far from the cylinder. The solution that diverges least rapidly when \( r \to \infty \) is (Rosenhead, 1963)

\[
\begin{align*}
    u_r &= u_\infty A_f \cos \theta \left( 1 - 2 \ln \frac{2r}{D_f} - \frac{D_f^2}{4r^2} \right) \\
    u_\theta &= u_\infty A_f \sin \theta \left( 1 + 2 \ln \frac{2r}{D_f} - \frac{D_f^2}{4r^2} \right)
\end{align*}
\]

(7.95)  (7.96)

where \( A_f = [2(2.0 - \ln \text{Re})]^{-1} \). The velocity field defined by (7.95) and (7.96) is accurate at distances for which the following condition holds:

\[
A_f \text{Re} \left( \frac{2r}{D_f} \right)^2 \ln \left( \frac{2r}{D_f} \right) \ll 1.
\]

For \( \text{Re} = 0.1 \), this condition is satisfied as long as \( 2r/D_f \ll 10 \).

The velocity field (7.95) and (7.96) pertains to low Reynolds number flow around an isolated cylinder. Our ultimate interest is in cylinders that are elements of a filter bed. Experimental pressure drop data for fibrous beds show that the drag force per filter element increases as the packing density is increased (Happel and Brenner, 1965). Thus it is advantageous to develop a velocity field that depends on the fiber solid fraction \( \alpha \).

A number of investigators have derived velocity fields around a cylinder assuming that the cylinder is contained in a fluid "cell" with a radius determined by requiring the volume of fluid to be in the same ratio to the cylinder volume as the fluid-to-fiber volume ratio in the fibrous medium (Happel, 1959; Kuwabara, 1959; Spielman and Goren, 1968). These cell models endeavor to account for the interference effect of neighboring cylinders on the flow field near a representative cylinder in an approximate way, and the resulting velocity fields are expected to apply best near the cylinder surface. Fortunately, since most mechanisms of particle capture are dominated by phenomena near the collector surface, these models are useful for providing flow fields within which to compute particle removal. The Kuwabara solution is

\[
\begin{align*}
    u_r &= \frac{u_\infty}{2 \text{Ku}} \left[ 1 - 2 \ln \frac{2r}{D_f} - \alpha \frac{D_f^2}{4r^2} \left( 1 - \frac{\alpha}{2} \right) + \frac{2\alpha r^2}{D_f^2} \right] \cos \theta \\
    u_\theta &= \frac{u_\infty}{2 \text{Ku}} \left[ 1 + 2 \ln \frac{2r}{D_f} + \alpha \frac{D_f^2}{4r^2} \left( 1 - \frac{\alpha}{2} \right) + \frac{6\alpha r^2}{D_f^2} \right] \sin \theta
\end{align*}
\]

(7.97)  (7.98)

where \( \text{Ku} = \alpha - \frac{3}{4} - \frac{\alpha^2}{4} - \frac{1}{2} \ln \alpha \). The stream function for this velocity field is

\[
\Psi = \frac{u_\infty r}{2 \text{Ku}} \left[ 1 - 2 \ln \frac{2r}{D_f} - \alpha \frac{D_f^2}{4r^2} \left( 1 - \frac{\alpha}{2} \right) + \frac{2\alpha r^2}{D_f^2} \right] \sin \theta
\]

(7.99)

The effect of crowding by neighboring fibers is to compress the streamlines and increase the fluid speed close to the central cylinder since \( \text{Ku} \) decreases as \( \alpha \) increases. Streamlines of the Kuwabara flow field, expressed as \( \Psi = \Psi / u_\infty D_f \), are shown in Figure
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7.19 for \( \alpha = 0.001, 0.01, \) and 0.1. Streamlines corresponding to constant values of \( \Psi = 0.01 \) and 0.3 are plotted. The packing density \( \alpha \) plays a role similar to that of the Reynolds number: that is, as \( \alpha \) increases, the streamlines are compressed toward the cylinder and toward the line of symmetry at \( y = 0 \).

At present there is considerable disagreement as to which of the available flow models for fibrous beds is best suited to predict particle capture. For our purposes it will be sufficient to employ the Kuwabara solution above. For more in-depth discussions of the flow fields, we refer the reader to Spielman (1977) and Adamczyk and van de Ven (1981).

7.5.4 Deposition of Particles on a Cylindrical Collector by Brownian Diffusion

When analyzing the transport of particles by Brownian diffusion, the particles are treated as if they are gas molecules (see Section 5.4), and under steady-state conditions the number concentration of particles obeys the convective diffusion equation,

\[
\mathbf{u} \cdot \nabla N = D \nabla^2 N \tag{7.100}
\]

where \( D \) is the Brownian diffusivity. By defining \( \mathbf{u}^* = \mathbf{u}/u_\infty, N^* = N/N_\infty \), and the Peclet number \( Pe = D_f u_\infty/D \), (7.100) can be placed in dimensionless form,

\[
\mathbf{u}^* \cdot \nabla N^* = \frac{1}{Pe} \nabla^2 N^* \tag{7.101}
\]

The Peclet number is the product of the Reynolds number, \( Re = D_f u_\infty \rho / \mu \), and the Schmidt number, \( Sc = \mu / \rho D \), and represents the ratio of convective to diffusive transport. The boundary conditions on (7.101) are \( N^* = 1 \) far upstream of the cylinder and \( N^* = 0 \) at its surface.

Since the dimensionless velocity field \( \mathbf{u}^* \) is itself a function of \( Re \) from the dimensionless Navier-Stokes equations, dimensional analysis implies immediately that \( N^* = N^*(r^*, Re, Pe) \), where \( r^* \) denotes the dimensionless position. Our interest is in the dependence of the flux of particles to the surface of the cylinder. The local flux of particles to the cylinder surface is

\[
-D \left( \frac{\partial N}{\partial r} \right)_{r=D_f/2} = -\frac{D N_\infty}{D_f} \left( \frac{\partial N^*}{\partial r^*} \right)_{r^*=1/2} \tag{7.102}
\]

The deposition of particles over the entire surface of the cylinder can be represented in terms of an average mass transfer coefficient \( k_{m,v} \), such that the product of the mass transfer coefficient, the surface area of the cylinder (\( \pi D f L_f \)) and the "driving force" \( (N_\infty - 0) \) is equal to the local flux from the solution of the convective diffusion equation, (7.101), integrated over the surface of the cylinder.

From the Brownian diffusion coefficients shown in Figure 5.7 we calculate that for particles in air larger than about 0.01 \( \mu m \) in diameter, \( Sc \gg 1 \). (The Schmidt number for particles in air is the same order of magnitude as for molecules of a liquid.) Thus, even though \( Re \) is generally of order 1 or smaller, \( Sc \) is sufficiently large that in
the cases of interest to us, \( Pe >> 1 \). Physically, the large Peclet number implies that convective transport greatly exceeds diffusive transport, and the only region in which the two are of an equal order of magnitude is in a boundary layer close to the surface of the body. Thus the mainstream flow carries most of the particles past the cylinder and only in the immediate neighborhood of the cylinder is the diffusional process important. In this concentration boundary layer the particle number concentration drops sharply from the free stream value of \( N^* = 1 \) to \( N^* = 0 \).

The collection efficiency for a cylinder is defined as the number of particles contained in the projected area of the cylinder deposited per unit time divided by the total flow of particles in that area. The number deposited per unit time on the surface of a cylinder of length \( L_f \) is \( k_{av}(\pi D_f L_f ) (N_\infty - 0) \), and the total flow in the projected area is \( u_\infty N_\infty (D_f L_f ) \). Thus

\[
\eta = \frac{k_{av} \pi}{u_\infty} \tag{7.103}
\]

Friedlander (1977) presents the detailed solution of the convective diffusion problem to a cylinder, yielding the collection efficiency

\[
\eta = 3.68 A_f^{1/3} Pe^{-2/3} \tag{7.104}
\]

where the value of \( A_f \) depends on the particular flow field used. For the flow field of (7.95) and (7.96), \( A_f = \left[ 2 (2.0 - \ln \text{Re} ) \right]^{-1} \), and for the Kuwabara flow field, (7.97) and (7.98), \( A_f = (2 \text{ Ku})^{-1} \).

Since \( D = kTC_c/3\pi\mu D_p \), the collection efficiency of Brownian diffusion decreases as \( D_p \) increases according to \( D_p^{-2/3} \). Thus a plot of the logarithm of the efficiency versus the logarithm of particle diameter should exhibit a slope of \( -\frac{2}{3} \).

### 7.5.5 Deposition of Particles on a Cylindrical Collector by Interception

Collection by interception occurs because the particle has a finite size. Thus if the particle center approaches within a distance of \( D_p/2 \) of the collector surface, then collection occurs. To calculate the efficiency of collection by interception we need to determine what fraction of the particles approaching the collector will pass within a distance \( D_p/2 \) of the collector surface. The usual approaches to doing this ignore the hydrodynamic interaction between the particle and the collector that results from forced drainage of the fluid from the narrowing gap during approach and also neglect the effect of intermolecular forces of attraction between the collector and the particle (Spielman, 1977). (Electrostatic forces due to charging of the particle and collector, if present, are accounted for.)

Collection by interception can be approximated by neglecting any particle inertia and assuming that incoming particles simply follow the streamlines of the flow exactly. In so doing, we need only to determine the fraction of the flow in the projected upstream area of the collector that passes within a distance \( D_p/2 \) of the collector surface. For the
velocity fields we have been considering, the collection efficiency is just

\[ \eta = 2A_f \left( \frac{D_p}{D_f} \right)^2 \] (7.105)

where, due to the neglect of particle inertia, this expression is most applicable if \( D_p/D_f \ll 1 \).

Actually, Brownian diffusion and interception can be treated simultaneously using (7.100) with the modified boundary condition that \( N = 0 \) at the collision envelope \( r = D_f/2 + D_p/2 \) rather than at \( r = D_f/2 \). Analysis of (7.100) under this condition gives (Friedlander, 1977)

\[ \eta \frac{D_p}{D_f} \text{Pe} = f \left[ A_f \left( \frac{D_p}{D_f} \right)^3 \text{Pe} \right] \] (7.106)

which includes both (7.104) and (7.105) as special cases, since (7.104) can be expressed as \( \eta (D_p/D_f) \text{Pe} = 3.68 \left[ A_f (D_p/D_f)^3 \text{Pe} \right]^{1/3} \), and (7.105) can be rewritten as \( \eta (D_p/D_f) \text{Pe} = 2 \left[ A_f (D_p/D_f)^3 \text{Pe} \right] \). We reiterate that both (7.105) and (7.106) neglect any particle inertia (except that inherent in the concept of a particle’s Brownian motion).

### 7.5.6 Deposition of Particles on a Cylindrical Collector by Inertial Impaction and Interception

The final mechanism of particle collection that we consider is inertial impaction. As we described earlier, inertial impaction results because sufficiently massive particles are unable to follow curvilinear fluid motion and tend to continue along a straight path as the fluid curves around the collector. Therefore, when one accounts for particle inertia, the collection efficiency will exceed that calculated for interception alone without particle inertia (7.105), because some particles assumed to follow the flow streamlines around the collector cannot do so because of this mass. The basic approach to analyzing inertial impaction is to compute the trajectories of particles that approach the collecting cylinder and to determine those upstream locations from which particles are collected. Figure 7.20 shows the geometry of the collection of a particle by inertial impaction and interception. The trajectory of a particle initially at a distance \( y_1 \) from the centerline is shown. If all particles between the centerline and \( y_1 \) are captured and all particles farther from the centerline than \( y_1 \) escape collection, the flow streamline through \( y_1 \) is the limiting streamline and the particle trajectory through \( y_1 \) is the limiting or critical trajectory. Once \( y_1 \) has been determined, the collection efficiency is just \( \eta = 2y_1/D_f \). If the critical trajectory is taken as that passing within a distance \( D_p/2 \) of the cylinder surface rather than touching the surface, interception is automatically included within the analysis. Our object, then, is, by calculating particle trajectories, to identify the distance \( y_1 \) and thereby the collection efficiency for combined impaction and interception.

In the absence of external forces on the particle, its equation of motion is given
Collection of a particle by a cylinder placed transverse to the flow carrying the particles by the mechanisms of inertial impaction and interception.

by (5.19)

\[ \frac{\tau}{dt} \frac{dv}{dt} = u - v \]  

(7.107)

where \( \tau = \rho_p C_v D_p^2 / 18 \mu \). The particle trajectory is determined by integrating (7.107). It is advantageous to address the problem in Cartesian coordinates, so the form of (7.107) that we integrate to determine the particle trajectory, \((x(t), y(t))\), is

\[ \tau \frac{d^2 x}{dt^2} + \frac{dx}{dt} = u_x \]  

(7.108)

\[ \tau \frac{d^2 y}{dt^2} + \frac{dy}{dt} = u_y \]  

(7.109)

With reference to Figure 7.20, the initial conditions on (7.108) and (7.109) are

\[ x(0) = b \quad \frac{dx}{dt} \bigg|_{t=0} = u_x(b, y_1) \]  

(7.110)

\[ y(0) = y_1 \quad \frac{dy}{dt} \bigg|_{t=0} = u_y(b, y_1) \]  

(7.111)

At this point we need to specify the velocity field in Cartesian coordinates. Upon transforming the Kuwabara velocity field (7.97) and (7.98) into Cartesian coordinates, it is clear that a numerical solution of (7.108) and (7.109) is necessary. We can obtain an approximate solution by replacing the exact expressions for \( u_x \) and \( u_y \) by appropriate average velocities between points 1 and 2 in Figure 7.20 (Crawford, 1976). The average velocity in the \( x \)-direction between points 1 and 2 is

\[ \bar{u}_x = \frac{1}{2} (u_x \bigg|_1 + u_x \bigg|_2) \]  

(7.112)
Now, \( b \) is chosen so that \( u_x \big|_1 = -u_\infty \). Equating mass flows at planes 1 and 2, \( -u_\infty y_1 = u_x \big|_2 y_2 \). Thus

\[
\bar{u}_x = -\frac{u_\infty}{2} \left( 1 + \frac{y_1}{y_2} \right)
\]

(7.113)

The average velocity in the \( y \)-direction is obtained by noting that the fluid must travel the vertical distance from 1 to 2 in the time during which it travels horizontally,

\[
\bar{u}_y = \frac{y_2 + D_f/2 - y_1}{-b/\bar{u}_x} = \frac{u_\infty}{2b} \left[ \left( y_2 + \frac{D_f}{2} - y_1 \right) \left( 1 + \frac{y_1}{y_2} \right) \right]
\]

(7.114)

Now we need to obtain a relation between \( y_1 \) and \( y_2 \). We see that \( y_1 \) and \( y_2 \) lie on the same fluid streamline. Streamlines in a flow are defined by the stream function \( \Psi \), such that lines of constant \( \Psi \) are the streamlines. The stream function for the Kuwabara flow field was given by (7.99). To obtain a relation between \( y_1 \) and \( y_2 \), we note that the stream function at point 2 where \( \theta = \pi /2 \) and \( r = D_f/2 + y_2 \),

\[
\Psi \big|_2 = -\frac{u_\infty \left( (D_f/2) + y_2 \right)}{2Ku} \left[ 2 \ln \frac{D_f + 2y_2}{D_f} - 1 + \alpha \right]
\]

\[
+ \frac{D_f^2}{4 \left( (D_f/2) + y_2 \right)^2} \left( 1 - \frac{\alpha}{2} \right) - \frac{2\alpha}{D_f^2} \left( D_f^2 + y_2 \right)^2 \]

(7.115)

The value of the stream function at point 1 is \( \Psi \big|_1 = -u_\infty y_1 \).* Equating the values of \( \Psi \) at the two points gives the desired relation between \( y_1 \) and \( y_2 \),

\[
\frac{2y_1}{D_f} = \frac{1}{2Ku} \left( 1 + \frac{2y_2}{D_f} \right) \left[ 2 \ln \left( 1 + \frac{2y_2}{D_f} \right) - 1 \right]
\]

\[
+ \alpha + \frac{1 - \alpha/2}{\left( 1 + 2y_2/D_f \right)^2} - \frac{\alpha}{2} \left( 1 + \frac{2y_2}{D_f} \right)^2 \]

(7.116)

This equation gives us the relationship between any two points at planes 1 and 2, as expressed by the distance \( y_1 \) from the \( y = 0 \) line at plane 1 and by the distance \( y_2 \) from the cylinder surface at plane 2, that lie on the same streamline of the Kuwabara flow field. Now we need to find that particular streamline on which a particle starting at position \( y_1 \) at plane 1 is just captured, that is such that its trajectory comes within a distance \( D_p/2 \) of the cylinder surface at plane 2. To find the starting location \( y_1 \) such that a particle is just captured, we turn to the equations of motion of the particle.

As noted, in order to make the problem more tractable we will replace \( u_x \) and \( u_y \),

*To see this, note that \( u_x = -\partial \Psi / \partial y \). At point 1, \( u_x = -u_\infty \). Integrating from the \( y = 0 \) streamline along which \( \Psi = 0 \) to \( y = y_1 \) gives \( \Psi = u_\infty y_1 \).
in (7.108) and (7.109) by $\bar{u}_x$ and $\bar{u}_y$,

$$\frac{\tau}{\bar{u}_x} \frac{d^2x}{dt^2} + \frac{dx}{dt} = \bar{u}_x$$  \hspace{1cm} (7.117)$$

$$\frac{\tau}{\bar{u}_y} \frac{d^2y}{dt^2} + \frac{dy}{dt} = \bar{u}_y$$  \hspace{1cm} (7.118)$$
to be solved subject to

$$x(0) = b \hspace{1cm} \frac{dx}{dt} \bigg|_{t=0} = \bar{u}_x$$  \hspace{1cm} (7.119)$$

$$y(0) = y_1 \hspace{1cm} \frac{dy}{dt} \bigg|_{t=0} = 0$$  \hspace{1cm} (7.120)$$
The solution of (7.117) and (7.118) subject to (7.119) and (7.120) is

$$x(t) = b + \bar{u}_x t$$  \hspace{1cm} (7.121)$$

$$y(t) = y_1 - \bar{u}_y \tau (1 - e^{-t/\tau}) + \bar{u}_y t$$  \hspace{1cm} (7.122)$$
For capture to occur, $y = (D_f + D_p)/2$ when $x = 0$. This occurs at $t = -b/\bar{u}_x$. Thus (7.122) becomes

$$y_1 = \frac{D_f + D_p}{2} + \bar{u}_y \tau (1 - e^{b/\bar{u}_x \tau}) + \frac{\bar{u}_y b}{\bar{u}_x}$$  \hspace{1cm} (7.123)$$
This equation provides a relationship among the starting position $y_1$, the particle properties, $D_p$ and $\tau$, and the average flow field, $\bar{u}_x$ and $\bar{u}_y$, for a particle that is just captured at $x = 0$. Equation (7.123) therefore defines the \textit{limiting trajectory}. All particles that start out at plane 1 with $y \leq y_1$ are collected, and vice versa.

We now have (7.116), which relates $y_1$ and $y_2$ along any streamline of the flow, and (7.123), which specifies that particular $y_1$ for which capture is just attained. Note that these two equations are coupled since $\bar{u}_x$ and $\bar{u}_y$ depend on both $y_1$ and $y_2$ through (7.113) and (7.114). Thus we now have two equations (7.116) and (7.123), in the two unknowns $y_1$ and $y_2$. (We are not really interested in the value of $y_2$.)

At this point we need to address the upstream distance $b$. Recall that we related the fiber volume fraction $\alpha$ to the length of fiber per unit volume of filter by (7.89). Let the cylinder of radius $b$ be the void region around the cylinder. This idea is consistent with the choice of $b$ as that point where the approach velocity can be taken as $u_\infty$. Thus $\alpha = D_f^2/4b^2$ or

$$b = \frac{D_f}{2\sqrt{\alpha}}$$  \hspace{1cm} (7.124)$$
It is useful to place our results in dimensionless form so that the problem need not be resolved for every different combination of variables. The natural length scale to use is the diameter of the cylinder $D_f$, and the characteristic velocity is the approach velocity
Thus the two components of the average velocity can be expressed as

\[
\begin{align*}
\frac{\bar{u}_x}{u_\infty} &= -\frac{1}{2} \left( 1 + \frac{2y_1/D_f}{2y_2/D_f} \right) \\
\frac{\bar{u}_y}{u_\infty} &= \frac{\sqrt{\alpha}}{2} \left[ \left( 1 + \frac{2y_1/D_f}{2y_2/D_f} \right) \left( 1 + \frac{2y_2}{D_f} - \frac{2y_1}{D_f} \right) \right]
\end{align*}
\] (7.125) (7.126)

Using (7.125) and (7.126) in (7.123), we obtain

\[
\frac{2y_1}{D_f} = \left( 1 + \frac{D_p}{D_f} \right) + St \sqrt{\alpha} \left[ \left( 1 + \frac{2y_1/D_f}{2y_2/D_f} \right) \left( 1 + \frac{2y_2}{D_f} - \frac{2y_1}{D_f} \right) \right]
\times \left\{ 1 - \exp \left[ -\frac{1}{St \sqrt{\alpha}} \left( 1 + \frac{2y_1/D_f}{2y_2/D_f} \right) ^{-1} \right] \right\} - \left( 1 + \frac{2y_2}{D_f} - \frac{2y_1}{D_f} \right)
\] (7.127)

where the dimensionless distances, \(2y_1/D_f\) and \(2y_2/D_f\), are seen to depend on the interception parameter, \(D_p/D_f\), the packing fraction \(\alpha\), and the Stokes number, \(St = u_\infty \tau / D_f = \rho_p C_c D^2 p u_\infty / 18 \mu D_f\), the ratio of the stop distance to the diameter of the fiber. The collection efficiency is just \(\eta = 2y_1/D_f\). To determine the efficiency, (7.116) and (7.127) must be solved simultaneously for \(2y_1/D_f\) and \(2y_2/D_f\). It is easiest to eliminate \(2y_1/D_f\) from (7.127) using (7.116) to obtain a single equation for \(2y_2/D_f\) that can be solved numerically.

We see that the collection efficiency depends on three dimensionless parameters, \(D_p/D_f\), \(\alpha\), and \(St\). Actually, we could have anticipated the dependence of the collection efficiency on these three dimensionless parameters by initially making the particle equation of motion (7.107) dimensionless at the start of the analysis by letting \(u^* = u/u_\infty\), \(v^* = v/u_\infty\), and \(t^* = t u_\infty/D_f\). The result is

\[
St \frac{dv^*}{dt^*} = u^* - v^*
\] (7.128)

Since \(u^*\) is a function of \(\alpha\), and the solution of (7.128) is evaluated at \(y^* = (D_p/D_f + 1)/2\), the dimensionless particle trajectory is seen to depend only on \(St\), \(\alpha\), and \(D_p/D_f\).

The collection efficiency for combined impaction and interception is a function of the Stokes number, \(St = \rho_p C_c D^2 p u_\infty / 18 \mu D_f\), the packing density \(\alpha\), and the ratio of particle to fiber diameter, \(D_p/D_f\). Figure 7.21 shows \(\eta\) as a function of \(St\) for \(\alpha = 0.001, 0.01,\) and \(0.1\), and \(D_p/D_f = 0.1\) calculated based on our approximate analysis. The results clearly show the effect of changing \(\alpha\). For the larger values of \(\alpha\) the streamlines lie closer to the cylinder than at smaller \(\alpha\) (recall Figure 7.19). Thus, at fixed Stokes number, increasing \(\alpha\) leads to increasing collection efficiency since the streamlines are crowded closer and closer to the cylinder, allowing fewer particles to escape past the cylinder. Conversely, to attain the same collection efficiency at a smaller value of \(\alpha\) requires greater particle inertia (i.e., Stokes number).

The parameter \(b\) is related to \(\alpha\) by (7.124) and is treated as the distance along the \(x\)-axis at which the flow is undisturbed by the presence of the cylinder. For \(\alpha = 0.1\),
the value of $b/D_f$ is calculated to be 1.58. The solution for the stream function actually begins to break down for $x/D_f$ values exceeding this value; fortunately, we only need the flow field in the vicinity of the cylinder. For $\alpha = 0.01$, $b/D_f = 5$, and for $\alpha = 0.001$, $b/D_f = 15.8$.

Continuing with Figure 7.21, we see that at a fixed value of $\alpha$ the collection efficiency increases with increasing Stokes number, eventually reaching a value of unity. Physically, a convenient way to think of increasing St is to imagine the particle density $\rho_p$ increasing at fixed size $D_p$ and approach velocity $u_\infty$. Thus, as St increases the particle becomes heavier and heavier and is less able to follow any changes in the flow field. A point is eventually reached as St increases where all the particles contained in the upstream projected area of the cylinder are collected; in fact, we see that $\eta$ values slightly larger than 1.0 are obtained, reflecting the interception contribution from particles even initially outside the upstream projected area of the cylinder but within the collection envelope at $r = (D_f + D_p)/2$.

**Example 7.8 Collection Efficiency by Inertial Impaction and Interception**

In this section we presented an approximate solution to the determination of the collection efficiency of a cylinder by combined inertial impaction and interception. The approximation arose in using the average velocity components $\bar{u}_s$ and $\bar{u}_r$ as given by (7.113) and (7.114), and by assuming that the critical particle trajectory for capture is that which passes within a distance $D_p/2$ of the cylinder surface at $\theta = \pi/2$. In this example we want to integrate
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(7.108) and (7.109) subject to (7.110) and (7.111) using the exact Kuwabara velocity field and compare the calculated collection efficiencies to those obtained using the approximate analysis.

The Kuwabara velocity field in Cartesian coordinates is found from (7.97) and (7.98) by using

\[ u_x = u_r \cos \theta - u_\theta \sin \theta \]
\[ u_y = u_r \sin \theta + u_\theta \cos \theta \]

and \( r^2 = x^2 + y^2 \). Doing so, we find

\[ u_x = \frac{u_\infty}{2} Ku \left[ \ln \frac{4(x^2 + y^2)}{D_f^2} + \alpha - \alpha \frac{x^2 + 3y^2}{D_f^2/2} + \frac{y^2 - x^2}{x^2 + y^2} \right] + \frac{D_f^2}{4} \left( 1 - \frac{\alpha}{2} \right) \frac{x^2 - y^2}{(x^2 + y^2)^2} \]
\[ u_y = \frac{u_\infty}{2} Ku \left[ \frac{2xy}{x^2 + y^2} - \frac{D_f^2 xy}{2(x^2 + y^2)^2} \left( 1 - \frac{\alpha}{2} \right) - \frac{4\alpha xy}{D_f^2} \right] \]

To determine the trajectory of a particle we solve (7.108) and (7.109) using this \( u_x \) and \( u_y \) subject to (7.110) and (7.111). At this point it is advantageous to put everything in dimensionless form. Let \( z_1 = x/D_f, z_2 = y/D_f, t^* = t/\tau, \) and \( St = u_\infty \tau/D_f, \) and we obtain

\[ \frac{d^2z_1}{dt^{*2}} + \frac{dz_1}{dt^*} = -\frac{St}{2 Ku} \left[ \ln \left( \frac{4(z_1^2 + z_2^2)}{D_f^2} \right) + \alpha - 2\alpha(z_1^2 + 3z_2^2) \right] + \frac{z_2^2 - z_1^2}{z_1^2 + z_2^2} + \frac{1}{4} \left( 1 - \frac{\alpha}{2} \right) \frac{z_1^2 - z_2^2}{(z_1^2 + z_2^2)^2} \]
\[ \frac{d^2z_2}{dt^{*2}} + \frac{dz_2}{dt^*} = \frac{St}{2 Ku} \left[ \frac{2z_1 z_2}{z_1^2 + z_2^2} - \left( 1 - \frac{\alpha}{2} \right) \frac{z_1 z_2}{2(z_1^2 + z_2^2)^2} - 4\alpha z_1 z_2 \right] \]

to be solved subject to

\[ z_1(0) = \frac{1}{2\sqrt{\alpha}} \quad \frac{dz_1}{dt^*} \big|_{t^* = 0} = \frac{u_x(b, y_1) \tau}{D_f} \]
\[ z_2(0) = \frac{y_1}{D_f} \quad \frac{dz_2}{dt^*} \big|_{t^* = 0} = 0 \]

These two second-order ordinary differential equations must be solved numerically. A convenient way to do so is to convert the two second-order ordinary differential equations into four first-order ordinary differential equations by letting \( w_1 = z_1, w_2 = z_2, w_3 = dz_1/dt^*, \) and \( w_4 = dz_2/dt^*. \) The four new dependent variables, \( w_1, w_2, w_3, \) and \( w_4 \) now satisfy four first-order ordinary differential equations, the first two of which are

\[ \frac{dw_1}{dt^*} = w_3 \]
\[ \frac{dw_2}{dt^*} = w_4 \]
and the second two of which are obtained from the original equations by noting that \(dw_3/dt^* = d^2z_1/dt^{*2}\) and \(dw_4/dt^* = d^2z_2/dt^{*2}\). We have solved these four coupled differential equations using a fourth-order Runge–Kutta method.

The solution gives the trajectory of a particle starting at \(t^* = 0\) at \(z_1 = (2\alpha^{1/2})^{-1}\) and \(z_2 = y_1/D_f\). To determine the collection efficiency it is necessary to solve the equations repeatedly for ever-increasing values of \(y_1\) to find that value of \(y_1\) for which the particle just escapes capture. This entire procedure is then repeated for a series of different Stokes numbers.

Figure 7.22 shows the trajectory of a particle and the streamline on which it starts for \(St = 1\), \(\alpha = 0.1\), \(D_p/D_f = 0.1\), and \(y_1/D_f = 0.2\). The approximate and exact collection efficiencies are compared in Figure 7.23 for \(\alpha = 0.1\) and \(D_p/D_f = 0.1\). The maximum difference between the two efficiencies is about 75\%, occurring in the vicinity of \(St = 0.1\).

**Figure 7.22** Particle trajectory approaching a cylinder in the Kuwabara flow field for \(St = 1\), \(\alpha = 0.1\), \(D_p/D_f = 0.1\), and \(y_1/D_f = 0.2\).

**Figure 7.23** Approximate and exact collection efficiencies for inertial impaction and interception by a cylinder. The Kuwabara flow field is assumed with a filter solid fraction \(\alpha = 0.1\). The approximate efficiency is that already given in Figure 7.21; the exact is that determined from numerical solution of the particle trajectories.
In view of the approximate nature of the Kuwabara velocity field, this discrepancy is probably not large enough to invalidate the approximate result.

In deriving the collection efficiency by the approximate approach, we assumed that the critical angle for capture is $\theta = \pi/2$. We can evaluate this assumption by plotting the angle of impaction obtained from the exact solution as a function of Stokes number (Figure 7.24). The results of the exact solution show that the critical angle for impaction can get as low as $30^\circ$ instead of the $90^\circ$ assumed in the approximate solution. We see that $\theta \to 90^\circ$ as $St \to 0$ and also as $St \to \infty$. In the case of $St \to 0$, the particle merely follows the flow streamlines, and the critical angle for collection will be $90^\circ$. On the other hand, in the limit of $St \to \infty$, the details of the flow streamlines around the cylinder are irrelevant since the particle proceeds on a straight line to the cylinder. Thus the critical angle for capture will also be $90^\circ$, reflecting particles starting out at $y_1 = (D_p + D_r)/2$.

The analysis we have presented strictly applies when the size of the particle is much smaller than the diameter of the collecting cylinder. The Kuwabara velocity field has been modified in the case where the particle and cylinder are the same order in size by Yeh and Liu (1974).

### 7.5.7 Collection Efficiency of a Cylindrical Collector

We can now summarize and evaluate the collection efficiencies by Brownian diffusion, interception, and inertial impaction. The collection efficiency for deposition by Brownian diffusion is given by (7.104). As we noted, by this mechanism the efficiency decreases as $D_p$ increases according to the two-thirds power of $D_p$. Interception efficiency was calculated simply by determining those flow streamlines that fall within a “collision envelope” at a distance of $D_p/2$ from the front half of the cylinder. In doing so, particle diffusion and inertia are neglected; only the velocity field is needed. Such an analysis gives the interception collection efficiency of (7.105). We see that this efficiency in-
creases as particle size increases according to $D_p^2$. Finally, a combined interception and inertial impaction efficiency was obtained by determining the particle trajectory, called the limiting trajectory, that comes within a distance of $D_p/2$ of the cylinder. Since the latter analysis includes both interception and inertial impaction, the collection efficiency will be larger than that predicted on the basis of interception alone. Similarly, by setting $D_p/D_f = 0$, a pure inertial impaction efficiency can be determined. Although there does not exist a closed-form expression for the inertial impaction efficiency, we expect that impaction efficiency should increase with increasing particle size and fluid velocity.

Figure 7.25 shows $\eta$ for combined impaction and interception as a function of St for $\alpha = 0.001, 0.01,$ and $0.1,$ and $D_p/D_f = 0.001, 0.01,$ and $0.1$ as calculated from the approximate analysis in the preceding section. In studying the effect of varying $D_p/D_f$ we see clearly the influence of interception on the collection efficiency. As St → 0, the collection efficiency reflects pure interception only. (Again, think of St → 0 as reflecting $\rho_p \rightarrow 0$.) Assuming that the effect of impaction is largely negligible at St = 0.001, the intercepts of the $\eta$ curves show how the collection efficiency by interception varies with both $\alpha$ and $D_p/D_f$. At very small $D_p/D_f$ (e.g., 0.001) we expect very low collection efficiency due to interception. The efficiency from pure interception was given by (7.105), so that $\eta$ increases as $(D_p/D_f)^2$. The intercept values shown for $D_p/D_f = 0.001$ and 0.1 can be confirmed to adhere to (7.105). At large enough Stokes number the efficiency curves for the different values of $D_p/D_f$ converge as impaction replaces interception as the principal mechanism of collection.

It is of interest to compare the three mechanisms of collection: Brownian diffusion, impaction, and interception. The collection efficiency for Brownian diffusion (7.104) depends on the Peclet number. To represent the collection efficiency on a single plot as a function of the Stokes number, we need only specify the approach velocity $u_\infty$. Figure 7.26 shows $\eta$ as a function of St for $\alpha = 0.1, D_p/D_f = 0.1, \text{ and } u_\infty = 1.0 \text{ cm s}^{-1}$. The curve for diffusion efficiency shows the expected decrease in efficiency for increasing Stokes number (heavier, less mobile particles).

Figure 7.27 is the more commonly used representation of collection efficiency versus particle diameter. The values of $\alpha$ and $u_\infty$ are the same as in Figure 7.26, but $D_p$ is now allowed to vary, with $D_f$ fixed at $1.0 \mu m$. Thus, now the ratio $D_p/D_f$ varies along the curve. As $D_p/D_f$ approaches unity, the assumptions made in the impaction/interception theory begin to break down. For example, hydrodynamic interactions between the particle and the cylinder become important. Therefore, the portion of the curve that exceeds $\eta = 1.0$ is not correct. (For example, for $\alpha = 0.1$, $D_f = 1.0 \mu m$, $b = 1.58 \mu m$, and the particle actually starts at a distance from the cylinder less than its diameter.) The overall collection efficiency versus particle diameter curve shown in Figure 7.27 exhibits a minimum in the efficiency between 0.1 and 1.0 $\mu m$ in diameter. In this range the particle is large enough so that its Brownian diffusivity is too small to lead to a substantial collection efficiency by that mechanism, and at the same time, it is too small for its inertia to be large enough so that impaction can be a strong contribution. In fact, by now we are not surprised to see this type of aerosol collection behavior, wherein a minimum in the collection occurs in the range 0.1 to 1.0 $\mu m$.  


Figure 7.25 Collection efficiency for combined impaction and interception for a cylinder placed transverse to the flow as a function of Stokes number for $D_p/D_f = 0.001, 0.01,$ and $0.1$ and $\alpha = 0.001, 0.01,$ and $0.1$. 
Figure 7.26 Collection efficiencies by Brownian diffusion and impaction/interception for a cylinder placed transverse to the flow as a function of Stokes number for $\alpha = 0.1$, $D_p/D_f = 0.1$, and $u_\infty = 1.0 \text{ cm s}^{-1}$.

7.5.8 Industrial Fabric Filters

Industrial fabric filtration is usually accomplished in a so-called baghouse, in which the particle-laden gases are forced through filter bags. Particles are generally removed from the bags by gravity. Figure 7.28 shows three baghouse designs, in which cleaning is accomplished by vibration [Figure 7.28(a)], air jet [Figure 7.28(b)], or traveling ring [Figure 7.28(c)].

The fabric filtration process consists of three phases. First, particles collect on individual fibers by the mechanisms we have already considered. Then an intermediate stage exists during which particles accumulate on previously collected particles, bridging the fibers. Finally, the collected particles form a cake in the form of a dust layer that acts as a packed bed filter for the incoming particles. As the dust layer accumulates, the pressure drop across the filter increases, and periodically the dust layer must be dislodged into the hopper at the bottom to "regenerate" the fabric bag. High efficiencies are attainable with fabric filters, particularly in treating combustion gases from coal-fired boil-
Sec. 7.5  Filtration of Particles from Gas Streams

7) diffusion

7) imp + int

Figure 7.27  Individual collection efficiencies due to Brownian diffusion and impact/interception, together with total collection efficiency as a function of particle diameter. The other parameters are \( \alpha = 0.1, \) \( u_\infty = 1.0 \text{ cm s}^{-1}, \) and \( D_f = 1.0 \mu\text{m}. \)

ers. To the extent that effective operation of an electrostatic precipitator depends on the presence of \( \text{SO}_2 \) in the gas as an ionizable species, fabric filters can operate with no loss of efficiency with low-sulfur fuel.

Fabric filters consist of semipermeable woven or felted materials that constitute a support for the particles to be removed. A brand-new woven filter cloth has fibers roughly 100 to 150 \( \mu\text{m} \) in diameter with open spaces between the fibers of 50 to 75 \( \mu\text{m} \). Initially, the collection efficiency of such a cloth is low because most of the particles will pass directly through the fabric. However, deposited particles quickly accumulate, and it is the deposited particle layer that enables the high-efficiency removal once a uniform surface layer has been established. Although fiber mat filters are similar in some respects to fabric filters, they do not depend on the layer of accumulated particles for high efficiency. Fiber mat filters generally are not cleaned but are discarded. They are ordinarily used when particle concentrations are low, so that reasonable service life can be achieved before discarding.

Fabric filters offer the following advantages: (1) they can achieve very high collection efficiencies even for very small particles; (2) they can be used for a wide variety of particles; (3) they can operate over a wide range of volumetric flow rates; and (4) they require only moderate pressure drops. The limitations of fabric filters are: (1) operation must be carried out at temperatures lower than that at which the fabric is destroyed, or its life is shortened to an uneconomical degree; (2) gas or particle constituents
that attack the fabric or prevent proper cleaning, such as sticky particles difficult to dislodge, are to be avoided; and (3) baghouses require large floor areas. The advantages of fabric filter baghouses clearly outweigh their limitations, as they currently represent close to 50% of the industrial gas-cleaning market.

In a fabric filter the particle layer performs the removal task. As the layer of collected particles grows in thickness, the pressure drop across the particle layer and the underlying fabric increases. The two major considerations in the design of a fabric filter assembly are the collection efficiency and the pressure drop as a function of time of operation (since the last cleaning). Dennis and Klemm (1979) (see also Turner and McKenna, 1984) developed a series of equations for predicting outlet concentration through a fabric filter. The collection efficiency depends on the local gas velocity and the particle loading on the fabric. Empirical correlations for the pressure drop through a
combined fabric-dust layer are available in Turner and McKenna (1984) and Cooper and Alley (1986).

7.5.9 Filtration of Particles by Granular Beds

An alternative to filtration in fibrous beds is the use of granular beds. The granular bed can be a fixed (packed), fluidized, or moving assemblage of inert particles. In the analysis of a granular bed filter, the bed is usually assumed to consist of an array of spherical elements through which the particle-laden gas flows. As before, the essential component of determining overall collection efficiency is the efficiency for particle capture by a single filter element, in this case a sphere. And, as before, collection occurs by the mechanisms of inertial impaction, interception, and diffusion. Gravity may also be important. A comprehensive experimental study of packed-bed filtration was reported by Gebhart et al. (1973), and their data were subsequently correlated by Balasubramanian and Meisen (1975). Given the single-sphere collection efficiency \( \eta \), the overall collection efficiency of a granular bed of length \( L \) can be derived as follows.

Let \( D_s \) be the uniform diameter of each sphere comprising the bed. The collection efficiency of a single sphere is defined as the ratio of the number of particles collected per unit time to that in the projected upstream area, \( \pi D_s^2 / 4 \), of the sphere. As in the case of the fibrous bed, the interstitial gas velocity in the bed, \( u_\infty \), is greater than that approaching the filter, \( \bar{u} \), due to the volume of the flow excluded by the spheres. The volumetric flow rate of air through the filter is \( Q = \bar{u} A_c \), so, as before, \( Q = u_\infty A_c (1 - \alpha) \), so \( u_\infty = \bar{u} / (1 - \alpha) \). If the number of spheres per unit volume of the bed is \( N_s \), the solid fraction \( \alpha \) of the bed is \( \alpha = (\pi / 6) D_s^3 N_s \).

We now perform the customary balance on the number concentration of particles over a differential element of bed depth \( dx \). The flows into and out of the element \( dx \) are \( Q N|_x \) and \( Q N|_{x+dx} \), respectively. The number of particles removed per unit time in the element \( dx \) is the product of the flow rate of particles into the element and the fraction that is removed, \( \eta \).

Thus the balance over \( dx \) is

\[
A_c \bar{u} (N|_x - N|_{x+dx}) = \left( \frac{\pi}{4} D_s^2 \eta N_s \right) (u_\infty N|_x) (A_c \, dx)
\]

Eliminating \( N_s \) in terms of \( \alpha \), using the relation between \( \bar{u} \) and \( u_\infty \), and taking the limit of \( dx \to 0 \), we obtain

\[
\frac{dN}{dx} = -\frac{3}{2} \left( \frac{\alpha}{1 - \alpha} \right) \frac{\eta}{D_s} N
\]

(7.129)

to be solved subject to \( N(0) = N_0 \). The overall bed efficiency \( \eta_b = 1 - N(L)/N_0 \), so

\[
\eta_b = 1 - \exp \left[ -\frac{3}{2} \left( \frac{\alpha}{1 - \alpha} \right) \frac{\eta L}{D_s} \right]
\]

(7.130)
A number of authors have considered the efficiency of collection of particles by spheres (Michael and Norey, 1969; Paretsky et al., 1971; Nielsen and Hill, 1976a, b; Rajagopalan and Tien, 1976; Tardos et al., 1976, 1978; Tardos and Pfeffer, 1980). Tardos and Pfeffer (1980) have derived an expression for the collection efficiency of a single sphere by interception and gravitational effects when $D_p/D_s \ll 1$,

$$\eta = \left(1 + \frac{D_p}{D_s}\right)^2 \eta_G + \frac{\eta_R}{1 + \text{Gr St}}$$  \hspace{1cm} (7.131)$$

where the efficiency for gravitational collection,

$$\eta_G = \frac{\text{Gr St}}{1 + \text{Gr St}}$$  \hspace{1cm} (7.132)$$

with $\text{Gr} = \frac{D_s g}{2u_\alpha^2}$ and $\text{St} = \frac{\rho_p u_\alpha C_c D_p^2}{9\mu D_s}$, and where the efficiency for interception is

$$\eta_R = \frac{3}{2} \left(\frac{1.31}{1 - \alpha}\right)^3 \left(\frac{D_p}{D_s}\right)^2$$  \hspace{1cm} (7.133)$$

Note that the collection efficiency due to gravitational effects is independent of the flow field and is therefore independent of the bed solid fraction $\alpha$. The efficiency expression (7.131) has been shown by Tardos and Pfeffer (1980) to be applicable for values of the Stokes number smaller than about $\text{St} = 0.05$. For larger values of $\text{St}$, a combined inertial, interception, and gravitational efficiency must be computed using the limiting trajectory.

### 7.6 WET COLLECTORS

Wet collectors, or scrubbers, employ water washing to remove particles directly from a gas stream. Scrubbers may be grouped broadly into two main classes: (1) those in which an array of liquid drops (sprays) form the collecting medium, and (2) those in which wetted surfaces of various types constitute the collecting medium. The first class includes spray towers and venturi scrubbers, while the second includes plate and packed towers. In this book we concentrate on the first class of devices.

Scrubbing is a very effective means of removing small particles from a gas. Removal of particles results from collisions between particles and water drops. In the humid environment of a scrubber, small, dry particles also grow in size by condensation of water and thereby become easier to remove. Reentrainment of particles is avoided since the particles become trapped in droplets or in a liquid layer. A scrubber also provides the possibility of simultaneously removing soluble gaseous pollutants. The particle-laden scrubbing liquid must be disposed of—a problem not encountered in dry methods of gas cleaning.

A spray scrubber is a device in which a liquid stream is broken into drops, approximately in the range 0.1 to 1.0 mm in diameter, and introduced into the particle-
laden gas stream. The array of moving drops becomes a set of targets for collection of the particles in the gas stream. Collection efficiency is computed by considering the efficiency of a single spherical collector and then summing over the number of drops per unit volume of gas flow. The relative motion between the drops and particles is an important factor in the collection efficiency because capture occurs by impaction and direct interception. (Diffusion is also important for smaller particles.)

There are two general types of spray scrubbers. The first class comprises those having a preformed spray where drops are formed by atomizer nozzles and sprayed into the gas stream. These include:

1. Countercurrent gravity tower, where drops settle vertically against the rising gas stream
2. Cross-current tower, where drops settle through a horizontal gas stream
3. Cocurrent tower, where spray is horizontal into a horizontal gas stream

The second class comprises those in which the liquid is atomized by the gas stream itself. Liquid is introduced more or less in bulk into a high-velocity gas flow that shatters the liquid into drops. Devices in this class are called venturi scrubbers since the high-velocity gas flow is achieved in a venturi (a contraction).

Figure 7.29 illustrates four types of wet collection equipment. The simplest type of wet collector is a spray tower into which water is introduced by means of spray nozzles [Figure 7.29(a)]. Gas flow in a spray chamber is countercurrent to the liquid, the configuration leading to maximum efficiency. Collection efficiency can be improved over the simple spray chamber with the use of a cyclonic spray tower, as shown in Figure 7.29(b). The liquid spray is directed outward from nozzles in a central pipe. An unsprayed section above the nozzles is provided so that the liquid drops with the collected particles will have time to reach the walls of the chamber before exit of the gas. An impingement plate scrubber, as shown in Figure 7.29(c), consists of a tower containing layers of baffled plates with holes (5000 to 50,000 m⁻²) through which the gas must rise and over which the water must fall. Highest collection efficiencies of wet collectors are obtained in a venturi scrubber, shown in Figure 7.29(d), in which water is introduced at right angles to a high-velocity gas flow in a venturi tube, resulting in the formation of very small water droplets by the flow and high relative velocities of water and particles. The high gas velocity is responsible for the breakup of the liquid. Aside from the small droplet size and high impingement velocities, collection is enhanced through particle growth by condensation. Table 7.1 summarizes particle scrubbing devices.

The collection efficiency of wet collectors can be related to the total energy loss in the equipment; the higher the scrubber power, per unit volume of gas treated, the better is the collection efficiency. Almost all the energy is introduced in the gas, and thus the energy loss can be measured by the pressure drop of gas through the unit.

The major advantage of wet collectors is the wide variety of types, allowing the selection of a unit suitable to the particular removal problem. As disadvantages, high-pressure drops (and therefore energy requirements) must be maintained, and the handling and disposal of large volumes of scrubbing liquid must be undertaken.
Figure 7.29 Wet collectors: (a) spray tower; (b) cyclone spray tower; (c) impingement scrubber; (d) venturi scrubber.
TABLE 7.1 PARTICLE SCRUBBERS

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate scrubber</td>
<td>A vertical tower containing one or more horizontal plates (trays). Gas enters the bottom of the tower and must pass through perforations in each plate as it flows countercurrent to the descending water stream. Plate scrubbers are usually named for the type of plates they contain (e.g., sieve plate tower). Collection efficiency increases as the diameter of the perforations decreases. A cut diameter, that collected with 50% efficiency, of about 1 ( \mu m ) aerodynamic diameter can be achieved with 3.2-mm-diameter holes in a sieve plate.</td>
</tr>
<tr>
<td>Packed-bed scrubber</td>
<td>Operates similarly to packed-bed gas absorber (see Chapter 8). Collection efficiency increases as packing size decreases. A cut diameter of 1.5 ( \mu m ) aerodynamic diameter can be attained in columns packed with 2.5-cm elements.</td>
</tr>
<tr>
<td>Spray scrubber</td>
<td>Particles are collected by liquid drops that have been atomized by spray nozzles. Horizontal and vertical gas flows are used, as well as spray introduced concurrent, countercurrent, or cross-flow to the gas. Collection efficiency depends on droplet size, gas velocity, liquid/gas ratio, and droplet trajectories. For droplets falling at their terminal velocity, the optimum droplet diameter for fine-particle collection lies in the range 100 to 500 ( \mu m ). Gravitational settling scrubbers can achieve cut diameters of about 2.0 ( \mu m ). The liquid/gas ratio is in the range 0.001 to 0.01 ( m^3 ) ( m^3 ) of gas treated.</td>
</tr>
<tr>
<td>Venturi scrubber</td>
<td>A moving gas stream is used to atomize liquids into droplets. High gas velocities (60 to 120 m s(^{-1})) lead to high relative velocities between gas and particles and promote collection.</td>
</tr>
<tr>
<td>Cyclone scrubber</td>
<td>Drops can be introduced into the gas stream of a cyclone to collect particles. The spray can be directed outward from a central manifold or inward from the collector wall.</td>
</tr>
<tr>
<td>Baffle scrubber</td>
<td>Changes in gas flow velocity and direction induced by solid surfaces.</td>
</tr>
<tr>
<td>Impingement-entrainment scrubber</td>
<td>The gas is forced to impinge on a liquid surface to reach a gas exit. Some of the liquid atomizes into drops that are entrained by the gas. The gas exit is designed so as to minimize the loss of entrained droplets.</td>
</tr>
<tr>
<td>Fluidized-bed scrubber</td>
<td>A zone of fluidized packing is provided where gas and liquid can mix intimately. Gas passes upward through the packing, while liquid is sprayed up from the bottom and/or flows down over the top of the fluidized layer of packing.</td>
</tr>
</tbody>
</table>


7.6.1 Spray Chamber

We begin our analysis of spray scrubbing with the conceptually simplest of the devices, a gravity spray chamber. Water droplets are introduced at the top of an empty chamber through atomizing nozzles and fall freely at their terminal settling velocities countercurrently through the rising gas stream. The particle-containing liquid collects in a pool at the bottom and must be pumped out for treatment to remove the solids, and the cleaned liquid is usually recycled to the tower. A schematic of a spray chamber is given in Figure
7.30. We assume that all the falling drops have the same diameter $D_s$. The volumetric flow rate of water fed to the top of the chamber is $W$ (m$^3$ s$^{-1}$). If every drop has diameter $D_s$, the number of drops per second fed to the top of the chamber and passing any point in the chamber is $W/[(\pi/6) D_s^3]$. The drop concentration at any point in the chamber is $W/[(\pi/6) D_s^3 A_c \nu]$, where $A_c$ is the cross-sectional area of the chamber and $\nu$ is the fall velocity of the drops. We assume that $D_s$ remains constant in the chamber.

If a drop of diameter $D_s$ is falling in still air, its terminal velocity $v_t$ is such that the drag force is just balanced by the gravitational force on the drop. Now in the spray chamber the drop is falling at a fall velocity $v$ relative to a fixed coordinate system in the presence of a rising gas velocity $v_g$. Thus $v$ is not the same as $v_t$, due to the influence of the rising gas. The new fall velocity $v$ is still determined by the equality of the drag and gravity forces. The drag force can be expressed from (5.8) as

$$F_{\text{drag}} = \frac{\pi}{8} C_D \rho \frac{D_s^2}{2} (v + v_g)^2$$

(7.134)

Figure 7.30 Spray chamber.
where the drag force depends on the relative velocity between the drop and the gas, \( v + v_g \). By equating this drag force to the gravity force,

\[
F_{\text{gravity}} = \frac{\pi}{6} D_s^3 (\rho_p - \rho_g) g
\]

we find that the sum of the new fall velocity and the rising gas velocity equals the terminal velocity in still air,

\[
v + v_g = v_t
\]

Given the drop size, we can compute (or find correlations for) \( v_t \), and given the volumetric flow rate of gas through the unit, we can calculate \( v_g \). Thus we can compute the water drop fall velocity from (7.136).

We now wish to derive an equation governing the overall collection efficiency of a spray tower. Let \( \eta \) be the collection efficiency of particles on an individual droplet, defined as the ratio of the cross-sectional area of the hypothetical tube of gas from which the particles are all removed to the frontal area of the droplet. Consider a differential section of chamber height as shown in Figure 7.30. The number of particles removed per second from the gas stream over \( dx \) is just

\[
v_g A_c (N \big|_x - N \big|_{x + dx})
\]

This quantity is equated to the product of the fraction of the volumetric flow of gas through \( dx \) from which all particles are removed and the total incoming number of particles per second. The total incoming number of particles per second is \( N \big|_x v_g A_c \). Thus we need to obtain an expression for the fraction of the volumetric flow of gas through \( dx \) from which all particles are removed.

The distance \( dx \) is fixed as the distance a drop falls in time \( dt \) relative to the chamber,

\[
dx = v \, dt
\]

During the time \( dt \) the volume of air that flows through the hypothetical tube having the frontal area of the droplet is

\[
\left( \frac{\pi}{4} D_s^2 \right) v_t \, dt
\]

where \( v_t \) is the relative velocity between the droplet and the gas. This quantity can be expressed in terms of \( dx \) as

\[
\left( \frac{\pi}{4} D_s^2 \right) \frac{v_t}{v} \, dx
\]

Now the volume from which all particles are removed by the single drop is

\[
\eta \left( \frac{\pi}{4} D_s^2 \right) \frac{v_t}{v} \, dx
\]
Thus the total volume of gas swept clean per second by all the droplets in $dx$ is

$$\eta \left( \frac{\pi}{4} D_s^2 \right) \frac{v_t}{v} \frac{W}{(\pi/6) D_s^3}$$

and the fraction of the volumetric flow of gas through $dx$ from which all particles are removed is

$$\left[ \frac{\eta(\pi/4) D_s^2 (v_t/v)}{v_g A_c} \right] \left\{ \frac{W}{[(\pi/6) D_s^3]} \right\}$$

Then the number of particles removed per second from $dx$ is

$$\left[ \frac{\eta(\pi/4) D_s^2 (v_t/v)}{v_g A_c} \right] \left\{ \frac{W}{[(\pi/6) D_s^3]} \right\} \frac{N}{v_g A_c}$$

Thus

$$N \bigg|_x - N \bigg|_{x+dx} = \left[ \frac{\eta(\pi/4) D_s^2 (v_t/v)}{v_g A_c} \right] \left\{ \frac{W}{[(\pi/6) D_s^3]} \right\} N \bigg|_x$$

Taking the limit as $dx \to 0$ gives

$$\frac{dN}{dx} = -\left[ \frac{3}{2} \eta \left( \frac{v_t}{v} \right) \frac{W}{v_g A_c D_s} \right] N \quad (7.137)$$

Integrating (7.137) subject to $N(0) = N_0$ gives

$$N(L) = N_0 \exp \left[ -\frac{3}{2} \eta \left( \frac{v_t}{v} \right) \frac{W}{v_g A_c D_s} L \right] \quad (7.138)$$

The overall spray chamber efficiency is

$$\eta_s = 1 - \frac{N(L)}{N_0}$$

$$= 1 - \exp \left[ -\frac{3}{2} \eta \left( \frac{v_t}{v} \right) \frac{W}{v_g A_c D_s} L \right] \quad (7.139)$$

The quantity $(W/v_g A_c)$ is the ratio of the volumetric flow rate of water to the volumetric flow rate of air. This ratio is of basic importance in the operation of a scrubber. Liquid-to-gas volumetric ratios usually fall in the range 2 to 20 gal of liquid per thousand cubic feet of gas, or $0.27 \times 10^{-3}$ to $2.7 \times 10^{-3}$ m$^3$ of liquid per m$^3$ of gas. This ratio determines the number of drops per unit volume of gas, $6W/\pi D_s^3 G$. For example, the number of droplets per cm$^3$ at the two limits of the flow rate ratio for different droplet diameters is
If too small a $D_s$ is attempted at a high $W/G$ ratio, the drop concentration would be so large that collision and coalescence would probably occur, driving the droplet population to larger sizes and lower concentration.

In summary, the overall efficiency of a spray tower increases as the collection efficiency of a single drop increases, as the length of the chamber increases, and as the ratio of the volumetric flow rate of water to that of air increases. It increases as the diameter of the drops decreases.

### 7.6.2 Deposition of Particles on a Spherical Collector

The collection efficiency of a sphere is equal to the ratio of the total number of collisions per second occurring between particles and the spherical collector to the total number of particles per second flowing into the tube having the cross-sectional area of the sphere. We can follow exactly the same approach as we did in determining the collection efficiency of a cylinder by Brownian motion, impaction, and interception; only here we need the flow field around a sphere. However, the current problem is somewhat more complicated than just being the spherical analog of the cylindrical collector. The collecting spheres are falling water drops, which may develop internal circulations that influence the flow field of the gas in their vicinity. Also, drops of sufficiently large size may no longer be spherical as they fall, although we will not include this aspect in our analysis. An alternative to the approaches in Section 7.5 is to rely on dimensional analysis to suggest the dimensionless variables on which the collection efficiency should depend. To formulate a correlation for $\eta$ based on dimensional analysis, we must identify the dimensionless groups that arise in the dimensionless equations of motion of a particle. We are interested specifically in the case of falling water droplets. Allowing for the possibility of internal circulations in the drop that may affect the flow field around it, we find that $\eta$ depends on eight variables: $D_p, D_s, v, v_g, \mu_v, \mu_{air}, D_s, \rho_{air}$. These eight variables have three dimensions. By the Buckingham pi theorem, there are eight minus three, or five, independent dimensionless groups. The actual groups can be obtained by nondimensionalizing the equations of motion for the fluid and the particles. The five dimensionless groups are:

$$\text{Re} = \frac{D_s v \rho_{air}}{\mu_{air}} \quad \text{Reynolds number of sphere}$$

$$\text{Sc} = \frac{\mu_{air}}{\rho_{air} D_s} \quad \text{Schmidt number of particles}$$
Stokes number of particles
\[ \text{St} = \frac{C_c \rho_p D_p^2 v_t}{18 \mu D_s} \]

Ratio of diameters of particle and drop
\[ \kappa = \frac{D_p}{D_s} \]

Viscosity ratio of water to air
\[ \omega = \frac{\mu_w}{\mu_{\text{air}}} \]

Note that the Reynolds and Stokes numbers of the falling drop are based on the relative velocity to the air, which is just its terminal settling velocity, \( v_t \).

Slinn (1983) has presented the following general equation for the collection efficiency of a sphere
\[
\eta = \frac{8}{\text{Re Sc}} \left[ 1 + \frac{0.4}{\sqrt{2}} \text{Re}^{1/2} \text{Sc}^{1/3} + \frac{0.16}{\sqrt{2}} \text{Re}^{1/2} \text{Sc}^{1/2} \right] \\
+ 4\kappa \left[ \omega^{-1} + (1 + \sqrt{2} \text{Re}^{1/2}) \kappa \right] + \left[ \frac{2 \text{St} - S_\star}{2 \text{St} - S_\star + \frac{2}{3}} \right]^{3/2} \tag{7.140}
\]

The first term in (7.140) is the contribution from Brownian diffusion, the second is that due to interception, and the third accounts for impaction. In (7.140) it is assumed that both the collector drop and the collected particles have unit density. For particles of density different from 1.0 g cm\(^{-3}\), the last term in (7.140) should be multiplied by \((\rho_w/\rho_p)^{1/2}\).

Figure 7.31 shows the single-sphere collection efficiency \( \eta \) as a function of \( D_p \) for \( D_s = 0.5, 1.0, 2.0, \) and 4.0 mm as predicted by (7.140). At a fixed value of \( D_s \), at the lower end of the size spectrum \( \eta \) decreases with increasing \( D_p \) due to the decreased importance of Brownian diffusion. At the large particle end of the spectrum \( \eta \) increases as \( D_p \) increases due to the predominant role of inertial impaction and interception. A minimum in the collection efficiency is seen to exist between 0.5 and 1.0 \( \mu \text{m} \) diameter. At a fixed value of \( D_p \), \( \eta \) decreases as \( D_s \) increases due to the decreased importance of interception. The empirical nature of (7.140) is evident in the rather abrupt increase in the efficiency at about \( D_p = 4 \mu \text{m} \) due to the impaction contribution. This abrupt change is the result of attempting to fit two different physical phenomena into a single empirical equation. Figure 7.32 shows \( \eta \) as a function of \( D_s \) for \( D_p \) ranging from 1 to 7 \( \mu \text{m} \). The increase in \( \eta \) for \( D_p \) larger than 4 \( \mu \text{m} \) is due to the impaction contributions as predicted by (7.140).

Equation (7.140) is quite general in that it accounts for all three collection mechanisms. In many scrubber applications inertial impaction is the predominant removal mechanism, especially for particles larger than 1 \( \mu \text{m} \) in diameter. In that case Calvert (1984) has suggested an alternative to (7.140) for the collection efficiency due to im-
Figure 7.31 Collection efficiency (7.140) for a single sphere as a function of collected particle diameter $D_p$ at collector water droplet diameter $D_s = 50$, 100, 500, and 1000 $\mu$m. Conditions are for water droplets falling in still air at 298 K, 1 atm, collecting particles of $\rho_p = 1$ g cm$^{-3}$.

Figure 7.32 Collection efficiency (7.140) for a single sphere as a function of collector water droplet diameter $D_s$ at collected particle diameters $D_p = 1, 2, 3, 4, 5, 6, 7$ $\mu$m. Conditions are for water droplets falling in still air at 298 K, 1 atm, $\rho_p = 1$ g cm$^{-3}$. 
paction only,

$$\eta = \left( \frac{St}{St + 0.35} \right)^2 \quad (7.141)$$

Figure 7.33 shows $\eta$ from (7.141) as a function of $D$, for $D_p = 1$ to 7 $\mu$m for $\rho_p = 1$ g cm$^{-3}$. For small droplets in the Stokes law regime, $V_1 \sim D_s^2$, so as $D_s$ increases, St increases proportional to $D_s$. Thus, as $D_s$ increases, $\eta$ increases. At intermediate sizes $V_1 \sim D_s$, St is constant, and $\eta$ is constant. For large sizes $V_1$ increases less rapidly than $D_s$, so St decreases as $D_s$ increases, leading to a decrease in $\eta$. Thus there is a value of droplet diameter $D_s$ for which $\eta$ is a maximum. The peak value of $\eta$ occurs at about $D_s = 600 \mu$m regardless of particle diameter $D_p$. The value of $\eta$ at its peak is larger for larger particles and is rather flat, extending for 200 or 300 $\mu$m on either side of $D_s \approx 600 \mu$m. By comparing (7.141) to (7.140), we see that (7.141) should be valid for $D_p \geq 6 \mu$m when $100 \mu$m $\leq D_s \leq 1000 \mu$m.

The total effect of $D_s$ on the overall spray chamber efficiency is a result of the variation of $v_1$ and $\eta$ in (7.139). Since $\eta$ is relatively constant with $D_s$ in the range around $D_s = 600 \mu$m, the net effect on the factor $\eta v_1 / (v_1 - v_1^s) D_s$ is to make $\eta$ a maximum at the low end of the $D_s$ range, around 300 to 400 $\mu$m.

**Example 7.9 Overall Efficiency of a Spray Chamber**

We desire to calculate the overall efficiency of a spray scrubber as a function of water droplet diameter, particle diameter, and ratio of water to gas volumetric flow rates. Assume

![Figure 7.33](image_url)
that the collected particles have a density of 1 g cm\(^{-3}\). The chamber has a diameter of 1 m, is 5 m high, and operates at 298 K and 1 atm. Plot the overall collection efficiency as a function of droplet diameter over the range \(D_s = 50 \text{ \(\mu\)m (0.05 mm)} to 10^4 \text{ \(\mu\)m (10 mm)}\) for particle sizes from \(D_p = 1\) to 5 \(\mu\)m and for water and gas flow rates of \(W = 0.001 \text{ m}^3\text{s}^{-1}\) and \(G = 1 \text{ m}^3\text{s}^{-1}\).

The following empirical equation can be used for the terminal velocity of water droplets for \(D_s \geq 50 \text{ \(\mu\)m},\)

\[
v_t = 958 \left[ 1 - \exp \left( -\left( \frac{D_s}{0.171} \right)^{1.147} \right) \right]
\]

where \(v_t\) is in cm s\(^{-1}\) and \(D_s\) is in cm. Figure 7.34 shows the overall efficiency \(\eta_t\) for the spray chamber as a function of \(D_s\) and \(D_p\) ranging from 1 to 5 \(\mu\)m. The individual sphere collection efficiency is that predicted by (7.140). Although droplet diameters exceeding 1 mm (1000 \(\mu\)m) are unlikely, we have calculated \(\eta_t\) for \(D_s\) values up to 10 mm (1 cm) to show that a maximum in efficiency is achieved for a particular range of \(D_s\) values when impaction is the controlling collection mechanism. The explanation for that maximum is that at a fixed \(W/G\), larger droplets imply fewer droplets and thus a decreased target area for particle collection. As the droplets get very small, on the other hand, the Stokes number decreases and the individual sphere impaction contribution decreases.

### 7.6.3 Venturi Scrubbers

Venturi scrubbers are employed when high collection efficiencies are required and when most of the particles are smaller than 2 \(\mu\)m in diameter. There are a number of instances, in fact, where a venturi scrubber is the only practical device for a gas-cleaning appli-
cation. If the particles to be removed are sticky, flammable, or highly corrosive, for example, electrostatic precipitators and fabric filters cannot be used. Venturi scrubbers are also the only high-efficiency particulate collectors that can simultaneously remove gaseous species from the effluent stream.

The distinguishing feature of a venturi scrubber is a constricted cross section or throat through which the gas is forced to flow at high velocity. A typical venturi configuration is shown in Figure 7.35. The configuration includes a converging conical section where the gas is accelerated to throat velocity, a cylindrical throat, and a conical expander where the gas is slowed down. Liquid can be introduced either through tangential holes in the inlet cone or in the throat itself. In the former case, the liquid enters the venturi as a film on the wall and flows down the wall to the throat, where it is atomized by the high-velocity gas stream. In the latter, the liquid is injected perpendicular to the gas flow in the throat, atomized, and then accelerated. Gas velocities in the range 60 to 120 m s\(^{-1}\) are achieved, and the high relative velocity between the particle-laden gas
flow and the droplets promotes collection. The collection process is essentially complete by the end of the throat. Because they operate at much higher velocities than electrostatic precipitators or baghouses, venturi scrubbers are physically smaller and can be economically made of corrosion-resistant materials. Venturis have the simplest configuration of the scrubbers and are the smallest in size.

A typical range of liquid to gas flow rate ratios for a venturi scrubber is 0.001 to 0.003 m$^3$ liquid per m$^3$ gas. At the higher liquid/gas ratios, the gas velocity at a given pressure drop is reduced, and at lower ratios, the velocity is increased. For gas flow rates exceeding about 1000 m$^3$ min$^{-1}$ venturi scrubbers are generally constructed in a rectangular configuration in order to maintain an equal distribution of liquid over the throat area.

In essence, venturi scrubbers are cocurrent flow devices for which the incremental collection along the axis of the venturi can be described by

$$\frac{dN}{dx} = -\left[\frac{3}{2} \eta \left(\frac{W}{G}\right) \frac{v_g - v_d}{v_d} \frac{1}{D_s}\right] N$$

(7.142)

where $v_g$ and $v_d$ are the gas (particle) and droplet velocities, respectively. One may integrate (7.142) together with an inertial impaction expression for $\eta$ to obtain the overall efficiency of a venturi scrubber as (Calvert, 1984)

$$\eta_i = 1 - \exp \left[\frac{1}{55} \frac{W \rho_l D_s}{\mu_g} F(K_p f)\right]$$

(7.143)

where

$$F(K_p f) = \frac{1}{K_p} \left[-0.7 - K_p f + 1.4 \ln \left(\frac{K_p f + 0.7}{0.7}\right) + \frac{0.49}{0.7 + K_p f}\right]$$

and $K_p = 2$ St. $f$ is an empirical parameter that accounts for collection by means other than impaction, such as particle growth due to condensation. It has been found that the performance of a variety of large-scale venturi and other gas-atomized spray scrubbers can be correlated with (7.143) using $f = 0.5$. For hydrophobic particles in smaller units $f = 0.25$.

### 7.7 SUMMARY OF PARTICULATE EMISSION CONTROL TECHNIQUES

Table 7.2 presents a summary of particulate emission control techniques, including minimum particle sizes, ranges of efficiency, and advantages and disadvantages of each type of unit. In selecting a method to meet a particular gas cleaning need, the most important consideration is the total cost (operating and equipment) of the method. The advantages and disadvantages listed in Table 7.2 give an indication of the considerations that enter into a determination of the cost of a particular device. Figure 7.36 shows typical collection efficiency curves for the devices considered in this chapter.
### TABLE 7.2 SUMMARY OF PARTICULATE EMISSION CONTROL TECHNIQUES

<table>
<thead>
<tr>
<th>Device</th>
<th>Minimum particle size (µm)</th>
<th>Efficiency (% basis)</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational settler</td>
<td>&gt;50</td>
<td>&lt;50</td>
<td>Low-pressure loss, Simplicity of design and maintenance</td>
<td>Much space required, Low collection efficiency</td>
</tr>
<tr>
<td>Cyclone</td>
<td>5-25</td>
<td>50-90</td>
<td>Simplicity of design and maintenance, Little floor space required, Dry continuous disposal of collected dusts, Low-to-moderate pressure loss, Handles large particles, Handles high dust loadings, Temperature independent</td>
<td>Much head room required, Low collection efficiency of small particles, Sensitive to variable dust loadings and flow rates</td>
</tr>
<tr>
<td>Wet collectors</td>
<td></td>
<td></td>
<td>Simultaneous gas absorption and particle removal</td>
<td>Corrosion, erosion problems, Added cost of wastewater treatment and reclamation, Low efficiency on submicron particles, Contamination of effluent stream by liquid entrainment, Freezing problems in cold weather, Reduction in buoyancy and plume rise</td>
</tr>
<tr>
<td>Spray towers</td>
<td>&gt;10</td>
<td>&lt;80</td>
<td>Ability to cool and clean high-temperature, moisture-laden gases</td>
<td></td>
</tr>
<tr>
<td>Cyclonic</td>
<td>&gt;2.5</td>
<td>&lt;80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impingement</td>
<td>&gt;2.5</td>
<td>&lt;80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Venturi</td>
<td>&gt;0.5</td>
<td>&lt;99</td>
<td>Corrosive gases and mists can be recovered and neutralized, Reduced dust explosion risk, Efficiency can be varied</td>
<td></td>
</tr>
<tr>
<td>System</td>
<td>Collection Efficiency</td>
<td>Remarks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------------</td>
<td>-------------------------------------------------------------------------</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Electrostatic precipitator | < 1 95-99 99+ %        | Efficiency obtainable  
Very small particles can be collected  
Particles may be collected wet or dry  
Pressure drops and power requirements are small compared with other high-efficiency collectors  
Maintenance is nominal unless corrosive or adhesive materials are handled  
Few moving parts  
Can be operated at high temperatures (573 to 723 K)  
Water vapor contributes to visible plume under some atmospheric conditions  
Relatively high initial cost  
Precipitators are sensitive to variable dust loadings or flow rates  
Resistivity causes some material to be economically uncollectable  
Precautions are required to safeguard personnel from high voltage  
Collection efficiencies can deteriorate gradually and imperceptibly |
| Fabric filtration        | < 1 > 99              | Dry collection possible  
Decrease of performance is noticeable  
Collection of small particles possible  
High efficiencies possible  
Sensitivity to filtering velocity  
High-temperature gases must be cooled  
Affected by relative humidity (condensation)  
Susceptibility of fabric to chemical attack |
Figure 7.36 Collection efficiencies for gas cleaning devices (Licht, 1980).

PROBLEMS

7.1. Derive expressions for the overall efficiencies with respect to number, surface area, and mass of a device that has a collection efficiency

\[ \eta(D_p) = \begin{cases} 1 & D_p > D_{p_0} \\ 0 & D_p \leq D_{p_0} \end{cases} \]

with a log-normal size distribution entering the device. Note: You may find the following integral useful:

\[ \int_{L_1}^{L_2} e^{\mu} \exp \left[ -\frac{(u - \bar{u})^2}{2\sigma_u^2} \right] \, du = (\pi/2)^{1/2} \sigma_u e^{\bar{u}} e^{\sigma_u^2/2} \left\{ \operatorname{erf} \left[ \frac{L_2 - (\bar{u} + r\sigma_u^2)}{\sqrt{2}\sigma_u} \right] \right. 
\left. - \operatorname{erf} \left[ \frac{L_1 - (\bar{u} + r\sigma_u^2)}{\sqrt{2}\sigma_u} \right] \right\} \]
7.2. A particulate collector has a collection efficiency

\[ \eta(D_p) = \begin{cases} 
1 - D_p^{-1} & D_p > 1 \mu m \\
0 & D_p \leq 1 \mu m 
\end{cases} \]

An aerosol with a log-normal size distribution with \( \bar{D}_{pg} = 2 \mu m \) and \( \sigma_g = 1.35 \) is passed through the device. Compute the overall efficiency of the device with respect to particle number, surface area, and mass. Note: The formula given in Problem 7.1 is needed.

7.3. Several particulate collection devices are often operated in series, with each succeeding device used to collect smaller and smaller particles. Consider \( n \) particulate removal devices connected in series, such that the outlet stream from unit 1 is the inlet stream to unit 2, and so on. If the efficiencies of the \( n \) devices are \( \eta_1(D_p) \), \( \eta_2(D_p) \), \ldots , \( \eta_n(D_p) \), show that the total efficiency of the \( n \) units is

\[ E_n = \int_0^\infty \left[ \eta_1(D_p) + \eta_2(D_p) \left[ 1 - \eta_1(D_p) \right] + \cdots + \eta_n(D_p) \left[ 1 - \eta_{n-1}(D_p) \right] \right] n(D_p) \, dD_p \]

7.4. Design a plate-type settling chamber for a stream of 100 m\(^3\) s\(^{-1}\) of air at 298 K and 1 atm to collect particles of density 2.0 g cm\(^{-3}\). The chamber must not exceed 5 m in width or 6 m in height and must collect particles of 50 \( \mu m \) with 99.5% efficiency. Determine the length of the chamber required if 100 trays are used. For simplicity, assume that Stokes' law may be used to calculate the settling velocity of 50-\( \mu m \) particles.

7.5. Air at 298 K and 1 atm laden with acid fog is led from a process to a square horizontal settling chamber 8 m long and 50 cm high. The fog can be considered to consist of spherical droplets of diameter 0.8 mm and density 1 g cm\(^{-3}\). It is desired to remove 90% of the fog from the stream. Find the volumetric flow rate, in cubic meters per hour, which will allow 90% removal.

7.6. Settling chambers are commonly used in a sinter plant to remove large particles of quartz and iron oxide from effluent gas streams. A settling chamber 3 m high and wide and 6 m long is available. The volumetric flow rate of air through the chamber is 5000 m\(^3\) h\(^{-1}\). The densities of quartz and iron oxide particles are 2.6 and 4.5, respectively. Compute and plot efficiency curves for this unit at the given gas flow rate at 298 K for both types of particles as a function of particle diameter over the diameter range 1 to 60 \( \mu m \).

7.7. Consider a settling chamber that is so well mixed internally that at any instant all uncollected particles are uniformly mixed throughout the entire volume of the chamber. Show that the collection efficiency for such a device is given by

\[ \eta(D_p) = \frac{v_p W L / Q}{1 + v_p W L / Q} \]

7.8. Consider a cyclone flow having an inlet velocity of 10 m s\(^{-1}\), an angle of turn of 15\( \pi \), and inner and outer radii of 5.0 and 10.0 cm, respectively. Assume that the particles have density \( \rho_p = 1.5 \) g cm\(^{-3}\). Plot the collection efficiency as a function of particle size over the range 0.5 \( \mu m \) \( \leq D_p \leq 10 \) \( \mu m \) for \( T = 323 K \) and \( W = 0.4 \) m.

(a) assuming laminar flow conditions.
(b) assuming turbulent flow conditions.
(c) using (7.42).

Discuss your results.
Design a cyclone to remove 99% of particles of 20 μm diameter and density 1.5 g cm⁻³ from a stream of 20 m³ s⁻¹ of air at 298 K and 1 atm. Determine suitable values for the major dimensions of the unit. Plot the collection efficiency as a function of particle size over the range 1 μm ≤ D_p ≤ 25 μm.

Determine the collection efficiency versus D_p curve over the range 0.1 μm ≤ D_p ≤ 10 μm for the following single-stage electrostatic precipitator:

\[ Q = 0.1 \text{ m}^3 \text{s}^{-1} \quad T = 300 \text{ K} \]
\[ \rho_p = 1.5 \text{ g cm}^{-3} \quad \kappa = 4 \]
\[ r_c = 0.25 \text{ m} \quad \mu = 1.8 \times 10^{-4} \text{ g cm}^{-1} \text{s}^{-1} \]
\[ r_0 = 0.002 \text{ m} \]
\[ f = 0.6 \]
\[ L = 30 \text{ m} \]

A cylindrical single-stage electrostatic precipitator for gas sampling is to provide 95% efficiency for a flow rate of 0.01 m³ s⁻¹ for particles of 3 μm diameter, \( \rho_p = 1.5 \text{ g cm}^{-3} \), \( \kappa = 4 \), and inlet concentrations of \( 5 \times 10^4 \mu g \text{ m}^{-3} \). Use a velocity of 100 cm s⁻¹ in a single tube with \( r_0 = 0.1r_c \) and \( f = 0.65 \). Determine the required length of the tube.

Consider a filter of packing density 0.04 consisting of fibers of diameter 7 μm in an airstream with an approach velocity of 0.55 m s⁻¹ at 298 K and 1 atm, containing particles of \( \rho_p = 1.5 \text{ g cm}^{-3} \). Compute the collection efficiency of the fiber as a function of \( D_p \) from diffusion, interception, and impaction.

A filter bed of packing density 0.1 and fiber diameter 4 μm for use in removing radioactive particles from a gas stream must provide an overall collection efficiency of at least 99.99% for particles of any size. Given a flow rate of air at 298 K, 1 atm of 10 m³ s⁻¹, \( \rho_p = 1.2 \text{ g cm}^{-3} \), and filter width and height of 1.5 and 1.0 m, respectively, determine the necessary depth of the filter.

Compute the collection efficiency of a cigarette filter which is a fiber layer of thickness 1 cm and void fraction 0.5. Assume that the smoke particles are a monodisperse aerosol of diameter 0.2 μm and density 1 g cm⁻³ and that the fiber filaments have a diameter of 50 μm. Smoke is inhaled at a velocity of 3 cm s⁻¹ and at 298 K and 1 atm.

Determine the overall efficiency of a spray chamber as a function of particle size for particles of density 1.5 g cm⁻³ over the range 0.1 μm ≤ D_p ≤ 10 μm. The cylindrical chamber has a diameter of 1 m, is 5 m high, and operates with air and water flow rates of 1 m³ s⁻¹ and 0.01 m³ s⁻¹, respectively. The air is at 298 K, 1 atm, and the water drop diameter is 1 mm, for which the terminal velocity in still air is 4 m s⁻¹.

Derive an expression for the overall efficiency of a settling chamber in which a horizontal flow of gas is contacted by a vertical settling spray.

With increasingly stringent fuel economy standards, diesel engines appear to be attractive alternative power plants for passenger vehicles. However, despite its superior fuel economy, a diesel engine presents a challenging emission control problem. As the engine is operated in a manner to maintain low NOx emissions, particulate emissions exceed the exhaust emission standard. Thus an additional scheme for controlling particulate emissions is necessary. One such scheme that has received extensive study is the filtration of exhaust particulate matter by a filter bed placed in the exhaust system. In this problem we consider
the design of a fibrous filter bed for this purpose (Oh et al., 1981). (Disposal of the collected particulate matter is a key problem, although we do not consider it here.) We will focus on the initial performance of the filter, that is, on the period during which the deposition of particles is not influenced by those already collected.

The following conditions can be assumed for the exhaust filtration problem:

\[
\alpha = 0.05 \quad T = 473 \text{ K} \\
D_f = 10 \mu m \quad u_\infty = 8 \text{ cm s}^{-1} \\
\rho_p = 1 \text{ g cm}^{-3}
\]

(a) Calculate the Stokes number for particles ranging in size from 0.02 to 1.0 \(\mu m\) in diameter. Show that the Stokes number is sufficiently small that inertial impaction can be neglected as a significant mechanism of collection.

(b) Plot the single fiber collection efficiency as a function of particle diameter over the range 0.02 to 1.0 \(\mu m\) showing the individual contributions of diffusion and interception. Assume the Kuwabara flow field to be applicable.

(c) For a given set of operating conditions there are four filter design parameters that can be varied: filter face area, filter thickness, fiber size, and packing density. These design parameters can be divided into two groups; the first two parameters are related to the size and shape of the filter, and the last two refer to the properties of the filter medium. Consider the operating conditions corresponding to an Oldsmobile 5.7-liter diesel engine automobile being driven at 40 mph (64 km h\(^{-1}\)):

Exhaust flow rate = 2832 l min\(^{-1}\) (at 293 K, 1 atm)

Exhaust temperature = 473 K

Mass median particle diameter = 0.2 \(\mu m\)

Particle density = 1 g cm\(^{-3}\)

With filter thickness as the ordinate (in cm) and filter face area as the abscissa (in cm\(^2\)), assuming that \(\alpha = 0.05\) and \(D_f = 10 \mu m\), plot the curve of overall bed efficiency = 90% based on particle mass. (If we require at least 90% efficiency, all points above this line are candidates for a filter design.)

(d) Now assuming a filter face area of 2500 cm\(^2\) and a thickness of 3.4 cm, with the operating conditions above, plot the overall bed efficiency as a function of packing density \(\alpha\) over the range 0 < \(\alpha\) < 0.1 for \(D_f\) = 5, 10, 20, and 40 \(\mu m\). Discuss your results. (Note that an important design consideration that we have not included here is the pressure drop across the filter bed. Just as it is desired to maintain the efficiency higher than a certain level, it is sought to keep the pressure drop across the bed below a certain level.)

7.18. In most real fibrous filtration problems there is not only a distribution of particles by size to be filtered but also a distribution of sizes of the filter elements. In this problem we wish to extend the treatment given in the text on the collection efficiency of a filter bed to include distributions in size of both the particles and the cylindrical filter elements.

(a) Assuming that the particle volume distribution is log-normally distributed by particle diameter

\[
n_v(D_p) = \frac{V_I}{\sqrt{2\pi} \log \sigma_d D_p} \exp \left[ - \frac{(\log D_p - \log D_{p2})^2}{2 \log^2 \sigma_d} \right]
\]
where $V_i$ is the total particle volume concentration; and assuming that the fiber size distribution is normally distributed, such that $n_f(D_p)dD_p$ is the fraction of fibers having diameters in the range $[D_p, D_p + dD_p]$,

$$n_f(D_p) = \frac{1}{\sqrt{2\pi}\sigma_f} \exp\left[\frac{(D_f - D_p)^2}{2\sigma_f^2}\right]$$

derive an equation for the overall filter efficiency in terms of particle mass. (Since fibers cannot have negative diameters, in using the normal distribution for $n_f$ one needs to assume that the distribution is rather sharply peaked about $D_f$.)

(b) Let us apply the result of part (a) to the filtration of diesel exhaust particles (Oh et al., 1981). The diesel exhaust particles are characterized by $\rho_p = 1 \text{ g cm}^{-3}$, $V_i = 50,700 \mu m^3 \text{ cm}^{-3}$, $D_{ps} = 0.17 \mu m$, and $\sigma_g = 1.74$. The filter medium consists of medium-fine grade commercial steel wool for which $D_f = 22.5 \mu m$ and $\sigma_f = 7.5 \mu m$. Assuming a cylindrical bed with a face diameter of 20 cm, a depth of 5.1 cm, and a packing density of 0.03, calculate the mass removal efficiency of the bed when operated at 473 K and an exhaust flow rate of 300 l min$^{-1}$ (at 293 K, 1 atm).

**REFERENCES**


