EXPERIMENTAL DEMONSTRATION OF THE EQUIVALENCE OF A MECHANICALLY OSCILLATED ELECTROSTATIC CHARGE TO AN ALTERNATING CURRENT

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Abstract

Method of testing equivalence of a mechanically oscillated charge to an alternating current.—The apparatus consisted of a cylinder approximately 4 inches in diameter by 9 inches long, constructed of insulating material, but coated on the outside with a thin sheet of copper foil which could be charged to potentials which varied from 1400 to 2600 volts. The cylinder was arranged to oscillate around its axis at frequencies in the neighborhood of 20 cycles per second through an amplitude of something over 180°. It was surrounded by an "effect coil," having 260,772 turns of fine wire, which was connected through a three-stage amplifier with a tuned vibration galvanometer for detecting the electromotive force which would presumably be induced by the oscillation of the charged cylinder. The "effect coil" was shielded by a grounded covering of sheet copper, the inner portion of this covering and the coating of copper foil on the cylinder thus forming the two plates of an electrostatic condenser. Connected in series with the "effect coil" was a similar "compensating coil" having the same number of turns but connected in opposition so as to neutralize the effect of external magnetic disturbances. The electromotive force induced in the "effect coil" was measured by balancing out with the help of an auxiliary current flowing in a current sheet in such a way as to induce an opposing electromotive force in the "compensating coil," this auxiliary current having the same frequency as the electromotive force to be balanced and being adjustable as to phase and amplitude. The apparatus was calibrated by replacing the charged oscillating cylinder by a current sheet of known dimensions and carrying a known current. Accidental effects coming from a number of sources were studied and as far as possible reduced or eliminated.

Results of the test and conclusions.—A total of 64 effect runs were made, together with the necessary blank runs for correcting for the small residual effect. The runs were made in 16 different groups of 4 runs each under a given set of conditions. The changes in conditions consisted in changes in the sign and magnitude of the potential applied to the cylinder, changes in the method of connecting the "effect coil" and "compensating coil" to the grid and filament of the amplifier, changes in the neutral position around which the cylinder was oscillated, and change to a different frequency of oscillation. The phase of the effect obtained depended on the conditions of operation in the way predicted by theory. It was changed through 180° by a reversal of the sign of the applied potential, depended on the connections used and on the frequency of oscillation in the manner predicted from the calibration runs, and was independent of the neutral position around which the cylinder was oscillated. The magnitude of the effect obtained was also in agreement with theory. It was closely proportional to the potential applied to the cylinder and to the velocity of motion of the cylinder, and had very approximately the absolute value predicted from the measured capacity of the cylinder, the applied potential and the velocity of motion.

The result of the experiment is to show that a mechanically oscillated electrostatic charge of electricity surrounds itself with an alternating magnetic field, accompanied at right angles by an alternating electric field capable of producing an alternating electromotive force in a suitably placed secondary. It further shows that at right angles to both the magnetic and electric fields there must be a Poynting-vector field corresponding to the observed transmission of energy to the secondary, and that these quantities all have the magnitude and time dependence predicted by electromagnetic theory. The present experiment must be regarded as testing a more extended portion of the fundamental basis of electromagnetic theory than the original Rowland experiment, and as giving a more clear-cut demonstration of the equivalence of a mechanically oscillated charge of electricity to an ordinary alternating current than have previous experiments.

I. Introduction

NATURE of the problem. It was shown over fifty years ago by the Rowland experiment that an electrostatic charge of electricity maintained in uniform mechanical motion produces in its surroundings a steady magnetic field of the strength that is calculated on the assumption that what we ordinarily call a direct current of electricity really does consist of electric charges in uniform motion, and that the velocity of light gives the ratio of the electromagnetic unit to the electrostatic unit of electricity. The result may be taken as evidence for the correctness of the usual assumptions as to the nature of a direct current of electricity, and hence as a partial confirmation of the fundamental notions upon which electromagnetic theory is based.

It was the purpose of the experimental work to be described in this article to investigate the fundamental notions of electromagnetic theory somewhat further, by testing for the equivalence which we should also expect between an ordinary alternating current and an electrostatic charge of electricity maintained in a state of mechanical oscillation, by determining if the latter will produce by induction the expected effects in a suitably placed secondary coil. As will be shown in the sequel the predicted equivalence has been satisfactorily demonstrated.

The present experiment must be regarded as providing a test for a somewhat more extended portion of the fundamental basis of electromagnetic theory than does the Rowland experiment in its original simple form. The result of the Rowland experiment is to show that a charge of electricity maintained in uniform motion surrounds itself with a steady magnetic field of the expected intensity. Using the language of ordinary electromagnetic theory, the result of the present experiment is to show that a mechanically oscillated charge of electricity surrounds itself with an alternating magnetic field accompanied at right angles by an alternating electric field, which produces an alternating electromotive force in the secondary coil, while at right angles to both the magnetic and electric fields there must be a Poynting-vector field which corresponds to the observed transmission of energy to the secondary, and these quantities must all have the magnitude and time-dependence predicted by electromagnetic theory.

It is felt that the more extended justification for the usual basis of electromagnetic theory, provided by the present experiment, should be of interest at a time when the facts that have led to the quantum theory have made it evident that the older formulation of electromagnetic theory has to be seriously modified, if we desire to apply it to behaviour within the atom. Thus, for example, we may contrast the expected transmission of energy from an oscillating electric charge to a secondary coil demonstrated by the present experiment, with the originally unexpected lack of emission of energy from accelerated electrons which had to be assumed in the original Bohr atom. It is, however, in agreement with our modern trend of physical thought to expect no necessity for modifying the basis of electromagnetic theory when applied to *macroscopic* phenomena such as those of the present experiment, and to believe that the necessary modifications of electromagnetic theory concern themselves primarily with *microscopic* phenomena, such for example as those of the electrons within the atom.

2. Previous work on the Rowland experiment. Before proceeding, a brief statement must be made as to previous work on the Rowland experiment and its modifications, since one of the latter had certain features in common with the present experiment.[1]

In the original Rowland[2] experiment, a circular ebonite disk, which was covered with gold foil and formed one of the plates of a condenser, was charged to a high potential relative to the other plates and rotated around its axis. The magnetic field produced by the motion of this charged disk was then measured by a magnetometer placed near the periphery of the disk. Essentially this same experiment, without modifications in principle, was also later performed by Röntgen[3], Lecher,[4] Rowland and Hutchinson,[5] Himstedt,[6] and Eichenwald,[7] and incidentally to other work by Pender and by Crémieu and Pender, while E. P. Adams[8] carried out a somewhat similar experiment rotating a series of charged balls instead of a charged plate. All of these experimenters, with the exception of Lecher, observed magnetic fields of at least approximately the expected magnitude.

A quite different method of attack, however, was employed by Crémieu,[9] by Pender,[10] by Crémieu and Pender[11] together in a combined investigation and later by Karpen.[12] The method used was to rotate uniformly in a given direction one or more disks, which were alternately charged and discharged, thus presumably leading to the production of an alternating current. The effects of this alternating current, in inducing an alternating current in a

[1] For an excellent summary of work on electric convection up to the end of 1913 see the report by Eichenwald in the second volume of Graetz, "Handbuch der Elektrizität und des Magnetismus," Leipzig 1921.

[2] Rowland, Amer. Jour. of Science **15**, 30 (1878).

[3] Röntgen, Berl. Ber. 198 (1885); Wied. Ann. **35**, 264 (1888); **40**, 93 (1890).

[4] Lecher, Repert. d. Phys. **20**, 151 (1884).

[5] Rowland and Hutchinson, Phil. Mag. **27**, 445 (1889).

[6] Himstedt, Wied. Ann. **38**, 560 (1889); **40**, 720 (1890).

[7] Eichenwald, Phys. Zeits. **2**, 703 (1901); **4**, 308 (1903); Ann. d. Phys. **11**, 1 (1903).

[8] Adams, Amer. Jour. of Science **11**, 155 (1901).

[9] Crémieu, Ann. d. chim. phys. **24**, 85, 145, 299 (1901).

[10] Pender, Phil. Mag. **2**, 179 (1901); **5**, 34 (1903).

[11] Crémieu and Pender, J. d. physique **2**, 641 (1903).

[12] Karpen, J. d. physique **2**, 667 (1903); Ann. chim. phys. **2**, 465 (1904).

suitably placed secondary, were then looked for by connecting the secondary through a commutator with a sensitive direct current galvanometer. Crémieu failed to obtain the expected effect, but Pender and Karpen both obtained effects of approximately the predicted magnitude.

In the combined work of Crémieu and Pender, carried out at the suggestion of Poincare, the attempt was made to discover the cause of the above mentioned discrepancy in their results It was first satisfactorily demonstrated that the effect did not occur when the original Crémieu apparatus was operated, but did occur with the original Pender apparatus. It was then found that the physical difference which accounted for Crémieu's negative result was the presence of a thin layer of insulating material with which he had covered his disks, and it was actually demonstrated that the galvanometer deflection obtained from the Pender apparatus was reduced from 140 to 15 scale divisions by covering his rotating and stationary plates by a thin layer of mica, while the expected magnetic effect appeared when the insulation was removed from the Crémieu apparatus. The theoretical reason for this, however, is not clear and later tests made by Karpen and Eichenwald showed no such shielding effect from insulating materials. It is evident that the combined work of Crémieu and Pender still left something to be explained, and their article closes with the equivocal statement: *Il ne nous appartient pas de dire si ces effets magnêtiques sont bien rêellement ceux prêvus pour une convection d'êlectricitê dans le sens où Faraday et Maxwell entendaient cette expression, ni de dêcider s'ils sont d'accord avec les hypothèses fondamentales des thêories actuelles,*"

3. Relation of present experiment to those of Crêmieu, Pender and Karpen. There is a certain similarity between the present experiment and those just described above. There are also, however, important theoretical and experimental differences.

From a theoretical point of view, the foregoing experiments consisted in an attempt to demonstrate the equivalence between an ordinary alternating current and a body in uniform motion but subjected to an alternating change in the charge of electricity which it carries. Our experiments on the other hand are an attempt to demonstrate the equivalence between an ordinary alternating current and a body carrying a steady charge of electricity, but maintained in an oscillatory motion. Our experiments would thus seem to give a clearer answer to the particular question as to whether an ordinary alternating current really consists in the oscillatory motion of electricity, since the process of charging and discharging in the other method might introduce unsuspected factors.

On the experimental side also, it should be noted that the use of a charging current which has the same period as that which is to be detected is a danger which is avoided in our method. Experimentally, our work also differs from the foregoing in using an amplifier and vibration galvanometer for determining the effect in the secondary. This has the advantage of doing away with the use of commutators, which do not always operate satisfactorily. Our set up also differed from previous ones in using a cylinder instead of a disk for the charged body, which is in some ways a more efficient shape.

4. *Relation of present experiment to those of Tolman, Karrer and Guernsey, and of Tolman and Mott-Smith.* The present experiment also has a certain similarity to previous experiments of Tolman, Karrer and Guernsey[13] and of Tolman and Mott-Smith,[14] in which an uncharged thick-walled copper cylinder was oscillated around its axis and a determination made of the alternating current which was presumably produced by the lag of the conduction electrons behind the motion of the cylinder. These experiments may also be regarded as giving evidence for the belief that an ordinary alternating current of electricity really does consist of charges of electricity in oscillatory motion. They do not, however, give a clear cut demonstration of this fact, since their theoretical interpretation involves hypotheses as to the nature of the conduction process in metals, and assumptions as to the mass and charge of the conducting particles, and since they do not exclude the possibility that an ordinary alternating current requires the relative motion of positive and negative charges within the material of a conducting substance. Furthermore, the results obtained from them did not precisely agree with the predictions of the elementary theory which was available for their interpretation.

II. Description of the Apparatus.

In order to permit an intelligent criticism of the measure of success that we have had in eliminating accidental effects and measuring solely the effect of interest, we shall describe the apparatus and experimental work in considerable detail. The apparatus will first be described in its final form, and in later sections a discussion will be given of successive changes that were made in order to eliminate various accidental effects which were encountered.

5. *The location.* Work of the kind to be described cannot be carried out in an ordinary laboratory because of unsteadiness in electromagnetic conditions, and hence the oscillating apparatus was located in a small wooden building (on the Campus of the California Institute of Technology), which had been built for the work of Tolman and Mott-Smith on the inertia of the electric carrier in copper.

The building had a floor space of 16 by 12 feet with its axis pointing toward the magnetic north, and was provided with a cement pier and cement lined pit, arranged so that the oscillating apparatus could be mounted, to reduce accidental effects, with its axis approximately parallel to the earth's magnetic field as shown in Fig. 1. The pier and pit were constructed of neat cement which was only very slightly paramagnetic, but the pier rested on an ordinary concrete foundation. The pier was 9 feet 4 inches tall measured from the floor level and the pit 4 feet 6 inches deep as described more in detail in the article of Tolman and Mott-Smith.

The horizontal component, declination and dip of the earth's magnetic field at the site of the building was determined previous to construction by

[13] Tolman, Karrer and Guernsey, Phys. Rev. **21,** 525 (1923).

[14] Tolman and Mott-Smith, Phys. Rev. **28,** 794 (1926).

Mr. Wallace M. Hill of the U. S. Coast and Geodetic Survey in the spring of 1923. The strength of the horizontal component at that time was 0.26753 gauss and the dip 59° 25′ giving us for the total intensity of the earth's magnetic field H = 0.52581 gauss which is the figure that we use in computing the results of our calibrations made against a rotating earth inductor. Since the above date, a not very distant car line has been discontinued and a new building has been built nearby on the campus, but it is not believed

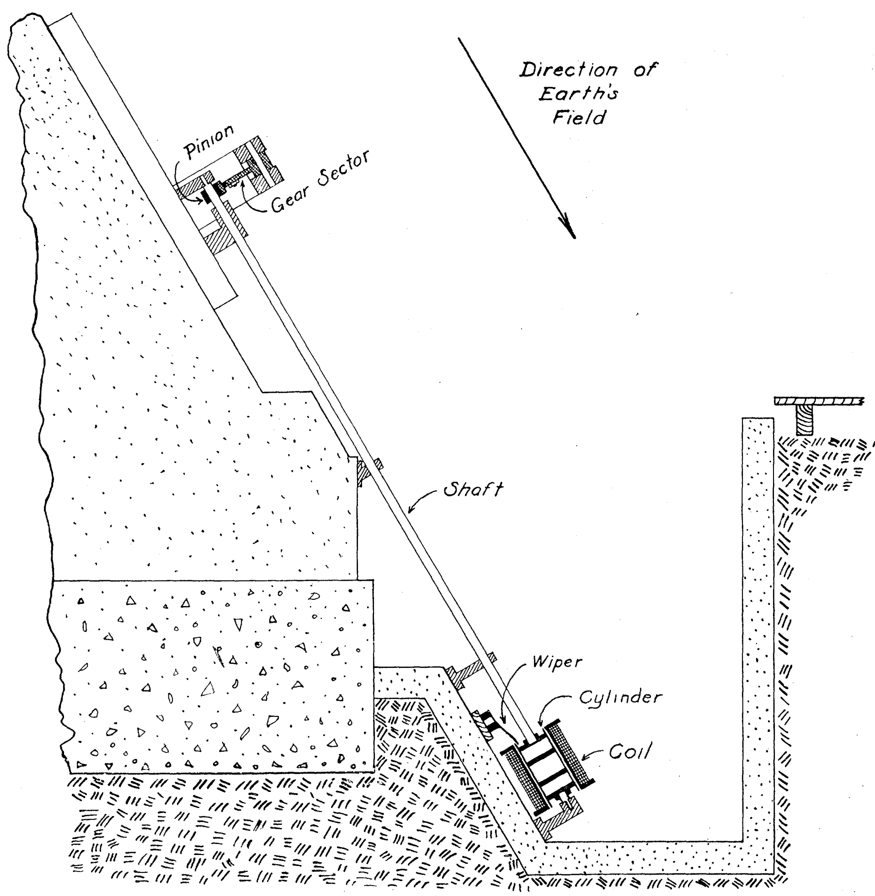

FIG. 1

that these changes have seriously affected the steady part of the earth's magnetic field.

6. *The motor drive.* The oscillating apparatus was driven by a five horse power direct current motor, which was mounted west of our building at a distance of approximately 42 feet from the axis of the oscillating apparatus. The motor was enclosed in a series of four magnetic shields made from sheet iron having a thickness of 1/16 inch.

The direct current for operating the motor came from a motor-generator set, located in a neighboring building, and was carried out to the motor in a duplex cable which passed at a considerable height above the small building containing the apparatus.

The main switch and starting rheostat for the motor were located on a pole near the motor and were controlled from inside the building by cords passing over pulleys. It was found possible, however, to have the rheostat for the field control of the motor inside the building containing the apparatus.

The power necessary to operate the apparatus at the speed where most of the final runs were made was about 2 kilowatts measured at the switch board.

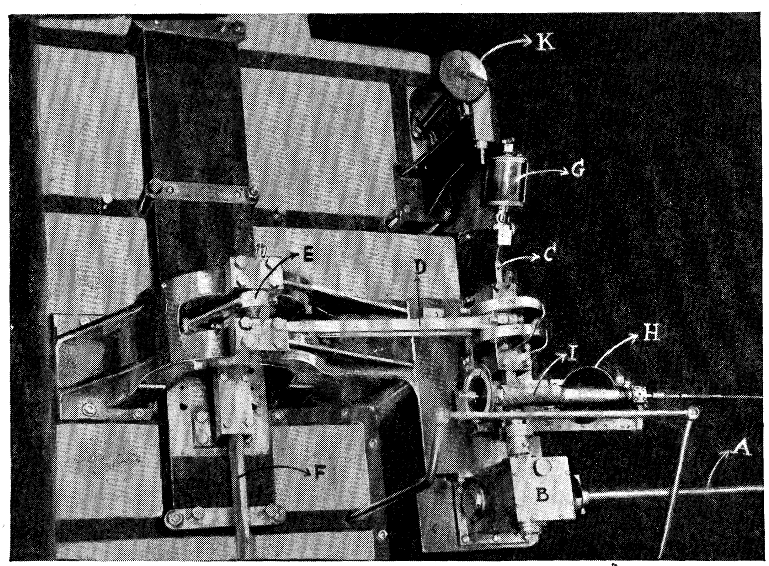

FIG. 2

7. *The oscillating apparatus.* The main features of the oscillating apparatus are shown in the drawing, Fig. 1, and in the retouched photographs, Fig. 2 and Fig. 3.

The power from the motor was brought to the apparatus by a one-inch Tobin bronze shaft connected directly to the shaft of the motor, and supported by eight iron hangers spaced at intervals between the motor and the building containing the apparatus. The end of this shaft is seen at A at the right of Fig. 2.

The power was transmitted to the Tobin bronze crank shaft C through a pair of 45° bronze spiral gears running in oil in the gear box shown at B. These gears had a reducing ratio of 16 to 13. The crank shaft C was supported by brass pillow blocks mounted on a heavy brass casting which was itself bolted and grouted in position on the sloping northern face of the cement pier. The shaft carried a small Tobin bronze 45° spiral gear for operating an earth inductor, as will be described later, and a pair of brass flywheels, one on each side of the crank pin, to improve the steadiness of running.

The Tobin bronze crank pin was 1.25 inches long, 0.972 inches in diameter and was offset from the center by 1.133 inches. At first the crank pin gave considerable trouble by excessive heating, but finally, using continuous lubrication from the oil reservoir *G* seen in Fig. 2 above the end of the crank shaft, and using a connecting rod with a loose fitting fiber lined bearing, the difficulty was overcome.

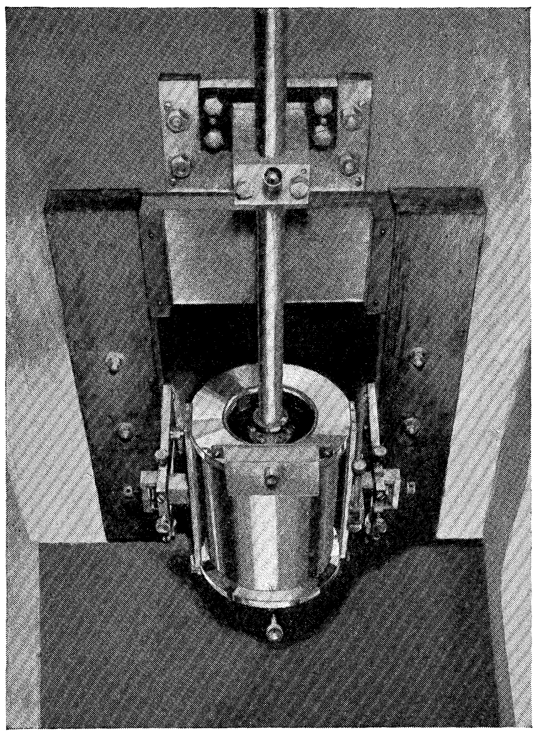

FIG. 3

The connecting rod *D* was made from a heat treated aluminum alloy (Dural) casting and transmitted the power to a wrist pin on the gear sector shown in section in Fig. 1 and at *E* in Fig. 2. The length between the centers of the connecting rod bearings was 20.147 inches.

The gear sector was made from a casting of the same aluminum alloy, and was mounted on a 7/8 inch Tobin bronze shaft, running in brass bearings which were supported by the heavy yoke which formed part of the main casting. The Tobin bronze wrist pin had a diameter of 0.747 inches, a bearing length of 11/16 inches, and was offset from the center by 4.156 inches. The distance between the centers of the crank shaft and the gear sector shaft was 20.174 inches, which, combined with the dimensions stated above, gave the gear sector a motion through an angle of a little more than 30°.

The gear sector engaged with a fiber pinion on the upper end of the shaft that drove the cylinder. The length of face of the gear sector and pinion was 2 inches. The pitch diameters of gear sector and pinion were 12.187 and 2 inches respectively, with 16 pitch involute teeth, giving an angular motion of somewhat over 180° to the cylinder.

The shaft that drove the cylinder was made of brass tubing 1 inch outside diameter with a 1/32 inch wall, and was 6 feet 8 inches long from pinion to cylinder. It was supported by four brass bearings as shown in Fig. 1, and in addition there was a step bearing 3/8 inch in diameter at the lower end of the cylinder. The shaft was purposely made light for obvious reasons, and actually turned out to be so light that the "dynamic amplitude" of oscillation of the cylinder obtained under running conditions was appreciably larger than the "static amplitude" obtained when the apparatus was turned over slowly by hand.

8. The cylinder. The cylinder was located in the pit as seen in Figs. 1 and 3. It was constructed to be as light as possible out of a tube of pressed insulating material known in the trade as bakelite tubing. The tubing used had an outside diameter of 4 inches and a wall thickness of 1/16 inch, a light cut being taken off the outside after assembly. It was reinforced inside with four bakelite disks as shown in section in Fig. 1, the top disk being permanently connected with the driving shaft and the bottom disk carrying a short spindle which fitted into the step bearing.

The cylinder was coated on the outside with a layer of copper foil approximately 0.001 inch thick to carry the charge of electricity. The foil was soldered at the two ends to light copper rings, which were fastened to the bakelite cylinder, and was cemented on the cylinder with a mixture of 5 parts rosin to 1 of beeswax using a hot roller. The copper rings on the ends of the cylinder were cut and a very narrow longitudinal slit was left in the copper coating in such a way that electricity could not flow circumferentially around the cylinder. The total length of the finished cylinder was 23.42 cm and the average outside diameter 10.141 cm.

9. Arrangement for charging the cylinder. The direct current high voltage necessary for charging the cylinder was generated in a small building 100 feet away from the apparatus, from 110 volt 50 cycle alternating current, by transforming up to the desired potential and rectifying with a hot filament vacuum tube rectifier. The pulsating rectified potential thus obtained was used to maintain a steady charge in a condenser (Westinghouse, Style 376093) having a capacity of 0.97 microfarads.

The potential from this condenser was connected, for protection through a high resistance, to an overhead cable leading to the building containing the apparatus. Here connection was made with a Kelvin electrostatic voltmeter (Kelvin and James White, Ltd.) for measuring the applied potential, and with a double throw switch arranged so that the cylinder could be grounded or charged at will. The potential for charging the cylinder was lead in through a light spring-brass wiper which made rubbing contact with the inside face of the copper ring at the upper end of the cylinder, as shown in Fig. 1.

10. The layout of the electrical measuring circuits. The general layout of the electrical measuring circuits is shown diagrammatically in Fig. 4, in the form which it took for calibrating the apparatus. For making the effect runs, the calibration current sheet was replaced by the oscillating charged cylinder. The layout can be regarded as composed of four individual circuits.

Circuit No. 1 contains the calibration earth inductor, the calibration current sheet which is arranged to induce electromotive forces in the effect

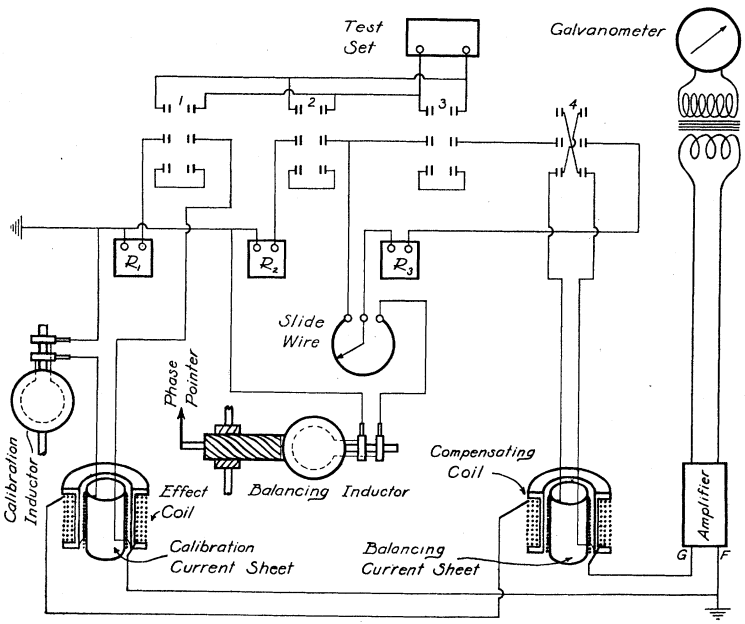

FIG. 4

coil, and the resistance boxes R_1 for regulating the current through the current sheet. The circuit is thrown into action by putting switch No. 1 in the down position, while its resistance can be measured with the help of the Leeds and Northrup test set by putting switch No. 1 in the up position.

Circuit No. 2 contains the balancing earth inductor, the slide wire and the resistance box R_2. The circuit is thrown into action by putting switch No. 2 in the down position, while the resistance can be measured with switch No. 2 in the up position.

Circuit No. 3 contains that portion of the slide wire which it is desired to tap off, the balancing current sheet which is arranged to induce electromotive forces in the compensating coil, and the resistance box R_3. The circuit is thrown into action by putting switch No. 3 in the down position, while its resistance can be measured with switch No. 3 in the up position. An auxiliary reversing switch, No. 4, is provided for changing by 180° the phase of the current going to the balancing current sheet, while smaller changes in phase can be made by adjusting the longitudinal position of the balancing inductor as will be described more in detail later.

Circuit No. 4 contains the effect coil and compensating coil connected in series, and then connected to the grid and filament of the vacuum tube amplifier, which in turn was connected through a transformer with the vibration galvanometer. The effect coil and compensating coil were made as nearly alike as possible and were connected in opposition in order to reduce the effect of accidental magnetic disturbances coming in from outside.

The circuits were grounded as indicated in Fig. 4. The ground consisted of a pair of iron pipes driven into the earth just outside our building to a distance of several feet. Water was poured around the pipes and salt water poured into them at the time of installation. When first set up there was a resistance of a few hundred ohms between this ground and the pipes of the city water system, and after the end of our runs this had sunk to less than a hundred ohms.

It will be noted that the method of balancing the electromotive force in the effect coil by an induced electromotive force in the compensating coil, as adopted for these experiments, is different from the method of Tolman and Mott-Smith in which an electromotive force, tapped off from the slide wire, was directly opposed to that generated in the effect coil. The present method has the advantage of not introducing possible spurious effects from the balancing inductor and slide wire directly into the circuit that contains such a sensitive element as the amplifier grid, and also has the possible advantage of providing a certain measure of symmetry in the way in which the effect and balancing electromotive forces enter the detecting circuit. It has the disadvantage, however, of using the tapped off portion of the slide wire to furnish current instead of merely potential through the sliding contacts.

Certain parts of the apparatus connected with the electrical measuring circuits will now be described in more detail.

11. The earth inductors and the phase and speed measuring devices. The calibration earth inductor, which is shown diagrammatically in Fig. 4, will be seen in its actual position near K in Fig. 2. When the calibration runs were made, the connecting rod D and the oil reservoir G were removed and a direct connection was made between the end of the crank shaft C and the shaft which operated the calibration inductor through a pair of 45° brass spiral gears.

The axis of the calibration inductor was adjusted with great care so as to be perpendicular to the total earth's field, determined by Mr. Hill as already described. The wooden spool for the inductor was wound with 12 layers of 11 turns each of No. 20 enamel covered copper wire, having a diameter of 0.85 mm. The inside diameter of the coil was 9.000 cm, the outside diameter 10.740 cm, and the width of space filled by the winding 1 cm. The average radius of the coil for purposes of calculation was taken as the square root of the mean square radius of the turns and had the value 4.963 cm.

The two ends of the coil were brought out to brass slip rings on the axis of the coil, where contact was made with brushes. These brushes consisted of strips of spring brass sheet, which were slit into three longitudinal fingers which pressed against the rings. This arrangement proved to be very satisfactory provided it was frequently cleaned with kerosene while running.

The balancing earth inductor, which is shown diagrammatically in Fig. 4 will be seen in its actual location at H in Fig. 2. It was of similar construction to the calibration inductor, but its exact dimensions are not necessary since it functions in the same manner both in the calibration and the effect runs. It was provided with slip rings and brushes similar to those for the calibration inductor.

This balancing inductor was driven by a 45° brass spiral gear shown in Fig. 2 at I, which meshed with a similar shorter gear on the crank shaft. This made it possible to vary the phase of the balancing inductor by varying its longitudinal position, and this could be done, while the apparatus was running, with the help of a lead screw which moved the entire mounting of the inductor in ways which were provided for the purpose. One end of the shaft of the inductor was provided with a phase pointer and protractor scale, as will also be seen in Fig. 2, which were used in measuring the phase settings which had been found necessary to secure electrical balance.

To obtain the phase readings in the case of the calibration runs, the apparatus was first turned over by hand until the coil of the calibration inductor was parallel to the earth's field, as shown by a suitably adjusted auxiliary pointer and reference marks which will be seen at K in Fig. 2, and the position of the phase pointer on the shaft of the balancing inductor was then read. The readings were always taken in pairs corresponding to the "up" and "down" positions of the auxiliary pointer, the two readings when one of them was corrected by 180° usually agreeing within a degree.

To obtain the phase readings in the case of the effect runs, the apparatus was turned over until the gear sector was in the center of its stroke as shown by suitably placed reference marks on the top face of the upper fly wheel, and the position of the phase pointer was then read. These readings were also taken in pairs, corresponding to the center of the stroke moving both east and west. Owing to the geometry of the driving mechanism the two readings when one of them was corrected by 180° differed by about 6°.

At the opposite end of the shaft of the balancing inductor from that carrying the phase pointer, a light extension will be seen in Fig. 2, which operated a centrifugal tachometer as well as a vibrating reed tachometer, the latter proving to be the more satisfactory.

12. The slide wire and resistances. The slide wire from which the current was tapped off to operate the balancing current sheet was a Leeds and Northrup Kohlrausch bridge, Type 4258, No. 82123, with a total resistance of about 32 ohms. Such an instrument is of course not specially adapted for the purpose to which we put it, since the moving contacts are not designed for the transmission of current. Nevertheless, since the instrument was kept well cleaned with kerosene, and the resistance in circuit No. 3 which contained the balancing current sheet was made rather high, it is believed that the set-up was satisfactory for our purposes.

The resistances R_1, R_2 and R_3 for the three circuits Nos. 1, 2 and 3 were provided by Leeds and Northrup non-inductive resistance boxes, and should have the same impedance for direct current and for the low frequency currents (17 to 23 cycles) which we employed.

The total resistance in the calibration circuit No. 1 was made 10,000, 5000, 2500 or 1000 ohms according to the strength of the calibration current desired. The total resistance in the slide wire supply circuit No. 2 was always made 250 ohms, and that in the balancing circuit No. 3 was made 100 ohms with the slide wire set at zero. The resistances in circuits 1 and 2 were thus high enough to make small variations in the contact resistances at the brushes of the inductors negligible, and the resistance in circuit No. 3 high compared with small variations in the slide wire contact resistances.

13. The current sheets. The calibration current sheet was designed of course to have nearly the same dimensions as the cylinder. It consisted of 73 complete turns of No. 18 double-cotton-covered wire wound on the outside of an orangewood cylindrical block in a groove having a pitch of 1/8 inch, the total length of coil from the center of the wire at the beginning of the first turn to the center of the wire at the end of the last turn was 9.125 inches or 23.18 cm, this being the same length as the spool of the coil and a trifle shorter than the cylinder. The diameter of the coil measured to the center of the wire was 10.120 cm. A correction was made for the slight difference between the area of the coil and that of the cylinder.

The balancing current sheet was made from 605 turns of No. 28 single-silk-covered wire wound on a 4 inch bakelite tube. Its exact dimensions are not necessary since it functions in the same way in the calibration and effect runs.

14. The effect and compensating coils and their connections. The effect coil was the same one used in the work of Tolman and Mott-Smith. The spool for the coil was constructed of "micarta." It had an overall length of 9.125 inches, the end disks being 3/8 inch thick. The outside diameter was 7 inches, the inside diameter 4.125 inches, and the wall thickness of the central tube 1/16 inch. It was carefully wound in one length with soldered joints, with 18 lbs. of No. 38 enamel-coated wire, with a total of 260,772 turns, giving a resistance of 207,300 ohms and an approximate length of 67 miles. The compensating coil was made with a spool of the same dimensions and exactly the same number of turns of similar wire, and had approximately the same resistance and length.

The effect coil was located in the pit surrounding the cylinder as shown in Fig. 3. It was mounted in a cradle with adjusting screws for tipping its axis in any desired direction and for aligning it lengthwise and sidewise. It was adjusted parallel to the axis of the cylinder with the help of removable centering plates, which brought the upper end of the coil central with the cylinder shaft and the lower end central with the step bearing. The plates were removed when the apparatus was running.

The balancing coil was placed on the floor of the building at a distance of about 6 feet from the effect coil and set in a wooden cradle which held it approximately parallel to the effect coil, in order to balance out external magnetic disturbances.

Boths coils were electrostatically shielded by enclosing them in grounded coverings of sheet copper which fitted closely to the coil and end plates and to the inside of the spool. The copper covering on the inside of the spool of the effect coil and the coating of copper foil on the outside of the cylinder thus formed the two plates of a cylindrical condenser whose capacity determined the charge carried by the cylinder.

Further electrostatic shielding was provided by enclosing the binding posts for the coils in brass boxes as seen at the front upper end of the coil in Fig. 3, and connections with the coils were made with wires enclosed in a grounded lead sheath.

The lead-covered wires from the two coils and from the amplifier input were brought together to binding posts inside a grounded brass receptacle to permit the connection of the coils and amplifier in the different ways used. Four different methods of connection, keeping the two coils connected in opposition, were possible. These different connections were called A, B, C and D and are described in Table I. The different terminals are denoted by the following symbols:—G and F are the grid and filament of the amplifier input, E_i and E_o the inner and outer terminals for the effect coil and C_i and C_o those for the compensating coil. The symbols enclosed in brackets indicate the terminals connected together.

TABLE I. *Description of connections.*

Connection	Terminals connected together		
A	$(G\,E_0)$	(E_iC_i)	(C_0F)
B	$(G\,E_i)$	(E_0C_0)	(C_iF)
C	$(G\,C_0)$	(C_iE_i)	(E_0F)
D	$(G\,C_i)$	(C_0E_0)	(E_iF)

15. The amplifier. The amplifier was a three-stage resistance coupled amplifier of standard design. It was operated with a plate voltage of 135 volts obtained from a battery of dry cells, and used a filament heating current for all three stages of a little less than one ampere, obtained from an Edison storage battery. The whole amplifier including both batteries, was electrostatically shielded by completely enclosing it in a grounded brass box. The filament of the first tube was also grounded, and the input wires from the coils as well as the output wires to the transformer were enclosed in a grounded lead sheath. The first two stages of the amplifier were operated by high μ tubes while the third stage had a lower μ tube in order to provide the power output to operate the transformer and galvanometer.

The transformer was a 284-D General Radio Company transformer. It was placed near the galvanometer at a considerable distance from the amplifier and other apparatus. The electrostatic shielding of the connecting wires in grounded lead sheath was continued as far as this transformer.

16. The galvanometer. The galvanometer was a Leeds and Northrup vibration galvanometer with a flexible enough suspension so that it could be tuned to the low frequencies in the neighborhood of 20 cycles which were used. This possibility of using a tuned detecting instrument is of the greatest importance, since the accidental external disturbances which enter the coils are apparently many times greater than the effect of interest, but do not have the right frequency to correspond to the tuning. The direct current resistance of the galvanometer was 765 ohms. It was provided with a concave mirror which was illuminated with a single filament lamp, the distance from mirror to ground glass scale being about 14 feet.

III. Elimination of Accidental Effects

The foregoing description applies to the final form in which the apparatus was used for making the experimental runs which are to be reported. This final form, however, was partly the result of a series of changes which were made to eliminate various accidental effects and we must next describe the nature of these accidental effects and the methods by which they were reduced.

17. External electromagnetic disturbances. In work of this kind, with a powerful coil connected to a three stage amplifier, effects due to external electromagnetic disturbances are hard to eliminate, and, with the amplifier on, our galvanometer was always moving to some extent even though the apparatus itself was not operating. This residual motion of the vibration galvanometer produced a somewhat steady widening of the band of light from the galvanometer with occasional superimposed sudden "kicks." The steady widening appeared to be due at least partly to 50 cycle commercial circuits, but the cause of the sudden kicks is unknown.

The residual motion of the galvanometer was appreciably reduced by using a compensating coil in series with the effect coil as described above. This device for the elimination of external disturbances, by the connection of two similar coils in such a manner as to "buck" each other, had earlier been used by Tolman and Stewart[15] in somewhat similar work, but had not been used in the immediately preceding work of Tolman and Mott-Smith. The introduction of the device made the conditions of measurement appreciably better in the present work than in that of Tolman and Mott-Smith, even though the location of the apparatus is no longer as favorable as at that time.

The magnitude of the residual motion of the galvanometer varied somewhat from time to time, but was never large enough so that there was the slightest doubt as to the increased effect obtained from putting the charge on to the oscillating cylinder. The accuracy of the measurements was doubtless somewhat reduced by the presence of the residual motion of the galvanometer. There was, however, no reason to believe that this re-

[15] Tolman and Stewart, Phys. Rev. **8,** 97 (1916); **9,** 164 (1917).

sidual motion had any systematic effect on the results since a null method was used in which the effect of interest was balanced out, presumably without affecting the residual motion of the galvanometer. And as a matter of fact the appearance of the minimum deflection when balance was obtained was quite similar to the residual deflection when the apparatus was not running.

18. Location of high voltage generator. As already stated in Section 9, the direct current high voltage for charging the cylinder was generated in a separate building 100 feet away from that in which the experiments were performed. This was done since in earlier experiments it was found that the operation of a high voltage transformer near the apparatus produced extremely large galvanometer deflections. At a distance of 100 feet, however, there was no appreciable effect from the apparatus which produced the high voltage.

19. Location and shielding of motor. In undertaking the present experiments, the attempt was first made to adapt the apparatus of Tolman and Mott-Smith to the new purpose, by merely replacing the oscillating conducting cylinder, which they used to detect the inertial lag of the conduction electrons, by a charged cylinder of similar moment of inertia but made non-conducting by the presence of a longitudinal slit. A long series of measurements was made with this set up, but the calculated magnitude of the expected effect, which could be obtained in this way, proved to be considerably smaller than the accidental effects which were actually found to be produced by the oscillation of the uncharged cylinder. For this reason it became necessary to give up the previous method of maintaining at its natural period of oscillation a system consisting of a heavy cylinder restrained by torsion rods, and replace it by a very light cylinder driven through as large an angle as possible by a forced mechanical action. It was soon found, however, that this change involved the use of considerably more power than could be provided by the non-magnetic water motor previously used and it appeared desirable to experiment with electric motors as a source of power.

Direct current motors were chosen, since our apparatus is particularly sensitive to alternating current disturbances. A two horse power motor was first tried and found not to produce much magnetic disturbance when operating just outside the east wall of our building. It was found, however, that more power was necessary and a five horse power motor was next installed.

Unfortunately the larger motor produced a very much greater magnetic disturbance and at a distance of about 12 feet from the axis of our apparatus it produced very serious trouble when running This was much reduced but not entirely eliminated by enclosing the motor in four magnetic shields made of sheet iron 1/16 inch in thickness. Finally, as already stated, the motor was permanently installed at a distance of about 42 feet. In this latter position it still produced a slight but noticeable disturbance without the magnetic shields, but even this disappeared with the shields on. The motor was purposely arranged to run at a different speed from the apparatus (ratio 16 to 13), so as not to have the same period as the tuned galvanometer.

Since the motor was driven by direct current, it was at first hoped that the main switch and starting rheostat could be located for convenience inside the building containing the apparatus. It was found, nevertheless, that the direct current leads apparently carried a slight superimposed alternating or pulsating component which produced a small but definite disturbance as shown by the galvanometer. For this reason the main switch and starting rheostat were finally removed from the building as already described. It was possible, however, to retain the field control rheostat for the motor inside the building and thus maintain a continuous adjustment of the speed.

20. Location of the oscillating cylinder. The attempt was first made in this work to place the cylinder directly above the fiber pinion which was driven by the gear sector. With the cylinder and surrounding coil in this position, however, accidental effects were found when the apparatus was running which appeared to come from periodic currents flowing in the heavy brass yoke which supported the bearings for the shaft of the gear sector. These currents seemed to arise from the slight vibration of this yoke in the earth's magnetic field, since the effects were modified in very systematic ways by holding a bar of iron, while the apparatus was running, with its end near different portions of the yoke.

To avoid these accidental effects, the cylinder was finally mounted, as already described, in the pit at a considerable distance below the yoke, even though this involved the use of a long shaft which proved flexible enough so that the dynamic amplitude of oscillation of the cylinder was greater than the amplitude when the apparatus was turned over by hand.

21. Leakage current from the cylinder. The cylinder was specially designed, as will be noticed from the cross section in Fig. 1, so as to give a long leakage path from the charged coating to the central shaft. It is of course desirable to keep the leakage current as small as possible, since it is conceivable that when the apparatus is rotating, the leakage current might acquire a pulsating component which would have the frequency of rotation and affect the results.

The actual resistance from the cylinder to ground was approximately measured, with the apparatus rotating, by noting the time taken for the cylinder to discharge our high voltage condenser. The leakage current from cylinder to ground could then be calculated and was found to be about 1×10^{-5} amperes at 2000 volts. As this is only about one half the expected alternating current produced by the oscillating cylinder, it is believed that the effect of possible pulsations in the leakage current can be neglected.

22. Electrostatic shielding of the amplifier and connections. During a considerable part of the preliminary work, the amplifier and connecting wires were not electrostatically shielded although the effect and compensating coils were covered with sheet copper. With such an arrangement, however, conditions were not as good as with the more complete electrostatic shielding finally used as described above.

In the first place in the absence of the more complete shielding, it appeared to be undesirable to try to ground the filament of the first stage of the amplifier, since the grid then apparently responded more markedly to electrostatic disturbances and it was better to leave both grid and filament "floating". This, however, is not as satisfactory as to have proper electrostatic shielding with the potential of the filament definitely fixed.

In the second place even with grid and filament both "floating" the set up was still very sensitive to some electrostatic disturbances. Thus the mere rubbing of one's hair in the neighborhood of the apparatus would produce violent galvanometer motions; and with the high voltage merely coming into the building to the switch for throwing it on to the cylinder, a violent galvanometer kick would often be produced when the long wooden handle leading to the switch was merely touched with the hand or even when the wall of the building several feet away from the switch was touched. With the more complete shielding these annoying effects disappeared.

23. Elimination of a special electrostatic effect from the charged cylinder. We must finally turn our attention to the elimination of a particularly serious electrostatic effect coming from the charged cylinder itself, the cause of which was not discovered until after a long period of preliminary experimentation.

In originally constructing the copper electrostatic shields for the effect and compensating coils, the inside of the spools for these coils were lined with a sheet of copper which was rolled into a cylindrical form in such a way that the opposite edges came nearly but not quite together, leaving a very narrow longitudinal slit in the shielding. This was done partly because it was thought that this slit would be an advantage in breaking what would otherwise act as a very low resistance short-circuited turn in the transformer, and partly for reasons of ease of construction since it was felt that the shielding would not be seriously affected by such a narrow slit.

Nevertheless, the preliminary effect runs, made with this narrow slit in the shielding, lead to quite erroneous results. Although a reasonably definite and reproducible effect was obtained when the oscillating cylinder was charged, the magnitude and phase of the effect were found to be different for the different methods of connection A, B, C and D described in Section 14, and to be different when the neutral position around which the cylinder was oscillated was changed, thus changing the relative positions of the longitudinal slit in the shielding of the coil and the longitudinal slit in the copper foil coating of the cylinder. Furthermore, the magnitude of this effect was in general considerably greater than that which was predicted for the effect of interest, although with connections D, in which the inside of the effect coil was connected directly to the grounded filament, the results obtained were not far from the predicted ones.

These findings suggested the possibility that a periodic electric field from the charged oscillating cylinder was passing through the narrow slit in the shielding of the effect coil and affecting the grid of the amplifier by direct electrostatic induction. This conclusion was made more certain, however, by some special experiments which were carried out as follows.

A small rotating electrostatic inductor was constructed from a cylinder of fiber $1\frac{1}{2}$ inches long and $2\frac{1}{2}$ inches in diameter. This cylinder was provided with a cylindrical brass sheath, one quarter segment of which was kept charged to a high direct current potential by a brush which was connected to our high potential supply.. Arrangements were also made so that the cylinder could be rotated, at the frequency to which our galvanometer was tuned, through a flexible shaft which made it possible to hold the cylinder so as to produce a periodic electric disturbance at any point desired. With connections *A*, in which the inside of the effect coil did not go directly to the grounded filament, there was a very large effect (galvanometer off-scale) when this electrostatic inductor was rotated in a position near the slit in the shielding of the effect coil. There was, however, no effect when the conductor was held just as near to the continuous portion of the shielding. Furthermore there was no effect or only a very small one when the rotating inductor was held near the slit, provided connections *D* were used, in which the inside of the effect coil was connected directly to the grounded filament.

As a result of the conclusions made obvious by these experiments, a new inside shield for the effect coil was constructed out of copper tubing turned down so as just to fit the inside of the coil and have a wall thickness of about 0.01 inches, and this was soldered to the copper end coverings of the coil. With this new arrangement our electrostatic shielding appeared to be entirely satisfactory as will be evident in what follows.

It is of course a matter of vital importance in this work to have proper shielding from electrostatic effects, since from an experimental point of view the amplifier is very sensitive to electrostatic disturbances, and from a theoretical point of view we are only interested in the electromagnetic action of our primary current on the secondary effect coil.

IV. The Calibration of the Apparatus

We are now ready to consider the calibration of the apparatus in its final form. We must first state the results obtained in calibrating certain separate parts of the apparatus and then present the results of the calibration runs themselves.

24. *The electrostatic voltmeter.* To calibrate the electrostatic voltmeter (see Section 9) used for measuring the high voltage on the cylinder, we employed a Weston voltmeter, Type 45, No. 33873, together with the Weston Voltage Multiplier No. 33873 (ratio 20 to 1) which went with it. Corrections to the readings of the voltmeter No. 33873 itself were first obtained by comparing it with a Laboratory Standard Weston Voltmeter, No. 822 belonging to the Electrical Engineering Department of this Institute, and the readings of our electrostatic voltmeter were then compared with those of voltmeter No. 33873 in series with the multiplier, using high potential from the source described in Section 9.

Calibrations of the electrostatic voltmeter were thus made both before and after our series of runs, without any marked difference in the results. In our range of use, 1400 to 2600 volts, it was only possible at the utmost to estimate the readings of the electrostatic voltmeter to the nearest 5 to 10 volts and our final corrected readings are probably affected with about that amount of uncertainty.

In connection with our method of calibrating it should be noted that our source of potential was not strictly steady, since it came from a condenser kept charged by a rectified 50 cycle alternating potential. This gave a slight pulsation to the actual potential, since an appreciable current had to be supplied to operate the Weston voltmeter. It was shown by calculation, nevertheless, allowing for the fact that one voltmeter measures the mean voltage and the other the square root of the mean square voltage, that these pulsations would not have an appreciable effect on the results of the calibration.

25. The capacity of the cylinder. In order to calculate the charge carried by the cylinder, it was necessary to know the capacity of the condenser formed by the cylinder and its surroundings. To measure this, a 50 cycle electrically driven tuning fork was arranged to connect the cylinder alternately to the charged side of a half microfarad supply condenser and to ground, the voltage of the supply condenser being followed by a Cambridge and Paul string electrometer, and the time of discharge over a given range being taken with a stop watch. This time was then compared with that necessary to discharge the supply condenser over the same voltage range into a General Radio Company precision condenser, Type 222, Serial No. 482, set at 527 micro-microfarads (claimed accuracy 0.1%), which was nearly the same capacity as that to be measured.

Before making the measurements, the coil and cylinder were carefully lined up and measurements were taken with the cylinder turned to different positions, and with different adjustments of the tuning fork without any marked change in the results. In all 37 pairs of time readings on the two condensers were taken, and the final value for the capacity of the cylinder came out 538.5 micro-microfarads with a probable error[16] of 0.8.

26. The slide wire. The slide wire was calibrated over the range employed, and it was found that the resistance of the wire was indeed quite closely proportional to the readings of the instrument.

27. The tachometer. The speed of the apparatus was controlled with the help of a vibration reed tachometer. This was calibrated from time to time at the two speeds (23 cycles and 17 cycles per second) chosen for operation and was found to be approximately correct. A high order of accuracy in the control of the speed is not essential since the currents in the calibration current sheet, in the balancing current sheet, and that produced by the oscillating cylinder should all of them be closely proportional to the speed.

28. Method of making the calibration runs. We must now turn our attention to the actual calibration runs made, on the apparatus as a whole, to determine the slide wire readings and phase settings which corresponded to

[16] For the purposes of this article we are taking the "probable error" as the mean deviation divided by the square root of the number of observations. We give its values merely as a rough indication of the reproducibility of certain measurements.

a given flow of alternating current through the calibration current sheet. In making these runs, one operator controlled the speed of rotation of the apparatus and the other adjusted the phase and slide wire so as to secure the best balance. At the end of each run the apparatus was stopped, the slide wire was read, and the phase readings taken with the calibration inductor parallel to the earth's field in both the "up" and "down" positions.

The runs were taken in sets of five each. At the beginning of each set the slip rings for the earth inductors were cleaned with kerosene and the resistances R_1, R_2, and R_3 were adjusted so as to give the desired value to the total resistance in circuit No. 1, and so as to make the resistances in circuits No. 2 and No. 3, 250.0 and 100.0 ohms respectively. During the course of the set, the slip rings were again cleaned with kerosene, in general before the 3rd and 5th runs, and at the end of the set the three resistances were again measured. Usually the resistances at the end of the run agreed with those at the beginning within the accuracy of measurement and in only one case was there a difference as great as 0.2%. The resistances of circuits Nos. 1 and 2 were always measured with the apparatus running, so as to make sure that the brushes and slip rings were functioning properly, and the resistance in circuit No. 3 was always measured with a slide wire reading of zero.

29. Data obtained in the calibration runs. Calibration runs were made with the four different possible methods of connecting the coils and amplifier and with different resistances in circuit No. 1 containing the calibration current sheet. The average results, together with the probable errors[16] of those averages which are later used, are given below in Table II.

TABLE II. *Results of the calibration runs.*

Connection	Resistance in circuit 1 (ohms)	Number of runs made	Average phase (degrees)	P.E.	Average slide wire	P.E.	Current × 10^7 ÷ Slide wire (amperes)
	At 23 cycles per second						
A	10,000	20	201.6	0.5	83.3	0.8	4.313
A	5,000	5	201.3		172.0		
A	2,500	5	201.6		349.8		
A	1,000	5	201.6		878.6		
B	10,000	20	213.2	0.7	90.2	1.3	3.983
B	1,000	5	213.6		935.0		
C	10,000	20	156.9	0.5	142.2	2.3	2.526
C	1,000	5	156.1		1538.8		
D	10,000	20	150.3	0.2	135.7	0.6	2.647
D	1,000	5	149.9		1429.4		
	At 17 cycles per second						
D	10,000	20	164.1	0.4	118.8	0.6	2.235

The phase readings were taken to the nearest degree when 10,000 ohms was in circuit No.1, and to the nearest half-degree when the smaller resistances were in circuit No. 1, since the larger calibration currents then permitted a more precise setting of the balance. The value of the phase reading assigned to a given individual run was obtained by averaging the readings for the "up" and "down" positions of the calibration inductor after correcting the latter by 180°. The slide wire readings were made to the nearest 0.0001 part of the total length of wire, and the average results in Table II are given to one more figure than read.

Before discussing the use of the calibration runs in interpreting the effect runs, it will be desirable to show that the nature of the results obtained in the calibration runs is what would be expected from the nature of the set up, since this will increase our confidence in the method of measurement adopted.

In the first place, it will be noted from the table that, with a given method of connection, the slide wire readings for balance are not quite inversely proportional to the resistances in circuit No. 1, and hence not quite proportional to the currents in the calibration current sheet. This arises from the fact that the balancing current which flows through the balancing current sheet is itself not quite proportional to the slide wire setting, owing to the unavoidable nature of the design of circuits Nos. 2 and 3. By calculating from the various resistances involved, the currents which actually corresponded to the different slide wire settings, it was found for each method of connection that the balancing currents were themselves actually closely proportional to the calibrating currents as would be expected.

In the second place, it will be noted that the phase settings and the ratios between the balancing and calibration currents necessary to obtain balance vary as we go from one method of connection to another. This presumably arises from a lack of symmetry in the detecting circuit, and as a matter of fact calculations were made which indicated that the presence of a reasonable amount of capacity from the coils to ground, combined with the fact that the filament of the amplifier was itself grounded, would lead to results of the kind actually obtained. Furthermore, as a test of this explanation, it was found that the actual introduction of additional capacity between the ground and the wire connecting the two coils, led approximately to the expected change in the conditions of balance.

Finally, it will be noted that a change in frequency from 23 to 17 cycles had the effect of lowering the slide wire setting and retarding the phase setting necessary for balance. Calculation also showed, however, that behaviour of this kind could be expected from the lack of symmetry mentioned above. Hence in general the method of measurement appears to have behaved in a reliable manner.

30. Calculation of results of the calibration runs. We must now consider the use of the calibration runs in interpreting the effect runs. Since, as mentioned above, the slide wire settings were necessarily not quite proportional to the calibration currents the main calibration runs, upon which subsequent calculations are based, were carried out with a resistance of 10,000 ohms in circuit No. 1 which gave slide wire readings near enough to those obtained in the effect runs so that the lack of strict proportionality could be neglected.[17]

With 10,000 ohms in the circuit and a speed of revolution of 23 cycles per second, the *average* current flowing in the calibration circuit will evidently have the value

$$I = \frac{4\times 23\times \pi(4.963)^2\times 132\times 0.5258\times 10^{-8}}{10,000} = 4.941\times 10^{-7} \text{ amperes}$$

and at 17 cycles, the value

$$I = (17/23)\times 4.941\times 10^{-7} = 3.652\times 10^{-7} \text{ amperes}$$

where 4.963 cm is the average radius and 132 the number of turns of the calibration inductor as given in Section 11, 0.5258 is the strength of the earth's field in gauss as given in Section 5, and the total impedance is taken equal to the resistance since it was shown by calculation that the self inductances of the calibration inductor and current sheet, and the mutual inductances of the current sheet with the effect coil and its copper shielding produced a negligible effect on the primary current.[18]

The currents thus calculated are those which passed through the 73 turns of the calibration current sheet when the calibration runs were made, and we must now calculate the currents which would give the same effect when passed through one turn having the dimensions of our cylinder. These equivalent currents will evidently have at 23 cycles the value

$$I = 73\times 4.941\times 10^{-7}\times (10.120)^2/(10.141)^2 = 3.592\times 10^{-5} \text{ amperes}$$

and at 17 cycles the value

$$I = 73\times 3.652\times 10^{-7}\times (10.120)^2/(10.141)^2 = 2.656\times 10^{-5} \text{ amperes}$$

where 10.120 is the diameter of the calibration current sheet and 10.141 the diameter of the cylinder, as given in sections 8 and 13, and the slight difference in the lengths of the current sheet and cylinder can evidently be neglected.[19]

[17] If correction were made for the lack of strict proportionality between slide wire setting and the current in the balancing current sheet, it would affect the lowest slide wire reading obtained in the effect runs by less than 0.2 percent, and would affect the other slide wire readings by an even smaller amount.

[18] Calculation showed in the case of the effect coil that the resistance was too high to get any back action of secondary current on the primary, and in the case of the copper shielding that the mutual inductance was too low to get back action. This latter was roughly checked experimentally, since it was found that the sensitivity of the apparatus was not decreased when the original inner shield of the coil, which did not permit the circumferential flow of electricity, was replaced by a continuous tube.

[19] The two lengths were 23.18 cm and 23.42 cm respectively, and the coupling between primary and secondary is not sensitive to a small motion of the primary along its axis. In addition no allowance has in any case been made for the lack of uniformity in the surface density of the charge on the cylinder.

The ratios between these values of the current and the slide wire readings obtained at balance may now be used to interpret the effect runs. These ratios have been calculated and are given in the last column of Table II.

V. The Effect Runs

31. Method of making the effect runs. The effect runs were made in sets of five runs each, the slip rings being cleaned and the resistances in circuits Nos. 2 and 3 being adjusted and measured as described in the case of the calibration runs. Usually the resistances at the end of the set agreed with those at the beginning within the accuracy of measurement and in only two cases was there a difference as great as 0.2 percent.

In order to correct for a small residual effect which was present when the cylinder was oscillating without charge, the first, third and fifth runs of each set were blank runs made with the cylinder uncharged by connection to ground, while the second and fourth runs were made with the cylinder charged to the desired potential by connection with our source of high potential as described in Section 9. In making the second and fourth runs, the apparatus was first started up without any charge on the cylinder and without making any change in the previous slide wire or phase setting, in order to see if the residual effect had remained constant. This was always found to be approximately the case, and the potential was then thrown on the cylinder with the apparatus rotating and the new balance obtained. Thus the first and second runs, and the third and fourth runs of the set each formed a pair from which a corrected value of the slide wire and phase corresponding to the effect of putting on the charge could be obtained. The fifth run of the set was considered merely as an extra blank which showed that no sudden change in the residual effect had occurred.

At the end of each run before stopping the apparatus, the potential, if any, on the cylinder was read on the electrostatic voltmeter, and the apparatus was then stopped, the slide wire was read, and the phase readings taken corresponding to the center of the stroke moving both east and west. Two sets of runs were made without changing the potential, thus giving four values of the effect corresponding to each set of conditions.

For the complete interpretation of the results obtained in the effect runs it was also necessary to measure the amplitude of the motion of the cylinder in order to calculate the current which should have been produced by the motion of the charge which it carried, and since the galvanometer was tuned to the frequency of rotation, what is really wanted is the amplitude of the first harmonic in the Fourier analysis of its motion. On account of the flexibility of the shaft which drove the cylinder and probably also to some extent on account of play in the bearings and backlash in the gears, the total dynamic amplitude of motion obtained when the apparatus was actually running was greater than the static amplitude obtained by turning the apparatus over by hand. Since a graphical analysis of the slow motion of the cylinder when turned over by hand showed that the amplitude of its first harmonic was only very slightly less than the total static amplitude, it seems reasonable to expect that the actually desired amplitude of the first harmonic of the running motion will be somewhere in between what we have called the static and the dynamic amplitudes.

The static amplitude was carefully measured near the beginning of the effect runs and found to have the value 192.8°. The dynamic amplitude was measured by observing the extreme positions, while running, of a small white pointer which was temporarily attached to the upper end of the cylinder in such a way as to move back and forth over a protractor scale on the top of the coil. At 23 cycles, eight readings of the dynamic amplitude were taken during the course of the runs and eight more at the close of these runs, the final average being 234.0° with a probable error of 0.1°. At 17 cycles, eight readings were taken at the close of the runs with an average of 216.7° and a probable error of 0.1°.

32. Method of treating the data from the effect runs. The method of treating the data from the effect runs is shown by Tables III and IV, which contain

TABLE III. *Effect Runs Nos. 1–10.*

Run	Potential on cylinder (volts)	Dev.	Phase (degrees) E	Phase (degrees) W	Phase (aver.)	Phase (corr.)	Dev.	Slide wire	Slide wire (corr.)	Dev.
1	0		99	93	96			25		
2	−2110	8	344	338	341	330.8	0.5	113	126	0.5
3	0		97	90	93.5			26		
4	−2130	12	346	340	343	331.0	0.3	106	118	7.5
5	0		113	108	110.5			20		
6	0		113	107	110			14		
7	−2110	8	340	334	337	332.2	0.9	117	127	1.5
8	0		88	82	85			23		
9	−2120	2	344	338	341	331.0	0.3	123	131	5.5
10	0		108	102	105			12		
Aver.	−2118	8				331.3	0.5		125.5	3.8
Corr.	−2154									

TABLE IV. *Effect Runs Nos. 11–20.*

Run	Potential on cylinder (volts)	Dev.	Phase (degrees) E	Phase (degrees) W	Phase (aver.)	Phase (corr.)	Dev.	Slide wire	Slide wire (corr.)	Dev.
11	0		87	81	84			20		
12	+2290	6	147	141	144	152.5	1.6	126	117	7.5
13	0		77	71	74			24		
14	+2285	1	143	137	140	150.3	0.6	131	123	1.5
15	0		47	41	44			18		
16	0		53	48	50.5			20		
17	+2285	1	144	138	141	150.0	0.9	126	127	2.5
18	0		69	62	65.5			23		
19	+2275	9	144	138	141	151.0	0.1	135	131	6.5
20	0		57	51	54			13		
Aver.	+2284	4				150.9	0.8		124.5	4.5
Corr.	+2319									

the results from our first twenty runs made with approximately equal negative and positive potentials on the cylinder, and by Figs. 5 and 6 which illustrate the method of correcting for the residual effect.

The potential on the cylinder with a given adjustment of our high potential apparatus remained nearly enough constant so that the average value could be taken as applying to all the runs involved. The potentials as read are given in the second column of the tables, the average value, together with the corrected value obtained by applying the results of the calibration of the voltmeter, being given at the bottom of the column, and the deviations from the mean being given in the next column.

The fourth and fifth columns of the tables give the phase reading at the center of the stroke moving east and that at the center of the stroke moving west corrected by 180°. As already mentioned in Section 11, these two figures differed by an approximately constant amount of about 6°, owing

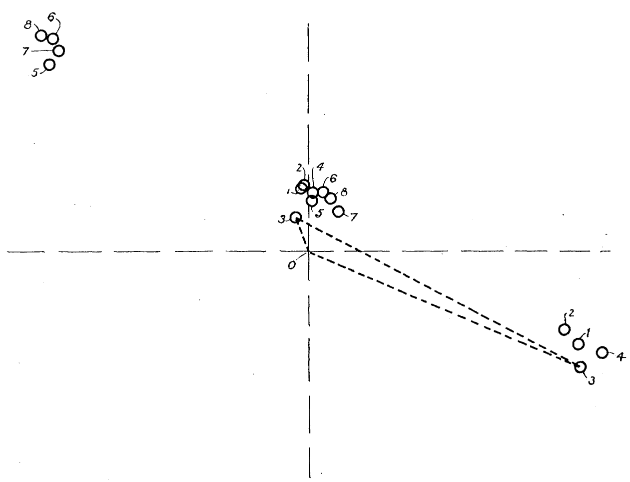

Fig. 5

to the geometry of the driving mechanism. Their average value is given in the sixth column of the table. The slide wire readings were read to the nearest 0.0001 part of the total length and are given to the number of figures read in the next but one to the last column of the tables.

The corrected values for the phase and slide wire readings, corresponding solely to the added effect produced by the charge on the cylinder, were obtained by subtracting the vectors which represented the phase and slide wire readings for the blank runs, from those which represented the results in the corresponding runs with the cylinder charged. This was done graphically as shown in Fig. 5, where the data have been plotted for the runs given in Tables III and IV.

The individual points in this plot are the termini of vectors plotted from the origin with a length equal to the slide wire reading and at an angle which equals that of the phase reading. The group of points just above the origin represent the readings obtained in the blank runs, those in the upper left hand quadrant represent the readings obtained with the positive voltage on the cylinder, and those in the lower right hand quadrant with the negative voltage.

The points in the plot are numbered in the order in which the runs were made, the pair of points, belonging to an individual blank run and the associated following run in which the cylinder was charged, being given the same number. The vectors which connect the two points of these pairs then represent the corrected slide wire readings and phase angles which correspond to the effect of putting the charge on the cylinder. Thus the shorter vector 03 in Fig. 5 represents the result obtained in the blank run No. 5 in Table III, the longer vector 03 represents run No. 6, and the vector 33 the corrected result.

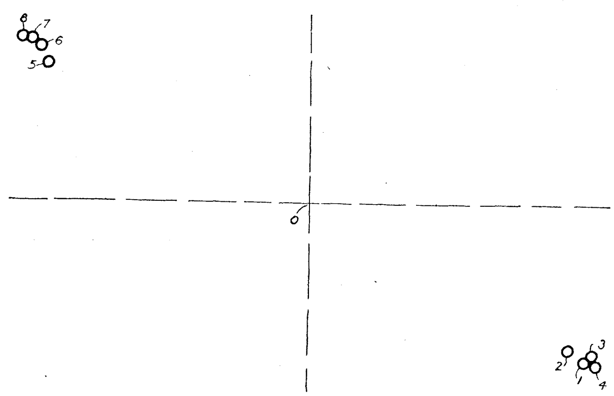

FIG. 6

The corrected results obtained in this way are shown in Fig. 6. plotted directly from the origin, the points being numbered as before. The method of correcting assumes that the residual effect given by the uncharged cylinder remains present in the same amount after the potential is applied. This is, of course, a reasonable assumption to make, especially as it seems plausible to suppose that the residual effect was due to small currents induced in the apparatus by motion through the earth's magnetic field. It will be seen by comparing Fig. 6 with Fig. 5, that the application of the correction gives a considerable improvement in the appearance of the results.

33. The results of the effect runs. With the methods described above, we made a total of 64 successful effect runs together with the blank runs which went with them. In addition three runs were tried at a potential so high that successful balance could not be made because of too frequent sparking, and two runs were tried at a moderate potential at a time, however, when the cylinder had recently been replaced in position, which appeared to produce unsteady conditions until the air gap between cylinder and coil had been "cleaned up" by treatment with higher potential.

The 64 runs consisted of 16 groups of 4 runs each, made under different sets of conditions. The average results for the 16 different sets of conditions are given in Table V in the order in which they were actually obtained. The first column gives the method of connecting the coils and amplifier as described in Section 14. The second column specifies the neutral position around which the cylinder was oscillated, by giving the direction of the longitudinal slit in the copper foil which coated the cylinder. The third and fourth columns give the average potential on the cylinder together with its average deviation. The next three columns give the average value of the phase setting corresponding to balance, together with its average deviation and the "expected" value of the phase based on the results of the calibration runs. The last columns give the average slide wire corresponding to balance, its average deviation, the calculated equivalent current flowing in a sheet having the dimensions of the cylinder which would produce this slide wire reading, and the ratio of this equivalent current to the potential of the cylinder.

34. Discussion of results obtained in the effect runs. The different conditions under which the foregoing effect runs were carried out were designed for the purpose of testing the results produced: *a*, by changing the potential on the cylinder; *b*, by changing the method of connecting the coils and amplifier; *c*, by changing the neutral position around which the cylinder was oscillated; and *d*, by changing the speed of oscillation of the apparatus. The first runs were made at the highest speed at which the apparatus would run with reasonable smoothness, using a moderate potential on the cylinder, and using connections *D*, which had given approximately the expected results in the preliminary runs without the final unjointed shield inside the coil and which had happened to give the most concordant results in the calibration runs. The potentials were then varied to as high values as could be used without too much sparking over between cylinder and shield and to as low values as would still give reasonable accuracy. This was followed by changes in the method of connection, by changes in the neutral position of the cylinder, and finally by change to an appreciably lower speed of oscillation which, however, was still high enough to give reasonable accuracy.

We must now examine the results thus obtained, as given in Table V, to see if they agree with what would be theoretically predicted. If so, the phase settings obtained at balance should agree with what would be predicted on the basis of the calibration runs; the effective current produced by the moving cylinder should be proportional to the potential of the cylinder, and to the speed of the peripheral motion; and the absolute magnitude of this effective current should agree with that calculated from the motion of the cylinder and the charge on it.

In order to compare the phase readings obtained in the effect and calibration runs, it was noted on the one hand that the average phase readings obtained in the effect runs corresponded to a position in which the cylinder was at the center of its motion moving clockwise viewed from above, and it was noted on the other hand, from a check made on the direction of motion of the calibration inductor through the earth's field, that the average phase readings obtained in the calibration runs corresponded to a position in which the current in the calibration current sheet was at a maximum and flowing clockwise viewed from above. Hence the average phase readings obtained in the effect runs with a positive charge on the cylinder are to be compared directly with those obtained in the corresponding calibration runs, and the readings obtained with a negative charge are to be compared with those obtained in the calibration runs after correcting the latter by 180°.

TABLE V. *Results of the effect runs.*

Connection	Orientation of cylinder	Potential of cylinder (volts)	A.D.	Average phase (degrees)	A.D.	Phase obtained in calibration (degrees)	Average slide wire	A.D.	Equivalent Current $\times 10^5$ (amperes)	Current ÷ Potential $\times 10^{10}$
				At 23 cycles per second						
D	North	−2154	8	331.3	0.5	330.3	125.5	3.8	3.322	154
D	North	+2319	4	150.9	0.8	150.3	124.5	4.5	3.296	142
D	North	+1474	2	148.0	2.8	150.3	82.8	5.8	2.192	149
D	North	−1533	2	335.1	2.2	330.3	81.8	6.3	2.166	141
D	North	−2639	10	337.1	0.7	330.3	145.5	2.0	3.852	146
D	North	+2665	19	156.1	2.5	150.3	156.0	2.0	4.130	155
D	North	+1886	6	154.6	2.8	150.3	111.5	8.8	2.951	156
D	North	−1781	4	334.6	2.4	330.3	98.0	2.5	2.594	146
A	North	−2224	2	30.8	2.4	21.6	77.8	2.2	3.356	151
B	North	−2234	2	32.8	1.5	33.2	81.8	0.9	3.258	146
C	North	−2219	2	341.2	0.9	336.9	130.0	6.5	3.284	148
D	East	−2220	15	334.4	1.8	330.3	121.3	4.3	3.210	145
D	South	−2218	9	333.8	1.8	330.3	123.3	5.3	3.263	147
D	West	−2214	2	333.2	1.3	330.3	118.5	3.0	3.138	142
				At 17 cycles per second						
D	North	−2204	4	341.5	2.5	344.1	103.3	7.3	(3.371)	(153)
D	North	+2266	4	164.6	2.8	164.1	107.0	2.5	(3.493)	(154)
									Average	148.4

To facilitate the comparison, the figures thus obtained from the calibration runs have been entered in the seventh column of Table V, and their approximate agreement with those obtained in the effect runs will be noted. Attention is particularly called to the expected change in phase by 180° produced by a reversal in the sign of the charge on the cylinder, to the expected change in phase which accompanied a change in connections, to the expected lack of any change from a mere change in the neutral position around which the cylinder was oscillated, and to the expected change in phase which accompanied the change in frequency.

To test the expected proportionality between the effective current produced by the moving cylinder and the potential on the cylinder, we have used the figures in the last column of Table II, to calculate the equivalent currents flowing in a sheet having the dimensions of the cylinder which would have produced the same slide wire readings as were obtained in the effect runs, and have entered these values of equivalent current in the next to the last column of Table V. These values have then been plotted against the potentials in Fig. 7, where they will be seen to fall as well as could be expected on a straight line. In addition the ratios of equivalent current to potential have been calculated and entered in the last column of Table V, and will also be seen to be as constant as could be expected.

In order to compare the two results obtained at 17 cycles with those obtained at 23 cycles, we have multiplied the values which they gave for equivalent current by the ratio of the two frequencies 23/17 and by the ratio of the two dynamic amplitudes[20] 234.0/216.7. The corrected values of equivalent current thus obtained have been inserted in brackets in the next to the last column of Table V, and are included in the plot in Fig. 7, and have also been used for calculating the ratios of equivalent current to potential which are given in brackets in the last column of Table V. The agreement thus shown between the values of equivalent current obtained at 17 to 23 cycles furnishes a satisfactory check of the theoretical dependence of the current on the velocity of the cylinder.

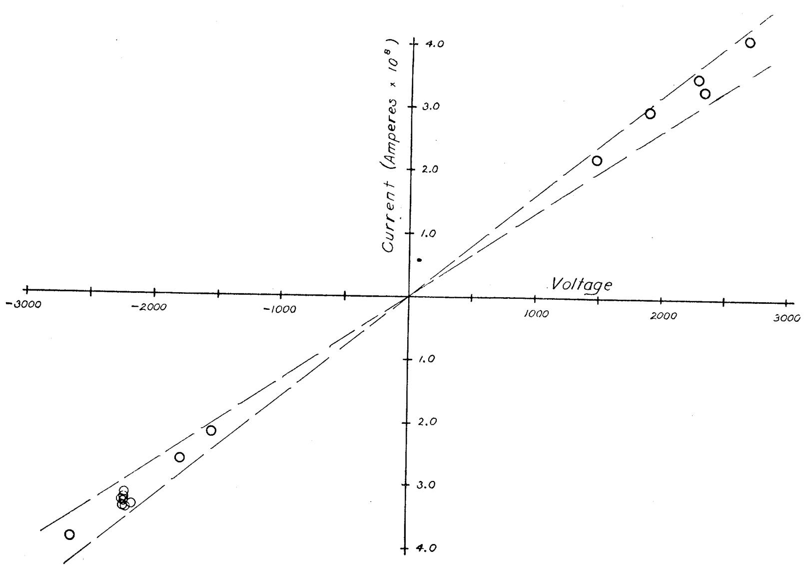

FIG. 7

Finally to test the absolute magnitudes of the equivalent currents we must calculate the theoretical current which would be produced by the motion of the charged cylinder, and in doing this shall use both the static and dynamic amplitudes, since, as stated in Section 31, it seems reasonable to expect that the first harmonic of the actual running motion of the cylinder would have an amplitude lying between these two. For the *average* value[21] of the theoretical current which would be produced by the moving cylinder when charged to a potential of one volt, we may evidently write for the two cases in question,

$$I=2\times 23\times\frac{192.8}{360.0}\times 5.385\times 10^{-10}=132.6\times 10^{-10}\text{ amperes}$$

[20] This assumes that the ratio of the total amplitudes of the two motions is the same as he ratio of the amplitudes of their first harmonics, which may not be exactly true.

[21] Note that the calibration runs were also calculated for the *average* value of the current

and

$$I = 2 \times 23 \times \frac{234.0}{360.0} \times 5.385 \times 10^{-10} = 161.0 \times 10^{-10} \text{ amperes}$$

where the values of the static and dynamic amplitudes 192.8 and 234.0 degrees are those given in Section 31, and 5.385×10^{-10} is the capacity of the cylinder in farads as given in Section 25.

Comparing now with the values in the last column of Table V, it will be at once seen that the experimental values of the equivalent currents do fall as expected between the two theoretical values, calculated using the static and dynamic amplitudes for the motion of the cylinder. This is also shown in the plot in Fig. 7, where the two diagonal lines through the origin have been drawn so as to give the two calculated relations between current and potential.

35. Conclusion. In view of all the foregoing correspondences, the conclusion can now be drawn that the experiments described above do give a very satisfactory demonstration of the equivalence of a mechanically oscillated electrostatic charge to an ordinary alternating current. Attention is specially called to the consistency of the effects obtained with different methods of connecting the coils and with different neutral positions for the cylinder, since this furnishes evidence for the elimination of accidental effects which were present before the apparatus was put into its final form. In summary, it can be said, that qualitatively the effects obtained from the oscillatory motion of our charged cylinder were in entire agreement with theory, and that quantitatively the correspondence between experimental and theoretical values was good and as close as could be expected.[22]

We desire to express our appreciation to the California Institute for the support which it gave to this expensive investigation, and to give our thanks to Mr. Julius Pearson and his assistants for their skill in the construction of apparatus.

[22] Using the language in which the results of the original Rowland experiment were expressed, this quantitative check would have been expressed as furnishing evidence that the ratio of the electromagnetic unit of current to the electrostatic unit of current is equal to the velocity of light. This method of expressing the quantitative check has, however, not occurred in the present article since we have from the start given the capacity and potential of the cylinder in farads and volts instead of in electrostatic units.

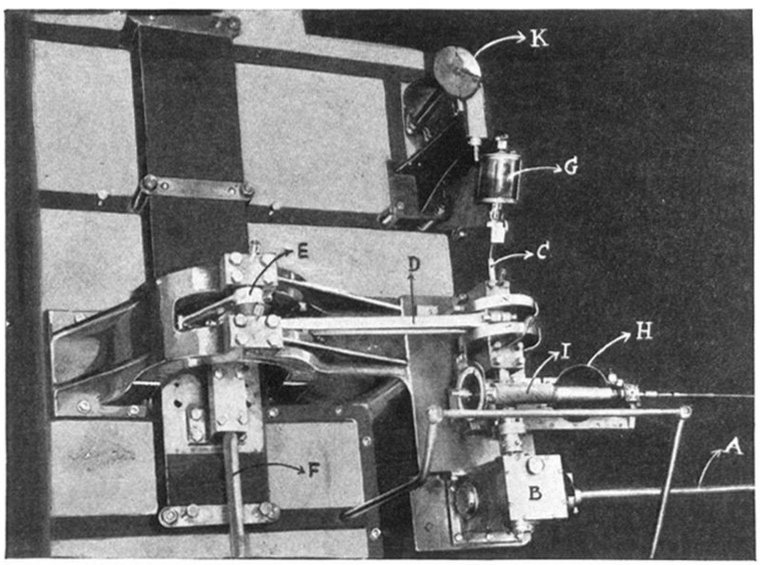

FIG. 2

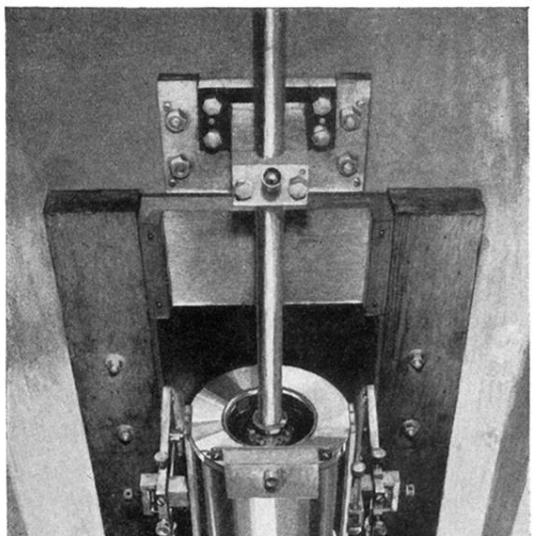

Fig. 3