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EARTHQUAKE ENGINEERING RESEARCH LABORATORY

DIGITAL CALCULATION OF RESPONSE SPECTRA FROM STRONG-MOTION EARTHQUAKE RECORDS

by

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ABSTRACT

This report presents a numerical method for computing response spectra from strong-motion earthquake records. The method is based on the exact solution to the governing differential equation and gives a three to four-fold saving in computing time compared to a third order Runge-Kutta method of comparable accuracy. An analysis was made of the errors introduced at various stages in the calculation of spectra so that allowable errors could be prescribed for the numerical integration. Using the proposed method of computing or other methods of comparable accuracy, example calculations show that the errors introduced by the numerical procedures are much less than the errors inherent in the digitization of the acceleration record.

Included as appendices to the report are computer programs in Fortran IV, with instructions for their use, for computing spectra, for correction of the baseline of the digitized record, and for the computation of ground velocity and displacement.

LIST OF SYMBOLS

Symbol	Explanation or Definition
$a(t)$	ground acceleration at time t;
A	2×2 matrix;
B	2×2 matrix;
c	damping coefficient for a simple oscillator;
C_1, C_2	constants;
e_A	percentage amplitude distortion in recording;
e_P	phase distortion in recording (degrees);
k	stiffness coefficient for a simple oscillator;
m	mass of the oscillator;
p	natural frequency of the element of an accelerograph;
S_a	acceleration spectrum value;
S_d	displacement spectrum value;
S_v	velocity spectrum value;
t	time;
T	natural period of a simple oscillator;
x	relative displacement of a simple oscillator;
y	relative displacement of the element of an accelerograph;
$\ddot{z}$	absolute acceleration of the mass of a simple oscillator;
α	input frequency to the accelerograph;
β	fraction of critical damping of a simple oscillator;
ω	natural frequency of a simple oscillator;
ξ	fraction of initial damping of the element of an accelerograph;
Δt_i	interval of digitization $t_{i+1} - t_i$
$\Delta\tau$	interval of integration; and
$(\Delta\tau)_m$	maximum interval of integration.

INTRODUCTION

Since their introduction in the 1930's and 1940's,[1,2,3] response spectra of strong-motion earthquakes have proved useful and informative in problems of design and analysis of structures subjected to strong earthquake motions. The spectra, calculated from the recorded ground acceleration, are plots of the maximum response to the earthquake of a simple oscillator over a range of values of its natural period and damping. These curves provide a description of the frequency characteristics of the ground motion and give the maximum response of simple structures to the earthquake. By superposition of different modes of response, spectrum techniques can be applied to the design and analysis of complex structures such as buildings and dams. Used in this manner the spectrum technique represents an approach intermediate between a design based on static loads and a complete integration of the equations of motion of the complex structure.

Strong-motion earthquake records have been obtained infrequently in the past and the reduction to digital form, or equivalent analog form, and subsequent calculation of spectra has been performed on more or less an individual basis. However, in recent years the number of strong-motion instruments placed in the seismic regions of the world, particularly California, Mexico and Japan, has increased to the point where a major earthquake in these areas will generate a large number of records. The potential volume of the data and the development of the tape-recording accelerograph indicate clearly that rapid and automated

data processing and spectrum calculation procedures are needed. In an effort to fulfill part of this need, this report presents a rapid, accurate method for computation of response spectra of strong-motion earthquakes and includes computer programs in Fortran IV for such calculations.

Response spectra were first obtained by Biot[1], using a direct mechanical analog and later by Housner et al.[4,5] using electric analog techniques. The availability of digital computers and a progressive increase in the speed of digital computation in recent years has led to an increasing use of digital computers in the calculation of spectra. The extensive application of the response spectrum to problems in earthquake engineering has sustained interest in methods of calculating spectra and has raised questions regarding accuracy, reproducibility and economy in such calculations.[6,7,8,9]

The digital computation of spectra requires the repeated numerical solution of the response of a simple oscillator to a component of recorded ground acceleration. The motion of the oscillator is described by a second order, linear, inhomogeneous differential equation, and if a digital description of the earthquake record is available, the response can be obtained by numerical integration. Several numerical integration techniques have been used for spectra calculations, for example, the third order Runge-Kutta scheme of integration, has been preferred by some investigators due to its accuracy, long range stability, self-starting feature and because it can be adapted easily to cases in which the excitation is not defined at regular intervals.[11,15] The truncation error in this method is proportional to $(\Delta\tau)^4$, where $\Delta\tau$

is the normalized interval of integration. Thus by a suitable choice of $\Delta\tau$ the calculations can be performed to an acceptable degree of accuracy.

An alternative approach to spectra calculations is based on obtaining the exact analytical solution to the governing differential equation for the successive linear segments of the excitation, then using this solution to compute the response at discrete time intervals in a purely arithmetical way.[6, 10] This method does not introduce numerical approximations in the integration other than those inherent in round-off, so in this sense it is an exact method.

In this report this exact method is used to develop a computation technique which leads to a three to four-fold saving in computing time compared to a third order Runge-Kutta method of comparable accuracy. If the earthquake record is digitized at equal time intervals, the proposed scheme gives spectral values which are exact in the sense mentioned above. If the record is digitized at arbitrary time intervals, however, it is necessary to introduce an approximation into the digitization. An analysis of the errors introduced at various stages in the preparation of response spectra and the results of digitization experiments show that the additional numerical approximations are not detrimental. In addition to the development of these numerical integration techniques, the report includes as appendices computer programs in Fortran IV for the calculation of the spectra, for baseline correction to the digitized record and for computation of ground velocity and displacement.

FORMULATION OF THE EXACT METHOD

Response spectra are plots of the maximum response of a simple oscillator to a component of the ground acceleration, plotted as functions of the natural period and damping of the oscillator. The digital computation of spectra requires the step-wise solution of the equations of motion of the oscillator and monitoring of the maximum values of the response parameters at each step of the integration.

Consider a viscously damped, simple oscillator subjected to the base acceleration $a(t)$ as shown in Fig. 1. The equation of motion of the oscillator is given by

$$m\ddot{x} + c\dot{x} + kx = -ma(t) \tag{2.1}$$

in which m = mass of the oscillator; c = damping coefficient, and k = stiffness of the restoring elements. Dividing by m and setting $\omega = \sqrt{k/m}$ and $c = 2m\omega\beta$ in Eq. 2.1 gives

$$\ddot{x} + 2\beta\omega\dot{x} + \omega^2 x = -a(t) \tag{2.2}$$

in which β = the fraction of critical damping, and ω = the natural frequency of vibrations of the oscillator.

Assuming that $a(t)$ may be approximated by a segmentally linear function as shown in Fig. 2, Eq. 2.2. may be written as

$$\ddot{x} + 2\beta\omega\dot{x} + \omega^2 x = -a_i - \frac{\Delta a_i}{\Delta t_i}(t - t_i); \quad t_i \leq t \leq t_{i+1} \tag{2.3}$$

with

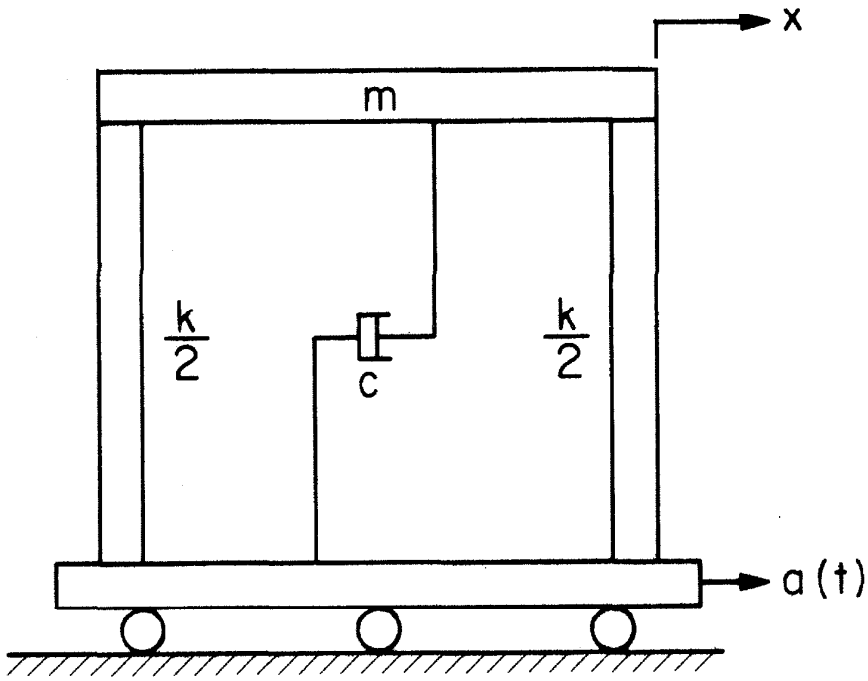

Fig. 1. -A SIMPLE OSCILLATOR

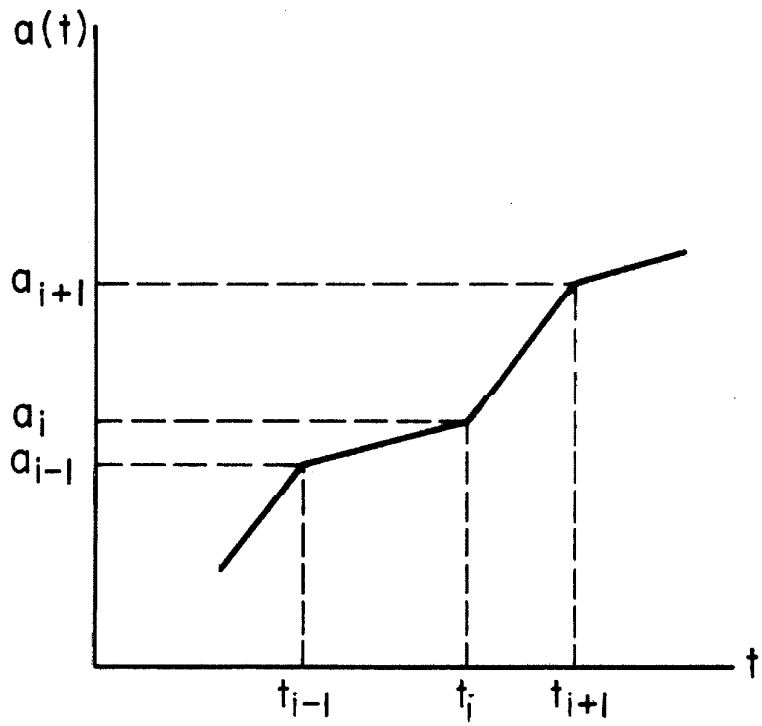

Fig. 2. - IDEALIZED BASE ACCELERATION

$$\Delta t_i = t_{i+1} - t_i$$
$$\Delta a_i = a_{i+1} - a_i \tag{2.4}$$

The solution of Eq. 2.3, for $t_i \leq t \leq t_{i+1}$ is given by

$$x = e^{-\beta\omega(t-t_i)}\left[C_1 \sin \omega\sqrt{1-\beta^2}\,(t-t_i) + C_2 \cos \omega\sqrt{1-\beta^2}\,(t-t_i)\right]$$
$$- \frac{a_i}{\omega^2} + \frac{2\beta}{\omega^3}\frac{\Delta a_i}{\Delta t_i} - \frac{1}{\omega^2}\frac{\Delta a_i}{\Delta t_i}(t - t_i) \tag{2.5}$$

in which C_1 and C_2 are constants of integration. Setting $x = x_i$ and $\dot{x} = \dot{x}_i$ at $t = t_i$ and solving for C_1 and C_2, it can be shown that

$$C_1 = \frac{1}{\omega\sqrt{1-\beta^2}}\left(\beta\omega x_i + \dot{x}_i - \frac{2\beta^2-1}{\omega^2}\frac{\Delta a_i}{\Delta t_i} + \frac{\beta}{\omega}a_i\right) \tag{2.6a}$$

$$C_2 = x_i - \frac{2\beta}{\omega^3}\frac{\Delta a_i}{\Delta t_i} + \frac{a_i}{\omega^2} \tag{2.6b}$$

Substitution of these values of C_1 and C_2 into Eq. 2.5 will show that x and $\dot{x}$ at $t = t_{i+1}$ are given by

$$\bar{x}_{i+1} = A(\beta,\omega,\Delta t_i)\,\bar{x}_i + B(\beta,\omega,\Delta t_i)\,\bar{a}_i \tag{2.7a}$$

in which

$$\bar{x}_i = \begin{Bmatrix} x_i \\ \dot{x}_i \end{Bmatrix} \qquad \bar{a}_i = \begin{Bmatrix} a_i \\ a_{i+1} \end{Bmatrix} \tag{2.7b}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{2.7c}$$

The elements of matrices A and B are given by

$$\left.\begin{aligned}
a_{11} &= e^{-\beta\omega\Delta t_i}\left(\frac{\beta}{\sqrt{1-\beta^2}}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i + \cos\omega\sqrt{1-\beta^2}\,\Delta t_i\right)\\
a_{12} &= \frac{e^{-\beta\omega\Delta t_i}}{\omega\sqrt{1-\beta^2}}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i\\
a_{21} &= -\frac{\omega}{\sqrt{1-\beta^2}}\,e^{-\beta\omega\Delta t_i}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i\\
a_{22} &= e^{-\beta\omega\Delta t_i}\left(\cos\omega\sqrt{1-\beta^2}\,\Delta t_i - \frac{\beta}{\sqrt{1-\beta^2}}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i\right)
\end{aligned}\right\}\quad(2.7d)$$

$$\left.\begin{aligned}
b_{11} &= e^{-\beta\omega\Delta t_i}\left[\left(\frac{2\beta^2-1}{\omega^2\Delta t_i}+\frac{\beta}{\omega}\right)\frac{\sin\omega\sqrt{1-\beta^2}\,\Delta t_i}{\omega\sqrt{1-\beta^2}}+\left(\frac{2\beta}{\omega^3\Delta t_i}+\frac{1}{\omega^2}\right)\cos\omega\sqrt{1-\beta^2}\,\Delta t_i\right]\\
&\quad -\frac{2\beta}{\omega^3\Delta t_i}\\
b_{12} &= -e^{-\beta\omega\Delta t_i}\left[\left(\frac{2\beta^2-1}{\omega^2\Delta t_i}\right)\frac{\sin\omega\sqrt{1-\beta^2}\,\Delta t_i}{\omega\sqrt{1-\beta^2}}+\frac{2\beta}{\omega^3\Delta t_i}\cos\omega\sqrt{1-\beta^2}\,\Delta t_i\right]\\
&\quad -\frac{1}{\omega^2}+\frac{2\beta}{\omega^3\Delta t_i}\\
b_{21} &= e^{-\beta\omega\Delta t_i}\left[\left(\frac{2\beta^2-1}{\omega^2\Delta t_i}+\frac{\beta}{\omega}\right)\left(\cos\omega\sqrt{1-\beta^2}\,\Delta t_i-\frac{\beta}{\sqrt{1-\beta^2}}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i\right)\right.\\
&\quad \left.-\left(\frac{2\beta}{\omega^3\Delta t_i}+\frac{1}{\omega^2}\right)\left(\omega\sqrt{1-\beta^2}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i+\beta\omega\cos\omega\sqrt{1-\beta^2}\,\Delta t_i\right)\right]\\
&\quad +\frac{1}{\omega^2\Delta t_i}
\end{aligned}\right\}\quad(2.7e)$$

$$b_{22} = -e^{-\beta\omega\Delta t_i}\left[\frac{2\beta^2-1}{\omega^2\Delta t_i}\left(\cos\omega\sqrt{1-\beta^2}\,\Delta t_i - \frac{\beta}{\sqrt{1-\beta^2}}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i\right)\right.$$

$$\left. - \frac{2\beta}{\omega^3\Delta t_i}\left(\omega\sqrt{1-\beta^2}\sin\omega\sqrt{1-\beta^2}\,\Delta t_i + \beta\omega\cos\omega\sqrt{1-\beta^2}\,\Delta t_i\right)\right] - \frac{1}{\omega^2\Delta t_i}$$

From Eq. 2.3, it follows that the absolute acceleration, $\ddot{z}_i$, of the mass at time t_i is given by

$$\ddot{z}_i = \ddot{x}_i + a_i = -(2\beta\omega\dot{x}_i + \omega^2 x_i) \tag{2.8}$$

Hence, if the displacement and velocity of the oscillator are known at some time t_o, the state of the oscillator at all subsequent times t_i can be computed exactly by a step-by-step application of Eqs. 2.7 and 2.8. The computational advantage of this approach lies in the fact that A and B depend only on β, ω and Δt_i. β and ω are constant during the calculation of each spectrum value, and if Δt_i is constant also, x_i, $\dot{x}_i$ and $\ddot{z}_i$ can be evaluated by the execution of only ten multiplication operations for each step of integration. Matrices A and B, defined by the rather complicated expressions, Eqs. 2.7d and 2.7e, need to be evaluated only at the beginning of each spectrum calculation. If varying time intervals are used, it is necessary, in general, to compute A and B at each step of integration. However, by rounding the time coordinates of the record, as discussed in the next section, the number of these matrices needed during the calculation can be reduced to only a few. These, too, can be computed at the beginning

of the calculation and called upon when needed, thereby saving computing time.

COMPUTATION OF SPECTRA

Introduction

To construct the response spectra, it is necessary to find the maximum values of the displacement, velocity and acceleration during a given excitation. This is done by computing the response at discrete time intervals through Eqs. 2.7 and 2.8 and monitoring the response parameters to retain the maximum values. This process of obtaining the maximum response is approximate because the response is found only at discrete points, whereas the true maxima may occur between such points. This error, called the error of discretization, is inherent in all numerical procedures, but can be bounded within acceptable limits by a suitable choice of the time interval, as discussed in Appendix I.

Under these conditions, the response spectra are given by

$$S_d(\omega,\beta) = \underset{i=1,N}{\text{Max}} \left[x_i(\omega,\beta) \right] \tag{3.1a}$$

$$S_v(\omega,\beta) = \underset{i=1,N}{\text{Max}} \left[\dot{x}_i(\omega,\beta) \right] \tag{3.1b}$$

$$S_a(\omega,\beta) = \underset{i=1,N}{\text{Max}} \left[\ddot{z}_i(\omega,\beta) \right] \tag{3.1c}$$

in which S_d, S_v and S_a are the spectral values of displacement, velocity, and acceleration respectively, for selected values of damping

and natural frequency; and N is the total number of discrete points at which the response is obtained.

The spectrum values can be obtained by direct application of Eqs. 2.7 and 2.8 to the digitized record. The details of such computation differ, however, depending on whether the earthquake record is digitized at equal or unequal time intervals.

Digitization of Earthquake Records

The digitization of earthquake records is usually carried out in one of the following two ways:

1. Digitization at equal time intervals. - The acceleration values are measured at regular time intervals and are assumed to be connected by straight line segments. If the interval of digitization, Δt, is sufficiently small, this procedure approximates the profile of the actual earthquake record quite closely. The choice of the interval of digitization depends upon the period range of interest and the nature of the earthquake record; commonly used values range from 0.01 to 0.04 seconds. Analytical methods for assessing the error induced by such a sampling process are available. [9, 12] Digitization at equal time intervals will be a virtual necessity for efficient processing of accelerograms recorded on magnetic tape, and in other instances where analog to digital conversion is made automatically.

2. Digitization at unequal time intervals. - The abscissae and ordinates of the points where changes of slope are judged to occur are

measured and the points are again assumed to be connected by straight line segments. This procedure leads to variable time intervals between successive points. The accuracy of this approach is difficult to determine analytically, but should be considerably greater than would be achieved by the same number of sample points at equal intervals. This method is the most convenient for manually controlled digitization techniques.

Baseline Correction

The processes of recording and digitization introduce distortions into the ground motion recorded during an earthquake. Such distortions may be defined by

$$a(t) = \hat{a}(t) + \epsilon(t) \tag{3.2}$$

in which $a(t)$ = the digitized acceleration record, $\hat{a}(t)$ = the true acceleration, and $\epsilon(t)$ = the error introduced during recording and digitization. A. G. Brady[7] has studied the problem of separating $\epsilon(t)$ from $a(t)$, and has concluded that this is accomplished reasonably well by the application of a parabolic baseline correction to the acceleration record with minimization of the mean square of the resulting velocity. Since this operation normally procedes spectral calculations, a computer program in Fortran IV, with instructions, is included as part of this report in Appendix II.

Computation of Ground Displacement and Velocity

Because of its direct application to structural dynamics, it is the ground acceleration that is recorded during strong earthquakes.

However, the ground velocity and displacement also contain valuable information. These motions can be obtained by direct integration of the adjusted ground acceleration record, using the recursion formulae given by Berg and Housner[13]. A computer program for this purpose is included in Appendix III of this report.

Spectra Computation for Equal Time Intervals

Equations 2.7 show that matrices A and B are functions of ω, β and Δt_1. If Δt_1 is constant, that is, the earthquake record is digitized at equal time intervals, these matrices need to be computed only once for each pair of ω and β and the chosen value of $\Delta\tau$, the interval of integration. Also, since the method does not involve truncation error, it is possible to use larger intervals of integration than in other methods. The choice of the interval of integration, $\Delta\tau$, is controlled by the interval of digitization $(\Delta\tau \leq \Delta t)$ and the error due to discretization. The bounds on error due to discretization are examined in Appendix I wherein it is shown that if $\Delta\tau \leq \frac{T}{20}$ (T = Natural Period), the error will be less than 1.2 per cent. Considering other errors in the computation of spectra, this choice of interval of integration seems reasonable. In fact, $\Delta\tau \leq T/10$ may be satisfactory for most purposes.

Since the third order Runge-Kutta scheme of integration often has been used for the digital computation of spectra, the proposed method is compared to it with regard to accuracy and computing time. To make the comparison, undamped velocity spectrum values were obtained for artificial earthquake No. 7[14] using a third order

Runge-Kutta scheme, for four values of the interval of integration: $\Delta\tau \leq T/10$, $T/20$, $T/40$, and $T/80$. The spectrum values also were obtained by the exact method for $\Delta\tau \leq T/10$ and $T/20$. The results of these computations are compiled in Table I, which shows the spectral values and the relative values of the computation time for different cases.

The following conclusions are drawn from the data in Table I:

1. For the third order Runge-Kutta scheme of integration, it is necessary to use $\Delta\tau \leq \frac{T}{80}$ to get accuracy to three significant figures.

2. For the exact scheme of integration, $\Delta\tau \leq T/10$, may be acceptable for most practical purposes. It is seen from the last two columns of Table I that in some cases the spectral values are the same for $\Delta\tau \leq T/10$ and $\Delta\tau \leq T/20$. This occurs when the maximum value has a time coordinate which is a common multiple of both step lengths.

3. For accuracy to three significant figures the exact method, with $\Delta\tau \leq T/20$, is about 3 to 4 times as fast as the third order Runge-Kutta method.

4. Using the IBM 7094 Computer at the California Institute of Technology Computing Center, the computing time for the proposed method with $\Delta\tau \leq T/20$ averages about 1.0 sec execution time per spectrum point for the 30 sec record used to compile Table I.

A Fortran IV computer program using the exact method, with $\Delta\tau \leq T/20$, is included in Appendix IV. The flow chart, instructions for the use of the program and sample programs controlling input and output are given also.

Spectra Computation for Unequal Time Intervals

If the earthquake record is digitized at unequal time intervals, matrices $A(\omega,\beta,\Delta t_i)$ and $B(\omega,\beta,\Delta t_i)$ in Eqs. 2.7 will, in general, change from step to step, making it necessary to compute them anew at each step of integration. From Eqs. 2.7d and 2.7e, which define the elements of these matrices, it is clear that this computation would take a large amount of time; in fact, it would make the method slower than a comparable third order Runge-Kutta scheme. However, if the inherent limitations of the digitized earthquake record are recognized, it is possible to develop an approximate method of satisfactory accuracy which gives a saving in computing time. The essential feature of this method is a rounding of the time coordinates of the original record to a predetermined accuracy. Then, choosing an appropriate maximum interval of integration, $(\Delta\tau)_m$, and subdividing each time interval of the rounded time record into time intervals equal to or less than $(\Delta\tau)_m$, the calculation proceeds much as before, except that $A(\omega,\beta,\Delta\tau_j)$ and $B(\omega,\beta,\Delta\tau_j)$ for four or five different values of $\Delta\tau_j$ must be calculated. This procedure is illustrated in Table II.

After the time record is rounded and subdivided, the values of the ground acceleration at the additional points created by subdivision are computed by linear interpolation from the original record. The modified record now consists of a set of points along the time axis, spaced at a limited number of known intervals, and the corresponding values of the acceleration. The number and size of the time intervals in any record depend on the way in which rounding of the time coordinates

is carried out and the choice of $(\Delta\tau)_m$. For the illustrative example presented in Table II, the possibilities are:

I $\Delta\tau_j$ = 0.04, 0.03, 0.02 and 0.01 (4 time-intervals)

IIa $\Delta\tau_j$ = 0.04, 0.03, 0.02, 0.01 and 0.005 (5 time-intervals)

IIb $\Delta\tau_j$ = 0.04, 0.035, 0.03, 0.025, 0.02, 0.015, 0.01, 0.005 (8 time-intervals)

Therefore, if the original record is modified as indicated above, the exact method needs only a limited number of matrices for each pair of ω and β, and these can be computed before the integration is started. By indexing each possible $\Delta\tau_j$ and the corresponding matrices, the appropriate matrices can be called at each step of integration.

The procedure outlined above requires that the time coordinates of the record to be rounded to a predetermined decimal fraction. Assuming that the original digital description of the earthquake record is exact, this is an approximation which must lead to random errors in the computation of spectra. However, a digital description of an earthquake at unequal time intervals is obtained by manual, or manually controlled, reading of a graphical record and is subject to certain unavoidable errors. If the round-off is carried out in such a way that effects of the approximation so introduced are well within the errors inherent in the process of digitization, the process outlined above will be acceptable.

Digitization Experiment

G. V. Berg[8] and A. G. Brady[7] have studied the errors introduced during the process of digitization and their effect on the computation of spectra. In the absence of gross personal errors or bias, the errors arise primarily from resolution of the scaling device and from the thickness of the trace which makes the choice of points at which a change of slope occurs somewhat arbitrary. To examine these errors an experiment on digitization was conducted by three persons (X, Y, Z) on the Benson-Lehner Data Reducer 099, at the California Institute of Technology. The first 5 seconds on the Taft, California N21E record were digitized independently by X (5 times), Y (2 times) and Z (2 times), at the highest resolution of the Data Reducer for the record used (0.0015 secs and 0.0001 g). In each case points were taken when a change in slope was judged to occur in the record. Taking one record of each person as a standard, corresponding time coordinates on the other records were subtracted from it to obtain what is called here the self-error of digitization. The mean and standard deviation for each case are shown in Table III. The standard deviations shown in Table III indicate that a careful reader will be consistent within 0.004 sec about 70 per cent of the time, with a mean error very close to zero.

To compare the digitized records obtained by different persons, one of the records of X was taken as an overall standard and the records of Y and Z were subtracted from it to obtain what is called the cross-error of digitization. The mean and standard deviation of the error are shown in Table IV. The standard deviations from the mean shown in Tables III and IV indicate that the cross error is about

four times the self error, thus showing the effect of personal preferences in reading. The consistent, negative means indicate a constant shift in the time axis of the record chosen as a standard as compared to the other records. Such a shift does not effect the deviation about the mean.

A comparable measure of the error introduced by rounding-off the time record can be obtained by subtracting the rounded record from the original record and computing the mean and standard deviation. These were done for one of the records digitized by X, and the results are shown in Table V, for rounding to 0.01 and 0.005 sec.

TABLE V

Error Due to Round-off in Time Coordinates

Error in Time Coordinates	Round-off to 0.01 secs	Round-off to 0.005 secs
Mean (sec)	0.00003	0.00004
Standard Deviation (sec)	0.00285	0.00149
Maximum Error	±0.005	±0.0025

The results of the experiment described above show that the error introduced by rounding-off the time coordinates of an earthquake record to 0.01 or 0.005 secs is well within the error inherent in the process of digitization. For round-off to 0.01 secs, the standard deviation of the error due to round-off is about half of the standard deviation of the self-error of digitization and about one sixth of the standard deviation

of the cross-error. For round-off to 0.005 secs, this margin is further increased by a factor of 2. From this it is concluded that a round-off to 0.01 or 0.005 secs is an acceptable approximation.

Effect of Rounding Upon Accuracy

G. V. Berg[8] has examined errors in spectra caused by random errors in time and acceleration coordinates and has concluded from an approximate analysis and from computer experimentation that a scatter of the order of 20 per cent in undamped spectra is to be expected from identical computing procedures applied to independently prepared digitizations of the same accelerogram. He attributed the scatter to random errors in reading the accelerogram. Using Berg's[8] approximate formulae for expected error in velocity spectra due to random errors in the time coordinates of the record, and assuming $a = 0.045$ g, (S_v) ave ≈ 20 ins/sec and $\Delta t = 0.05$ secs, it can be shown that for

	Expected Error in S_v	Expected Percentage Error in S_v (D = 30 secs)
Round-off to 0.01 secs ($\sigma \approx 0.003$ secs)	$0.12\sqrt{D}$	3.3
Round-off to 0.005 secs ($\sigma \approx 0.0015$ secs)	$0.06\sqrt{D}$	1.6

in which a = the r.m.s. value of the ground acceleration, σ = standard deviation of the expected error in the time coordinates and D = the duration of the earthquake. These numbers indicate that the error in spectral values due to round-off to 0.01 or 0.005 secs is much less than

the 17 per cent error Berg[8] found likely to occur as a result of random errors in reading the time coordinates during digitization.

The validity of the above analysis can be checked by comparing the spectra for unrounded and rounded time records. This was done for the N65°E component of the station 2 record of the Parkfield earthquake of June 27, 1966 and the N8°E component of the Lima, Peru earthquake of October 17, 1966. The undamped velocity spectra for the original (unrounded) record and for the records obtained after rounding-off the time coordinates to 0.01 secs and to 0.005 secs are shown in Figs. 3 and 4. For the Parkfield earthquake, the three curves are almost indistinguishable. For the Lima, Peru earthquake the average percentage errors are of the order expected on the basis of Berg's approximate analysis.

The computing time for spectra based on digitized accelerograms rounded to 0.005 secs was found to be only 10 to 15 per cent more than the computing time for records rounded to 0.01 secs. Considering the expected error in the spectral aalues in the two cases, it is concluded that to be consistent with the choice of $(\Delta\tau)_m$ ($\leq T/20$), round-off to 0.005 secs would be adopted as standard. Here again it may be pointed out that a choise of $(\Delta\tau)_m \leq T/10$ and round-off to 0.01 secs should prove acceptable for most purposes.

Intervals of Integration

The procedure for computing spectra outlined above requires the choice of a maximum interval of integration, $(\Delta\tau)_m$. Since truncation error is not involved, the choice is governed by the error introduced by

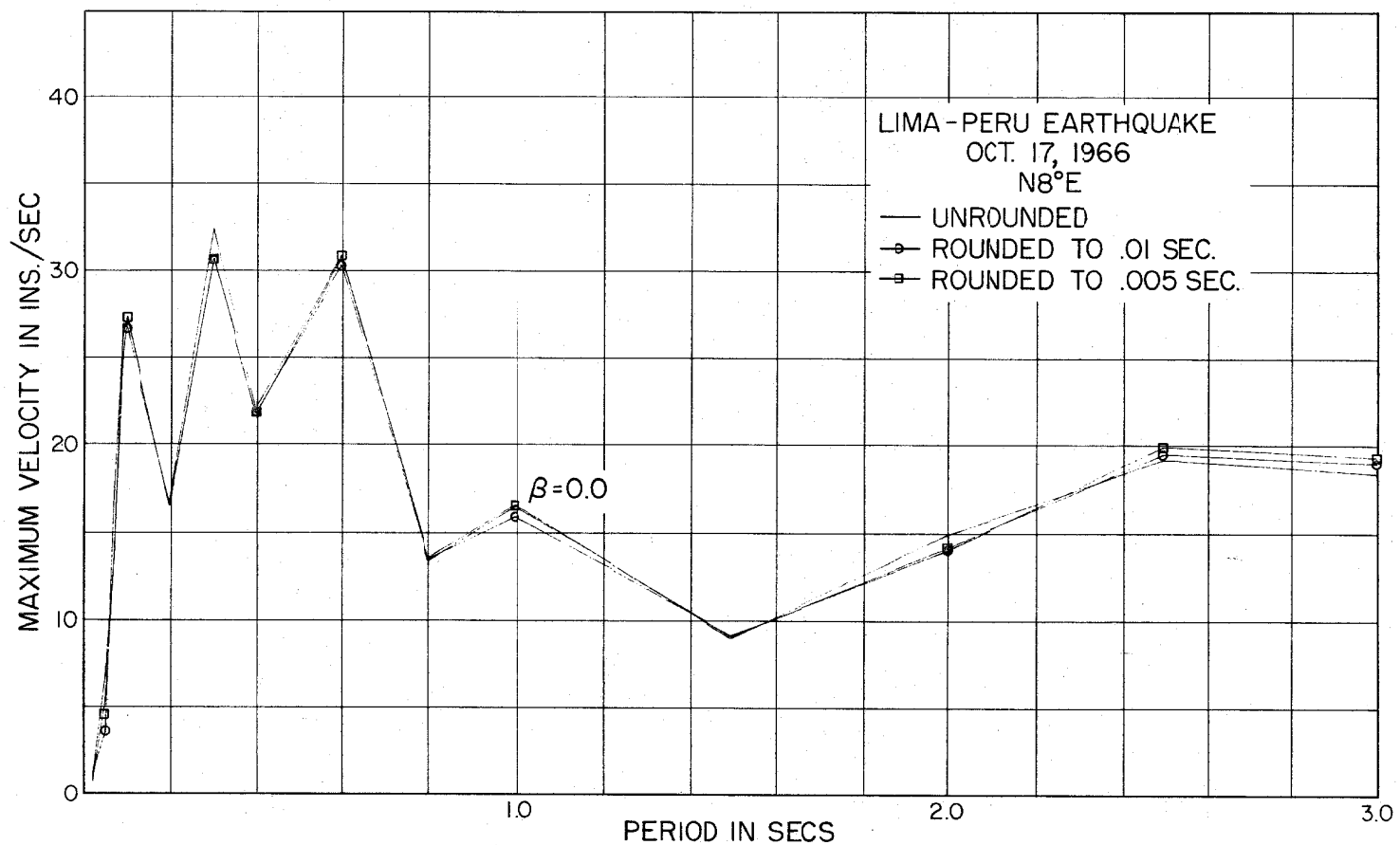

Fig. 3. - EFFECT OF ROUNDING THE TIME COORDINATES ON VELOCITY SPECTRA

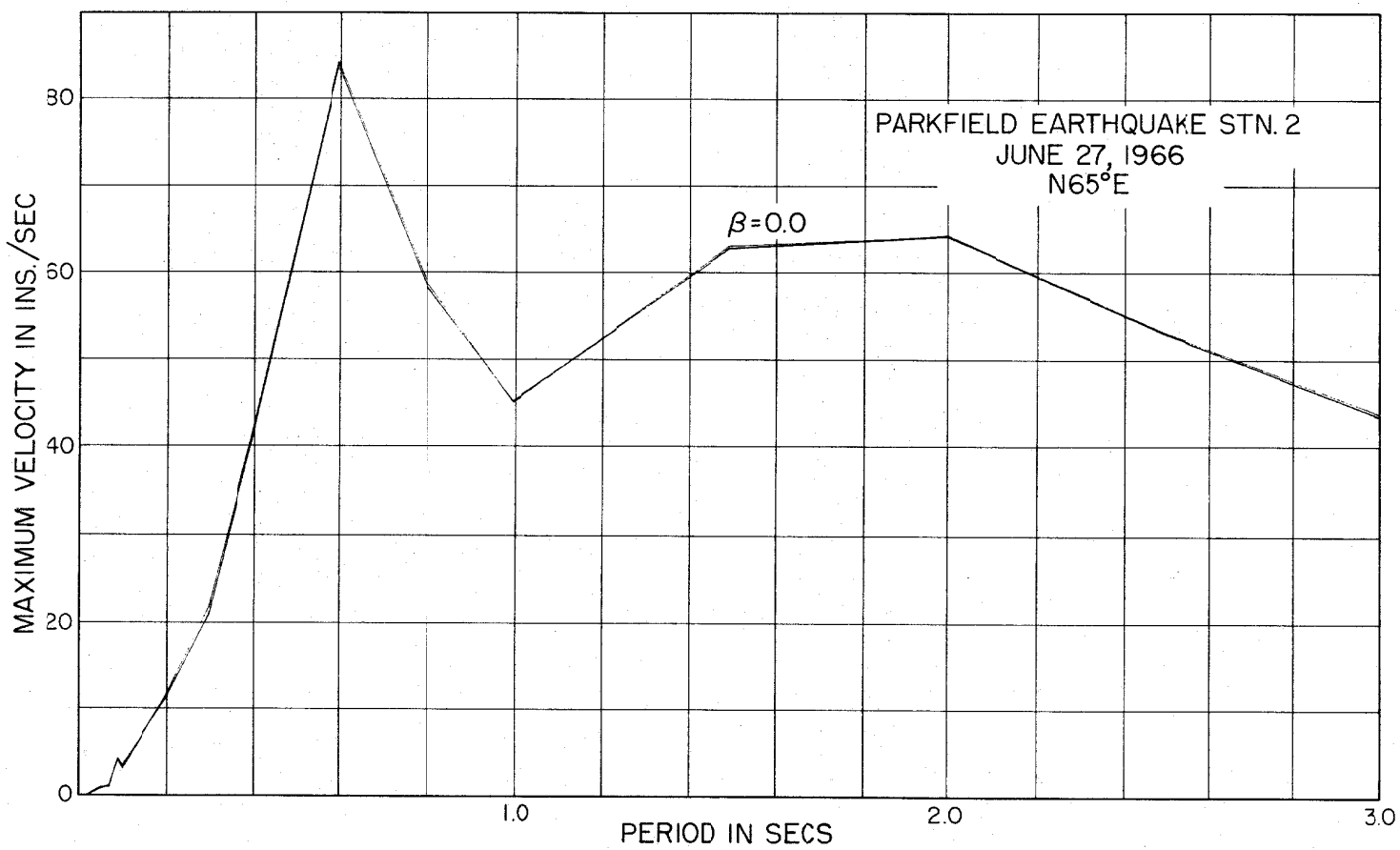

Fig. 4. - EFFECT OF ROUNDING THE TIME COORDINATES ON VELOCITY SPECTRA

discretization, and by computing time. As demonstrated in Appendix I, if $(\Delta\tau)_m \leq T/20$, the error due to discretization is less than 1.2 per cent. This may be treated as one constraint on the choice of $(\Delta\tau)_m$.

The effect of $(\Delta\tau)_m$ on the computing time is more complex in that a larger value of $(\Delta\tau)_m$ tends to reduce the number of integration steps but increases the number of matrices to be calculated. Smaller values of $(\Delta\tau)_m$ imply the reverse results, so a choice of $(\Delta\tau)_m$ should strike a balance between the two effects. The ideal choice of $(\Delta\tau)_m$ would depend on the nature of the accelerogram used, but a compromise satisfactory for most uses was achieved by the choice of $(\Delta\tau)_m = 0.05$ for spectrum periods equal or exceeding one second and $(\Delta\tau)_m \leq T/20$ for periods less than one second.

If the time-coordinates of the record are rounded to 0.005 secs, there is more than one way in which the intervals of digitization can be subdivided, as illustrated in Table II. The choice of subdividing method is governed by the computing time, and to examine this question a few trials were conducted using both the methods in Table II. It was found that in most cases, subdivision method IIa ($\Delta\tau_j = 0.04$, 0.03, 0.02, 0.01 and 0.005) required less computing time.

Comparison to a Third Order Runge-Kutta Method

To indicate the effect of truncation error on spectrum values obtained by a Runge-Kutta method and to compare the accuracy and computing time for that method with the techniques developed above, undamped velocity spectra were computed by both methods. The results of these computations are compiled in Table VI.

The results in Table VI show again that the accuracy of the proposed method is only achieved by the Runge-Kutta method with $\Delta\tau \leq T/80$, and that there is a three to four-fold savings in time by the use of the proposed technique.

Computer Program

A Fortran IV computer program using the exact method with round-off to 0.005 secs and $(\Delta\tau)_m \leq T/20$ or 0.05 secs, whichever is less, is included in Appendix V. The flow chart and instructions for the use of the program are included along with samples of input and output programs.

SUMMARY AND CONCLUSIONS

A numerical method for the calculation of response spectra, based on the exact solution to the governing differential equation, is developed in this report and presented along with computer programs for the application of the method. Computer programs for the baseline correction of accelerograms and for computing ground velocity and displacement are included also.

To adapt the exact solution to efficient computing, the solution is written in the form of two 2×2 matrices which operate upon the conditions at the beginning of the integration step and upon the acceleration at the beginning and end of the integration step to produce the exact velocity and displacement at the end of the time interval. Because the matrices are functions only of the damping and period of the oscillator

used to calculate spectra and of the magnitude of the time interval, only a limited number of the matrices need be evaluated. The method is applied directly to accelerograms digitized at equal time intervals, and by suitably rounding the time coordinates of records digitized at unequal time steps, the method can be applied to accelerograms digitized at unequal time intervals. It is shown that the errors introduced by rounding the time coordinates are much less than other errors inherent in the digitization of earthquake records.

The choice of using equal or unequal time intervals for digitization of accelerograms usually is not based on computational ease, but on the type of equipment available for the work. However, the choice of digitization method affects the calculation technique and the computing time. For the same number of sample points, digitization at unequal time intervals, usually represents the original record better than does digitization at equal intervals, and if digitization is done at equal intervals, a small time step usually must be used to avoid significant distortion of the record. Because for the usual integration methods the interval of integration must be less than or equal to the interval of digitization, a small digitization interval makes it necessary to use a small integration step where otherwise a larger interval could be used. This leads to an increase in computing time and is of particular significance to the method presented herein, because this technique does not involve truncation error and therefore can be used accurately with time intervals as large as $T/10$ to $T/20$. Thus, an accelerogram digitized at unequal intervals, with its smaller number of coordinates, can be processed more rapidly by the proposed method than can the same record digitized at equal time

intervals.

For the accelerograms used as examples in this report, the proposed method showed a three to four-fold saving in time over a third order Runge-Kutta method of comparable accuracy. This was found to be the case for records digitized at both equal and unequal time intervals. For accelerograms with durations of 30 seconds, spectra for 30 periods and 4 values of damping were calculated and plotted by an IBM 7094 computing system with an average computing time of 120 seconds, of which 100 seconds were used for execution of the calculations. If spectra for several earthquakes were to be computed in the same operation, a further savings in computing time could be achieved by storing the matrices used in the integration method rather than recomputing them at the beginning of the processing of each accelerogram.

Because of the increasing interest in the response of yielding structures to earthquake motions, it is appropriate to point out that the method presented herein can be adapted also to the calculation of spectra for bilinear hysteretic and elasto-plastic oscillators. To apply the method it would be necessary to compute two sets of matrices corresponding to the two stiffness coefficients of the structure.

ACKNOWLEDGMENT

Appreciation is extended to the National Science Foundation for partial support of this research under NSF Grant 1197X.

TABLE I.

Undamped Velocity Spectra and Computing Times for Records Digitized at Equal Time Intervals

(Artificial Earthquake No. 7, $\Delta t = 0.025$ secs)[14]

Period in secs.	S_v in ins/sec.					
	Third Order Runge-Kutta				Exact	
	$\Delta\tau \leq T/10$	$\Delta\tau \leq T/20$	$\Delta\tau \leq T/40$	$\Delta\tau \leq T/80$	$\Delta\tau \leq T/10$	$\Delta\tau \leq T/20$
0.05	0.126427	0.151835	0.167647	0.204386	0.213610	0.213611
0.1	1.175386	1.700338	1.938206	2.000499	2.021378	2.021378
0.2	5.892718	6.974802	7.951382	8.077665	8.129632	8.142673
0.3	8.149913	8.262451	7.951037	7.951037	7.927181	7.992554
0.4	16.399755	21.987390	22.985190	23.097921	23.174661	23.174610
0.5	20.716002	20.716002	22.897381	23.164824	23.352803	23.352803
0.7	25.625738	25.625738	26.732634	26.861891	26.966020	26.966020
0.9	14.911507	14.911507	15.019002	15.024426	15.059319	15.059319
1.0	13.712175	13.712175	13.712175	13.809191	13.841665	13.841665
1.6	23.740771	23.740771	23.740771	23.757607	23.772265	23.772265
2.0	13.137178	13.137178	13.137178	13.137178	13.148899	13.148899
Relative Computation Time	1.5	2	3	4	.75	1

Note: Spectral values below dotted lines were obtained at $\Delta\tau = 0.025$ secs, the interval of digitization.

TABLE II

Round-off and Subdivision of Time Record

[$(\Delta\tau)_m$ = 0.04 secs]

	Step	Original time	Rounded time	Subdivision into intervals of integration - $(\Delta\tau)_m$=0.04		Remarks
I	t_i	10.4267	10.43	0.04		Round-off to .01
	t_{i+1}	10.5213	10.52	0.04		
	Δt_i	0.0946	0.09	0.01		
				0.09		
II				(a)	(b)	
	t_i	10.4267	10.425	0.04	0.04	Round-off to .005
	t_{i+1}	10.5213	10.520	0.04	0.04	
	Δt_i	0.0946	0.095	0.01	0.015	
				0.005		
				0.095	0.095	

TABLE III

Self-Error of Digitization in Time Coordinates

Error in Time Coordinates	X					Y	Z
	1	2	3	4	Average	1	1
Mean (sec)	-0.00101	0.0001	-0.00021	0.00061	-0.00041	0.00013	0.00045
Standard Deviation (sec)	0.00432	0.00503	0.00412	0.00426	0.00443	0.00398	0.00556

TABLE IV

Cross-Error of Digitization in Time Coordinates

Error in Time Coordinates	Y			Z		
	1	2	Average	1	2	Average
Mean (sec)	-0.02525	-0.02538	-0.02532	-0.02212	-0.02257	-0.022235
Standard Deviation (sec)	0.01819	0.01794	0.01806	0.01520	0.01520	0.01655

TABLE VI

Undamped Velocity Spectra and Relative Computing Times for Records Digitized at Unequal Time Intervals

(N65°E Component the Station 2 Accelerogram of the Parkfield Earthquake, June 27, 1966, $(\Delta\tau)_m = 0.05$ secs)

Period in secs.	S_v in ins/sec.					
	Third Order Runge-Kutta				Exact	
	$\Delta\tau \leq T/10$	$\Delta\tau \leq T/20$	$\Delta\tau \leq T/40$	$\Delta\tau \leq T/80$	$\Delta\tau \leq T/10$	$\Delta\tau \leq T/20$
0.05	0.404177	0.547537	0.690496	0.781693	0.776551	0.802090
0.1	1.864953	2.749153	3.177131	3.391490	3.446407	3.446375
0.2	8.662501	10.587279	10.905353	10.929761	11.007311	11.007311
0.3	14.519660	20.197662	20.780575	20.880682	20.724359	20.954274
0.4	39.198212	40.554802	40.660100	40.677461	40.844893	40.844893
0.5	46.064539	50.050687	51.059478	51.129117	51.329105	51.329105
0.7	85.185941	86.373600	89.262830	89.655182	90.064254	90.063806
0.9	57.705933	59.043586	60.146103	60.228546	60.508116	60.488801
1.0	44.625701	44.625701	44.980583	44.990469	45.149522	45.149522
1.6	63.948129	63.948129	63.960555	63.941084	64.027008	64.027008
2.0	64.084503	64.084503	64.084503	64.134411	64.167577	64.167577
Relative Computation Time	1.25	1.5	2.25	3.5	0.75	1

Note: Spectrum values below dotted lines were calculated for $(\Delta\tau)_m = 0.05$ secs.

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APPENDIX I

ERRORS IN SPECTRA

Errors are introduced into response spectra at various stages in the processes leading to their final form. The sources of these errors and approximations for their magnitudes, where possible, are summarized here to estimate the accuracy of spectral calculations and to justify the approximations made in developing the computing technique presented in this report.

There are three broad phases in the computation of spectra: recording, digitization and computation. Some of the errors introduced in these phases are examined below. Errors such as the base line shifts and drifts caused by improper functioning of accelograph, vibration of pendulum suspension and distortion of recording paper prior to digitization are not discussed.

Errors in Recording

The trace produced by a strong-motion accelerograph is considered to be a record of ground acceleration during an earthquake. Since the basic element of an accelerograph is essentially a damped, simple oscillator, the recorded ground motion is governed by the equation

$$\ddot{y} + 2\xi p \dot{y} + p^2 y = - a(t) \qquad \text{(AI.1)}$$

where y = displacement of the element relative to base, p = natural period of the element, ξ = fraction of critical damping and $a(t)$ = ground acceleration during earthquake.

$$p^2 y \gg (\ddot{y} + 2\xi p \dot{y}) \tag{AI.2}$$

then

$$p^2 y \approx -a(t) \tag{AI.3}$$

and the graphical record may be regarded as a direct measure of the ground acceleration. Since an earthquake motion contains a range of frequencies, some distortions in amplitude and phase invariable occur, depending upon the damping of the recording element and the ratio of the natural frequency of the element to the input frequencies. From an analysis of the instrument response it can be shown that the percentage of amplitude distortion given by[16]

$$e_A = 100(\kappa - 1) \tag{AI.4}$$

and the phase distortion in degrees by

$$e_P - (\phi - 90r) \tag{AI.5}$$

in which e_A and e_P are the amplitude and phase errors, respectively, and

$$r = \frac{\alpha}{p} \tag{AI.6}$$

$$\kappa = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \tag{AI.7}$$

$$\phi = \tan^{-1} \frac{2\xi r}{1 - r^2} \tag{AI.8}$$

in which α = input frequency in radians per second.

For damping of the order of 60 per cent (usually provided in the accelerographs) and $0 < r < 1$, it can be shown[16] from Eqs. AI.4 and AI.5 that amplitude distortion is less than 10 per cent and phase distortion less than 5 degrees. Thus, the accelerograph will record with this accuracy frequency content from zero up to its natural frequency, which is typically about 15 cycles per second. If the ground motion contains frequencies higher than the natural frequency of the accelerograph, both the amplitude and the phase of this frequency components will be distorted significantly.

Errors in Digitization

1. Scaling Error. - This error arises from the inherent limitations of the resolving power of any scaling device. For most instruments now in use it is of the order of 0.01 in.

2. Random Error in Time and Acceleration Records. - The thickness of the line defining the record makes the choice of points at which discernible changes of slope occur and the scaling of magnitudes a matter of individual judgment. This leads to errors in both time and acceleration coordinates. If the same record is digitized by different persons, the standard deviations of the random errors in time and acceleration typically may be 0.018 secs and 0.001 g respectively. These reading errors may, in turn, cause errors up to 20 per cent in undamped spectra calculated from the records[6].

3. Baseline Correction. - The unknown distortion introduced into

the ground acceleration during recording and digitization is corrected to some degree by adjusting the baseline, herein by a technique which minimizes the resulting ground velocity.

4. Distortion of the Record. - For data sampling at equal time intervals, the cutoff frequency, called the Nyquist frequency, is given by[12]

$$f_c = \frac{1}{2\Delta t} \qquad \text{(AI.9)}$$

in which Δt is the sample interval. Such sampling causes aliasing error, since frequency content higher than f_c is folded into the lower frequency range 0 to f_c and confused with the data in this lower range. In earthquake records, with closely spaced, sharp peaks and many changes of slope this problem is important and it is necessary to use small intervals of digitization.

Errors in Computation

1. Straightline Approximation. - In the digital computation of spectra, the actual earthquake record is replaced by linear segments between the points of digitization. This is a minor approximation provided that the length of the time intervals is much shorter than the periods of interest.

2. Error Due to Discretization. - In any numerical method of computing the spectra, the response is obtained at a set of discrete points. Since spectral values represent maximum values

of response parameters which may not occur at these discrete points, discretization introduces an error which gives spectrum values lower than the true values. The error will be a maximum if the maximum response occurs exactly midway between two discrete points as shown in Fig. A.1. An estimate for the upper bound of this error can be found by noting that at the time of maximum displacement or velocity, the response of the oscillator is nearly sinusoidal at a frequency equal to its natural frequency. Under this assumption the error can be related to the maximum interval of integration $(\Delta\tau)_m$ and the period of the oscillator as shown below.

$(\Delta\tau)_m$	Maximum Error (per cent)
$\leq T/10$	≤ 4.9
$\leq T/20$	≤ 1.2
$\leq T/40$	≤ 0.3

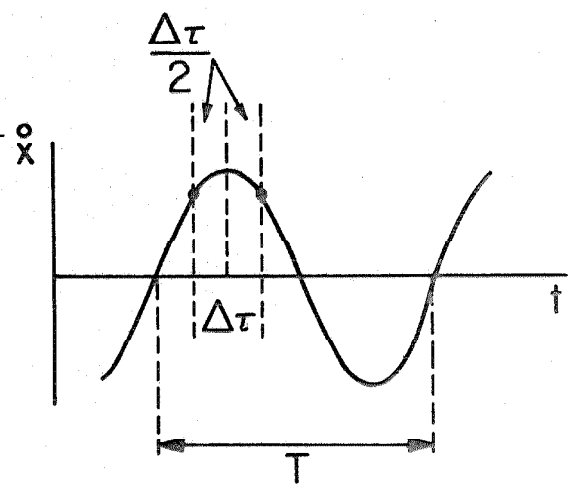


Fig. A.1

3. Truncation Error. - In general, a truncation error exists in numerical methods for integrating differential equations. For example, in third-order Runge-Kutta methods the error is proportional to $(\Delta\tau)^4$.

4. Error Due to Rounding the Time Record. - For earthquake records digitized at irregular time intervals, the integration technique proposed in this report requires rounding of the time record and the attendant error depends on the way the rounding is done. For round-off to 0.005 secs, the average error in spectrum values is expected to be less than 2 per cent.

APPENDIX II

SUBROUTINE FOR BASE LINE CORRECTION

Identification

PCN01 - Fortran IV Subroutine

Purpose

To perform a parabolic base line correction to an earthquake acceleration record, and to return corrected values of the ground acceleration along with the parameters of the parabolic correction.

Method

The corrected value of the acceleration at time t, is given by[7]

$$a^*(t) = a(t) - c_o - c_1 t - c_2 t^2 \qquad \text{(AII. 1)}$$

in which $a(t)$ = the digitized value of the ground acceleration at time t, $a^*(t)$ = the corrected value of the acceleration at time t, and c_o, c_1 and c_2 = are constants. The corrected velocity is given by

$$v^*(t) = v(t) - c_o t - \frac{1}{2} c_1 t^2 - \frac{1}{3} c_2 t^3 \qquad \text{(AII. 2)}$$

The mean square value of $v^*(t)$ is minimized if

$$\frac{\partial}{\partial c_i} \int_0^S [v^*(t)]^2 \, dt = 0 \quad ; \quad i = 0, 1, 2 \qquad \text{(AII. 3)}$$

in which S = the duration of the record.

Equation AII. 3 yields the coefficients c_o, c_1 and c_2. The corrected values of the acceleration are computed from Eq. AII. 1.

Usage

Call PCN01 (NDATA, T, GA, VEL, VELO, c_o, c_1, c_2, DELT, IFEQ)

where

NDATA	=	number of data points in the acceleration record;
T	=	an array containing time coordinates;
GA	=	an array containing acceleration coordinates, before and after correction;
VEL	=	an array containing velocity coordinates, generated by the subroutine;
VELO	=	the initial ground velocity (usually set equal to zero);
c_0, c_1, c_2	=	coefficients for parabolic correction;
DELT	=	the interval of digitization, if constant;
IFEQ	=	a control variable: If set to 1, it implies that input data is sampled at equal time intervals. In this case, only GA needs to be specified and the array T is generated by the subroutine. If IFEQ ≠ 1, it implies that input data is sampled at unequal time intervals and in this case both GA and T must be specified.

Input/Output

Input and Output are controlled by the calling program. Output of the corrected ground acceleration must be obtained in punched cards for subsequent use.

Computation Time

Execution time ≈ 1.5 secs for every 100 data points in the earthquake record.

```
C     ***************************************************************
C
C                SUBROUTINE FOR PARABOLIC BASE LINE CORRECTION
C                              PCN01
C
C     ***************************************************************
C
      SUBROUTINE PCN01(NDATA,T,GA,VEL,VELO,C0,C1,C2,DELT,IFEQ)
      DIMENSION T(1),GA(1),VEL(1)
      BE1=0.0
      BE2=0.0
      BE3=0.0
      T(1)=0.0
      VEL(1)=VELO
      M=NDATA-1
      DEL2=DELT*DELT
      DO 1 I=1,M
      IF(IFEQ.EQ.1) GO TO 2
      DELT=T(I+1)-T(I)
      DEL2=DELT*DELT
      GO TO 6
    2 T(I+1)=T(I)+DELT
    6 T2=T(I)*T(I)
      T3=T2*T(I)
      T4=T(I)*T(I+1)
      T5=T(I+1)*T(I+1)
      T6=T(I+1)*T5
      VEL(I+1)=VEL(I)+0.5*DELT*(GA(I)+GA(I+1))
      BE1=BE1+0.5*VEL(I)*DELT*(T(I)+T(I+1))+0.04166667*(GA(I)*(3.*T(I)
     1+5.*T(I+1))+GA(I+1)*(T(I)+3.*T(I+1)))*DEL2
      BE2=BE2+0.3333333*VEL(I)*DELT*(T2+T4+T5)+0.01666667*DEL2*(GA(I)*
     1(4.*T2+7.*T4+9.*T5)+GA(I+1)*(T2+3.*T4+6.*T5))
      BE3=BE3+0.25*VEL(I)*DELT*(T3+T2*T(I+1)+T(I)*T5+T6)+
     10.008333333*DEL2*(GA(I)*(5.*T3+9.*T2*T(I+1)+12.*T(I)*T5+14.*T3)
     2+GA(I+1)*(T3+3.*T2*T(I+1)+6.*T(I)*T5+10.*T6))
    1 CONTINUE
      XO=T(NDATA)
      BETA1=BE1/XO**3
      BETA2=BE2/XO**4
      BETA3=BE3/XO**5
      CA1=300.*BETA1-900.*BETA2+630.*BETA3
      CA2=-1800.*BETA1+5760.*BETA2-4200.*BETA3
      CA3=1890.*BETA1-6300.*BETA2+4725.*BETA3
      C0=CA1
      C1=CA2/XO
      C2=CA3/XO**2
      DO 5 I=1,NDATA
    5 GA(I)=GA(I)-C0-C1*T(I)-C2*T(I)*T(I)
      RETURN
       END
```

APPENDIX III

SUBROUTINE FOR COMPUTING GROUND VELOCITY AND DISPLACEMENT

Identification

PCN02 - Fortran IV Subroutine

Purpose

To compute the ground velocity and displacement for a component of an earthquake by integrating the digitized ground acceleration record.

Method

Starting with given initial conditions, ground velocity, $v(t)$ and displacement, $u(t)$ are computed by successive applications of the following formulae due to Berg and Housner[13].

$$u(t_{i+1}) = u_{i+1} = u_i + v_i \Delta t_i + \frac{1}{6}(\Delta t_i)^2 (2a_i + a_{i+1}) \qquad \text{(AIII. 1)}$$

$$v(t_{i+1}) = v_{i+1} = v_i + \frac{1}{2} \Delta t_i (a_i + a_{i+1}) \qquad \text{(AIII. 2)}$$

in which $u(t)$ = the ground displacement at time t, $v(t)$ = the ground velocity at time t, $a(t)$ = the ground acceleration at time t, and $\Delta t_i = t_{i+1} - t_i$.

Usage

Call PCN02 (NDATA, T, GA, VEL, DISP, VELO, DISPO, DELT, IFEQ)

where

NDATA	=	number of data points in the acceleration record;
T	=	an array containing time coordinates of the original accelerogram;
GA	=	an array containing acceleration coordinates;
VEL	=	an array containing the ground velocity coordinates computed by the subroutine;
DISP	=	an array containing the ground displacement

coordinates computed by the subroutine;

VELO	=	the initial velocity to be set by the user (usually VELO is set equal to zero);
DISPO	=	the initial displacement to be set by the user (usually DISPO is set equal to zero);
DELT	=	the interval of digitization, if constant;
IFEQ	=	a control variable: if set to 1, it implies that input data is sampled at equal time intervals. In this case, only GA needs to be specified and the array T is generated by the subroutine. If IFEQ ≠ 1 it implies that input data is sampled at unequal time intervals and in this case both GA and T must be specified.

Input/Output

Input and Output are controlled by the calling program.

Computation Time

Execution time ≈ 3.0 secs for every 100 data points in the earthquake record.

```
C     ***************************************************************
C
C           SUBROUTINE FOR GROUND DISPLACEMENT AND VELOCITY
C                              PCN02
C
C     ***************************************************************
C
      SUBROUTINE PCN02(NDATA,T,GA,VEL,DISP,VELO,DISPO,DELT,IFEQ)
      DIMENSION T(1),GA(1),VEL(1),DISP(1)
      DISP(1)=DISPO
      VEL(1)=VELO
      T(1)=0.0
      M=NDATA-1
      DO 2 I=1,M
      IF(IFEQ.EQ.1) GO TO 1
      DELT=T(I+1)-T(I)
      GO TO 3
    1 T(I+1)=T(I)+DELT
    3 DISP(I+1)=DISP(I)+VEL(I)*DELT+0.16666667*DELT*DELT*(2.*GA(I)+
     1GA(I+1))
    2 VEL(I+1)=VEL(I)+0.5*DELT*(GA(I)+GA(I+1))
      RETURN
       END
```

APPENDIX IV

SUBROUTINES FOR COMPUTING SPECTRA FROM EARTHQUAKE RECORDS DIGITIZED AT EQUAL TIME INTERVALS

Identification

PCN03
PCN04 } Fortran IV Subroutines

Purpose

To compute response spectra from earthquake records digitized at equal time intervals.

Method

The response of a damped, simple oscillator is calculated by successive application of Eqs. 2.7 and 2.8 and spectrum values are obtained by monitoring the maximum values of the response.

Usage

The computer program consists of the following:

1. Main program for Input/Output
2. Subroutine PCN03 for computation of spectra
3. Subroutine PCN04 for computation of matrices A and B

Subroutine PCN03 is called in the main program by the statement

Call PCN03 (ID, IP, N, DEL, DMP, PD, GA, SF, SD, SV, SA, FS)

where

ID	=	the number of damping values;
IP	=	the number of periods;
N	=	the number of sample points in the earthquake record

DEL	=	the interval of digitization;
DMP (I)	=	an array containing damping values as fractions of critical damping;
PD (I)	=	an array containing values of the periods;
GA (I)	=	an array containing acceleration coordinates;
SF	=	the scale factor for acceleration;
SD (I, J)	=	the displacement spectrum values for different values of damping (I) and periods (J), generated by the subroutine;
SV (I, J)	=	the velocity spectrum values for different values of damping (I) and period (J) generated by the subroutine
SA (I, J)	=	the acceleration spectrum values for different values of damping (I) and period (J), generated by the subroutine;
FS (I)	=	an array containing velocity at the end of the excitation, generated by the subroutine. For the undamped oscillator, this array gives the Fourier spectrum of the acceleration record.

Subroutine PCN04 is called within subroutine PCN03.

Input/Output

The input and output are controlled by the main program. A sample of such a program, including both printed and plotted output, is given below.

Remarks

The computer programs, in Fortran IV, for subroutines PCN03 and PCN04 are based on the following criteria:

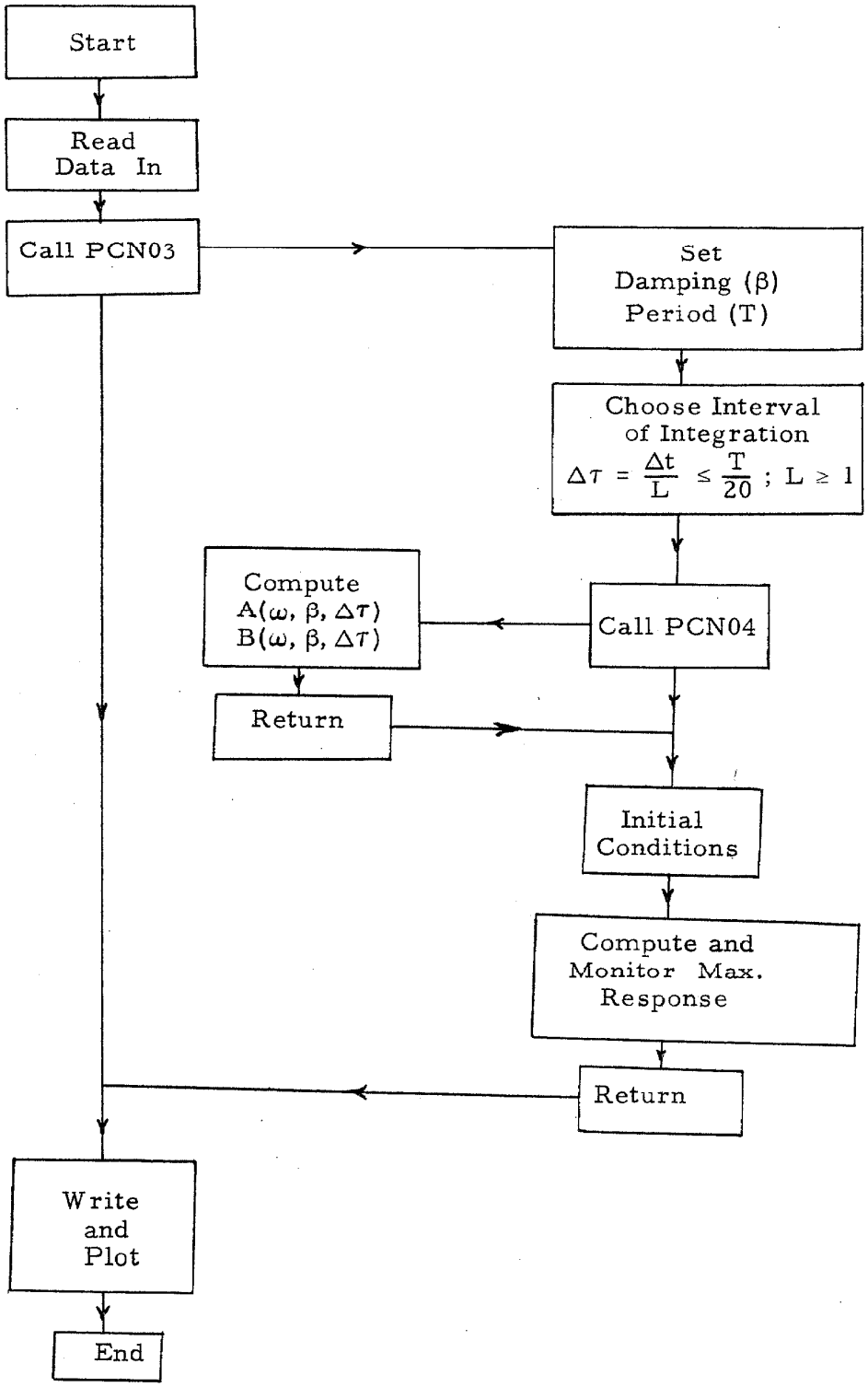
1. Interval of integration, $\Delta\tau \leq \frac{\text{Period}}{20}$ or interval of digitization, whichever is less.

2. The response of an undamped oscillator tends to build up with time during the excitation so that spectrum values often occur toward the end of the record. For this reason the record is assumed to continue with zero acceleration after the last data point until such time as the relative velocity had changed sign three times. This procedure insures that true maxima (for the record given) are found.

3. The user must prescribe the dimension statements in the main program (I/O) and the subroutines, depending upon the data points in the earthquake record and the number of damping and period values for which spectra are to be obtained. The dimension of the array GA, in the main program and subroutine PCN03 must be > N + 50 or N + 2*PMAX/DEL, whichever is greater, where PMAX = the maximum value of the period for which spectra are computed.

4. Average execution time = 1.0 sec per spectrum point for records of 30 second duration.

FLOW CHART


Start
Read Data In
Call PCN03
Set Damping (β) Period (T)
Choose Interval of Integration
$\Delta\tau = \frac{\Delta t}{L} \leq \frac{T}{20}$; $L \geq 1$
Compute $A(\omega, \beta, \Delta\tau)$ $B(\omega, \beta, \Delta\tau)$
Call PCN04
Return
Initial Conditions
Compute and Monitor Max. Response
Return
Write and Plot
End

```
C     ***************************************************************
C
C                 SAMPLE PROGRAM FOR INPUT/OUTPUT
C
C      SPECTRA CALCULATION FOR EARTHQUAKE RECORDS DIGITIZED
C                      AT EQUAL TIME INTERVALS
C
C     ***************************************************************
C
      DIMENSION DMP(5),PD(50),GA(3000),SD(5,50),SV(5,50),SA(5,50),
     1VM(50),FS(50)
      LOGICAL CL
C
C     *************** DATA READ-IN ***************
C
      CL=.FALSE.
      READ(5,100)ID,IP,N,DEL,SF,PMS
      READ(5,101)(DMP(J),J=1,ID)
      READ(5,102)(PD(J),J=1,IP)
      READ(5,103) (GA(J),J=1,N)
      CALL PCN03(ID,IP,N,DEL,DMP,PD,GA,SF,SD,SV,SA,FS)
C
C     ************************ OUTPUT-PRINT ************************
C
      DO 3 J=1,ID
      D=DMP(J)
      IF(D.GT.1.E-03) GO TO 24
      WRITE(6,104) D
      GO TO 23
24    WRITE(6,105) D
23    DO 4 I=1,IP
      P=PD(I)
      DMAX=SD(J,I)
      VMAX=SV(J,I)
      AMAX=SA(J,I)
      IF(D.GE.1.E-03) GO TO 25
      VEND=FS(I)
25    IF(D.GE.1.E-03) GO TO 26
      WRITE(6,106) P,DMAX,VMAX,VEND,AMAX
      GO TO 20
26    WRITE(6,107)P,DMAX,VMAX,AMAX
20    VM(I)=ABS(VMAX)
4     CONTINUE
C
C     ************************ OUTPUT-PLOT *************************
C
      IF(CL) GO TO 21
      CL=.TRUE.
      VMS=ABS(SV(1,1))
      DO 5 IX=2,IP
      IF(ABS(SV(1,IX)).LE.VMS) GO TO 5
      VMS=ABS(SV(1,IX))
5     CONTINUE
      IF(VMS.GT.100.) GO TO 51
      IF(VMS.GT.50.) GO TO 52
      VMS=50.
      GO TO 21
51    VMS=200.
      GO TO 21
52    VMS=100.
21    CALL XYPLOT(IP,PD,VM,0.0,PMS,0.0,VMS,DD,0)
```

```
3       CONTINUE                                                          I/O   62
100     FORMAT(3I4,3F10.4)                                                I/O   63
101     FORMAT(4F5.3)                                                     I/O   64
102     FORMAT(12F5.2)                                                    I/O   65
103     FORMAT(6F11.7)                                                    I/O   66
104     FORMAT(5X,8HDAMPING=,F4.2/5X,6HPERIOD,5X,4HDMAX,8X,4HVMAX,8X,     I/O   67
       14HVEND,10X,4HAMAX/6X,4HSEC.,6X,4HINS.,7X,8HINS./SEC,4X,8HINS./SEC,I/O   68
       44X12HINS./SEC/SEC)                                                I/O   69
105     FORMAT(5X,8HDAMPING=,F4.2/5X,6HPERIOD,5X,4HDMAX,8X,4HVMAX,10X,    I/O   70
       14HAMAX/6X,4HSEC.,6X,4HINS.,7X,8HINS./SEC,4X,12HINS./SEC/SEC)      I/O   71
106     FORMAT(6X,F4.2,4X,F7.4,5X,F8.4,4X,F7.4,7X,F8.4)                   I/O   72
107     FORMAT(6X,F4.2,4X,F7.4,5X,F8.4,7X,F8.4)                           I/O   73
        STOP                                                              I/O   74
         END                                                              I/O   75
C
C       ********** EXPLANATION **********
C
C       PMS=MAX. VALUE OF PERIOD (ABSCISSA) FOR PLOT
C       VMS=MAX. VALUE OF VELOCITY (ORDINATE) FOR PLOT
```

```
C     ***************************************************************
C
C       SUBROUTINE FOR COMPUTATION OF SPECTRA FROM EARTHQUAKE RECORD
C                DIGITIZED AT EQUAL TIME INTERVALS
C                          PCN03
C
C     ***************************************************************
C
      SUBROUTINE PCN03(ID,IP,N,DEL,DMP,PD,GA,SF,SD,SV,SA,FS)
      DIMENSION X(3),G(2),DMP(5),PD(50),A(2,2),B(2,2),GA(3000),TY(3),
     1SD(5,1),SV(5,1),SA(5,1),FS(5)
      DO4J=1,ID
      D=DMP(J)
      DO5K=1,IP
      P=PD(K)
      W=2.*3.141592654/P
C
C     *****CHOICE OF INTERVAL OF INTEGRATION*****
C
      DELP=P/20.
      L=DEL/DELP+1.-1.E-05
      DELT=DEL/FLOAT(L)
C
C     *****COMPUTATION OF MATRICES A AND B*****
C
      CALL PCN04(D,W,DELT,A,B)
C
C     **********INITIATION**********
C
      X(1)=0.0
      X(3)=0.0
      X(2)=0.0
      DMAX=0.0
      VMAX=0.0
      AMAX=0.0
      I=1
      DW=2.*W*D
      W2=W**2
      IA=2.*P/DELT+1.E-05
C
C     *****COMPUTATION OF RESPONSE*****
C
7     SL=(GA(I+1)-GA(I))/FLOAT(L)
      DO6M=1,L
      G(1)=(GA(I)+SL*FLOAT(M-1))*SF
      G(2)=(GA(I)+SL*FLOAT(M))*SF
      TY(1)=A(1,1)*X(1)+A(1,2)*X(2)-B(1,1)*G(1)-B(1,2)*G(2)
      TY(2)=A(2,1)*X(1)+A(2,2)*X(2)-B(2,1)*G(1)-B(2,2)*G(2)
      TY(3)=-(DW*TY(2)+W2*TY(1))
C
C     *****MONITORING THE MAX. VALUES*****
C
      IF(ABS(TY(1)).LE.ABS(DMAX)) GO TO 14
      DMAX=TY(1)
14    X(1)=TY(1)
      IF(ABS(TY(2)).LE.ABS(VMAX)) GO TO 15
      VMAX=TY(2)
15    X(2)=TY(2)
      IF(ABS(TY(3)).LE.ABS(AMAX)) GO TO 16
      AMAX=TY(3)
16    X(3)=TY(3)
```

```
6       CONTINUE                                                        03    62
C                                                                       03    63
C       *****TEST FOR END OF INTEGRATION*****                           03    64
C                                                                       03    65
32      I=I+1                                                           03    66
        IF(I.EQ.N) GO TO 18                                             03    67
        GO TO 19                                                        03    68
18      VEND = SQRT (X(2)**2 + W2*(X(1)**2) )                           03    69
19      IF(I.EQ.(N+IA)) GO TO 8                                         03    70
        IF(I.GE.N) GO TO 17                                             03    71
        GO TO 7                                                         03    72
17      GA(I+1)=0.0                                                     03    73
        GO TO 7                                                         03    74
8       IF(D.GE.1.E-03) GO TO 26                                        03    75
        FS(K) = VEND                                                    03    76
26      SD(J,K)=DMAX                                                    03    77
        SV(J,K)=VMAX                                                    03    78
        SA(J,K)=AMAX                                                    03    79
5       CONTINUE                                                        03    80
4       CONTINUE                                                        03    81
        RETURN                                                          03    82
         END                                                            03    83
```

```
C     ***************************************************************** 04    1
C                                                                       04    2
C          SUBROUTINE FOR COMPUTATION OF MATRICES A AND B               04    3
C                              PCN04                                    04    4
C                                                                       04    5
C     ***************************************************************** 04    6
C                                                                       04    7
      SUBROUTINE PCN04(D,W,DELT,A,B)                                    04    8
      DIMENSION A(2,2),B(2,2)                                           04    9
      DW=D*W                                                            04   10
      D2=D**2                                                           04   11
      A0=EXP(-DW*DELT)                                                  04   12
      A1=W*SQRT(1.-D2)                                                  04   13
      AD1=A1*DELT                                                       04   14
      A2=SIN(AD1)                                                       04   15
      A3=COS(AD1)                                                       04   16
      W2=W**2                                                           04   17
      A4=(2.*D2-1.)/W2                                                  04   18
      A5=D/W                                                            04   19
      A6=2.*A5/W2                                                       04   20
      A7=1./W2                                                          04   21
      A8=(A1*A3-DW*A2)*A0                                               04   22
      A9=-(A1*A2+DW*A3)*A0                                              04   23
      A10=A8/A1                                                         04   24
      A11=A0/A1                                                         04   25
      A12=A11*A2                                                        04   26
      A13=A0*A3                                                         04   27
      A14=A10*A4                                                        04   28
      A15=A12*A4                                                        04   29
      A16=A6*A13                                                        04   30
      A17=A9*A6                                                         04   31
      A(1,1)=A0*(DW*A2/A1+A3)                                           04   32
      A(1,2)=A12                                                        04   33
      A(2,1)=A10*DW+A9                                                  04   34
      A(2,2)=A10                                                        04   35
      B(1,1)=(-A15-A16+A6)/DELT-A12*A5-A7*A13                           04   36
      B(1,2)=(A15+A16-A6)/DELT+A7                                       04   37
      B(2,1)=(-A14-A17-A7)/DELT-A10*A5-A9*A7                            04   38
      B(2,2)=(A14+A17+A7)/DELT                                          04   39
      RETURN                                                            04   40
       END                                                              04   41
```

APPENDIX V

SUBROUTINES FOR COMPUTING SPECTRA FROM EARTHQUAKE RECORDS DIGITIZED AT UNEQUAL TIME INTERVALS

Identification

PCN05
PCN06 } Fortran IV Subroutines
PCN07

Purpose

To compute response spectra from earthquake records digitized at unequal time intervals.

Method

The response of a damped, simple oscillator is calculated by successive application of Eqs. 2.7 and 2.8 and spectrum values are obtained by monitoring the maximum values of the response.

Usage

The computer program consists of the following:

1. Main program for input/output
2. Subroutine PCN05 for computation of spectra
3. Subroutine PCN06 for subdivision and indexing of time coordinates
4. Subroutine PCN07 for computation of matrices A and B.

Subroutine PCN05 is called in the main program by the statement

Call PCN05 (ID, IP, N, DELM, DMP, PD, TY, RT, SFT, SFA, SD, SV, SA, FS)

where

ID	=	the number of damping values;
IP	=	the number of periods;
N	=	the number of sample points in the earthquake record;
DELM	=	the upper limit on the size of the maximum interval of integration (suggested value of DELM = 0.05 sec);
DMP (I)	=	an array containing damping values as fractions of critical damping;
PD (I)	=	an array containing values of periods;
TY (I)	=	an array containing acceleration coordinates;
RT (I)	=	an array containing time coordinates;
SFT	=	the scale factor for time;
SFA	=	the scale factor for acceleration;
SD (I, J)	=	the displacement spectrum values for different values at damping (I) and periods (J), generated by the subroutine;
SV (I, J)	=	the velocity spectrum values for different values of damping (I) and periods (J), generated by the subroutine;
SA (I, J)	=	the acceleration spectrum values for different values of damping (I) and periods (J), generated by the subroutine;
FS (I)	=	an array containing velocity at the end of the excitation, generated by the subroutine. For the undamped oscillator, this array gives the Fourier spectrum of the acceleration.

Subroutines PCN06 and PCN07 are called within subroutine PCN05.

Input/Output

The input and output are controlled by the main program. A sample of such a program is given below.

Remarks

The computer programs, in Fortran IV, for subroutines PCN05, PCN06 and PCN07 are based on the following criteria.

1. Maximum interval of integration, $(\Delta\tau)_m \leq \frac{\text{Period}}{20}$, the interval of digitization or 0.05 secs, whichever of the three is least.

2. The time coordinates of the accelerogram are rounded by the programs to the nearest 0.005 sec.

3. The record is assumed to continue with zero acceleration after the last data point until such time as the relative velocity had changed sign three times.

4. The user must prescribe the dimension statements in the main program (I/0) and the subroutines, depending upon the data points in the earthquake record and the number of damping and period values for which spectra are to be obtained. The dimensions of various arrays should be fixed as follows:

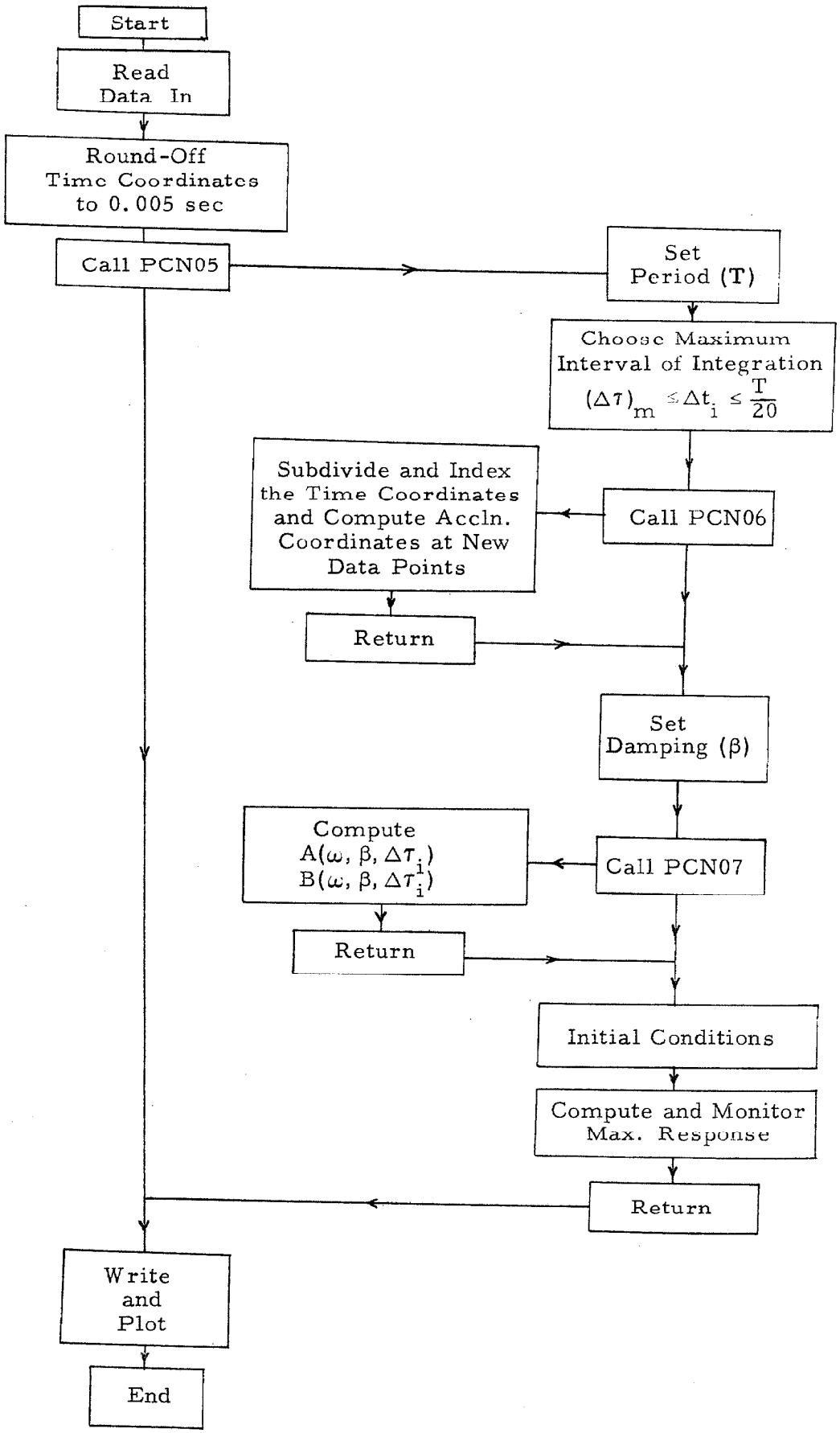
Array	Dimension	Remarks
RT,TY,TU	$\geq$ N	
GA,IND	$\geq$ (200*D+100) or (N+50)	For periods less than 0.2 secs.
	$\geq \underset{P}{\text{Max}}\left(\frac{20*D}{P} + 200\right)$ or $\left(\frac{D}{DELM} + \frac{2*PMAX}{DELM} + 100\right)$ or (N+50) whichever is greater	For periods greater than or equal to 0.2 secs.

where D = duration of the earthquake record in seconds, P = value of

the periods in seconds and PMAX = the maximum value of the periods.

5. Average execution time = 0.8 sec for spectrum point for a record of 30 sec duration.

FLOW CHART


Start
Read Data In
Round-Off Time Coordinates to 0.005 sec
Call PCN05
Set Period (T)
Choose Maximum Interval of Integration $(\Delta\tau)_m \leq \Delta t_i \leq \frac{T}{20}$
Call PCN06
Subdivide and Index the Time Coordinates and Compute Accln. Coordinates at New Data Points
Return
Set Damping (β)
Call PCN07
Compute $A(\omega, \beta, \Delta\tau_i)$ $B(\omega, \beta, \Delta\tau_i)$
Return
Initial Conditions
Compute and Monitor Max. Response
Return
Write and Plot
End

```
C     ***************************************************************  I/O
C                                                                      I/O
C                 SAMPLE PROGRAM FOR INPUT/OUTPUT                      I/O
C                                                                      I/O
C      SPECTRA CALCULATION FOR EARTHQUAKE RECORDS DIGITIZED            I/O
C                 AT UNEQUAL TIME INTERVALS                            I/O
C                                                                      I/O
C     ***************************************************************  I/O
C                                                                      I/O
      DIMENSION DMP(5),PD(50),RT(1000),TY(1000),SD(5,50),SV(5,50),     I/O
     1SA(5,50),FS(50),VMP(50)                                          I/O
      LOGICAL CL                                                       I/O
C                                                                      I/O
C     *************** DATA READ-IN ***************                     I/O
C                                                                      I/O
      CL=.FALSE.                                                       I/O
      READ(5,101)ID,IP,N,DELM,SFT,SFA,PMS                              I/O
      READ(5,102)(DMP(J),J=1,ID)                                       I/O
      READ(5,103)(PD(J),J=1,IP)                                        I/O
      READ(5,104)(RT(J),TY(J),J=1,N)                                   I/O
      CALL PCN05(ID,IP,N,DELM,DMP,PD,TY,RT,SFT,SFA,SD,SV,SA,FS)        I/O
C                                                                      I/O
C     ******** OUTPUT - PRINT ********                                 I/O
C                                                                      I/O
      DO10J=1,ID                                                       I/O
      D=DMP(J)                                                         I/O
      IF(D.GT.1.E-05) GO TO 12                                         I/O
      WRITE(6,108) D                                                   I/O
      GO TO 15                                                         I/O
12    WRITE(6,105) D                                                   I/O
15    DO 14 K=1,IP                                                     I/O
      DMAX=SD(J,K)                                                     I/O
      VMAX=SV(J,K)                                                     I/O
      AMAX=SA(J,K)                                                     I/O
      P=PD(K)                                                          I/O
      VMP(K)=ABS(VMAX)                                                 I/O
      IF(D.GT.1.E-05) GO TO 13                                         I/O
      VEND=FS(K)                                                       I/O
      WRITE(6,106) P,DMAX,VMAX,VEND,AMAX                               I/O
      GO TO 14                                                         I/O
13    WRITE(6,107) P,DMAX,VMAX,AMAX                                    I/O
14    CONTINUE                                                         I/O
C                                                                      I/O
C     ******** OUTPUT - PLOT ********                                  I/O
C                                                                      I/O
      IF(CL) GO TO 34                                                  I/O
      CL=.TRUE.                                                        I/O
      VMS=ABS(SV(1,1))                                                 I/O
      DO 5 IX=2,IP                                                     I/O
      IF(ABS(SV(1,IX)).LE.VMS) GO TO 5                                 I/O
      VMS=ABS(SV(1,IX))                                                I/O
5     CONTINUE                                                         I/O
      IF(VMS.LE.50.) GO TO 18                                          I/O
      IF(VMS.LE.100.) GO TO 57                                         I/O
      VMS=200.                                                         I/O
      GO TO 34                                                         I/O
18    VMS=50.                                                          I/O
      GO TO 34                                                         I/O
57    VMS=100.                                                         I/O
34    CALL XYPLOT(IP,PD,VMP,0.0,PMS,0.0,VMS,DD,0)                      I/O
10    CONTINUE                                                         I/O
```

```
101   FORMAT(3I4,4F10.4)                                                I/O   62
102   FORMAT(4F5.3)                                                     I/O   63
103   FORMAT(12F5.2)                                                    I/O   64
104   FORMAT((3X,4(F8.4,F9.6)))                                         I/O   65
108   FORMAT(5X,8HDAMPING=,F4.2/5X,6HPERIOD,5X,4HDMAX,8X,4HVMAX,8X,     I/O   66
     14HVEND,10X,4HAMAX/6X,4HSEC.,6X,4HINS.,7X,8HINS./SEC,4X,8HINS./SEC,I/O   67
     44X12HINS./SEC/SEC)                                               I/O   68
105   FORMAT(5X,8HDAMPING=,F4.2/5X,6HPERIOD,5X,4HDMAX,8X,4HVMAX,10X,    I/O   69
     14HAMAX/6X,4HSEC.,6X,4HINS.,7X,8HINS./SEC,4X,12HINS./SEC/SEC)      I/O   70
106   FORMAT(6X,F4.2,4X,F7.4,4X,F9.4,3X,F8.3,6X,F9.4)                   I/O   71
107   FORMAT(6X,F4.2,4X,F7.4,4X,F9.4,6X,F9.4)                           I/O   72
      STOP                                                              I/O   73
       END                                                              I/O   74
C
C     ********** EXPLANATION **********
C
C     PMS=MAX. VALUE OF PERIOD (ABSCISSA) FOR PLOT
C     VMS=MAX. VALUE OF VELOCITY (ORDINATE) FOR PLOT
```

```
C     ***************************************************************** 05
C                                                                       05
C       SUBROUTINE FOR COMPUTATION OF SPECTRA FROM EARTHQUAKE RECORD    05
C              DIGITIZED AT UNEQUAL TIME INTERVALS                      05
C                            PCN05                                      05
C                                                                       05
C     ***************************************************************** 05
C                                                                       05
      SUBROUTINE PCN05(ID,IP,N,DELM,DMP,PD,TY,RT,SFT,SFA,SD,SV,SA,FS)   05
      DIMENSION X(3),G(2),VMP(50),DMP(5),PD(50),A(10,2,2),B(10,2,2),    05
     1TY(1000),RT(1000),GA(7000),IND(7000),SD(5,50),SV(5,50),SA(5,50),  05
     2TX(3),TU(1000),FS(50)                                             05
      LOGICAL CL,FL                                                     05
      CL=.FALSE.                                                        05
      FL=.FALSE.                                                        05
C                                                                       05
C     *****SCALING AND ROUNDING-OFF THE TIME COORDINATES TO 0.005 SECS**05
C                                                                       05
      DO 20 J=1,N                                                       05
      RT(J)=RT(J)*SFT                                                   05
      IND(J)=RT(J)*100.+0.5                                             05
      TU(J)=FLOAT(IND(J))/100.                                          05
      DIF=RT(J)-TU(J)                                                   05
      IF(ABS(DIF).GE.0.0025) GO TO 91                                   05
      GO TO 20                                                          05
91    TU(J)=TU(J)+SIGN(1.,DIF)*0.005                                    05
20    TY(J)=TY(J)*SFA                                                   05
      DO1K=1,IP                                                         05
      P=PD(K)                                                           05
      W=6.28318/P                                                       05
C                                                                       05
C     *****CHOICE OF INTERVAL OF INTEGRATION AND SUBDIVISION            05
C     WITH INDEXING*****                                                05
C                                                                       05
      TY1=20.*DELM+1.E-05                                               05
      IF(FL) GO TO 26                                                   05
      IF(P.GT.TY1) GO TO 24                                             05
      IF(P.GT.(0.2-1.E-03)) GO TO 22                                    05
      IF(CL) GO TO 60                                                   05
      DELT=0.005                                                        05
      CL=.TRUE.                                                         05
      L=1                                                               05
      CALL PCN06(N,TU,TY,GA,IND,DELT,M)                                 05
60    IF(P.LT.(0.1-1.E-04)) GO TO 25                                    05
      DELT=0.005                                                        05
      IC=1                                                              05
      GO TO 26                                                          05
25    IF(P.LT.(0.05-1.E-04)) GO TO 61                                   05
      DELT=0.0025                                                       05
      IC=2                                                              05
      GO TO 26                                                          05
61    DELT=0.001                                                        05
      IC=5                                                              05
      GO TO 26                                                          05
22    IB=P*5.+6.E-03                                                    05
      DELT=0.01*FLOAT(IB)                                               05
      CL=.FALSE.                                                        05
      IF(L.EQ.(IB+1)) GO TO 26                                          05
      L=IB+1                                                            05
      GO TO 23                                                          05
24    CL=.FALSE.                                                        05
```

```
      FL=.TRUE.
      DELT=DELM
      L=100.*DELM+1.E-03+1.
23    CALL PCN06(N,TU,TY,GA,IND,DELT,M)
26    IA=2.*P/DELT+1.E-05
      DO3J=1,ID
      D=DMP(J)
C
C     *****COMPUTATION OF MATRICES A AND B*****
C
      CALL PCN07(D,W,L,A,B,DELT)
C
C     ********** INITIATION **********
C
      DO4IB=1,3
4     X(IB)=0.0
      DMAX=0.0
      VMAX=0.0
      AMAX=0.0
      DW=2.*D*W
      W2=W**2
      I=1
7     IB=IND(I)
C
C     *****COMPUTATION OF RESPONSE FOR PERIOD.LT.0.2 SEC.*****
C
      IF(CL) GO TO 56
      GO TO 27
56    SL=(GA(I+1)-GA(I))*DELT/0.005
      DO 28 IX=1,IC
      G(1)=GA(I)+FLOAT(IX-1)*SL
      G(2)=GA(I)+FLOAT(IX)*SL
      TX(1)=A(IB,1,1)*X(1)+A(IB,1,2)*X(2)-B(IB,1,1)*G(1)-
     1B(IB,1,2)*G(2)
      TX(2)=A(IB,2,1)*X(1)+A(IB,2,2)*X(2)-B(IB,2,1)*G(1)-
     1B(IB,2,2)*G(2)
      TX(3)=-(DW*TX(2)+W2*TX(1))
      IF(ABS(TX(1)).LE.ABS(DMAX)) GO TO 34
C
C     *****MONITORING THE MAX. VALUES*****
C
      DMAX=TX(1)
34    X(1)=TX(1)
      IF(ABS(TX(2)).LE.ABS(VMAX)) GO TO 35
      VMAX=TX(2)
35    X(2)=TX(2)
      IF(ABS(TX(3)).LE.ABS(AMAX)) GO TO 36
      AMAX=TX(3)
36    X(3)=TX(3)
28    CONTINUE
      GO TO 29
C
C     *****COMPUTATION OF RESPONSE FOR PERIOD.GE.0.2 SEC.*****
C
27    G(1)=GA(I)
      G(2)=GA(I+1)
      TX(1)=A(IB,1,1)*X(1)+A(IB,1,2)*X(2)-B(IB,1,1)*G(1)-
     1B(IB,1,2)*G(2)
      TX(2)=A(IB,2,1)*X(1)+A(IB,2,2)*X(2)-B(IB,2,1)*G(1)-
     1B(IB,2,2)*G(2)
      TX(3)=-(DW*TX(2)+W2*TX(1))
```

```
C                                                                       05   123
C     *****MONITORING THE MAX. VALUES*****                              05   124
C                                                                       05   125
      IF(ABS(TX(1)).LE.ABS(DMAX)) GO TO 44                              05   126
      DMAX=TX(1)                                                        05   127
44    X(1)=TX(1)                                                        05   128
      IF(ABS(TX(2)).LE.ABS(VMAX)) GO TO 45                              05   129
      VMAX=TX(2)                                                        05   130
45    X(2)=TX(2)                                                        05   131
      IF(ABS(TX(3)).LE.ABS(AMAX)) GO TO 46                              05   132
      AMAX=TX(3)                                                        05   133
46    X(3)=TX(3)                                                        05   134
C                                                                       05   135
C     *****TEST FOR END OF INTEGRATION*****                             05   136
C                                                                       05   137
29    I=I+1                                                             05   138
      IF(I.GE.(M+IA)) GO TO 8                                           05   139
      IF(I.LT.M) GO TO 7                                                05   140
      IF(I.GT.M) GO TO 9                                                05   141
      VEND = SQRT(X(2)**2 + W2*(X(1)**2) )                              05   142
      GA(I+1)=0.0                                                       05   143
      IND(I)=L                                                          05   144
      GO TO 7                                                           05   145
9     GA(I+1)=0.0                                                       05   146
      IND(I)=L                                                          05   147
      GO TO 7                                                           05   148
8     SD(J,K)=DMAX                                                      05   149
      SV(J,K)=VMAX                                                      05   150
      SA(J,K)=AMAX                                                      05   151
      IF(D.GE.1.E-03) GO TO 5                                           05   152
      FS(K)=VEND                                                        05   153
5     CONTINUE                                                          05   154
3     CONTINUE                                                          05   155
1     CONTINUE                                                          05   156
      RETURN                                                            05   157
       END                                                              05   158
```

```
C     ***************************************************************** 06    1
C                                                                       06    2
C          SUBROUTINE FOR SUBDIVISION AND INDEXING OF ROUNDED RECORD    06    3
C                              PCN06                                    06    4
C                                                                       06    5
C     ***************************************************************** 06    6
C                                                                       06    7
      SUBROUTINE PCN06(N,RT,TY,GA,IND,DELP,J)                           06    8
      DIMENSION RT(1),TY(1),GA(1),IND(1)                                06    9
      M=N-1                                                             06   10
      J=1                                                               06   11
      GA(1)=TY(1)                                                       06   12
      DO2I=1,M                                                          06   13
      DELM=DELP                                                         06   14
      DEL=RT(I+1)-RT(I)                                                 06   15
      SL=(TY(I+1)-TY(I))/DEL                                            06   16
      IF(DEL.LT.5.1E-03) GO TO 6                                        06   17
3     IF(DEL.GE.(DELM-2.E-04)) GO TO 1                                  06   18
      DELM=DELM-0.01                                                    06   19
      GO TO 3                                                           06   20
1     GA(J+1)=GA(J)+SL*DELM                                             06   21
      IND(J)=DELM*100.+1.E-03+1.                                        06   22
      J=J+1                                                             06   23
      DEL=DEL-DELM                                                      06   24
      IF(DEL.GT.5.1E-03) GO TO 3                                        06   25
6     IF(DEL.GT.1.E-03) GO TO 4                                         06   26
      GA(J)=TY(I+1)                                                     06   27
      GO TO 2                                                           06   28
4     GA(J+1)=TY(I+1)                                                   06   29
      IND(J)=1                                                          06   30
      J=J+1                                                             06   31
2     CONTINUE                                                          06   32
      RETURN                                                            06   33
       END                                                              06   34
```

```
C     ****************************************************************
C
C          SUBROUTINE FOR COMPUTATION OF MATRICES A AND B
C                              PCN07
C
C     ****************************************************************
C
      SUBROUTINE PCN07(D,W,L,A,B,DELT)
      DIMENSION A(10,2,2),B(10,2,2),TY(10)
      IF(DELT.GT.(0.005+1.E-04)) GO TO 3
      TY(1)=DELT
      GO TO 4
3     DEL=0.01
      TY(1)=0.005
      DO 1 J=2,L
      TY(J)=DEL
1     DEL=DEL+0.01
4     DO2J=1,L
      DEL=TY(J)
      DW=D*W
      D2=D**2
      A0=EXP(-DW*DEL)
      A1=W*SQRT(1.-D2)
      AD1=A1*DEL
      A2=SIN(AD1)
      A3=COS(AD1)
      W2=W**2
      A4=(2.*D2-1.)/W2
      A5=D/W
      A6=2.*A5/W2
      A7=1./W2
      A8=(A1*A3-DW*A2)*A0
      A9=-(A1*A2+DW*A3)*A0
      A10=A8/A1
      A11=A0/A1
      A12=A11*A2
      A13=A0*A3
      A14=A10*A4
      A15=A12*A4
      A16=A6*A13
      A17=A9*A6
      A(J,1,1)=A0*(DW*A2/A1+A3)
      A(J,1,2)=A12
      A(J,2,1)=A10*DW+A9
      A(J,2,2)=A10
      B(J,1,1)=(-A15-A16+A6)/DEL-A12*A5-A7*A13
      B(J,1,2)=(A15+A16-A6)/DEL+A7
      B(J,2,1)=(-A14-A17-A7)/DEL-A10*A5-A9*A7
      B(J,2,2)=(A14+A17+A7)/DEL
2     CONTINUE
      RETURN
       END
```