DIGITAL CALCULATION OF RESPONSE SPECTRA
FROM STRONG-MOTION EARTHQUAKE RECORDS

by

Navin C. Nigam and Paul C. Jennings

A report on research conducted under a grant from the National Science Foundation

Pasadena, California

June 1968
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ABSTRACT

This report presents a numerical method for computing response spectra from strong-motion earthquake records. The method is based on the exact solution to the governing differential equation and gives a three to four-fold saving in computing time compared to a third order Runge-Kutta method of comparable accuracy. An analysis was made of the errors introduced at various stages in the calculation of spectra so that allowable errors could be prescribed for the numerical integration. Using the proposed method of computing or other methods of comparable accuracy, example calculations show that the errors introduced by the numerical procedures are much less than the errors inherent in the digitization of the acceleration record.

Included as appendices to the report are computer programs in Fortran IV, with instructions for their use, for computing spectra, for correction of the baseline of the digitized record, and for the computation of ground velocity and displacement.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation or Definition</th>
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<tr>
<td>a(t)</td>
<td>ground acceleration at time t;</td>
</tr>
<tr>
<td>A</td>
<td>$2 \times 2$ matrix;</td>
</tr>
<tr>
<td>B</td>
<td>$2 \times 2$ matrix;</td>
</tr>
<tr>
<td>c</td>
<td>damping coefficient for a simple oscillator;</td>
</tr>
<tr>
<td>C_1, C_2</td>
<td>constants;</td>
</tr>
<tr>
<td>e_A</td>
<td>percentage amplitude distortion in recording;</td>
</tr>
<tr>
<td>e_P</td>
<td>phase distortion in recording (degrees);</td>
</tr>
<tr>
<td>k</td>
<td>stiffness coefficient for a simple oscillator;</td>
</tr>
<tr>
<td>m</td>
<td>mass of the oscillator;</td>
</tr>
<tr>
<td>$\nu$</td>
<td>natural frequency of the element of an accelerograph;</td>
</tr>
<tr>
<td>$S_a$</td>
<td>acceleration spectrum value;</td>
</tr>
<tr>
<td>$S_d$</td>
<td>displacement spectrum value;</td>
</tr>
<tr>
<td>$S_v$</td>
<td>velocity spectrum value;</td>
</tr>
<tr>
<td>t</td>
<td>time;</td>
</tr>
<tr>
<td>T</td>
<td>natural period of a simple oscillator;</td>
</tr>
<tr>
<td>x</td>
<td>relative displacement of a simple oscillator;</td>
</tr>
<tr>
<td>y</td>
<td>relative displacement of the element of an accelerograph;</td>
</tr>
<tr>
<td>$\ddot{z}$</td>
<td>absolute acceleration of the mass of a simple oscillator;</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>input frequency to the accelerograph;</td>
</tr>
<tr>
<td>$\beta$</td>
<td>fraction of critical damping of a simple oscillator;</td>
</tr>
<tr>
<td>$\omega$</td>
<td>natural frequency of a simple oscillator;</td>
</tr>
<tr>
<td>$\xi$</td>
<td>fraction of initial damping of the element of an accelerograph;</td>
</tr>
<tr>
<td>$\Delta t_1$</td>
<td>interval of digitization $t_{1+1} - t_1$</td>
</tr>
<tr>
<td>$\Delta \tau$</td>
<td>interval of integration; and</td>
</tr>
<tr>
<td>$(\Delta \tau)_m$</td>
<td>maximum interval of integration.</td>
</tr>
</tbody>
</table>
INTRODUCTION

Since their introduction in the 1930's and 1940's, response spectra of strong-motion earthquakes have proved useful and informative in problems of design and analysis of structures subjected to strong earthquake motions. The spectra, calculated from the recorded ground acceleration, are plots of the maximum response to the earthquake of a simple oscillator over a range of values of its natural period and damping. These curves provide a description of the frequency characteristics of the ground motion and give the maximum response of simple structures to the earthquake. By superposition of different modes of response, spectrum techniques can be applied to the design and analysis of complex structures such as buildings and dams. Used in this manner the spectrum technique represents an approach intermediate between a design based on static loads and a complete integration of the equations of motion of the complex structure.

Strong-motion earthquake records have been obtained infrequently in the past and the reduction to digital form, or equivalent analog form, and subsequent calculation of spectra has been performed on more or less an individual basis. However, in recent years the number of strong-motion instruments placed in the seismic regions of the world, particularly California, Mexico and Japan, has increased to the point where a major earthquake in these areas will generate a large number of records. The potential volume of the data and the development of the tape-recording accelerograph indicate clearly that rapid and automated
data processing and spectrum calculation procedures are needed. In an effort to fulfill part of this need, this report presents a rapid, accurate method for computation of response spectra of strong-motion earthquakes and includes computer programs in Fortran IV for such calculations.

Response spectra were first obtained by Dickey (1) using a direct mechanical analog and later by Housner et al. (4, 5) using electric analog techniques. The availability of digital computers and a progressive increase in the speed of digital computation in recent years has led to an increasing use of digital computers in the calculation of spectra. The extensive application of the response spectrum to problems in earthquake engineering has sustained interest in methods of calculating spectra and has raised questions regarding accuracy, reproducibility and economy in such calculations. (6, 7, 8, 9)

The digital computation of spectra requires the repeated numerical solution of the response of a simple oscillator to a component of recorded ground acceleration. The motion of the oscillator is described by a second order, linear, inhomogeneous differential equation, and if a digital description of the earthquake record is available, the response can be obtained by numerical integration. Several numerical integration techniques have been used for spectra calculations, for example, the third order Runge-Kutta scheme of integration, has been preferred by some investigators due to its accuracy, long range stability, self-starting feature and because it can be adapted easily to cases in which the excitation is not defined at regular intervals. (11, 15) The truncation error in this method is proportional to $(\Delta \tau)^4$, where $\Delta \tau$
is the normalized interval of integration. Thus by a suitable choice of \( \Delta \tau \) the calculations can be performed to an acceptable degree of accuracy.

An alternative approach to spectra calculations is based on obtaining the exact analytical solution to the governing differential equation for the successive linear segments of the excitation, then using this solution to compute the response at discrete time intervals in a purely arithmetical way.\(^{(6, 10)}\) This method does not introduce numerical approximations in the integration other than those inherent in round-off, so in this sense it is an exact method.

In this report this exact method is used to develop a computation technique which leads to a three to four-fold saving in computing time compared to a third order Runge-Kutta method of comparable accuracy. If the earthquake record is digitized at equal time intervals, the proposed scheme gives spectral values which are exact in the sense mentioned above. If the record is digitized at arbitrary time intervals, however, it is necessary to introduce an approximation into the digitization. An analysis of the errors introduced at various stages in the preparation of response spectra and the results of digitization experiments show that the additional numerical approximations are not detrimental. In addition to the development of these numerical integration techniques, the report includes as appendices computer programs in Fortran IV for the calculation of the spectra, for baseline correction to the digitized record and for computation of ground velocity and displacement.
FORMULATION OF THE EXACT METHOD

Response spectra are plots of the maximum response of a simple oscillator to a component of the ground acceleration, plotted as functions of the natural period and damping of the oscillator. The digital computation of spectra requires the step-wise solution of the equations of motion of the oscillator and monitoring of the maximum values of the response parameters at each step of the integration.

Consider a viscously damped, simple oscillator subjected to the base acceleration $a(t)$ as shown in Fig. 1. The equation of motion of the oscillator is given by

$$m\ddot{x} + c\dot{x} + kx = -ma(t) \quad (2.1)$$

in which $m =$ mass of the oscillator; $c =$ damping coefficient, and $k =$ stiffness of the restoring elements. Dividing by $m$ and setting $\omega = \sqrt{k/m}$ and $c = 2m\omega\beta$ in Eq. 2.1 gives

$$\dddot{x} + 2\beta\omega\dot{x} + \omega^2x = -a(t) \quad (2.2)$$

in which $\beta =$ the fraction of critical damping, and $\omega =$ the natural frequency of vibrations of the oscillator.

Assuming that $a(t)$ may be approximated by a segmentally linear function as shown in Fig. 2, Eq. 2.2, may be written as

$$\dddot{x} + 2\beta\omega\dot{x} + \omega^2x = -a_i - \frac{\Delta a_i}{\Delta t_i} (t - t_i); \quad t_i \leq t \leq t_{i+1} \quad (2.3)$$

with
Fig. 1. - A SIMPLE OSCILLATOR
Fig. 2. - IDEALIZED BASE ACCELERATION
\[ \Delta t_i = t_{i+1} - t_i \] (2.4)

\[ \Delta a_i = a_{i+1} - a_i \]

The solution of Eq. 2.3, for \( t_i \leq t \leq t_{i+1} \) is given by

\[
x = e^{-\beta \omega(t-t_i)} \left[ C_1 \sin \omega \sqrt{1-\beta^2} (t-t_i) + C_2 \cos \omega \sqrt{1-\beta^2} (t-t_i) \right]
- \frac{a_i}{\omega} + \frac{2\beta}{\omega^3} \frac{\Delta a_i}{\Delta t_i} - \frac{1}{\omega^2} \frac{\Delta a_i}{\Delta t_i} (t-t_i) 
\] (2.5)

in which \( C_1 \) and \( C_2 \) are constants of integration. Setting \( x = x_i \) and \( \dot{x} = \dot{x}_i \) at \( t = t_i \) and solving for \( C_1 \) and \( C_2 \), it can be shown that

\[
C_1 = \frac{1}{\omega \sqrt{1-\beta^2}} \left( \beta \omega x_i + \dot{x}_i - \frac{2\beta^2 - 1}{\omega^2} \frac{\Delta a_i}{\Delta t_i} + \frac{\beta}{\omega} a_i \right) 
\] (2.6a)

\[
C_2 = x_i - \frac{2\beta}{\omega^3} \frac{\Delta a_i}{\Delta t_i} + \frac{a_i}{\omega^2} 
\] (2.6b)

Substitution of these values of \( C_1 \) and \( C_2 \) into Eq. 2.5 will show that \( x \) and \( \dot{x} \) at \( t = t_{i+1} \) are given by

\[
\bar{x}_{i+1} = A(\beta, \omega, \Delta t_i) \bar{x}_i + B(\beta, \omega, \Delta t_i) \bar{a}_i 
\] (2.7a)

in which

\[
\bar{x}_i = \begin{pmatrix} x_i \\ \dot{x}_i \end{pmatrix}, \quad \bar{a}_i = \begin{pmatrix} a_i \\ \dot{a}_i+1 \end{pmatrix} 
\] (2.7b)

\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} 
\] (2.7c)
The elements of matrices $A$ and $B$ are given by

$$
a_{11} = e^{-\beta \omega \Delta t_1} \left( \frac{\beta}{\sqrt{1-\beta^2}} \sin \omega \sqrt{1-\beta^2} \Delta t_1 + \cos \omega \sqrt{1-\beta^2} \Delta t_1 \right) $$
$$
a_{12} = e^{\frac{-\rho \omega \Delta t_1}{\sqrt{1-\beta^2}}} \sin \omega \sqrt{1-\beta^2} \Delta t_1 $$
$$
a_{21} = -\frac{\omega}{\sqrt{1-\beta^2}} e^{-\beta \omega \Delta t_1} \sin \omega \sqrt{1-\beta^2} \Delta t_1 $$
$$
a_{22} = e^{-\beta \omega \Delta t_1} \left( \cos \omega \sqrt{1-\beta^2} \Delta t_1 - \frac{\beta}{\sqrt{1-\beta^2}} \sin \omega \sqrt{1-\beta^2} \Delta t_1 \right) $$

$$
b_{11} = e^{-\beta \omega \Delta t_1} \left[ \left( \frac{2\beta^2-1}{\omega^2 \Delta t_1} + \frac{\beta}{\omega} \right) \sin \omega \sqrt{1-\beta^2} \Delta t_1 + \left( \frac{2\beta}{\omega^2 \Delta t_1} + \frac{1}{\omega^2} \right) \cos \omega \sqrt{1-\beta^2} \Delta t_1 \right] $$
$$
- \frac{2\beta}{\omega^3 \Delta t_1} $$
$$
b_{12} = -e^{-\beta \omega \Delta t_1} \left[ \left( \frac{2\beta^2-1}{\omega^2 \Delta t_1} \right) \sin \omega \sqrt{1-\beta^2} \Delta t_1 + \frac{2\beta}{\omega^3 \Delta t_1} \cos \omega \sqrt{1-\beta^2} \Delta t_1 \right] $$
$$
- \frac{1}{\omega^2} + \frac{2\beta}{\omega^3 \Delta t_1} $$

$$
u_{21} = e^{-\beta \omega \Delta t_1} \left[ \left( \frac{2\beta^2-1}{\omega^2 \Delta t_1} \right) \frac{\beta}{\omega} \left( \cos \omega \sqrt{1-\beta^2} \Delta t_1 - \frac{\beta}{\sqrt{1-\beta^2}} \sin \omega \sqrt{1-\beta^2} \Delta t_1 \right) $$
$$
- \left( \frac{2\beta}{\omega^2 \Delta t_1} + \frac{1}{\omega^2} \right) \left( \omega \sqrt{1-\beta^2} \sin \omega \sqrt{1-\beta^2} \Delta t_1 + \beta \omega \cos \omega \sqrt{1-\beta^2} \Delta t_1 \right) \right] $$
$$
+ \frac{1}{\omega^2 \Delta t_1} $$

(2.7d)
$$b_{22} = -e^{-\beta \omega \Delta t_1} \left[ \frac{2\beta^2 - 1}{\omega^2 \Delta t_1} \left( \cos \omega \sqrt{1-\beta^2} \Delta t_1 - \frac{\beta}{\sqrt{1-\beta^2}} \sin \omega \sqrt{1-\beta^2} \Delta t_1 \right) \right]$$

$$- \frac{2\beta}{\omega \Delta t_1} \left( \omega \sqrt{1-\beta^2} \sin \omega \sqrt{1-\beta^2} \Delta t_1 + \beta \omega \cos \omega \sqrt{1-\beta^2} \Delta t_1 \right) - \frac{1}{\omega^2 \Delta t_1}$$

From Eq. 2.3, it follows that the absolute acceleration, $\ddot{z}_i$, of the mass at time $t_i$ is given by

$$\ddot{z}_i = \ddot{x}_i + a_i = - (2\beta \omega \dot{x}_i + \omega^2 x_i) \quad (2.8)$$

Hence, if the displacement and velocity of the oscillator are known at some time $t_0$, the state of the oscillator at all subsequent times $t_i$ can be computed exactly by a step-by-step application of Eqs. 2.7 and 2.8. The computational advantage of this approach lies in the fact that $A$ and $B$ depend only on $\beta$, $\omega$ and $\Delta t_1$. $\beta$ and $\omega$ are constant during the calculation of each spectrum value, and if $\Delta t_1$ is constant also, $x_i$, $\dot{x}_i$ and $\ddot{x}_i$ can be evaluated by the execution of only ten multiplication operations for each step of integration. Matrices $A$ and $B$, defined by the rather complicated expressions, Eqs. 2.7d and 2.7e, need to be evaluated only at the beginning of each spectrum calculation. If varying time intervals are used, it is necessary, in general, to compute $A$ and $B$ at each step of integration. However, by rounding the time coordinates of the record, as discussed in the next section, the number of these matrices needed during the calculation can be reduced to only a few. These, too, can be computed at the beginning.
of the calculation and called upon when needed, thereby saving computing time.

COMPUTATION OF SPECTRA

Introduction

To construct the response spectra, it is necessary to find the maximum values of the displacement, velocity and acceleration during a given excitation. This is done by computing the response at discrete time intervals through Eqs. 2.7 and 2.8 and monitoring the response parameters to retain the maximum values. This process of obtaining the maximum response is approximate because the response is found only at discrete points, whereas the true maxima may occur between such points. This error, called the error of discretization, is inherent in all numerical procedures, but can be bounded within acceptable limits by a suitable choice of the time interval, as discussed in Appendix I.

Under these conditions, the response spectra are given by

\[ S_d(\omega, \beta) = \max_{i=1,N} \left[ x_i(\omega, \beta) \right] \]  
\[ S_v(\omega, \beta) = \max_{i=1,N} \left[ \dot{x}_i(\omega, \beta) \right] \]  
\[ S_a(\omega, \beta) = \max_{i=1,N} \left[ \ddot{x}_i(\omega, \beta) \right] \]

in which \( S_d, S_v \) and \( S_a \) are the spectral values of displacement, velocity, and acceleration respectively, for selected values of damping
and natural frequency; and \( N \) is the total number of discrete points at which the response is obtained.

The spectrum values can be obtained by direct application of Eqs. 2.7 and 2.8 to the digitized record. The details of such computation differ, however, depending on whether the earthquake record is digitized at equal or unequal time intervals.

**Digitization of Earthquake Records**

The digitization of earthquake records is usually carried out in one of the following two ways:

1. **Digitization at equal time intervals.** - The acceleration values are measured at regular time intervals and are assumed to be connected by straight line segments. If the interval of digitization, \( \Delta t \), is sufficiently small, this procedure approximates the profile of the actual earthquake record quite closely. The choice of the interval of digitization depends upon the period range of interest and the nature of the earthquake record; commonly used values range from 0.01 to 0.04 seconds. Analytical methods for assessing the error induced by such a sampling process are available. (9, 12) Digitization at equal time intervals will be a virtual necessity for efficient processing of accelerograms recorded on magnetic tape, and in other instances where analog to digital conversion is made automatically.

2. **Digitization at unequal time intervals.** - The abscissae and ordinates of the points where changes of slope are judged to occur are
measured and the points are again assumed to be connected by straight line segments. This procedure leads to variable time intervals between successive points. The accuracy of this approach is difficult to determine analytically, but should be considerably greater than would be achieved by the same number of sample points at equal intervals. This method is the most convenient for manually controlled digitization techniques.

**Baseline Correction**

The processes of recording and digitization introduce distortions into the ground motion recorded during an earthquake. Such distortions may be defined by

\[ a(t) = \hat{a}(t) + \epsilon(t) \]  \hspace{1cm} (3.2)

in which \( a(t) \) = the digitized acceleration record, \( \hat{a}(t) \) = the true acceleration, and \( \epsilon(t) \) = the error introduced during recording and digitization. A. G. Brady\(^7\) has studied the problem of separating \( \epsilon(t) \) from \( a(t) \), and has concluded that this is accomplished reasonably well by the application of a parabolic baseline correction to the acceleration record with minimization of the mean square of the resulting velocity. Since this operation normally precedes spectral calculations, a computer program in Fortran IV, with instructions, is included as part of this report in Appendix II.

**Computation of Ground Displacement and Velocity**

Because of its direct application to structural dynamics, it is the ground acceleration that is recorded during strong earthquakes.
However, the ground velocity and displacement also contain valuable information. These motions can be obtained by direct integration of the adjusted ground acceleration record, using the recursion formulae given by Berg and Housner\(^{(13)}\). A computer program for this purpose is included in Appendix III of this report.

**Spectra Computation for Equal Time Intervals**

Equations 2.7 show that matrices \( A \) and \( B \) are functions of \( \omega, \beta \) and \( \Delta t_1 \). If \( \Delta t_1 \) is constant, that is, the earthquake record is digitized at equal time intervals, these matrices need to be computed only once for each pair of \( \omega \) and \( \beta \) and the chosen value of \( \Delta \tau \), the interval of integration. Also, since the method does not involve truncation error, it is possible to use larger intervals of integration than in other methods. The choice of the interval of integration, \( \Delta \tau \), is controlled by the interval of digitization (\( \Delta \tau \leq \Delta t \)) and the error due to discretization. The bounds on error due to discretization are examined in Appendix I wherein it is shown that if \( \Delta \tau \leq \frac{T}{70} \) (\( T = \) Natural Period), the error will be less than 1.2 per cent. Considering other errors in the computation of spectra, this choice of interval of integration seems reasonable. In fact, \( \Delta \tau \leq T/10 \) may be satisfactory for most purposes.

Since the third order Runge-Kutta scheme of integration often has been used for the digital computation of spectra, the proposed method is compared to it with regard to accuracy and computing time. To make the comparison, undamped velocity spectrum values were obtained for artificial earthquake No. 7\(^{(14)}\) using a third order
Runge-Kutta scheme, for four values of the interval of integration: 
\( \Delta \tau \leq T/10, T/20, T/40, \) and \( T/80. \) The spectrum values also were 
obtained by the exact method for \( \Delta \tau \leq T/10 \) and \( T/20. \) The results 
of these computations are compiled in Table I, which shows the 
spectral values and the relative values of the computation time for 
different cases.

The following conclusions are drawn from the data in Table I:
1. For the third order Runge-Kutta scheme of integration, it is 
necessary to use \( \Delta \tau \leq \frac{T}{80} \) to get accuracy to three significant figures.
2. For the exact scheme of integration, \( \Delta \tau \leq T/10, \) may be 
acceptable for most practical purposes. It is seen from the last two 
columns of Table I that in some cases the spectral values are the same 
for \( \Delta \tau \leq T/10 \) and \( \Delta \tau \leq T/20. \) This occurs when the maximum value 
has a time coordinate which is a common multiple of both step lengths.
3. For accuracy to three significant figures the exact method, 
with \( \Delta \tau \leq T/20, \) is about 3 to 4 times as fast as the third order 
Runge-Kutta method.
4. Using the IBM 7094 Computer at the California Institute of 
Technology Computing Center, the computing time for the proposed 
method with \( \Delta \tau \leq T/20 \) averages about 1.0 sec execution time per 
spectrum point for the 30 sec record used to compile Table I.

A Fortran IV computer program using the exact method, with 
\( \Delta \tau \leq T/20, \) is included in Appendix IV. The flow chart, instructions 
for the use of the program and sample programs controlling input and 
output are given also.
Spectra Computation for Unequal Time Intervals

If the earthquake record is digitized at unequal time intervals, matrices $A(\omega, \beta, \Delta t_i)$ and $B(\omega, \beta, \Delta t_i)$ in Eqs. 2.7 will, in general, change from step to step, making it necessary to compute them anew at each step of integration. From Eqs. 2.7d and 2.7e, which define the elements of these matrices, it is clear that this computation would take a large amount of time; in fact, it would make the method slower than a comparable third order Runge-Kutta scheme. However, if the inherent limitations of the digitized earthquake record are recognized, it is possible to develop an approximate method of satisfactory accuracy which gives a saving in computing time. The essential feature of this method is a rounding of the time coordinates of the original record to a predetermined accuracy. Then, choosing an appropriate maximum interval of integration, $(\Delta \tau)_m$, and subdividing each time interval of the rounded time record into time intervals equal to or less than $(\Delta \tau)_m$, the calculation proceeds much as before, except that $A(\omega, \beta, \Delta \tau_j)$ and $B(\omega, \beta, \Delta \tau_j)$ for four or five different values of $\Delta \tau_j$ must be calculated. This procedure is illustrated in Table II.

After the time record is rounded and subdivided, the values of the ground acceleration at the additional points created by subdivision are computed by linear interpolation from the original record. The modified record now consists of a set of points along the time axis, spaced at a limited number of known intervals, and the corresponding values of the acceleration. The number and size of the time intervals in any record depend on the way in which rounding of the time coordinates
is carried out and the choice of \( (\Delta \tau)_m \). For the illustrative example presented in Table II, the possibilities are:

I \( \Delta \tau_j = 0.04, 0.03, 0.02 \) and 0.01 (4 time-intervals)

IIa \( \Delta \tau_j = 0.04, 0.03, 0.02, 0.01 \) and 0.005 (5 time-intervals)

IIb \( \Delta \tau_j = 0.04, 0.035, 0.03, 0.025, 0.02, 0.015, 0.01, 0.005 \) (8 time-intervals)

Therefore, if the original record is modified as indicated above, the exact method needs only a limited number of matrices for each pair of \( \omega \) and \( \beta \), and these can be computed before the integration is started. By indexing each possible \( \Delta \tau_j \) and the corresponding matrices, the appropriate matrices can be called at each step of integration.

The procedure outlined above requires that the time coordinates of the record to be rounded to a predetermined decimal fraction. Assuming that the original digital description of the earthquake record is exact, this is an approximation which must lead to random errors in the computation of spectra. However, a digital description of an earthquake at unequal time intervals is obtained by manual, or manually controlled, reading of a graphical record and is subject to certain unavoidable errors. If the round-off is carried out in such a way that effects of the approximation so introduced are well within the errors inherent in the process of digitization, the process outlined above will be acceptable.
Digitization Experiment

G. V. Berg and A. G. Brady have studied the errors introduced during the process of digitization and their effect on the computation of spectra. In the absence of gross personal errors or bias, the errors arise primarily from resolution of the scaling device and from the thickness of the trace which makes the choice of points at which a change of slope occurs somewhat arbitrary. To examine these errors an experiment on digitization was conducted by three persons (X, Y, Z) on the Benson-Lehner Data Reducer 099, at the California Institute of Technology. The first 5 seconds on the Taft, California N21E record were digitized independently by X (5 times), Y (2 times) and Z (2 times), at the highest resolution of the Data Reducer for the record used (0.0015 secs and 0.0001 g). In each case points were taken when a change in slope was judged to occur in the record. Taking one record of each person as a standard, corresponding time coordinates on the other records were subtracted from it to obtain what is called here the self-error of digitization. The mean and standard deviation for each case are shown in Table III. The standard deviations shown in Table III indicate that a careful reader will be consistent within 0.004 sec about 70 per cent of the time, with a mean error very close to zero.

To compare the digitized records obtained by different persons, one of the records of X was taken as an overall standard and the records of Y and Z were subtracted from it to obtain what is called the cross-error of digitization. The mean and standard deviation of the error are shown in Table IV. The standard deviations from the mean shown in Tables III and IV indicate that the cross error is about
four times the self error, thus showing the effect of personal preferences in reading. The consistent, negative means indicate a constant shift in the time axis of the record chosen as a standard as compared to the other records. Such a shift does not effect the deviation about the mean.

A comparable measure of the error introduced by rounding-off the time record can be obtained by subtracting the rounded record from the original record and computing the mean and standard deviation. These were done for one of the records digitized by X, and the results are shown in Table V, for rounding to 0.01 and 0.005 sec.

<table>
<thead>
<tr>
<th>Error in Time Coordinates</th>
<th>Round-off to 0.01 secs</th>
<th>Round-off to 0.005 secs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (sec)</td>
<td>0.00003</td>
<td>0.00004</td>
</tr>
<tr>
<td>Standard Deviation (sec)</td>
<td>0.00285</td>
<td>0.00149</td>
</tr>
<tr>
<td>Maximum Error</td>
<td>±0.005</td>
<td>±0.0025</td>
</tr>
</tbody>
</table>

The results of the experiment described above show that the error introduced by rounding-off the time coordinates of an earthquake record to 0.01 or 0.005 secs is well within the error inherent in the process of digitization. For round-off to 0.01 secs, the standard deviation of the error due to round-off is about half of the standard deviation of the self-error of digitization and about one sixth of the standard deviation.
of the cross-error. For round-off to 0.005 secs, this margin is further increased by a factor of 2. From this it is concluded that a round-off to 0.01 or 0.005 secs is an acceptable approximation.

Effect of Rounding Upon Accuracy

G. V. Berg has examined errors in spectra caused by random errors in time and acceleration coordinates and has concluded from an approximate analysis and from computer experimentation that a scatter of the order of 20 per cent in undamped spectra is to be expected from identical computing procedures applied to independently prepared digitizations of the same accelerogram. He attributed the scatter to random errors in reading the accelerogram. Using Berg's approximate formulae for expected error in velocity spectra due to random errors in the time coordinates of the record, and assuming $a = 0.045 \text{ g}$, $(S_v)_{\text{ave}} \approx 20 \text{ ins/sec}$ and $\Delta t = 0.05 \text{ secs}$, it can be shown that for

<table>
<thead>
<tr>
<th>Round-off to 0.01 secs $(\sigma \approx 0.003 \text{ secs})$</th>
<th>Expected Error in $S_v$</th>
<th>Expected Percentage Error in $S_v$ $(D = 30 \text{ secs})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.12 \sqrt{D}$</td>
<td>3.3</td>
<td></td>
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<tr>
<td>Round-off to 0.005 secs $(\sigma \approx 0.0015 \text{ secs})$</td>
<td>$0.06 \sqrt{D}$</td>
<td>1.6</td>
</tr>
</tbody>
</table>

in which $a$ = the r.m.s. value of the ground acceleration, $\sigma$ = standard deviation of the expected error in the time coordinates and $D$ = the duration of the earthquake. These numbers indicate that the error in spectral values due to round-off to 0.01 or 0.005 secs is much less than
the 17 per cent error Berg\textsuperscript{8} found likely to occur as a result of random errors in reading the time coordinates during digitization.

The validity of the above analysis can be checked by comparing the spectra for unrounded and rounded time records. This was done for the N65°E component of the station 2 record of the Parkfield earthquake of June 27, 1966 and the N8°E component of the Lima, Peru earthquake of October 17, 1966. The undamped velocity spectra for the original (unrounded) record and for the records obtained after rounding-off the time coordinates to 0.01 secs and to 0.005 secs are shown in Figs. 3 and 4. For the Parkfield earthquake, the three curves are almost indistinguishable. For the Lima, Peru earthquake the average percentage errors are of the order expected on the basis of Berg's approximate analysis.

The computing time for spectra based on digitized accelerograms rounded to 0.005 secs was found to be only 10 to 15 per cent more than the computing time for records rounded to 0.01 secs. Considering the expected error in the spectral values in the two cases, it is concluded that to be consistent with the choice of $\Delta \tau_m \leq T/20$, round-off to 0.005 secs would be adopted as standard. Here again it may be pointed out that a choice of $\Delta \tau_m \leq T/10$ and round-off to 0.01 secs should prove acceptable for most purposes.

**Intervals of Integration**

The procedure for computing spectra outlined above requires the choice of a maximum interval of integration, $\Delta \tau_m$. Since truncation error is not involved, the choice is governed by the error introduced by
LIMA-PERU EARTHQUAKE
OCT. 17, 1966
N8°E

- UNRounded
→ ROUNDED TO .01 SEC.
→ ROUNDED TO .005 SEC.

MAXIMUM VELOCITY IN INS./SEC

PERIOD IN SECS

β=0.0

Fig. 3. - EFFECT OF Rounding THE TIME COORDINATES ON VELOCITY SPECTRA
discretization, and by computing time. As demonstrated in Appendix I, if \((\Delta \tau)_m \leq T/20\), the error due to discretization is less than 1.2 per cent. This may be treated as one constraint on the choice of \((\Delta \tau)_m\).

The effect of \((\Delta \tau)_m\) on the computing time is more complex in that a larger value of \((\Delta \tau)_m\) tends to reduce the number of integration steps but increases the number of matrices to be calculated. Smaller values of \((\Delta \tau)_m\) imply the reverse results, so a choice of \((\Delta \tau)_m\) should strike a balance between the two effects. The ideal choice of \((\Delta \tau)_m\) would depend on the nature of the accelerogram used, but a compromise satisfactory for most uses was achieved by the choice of \((\Delta \tau)_m = 0.05\) for spectrum periods equal or exceeding one second and \((\Delta \tau)_m \leq T/20\) for periods less than one second.

If the time-coordinates of the record are rounded to 0.005 secs, there is more than one way in which the intervals of digitization can be subdivided, as illustrated in Table II. The choice of subdividing method is governed by the computing time, and to examine this question a few trials were conducted using both the methods in Table II. It was found that in most cases, subdivision method IIa \((\Delta \tau_j = 0.04, 0.03, 0.02, 0.01\) and 0.005) required less computing time.

**Comparison to a Third Order Runge-Kutta Method**

To indicate the effect of truncation error on spectrum values obtained by a Runge-Kutta method and to compare the accuracy and computing time for that method with the techniques developed above, undamped velocity spectra were computed by both methods. The results of these computations are compiled in Table VI.
The results in Table VI show again that the accuracy of the proposed method is only achieved by the Runge-Kutta method with $\Delta \tau \leq T/80$, and that there is a three to four-fold savings in time by the use of the proposed technique.

**Computer Program**

A Fortran IV computer program using the exact method with round-off to 0.005 secs and $(\Delta \tau)_m \leq T/20$ or 0.05 secs, whichever is less, is included in Appendix V. The flow chart and instructions for the use of the program are included along with samples of input and output programs.

**SUMMARY AND CONCLUSIONS**

A numerical method for the calculation of response spectra, based on the exact solution to the governing differential equation, is developed in this report and presented along with computer programs for the application of the method. Computer programs for the baseline correction of accelerograms and for computing ground velocity and displacement are included also.

To adapt the exact solution to efficient computing, the solution is written in the form of two $2 \times 2$ matrices which operate upon the conditions at the beginning of the integration step and upon the acceleration at the beginning and end of the integration step to produce the exact velocity and displacement at the end of the time interval. Because the matrices are functions only of the damping and period of the oscillator
used to calculate spectra and of the magnitude of the time interval, only
a limited number of the matrices need be evaluated. The method is
applied directly to accelerograms digitized at equal time intervals, and
by suitably rounding the time coordinates of records digitized at unequal
time steps, the method can be applied to accelerograms digitized at
unequal time intervals. It is shown that the errors introduced by
rounding the time coordinates are much less than other errors inherent
in the digitization of earthquake records.

The choice of using equal or unequal time intervals for digitization
of accelerograms usually is not based on computational ease, but on the
type of equipment available for the work. However, the choice of digiti-
zation method affects the calculation technique and the computing time.
For the same number of sample points, digitization at unequal time
intervals, usually represents the original record better than does
digitization at equal intervals, and if digitization is done at equal intervals,
a small time step usually must be used to avoid significant distortion of
the record. Because for the usual integration methods the interval of
integration must be less than or equal to the interval of digitization, a
small digitization interval makes it necessary to use a small integration
step where otherwise a larger interval could be used. This leads to an
increase in computing time and is of particular significance to the method
presented herein, because this technique does not involve truncation
error and therefore can be used accurately with time intervals as large
as T/10 to T/20. Thus, an accelerogram digitized at unequal intervals,
with its smaller number of coordinates, can be processed more rapidly
by the proposed method than can the same record digitized at equal time
intervals.

For the accelerograms used as examples in this report, the proposed method showed a three to four-fold saving in time over a third order Runge-Kutta method of comparable accuracy. This was found to be the case for records digitized at both equal and unequal time intervals. For accelerograms with durations of 30 seconds, spectra for 30 periods and 4 values of damping were calculated and plotted by an IBM 7094 computing system with an average computing time of 120 seconds, of which 100 seconds were used for execution of the calculations. If spectra for several earthquakes were to be computed in the same operation, a further savings in computing time could be achieved by storing the matrices used in the integration method rather than recomputing them at the beginning of the processing of each accelerogram.

Because of the increasing interest in the response of yielding structures to earthquake motions, it is appropriate to point out that the method presented herein can be adapted also to the calculation of spectra for bilinear hysteretic and elasto-plastic oscillators. To apply the method it would be necessary to compute two sets of matrices corresponding to the two stiffness coefficients of the structure.

ACKNOWLEDGMENT

Appreciation is extended to the National Science Foundation for partial support of this research under NSF Grant 1197X.
<table>
<thead>
<tr>
<th>Period in secs.</th>
<th>$\Delta T = T/10$</th>
<th>$\Delta T = T/20$</th>
<th>$\Delta T = T/40$</th>
<th>$\Delta T = T/80$</th>
<th>Exact</th>
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<tr>
<td>0.05</td>
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<td>0.184353</td>
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<td>1.938206</td>
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<td>15.019002</td>
<td>15.024426</td>
<td>15.059319</td>
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Note: Spectral values below dotted lines were obtained at $\Delta T = 0.025$ sec. The interval of digitization: 1.5 sec.
**TABLE II**

Round-off and Subdivision of Time Record

\[ (\Delta \tau)_m = 0.04 \text{ secs} \]

<table>
<thead>
<tr>
<th>Step</th>
<th>Original time</th>
<th>Rounded time</th>
<th>Subdivision into intervals of integration - ((\Delta \tau)_m = 0.04)</th>
<th>Remarks</th>
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<tbody>
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<td>I</td>
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<td>10.43</td>
<td>(0.04)</td>
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<tr>
<td></td>
<td>(t_{i+1})</td>
<td>10.5213</td>
<td>10.52</td>
<td>(0.04)</td>
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<td></td>
<td>(\Delta t_i)</td>
<td>0.0946</td>
<td>0.09</td>
<td>(0.01)</td>
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<td>(0.09)</td>
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<td>II</td>
<td>(t_i)</td>
<td>10.4267</td>
<td>10.425</td>
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<td>10.5213</td>
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<td>(\Delta t_i)</td>
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<td>0.095</td>
<td>(0.01)</td>
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<td>(0.095)</td>
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### TABLE II
Self-Error of Digitization in Time Coordinates

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<tr>
<th>Error in Time Coordinates</th>
<th>X</th>
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<th></th>
<th></th>
<th></th>
<th>Y</th>
<th>Z</th>
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<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>Average</td>
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<tr>
<td>Mean (sec)</td>
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<td>Standard Deviation (sec)</td>
<td>0.00432</td>
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### TABLE IV
Cross-Error of Digitization in Time Coordinates

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<tr>
<th>Error in Time Coordinates</th>
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<th>Z</th>
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<tr>
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<tr>
<td>Mean (sec)</td>
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<td>Standard Deviation (sec)</td>
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<tr>
<td>Period in secs.</td>
<td>( S_v ) in ins/sec.</td>
<td>Third Order Runge-Kutta</td>
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<td>( \Delta \tau \leq T/10 )</td>
<td>( \Delta \tau \leq T/20 )</td>
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</table>

Relative Computation Time

| Relative Computation Time | 1.25 | 1.5  | 2.25 | 3.5  | 0.75 | 1    |

Note: Spectral values below dotted lines were calculated for \( (\Delta \tau)_m = 0.05 \) secs.
REFERENCES


APPENDIX I

ERRORS IN SPECTRA

Errors are introduced into response spectra at various stages in the processes leading to their final form. The sources of these errors and approximations for their magnitudes, where possible, are summarized here to estimate the accuracy of spectral calculations and to justify the approximations made in developing the computing technique presented in this report.

There are three broad phases in the computation of spectra: recording, digitization and computation. Some of the errors introduced in these phases are examined below. Errors such as the base line shifts and drifts caused by improper functioning of accelograph, vibration of pendulum suspension and distortion of recording paper prior to digitization are not discussed.

Errors in Recording

The trace produced by a strong-motion accelograph is considered to be a record of ground acceleration during an earthquake. Since the basic element of an accelograph is essentially a damped, simple oscillator, the recorded ground motion is governed by the equation

\[ \ddot{y} + 2\xi \omega_0 \dot{y} + \omega_0^2 y = -a(t) \]  \hspace{1cm} (AI.1)

where \( y \) = displacement of the element relative to base, \( \omega_0 \) = natural period of the element, \( \xi \) = fraction of critical damping and \( a(t) \) = ground acceleration during earthquake.
\[ p^2 y \gg (\ddot{y} + 2\xi \dot{y}) \quad (AI.2) \]

then

\[ p^2 y \approx -a(t) \quad (AI.3) \]

and the graphical record may be regarded as a direct measure of the ground acceleration. Since an earthquake motion contains a range of frequencies, some distortions in amplitude and phase invariably occur, depending upon the damping of the recording element and the ratio of the natural frequency of the element to the input frequencies. From an analysis of the instrument response it can be shown that the percentage of amplitude distortion given by \((16)\)

\[ e_A = 100(\kappa - 1) \quad (AI.4) \]

and the phase distortion in degrees by

\[ e_P = (\phi - 90x) \quad (AI.5) \]

in which \(e_A\) and \(e_P\) are the amplitude and phase errors, respectively, and

\[ r = \frac{\alpha}{\nu} \quad (AI.6) \]

\[ \kappa = \frac{1}{\sqrt{1-r^2}^2 + (\xi r)^2} \quad (AI.7) \]

\[ \phi = \tan^{-1} \left( \frac{2\xi r}{1 - r^2} \right) \quad (AI.8) \]

in which \(\alpha\) is input frequency in radians per second.
For damping of the order of 60 per cent (usually provided in the accelerographs) and $0 < r < 1$, it can be shown\(^{(16)}\) from Eqs. Al.4 and Al.5 that amplitude distortion is less than 10 per cent and phase distortion less than 5 degrees. Thus, the accelerograph will record with this accuracy frequency content from zero up to its natural frequency, which is typically about 15 cycles per second. If the ground motion contains frequencies higher than the natural frequency of the accelerograph, both the amplitude and the phase of this frequency components will be distorted significantly.

**Errors in Digitization**

1. **Scaling Error.** - This error arises from the inherent limitations of the resolving power of any scaling device. For most instruments now in use it is of the order of 0.01 in.

2. **Random Error in Time and Acceleration Records.** - The thickness of the line defining the record makes the choice of points at which discernible changes of slope occur and the scaling of magnitudes a matter of individual judgment. This leads to errors in both time and acceleration coordinates. If the same record is digitized by different persons, the standard deviations of the random errors in time and acceleration typically may be 0.018 secs and 0.001 g respectively. These reading errors may, in turn, cause errors up to 20 per cent in undamped spectra calculated from the records\(^{(6)}\).

3. **Baseline Correction.** - The unknown distortion introduced into
the ground acceleration during recording and digitization is corrected to some degree by adjusting the baseline, herein by a technique which minimizes the resulting ground velocity.

4. Distortion of the Record. - For data sampling at equal time intervals, the cutoff frequency, called the Nyquist frequency, is given by (12)

\[ f_c = \frac{1}{2\Delta t} \quad (A1.9) \]

in which \( \Delta t \) is the sample interval. Such sampling causes aliasing error, since frequency content higher than \( f_c \) is folded into the lower frequency range 0 to \( f_c \) and confused with the data in this lower range. In earthquake records, with closely spaced, sharp peaks and many changes of slope this problem is important and it is necessary to use small intervals of digitization.

**Errors in Computation**

1. Straightline Approximation. - In the digital computation of spectra, the actual earthquake record is replaced by linear segments between the points of digitization. This is a minor approximation provided that the length of the time intervals is much shorter than the periods of interest.

2. Error Due to Discretization. - In any numerical method of computing the spectra, the response is obtained at a set of discrete points. Since spectral values represent maximum values
of response parameters which may not occur at these discrete points, discretization introduces an error which gives spectrum values lower than the true values. The error will be a maximum if the maximum response occurs exactly midway between two discrete points as shown in Fig. A.1. An estimate for the upper bound of this error can be found by noting that at the time of maximum displacement or velocity, the response of the oscillator is nearly sinusoidal at a frequency equal to its natural frequency. Under this assumption the error can be related to the maximum interval of integration $(\Delta \tau)_m$ and the period of the oscillator as shown below.

<table>
<thead>
<tr>
<th>$(\Delta \tau)_m$</th>
<th>Maximum Error (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq T/10$</td>
<td>$\leq 4.9$</td>
</tr>
<tr>
<td>$\leq T/20$</td>
<td>$\leq 1.2$</td>
</tr>
<tr>
<td>$\leq T/40$</td>
<td>$\leq 0.3$</td>
</tr>
</tbody>
</table>

Fig. A.1

3. **Truncation Error.** In general, a truncation error exists in numerical methods for integrating differential equations. For example, in third-order Runge-Kutta methods the error is proportional to $(\Delta \tau)^4$. 
4. **Error Due to Rounding the Time Record.** - For earthquake records digitized at irregular time intervals, the integration technique proposed in this report requires rounding of the time record and the attendant error depends on the way the rounding is done. For round off to 0.005 sec, the average error in spectrum values is expected to be less than 2 per cent.
APPENDIX II

SUBROUTINE FOR BASE LINE CORRECTION

Identification

PCN01 - Fortran IV Subroutine

Purpose

To perform a parabolic base line correction to an earthquake acceleration record, and to return corrected values of the ground acceleration along with the parameters of the parabolic correction.

Method

The corrected value of the acceleration at time \( t \), is given by\(^{(7)}\)

\[
a^*(t) = a(t) - c_0 - c_1 t - c_2 t^2
\]  \hspace{1cm} (AII. 1)

in which \( a(t) \) = the digitized value of the ground acceleration at time \( t \), \( a^*(t) \) = the corrected value of the acceleration at time \( t \), and \( c_0, c_1 \) and \( c_2 \) = are constants. The corrected velocity is given by

\[
v^*(t) = v(t) - c_0 t - \frac{1}{2} c_1 t^2 - \frac{1}{3} c_2 t^3
\]  \hspace{1cm} (AII. 2)

The mean square value of \( v^*(t) \) is minimized if

\[
\frac{\partial}{\partial c_i} \int_0^S [v^*(t)]^2 \, dt = 0 \hspace{0.5cm} i = 0, 1, 2
\]  \hspace{1cm} (AII. 3)

in which \( S \) = the duration of the record.

Equation AII. 3 yields the coefficients \( c_0, c_1 \) and \( c_2 \). The corrected values of the acceleration are computed from Eq. AII. 1.

Usage

Call PCN01 (NDATA, T, GA, VEL, VELO, \( c_0, c_1, c_2 \), DELT, IFEQ)

where
NDATA = number of data points in the acceleration record;
T = an array containing time coordinates;
GA = an array containing acceleration coordinates, before and after correction;
VEL = an array containing velocity coordinates, generated by the subroutine;
VELO = the initial ground velocity (usually set equal to zero);
$c_0$, $c_1$, $c_2$ = coefficients for parabolic correction;
DELT = the interval of digitization, if constant;
IFEQ = a control variable: If set to 1, it implies that
input data is sampled at equal time intervals.
In this case, only GA needs to be specified and the array T is generated by the subroutine.
If IFEQ ≠ 1, it implies that input data is sampled at unequal time intervals and in this case both GA and T must be specified.

Input/Output

Input and Output are controlled by the calling program. Output of the corrected ground acceleration must be obtained in punched cards for subsequent use.

Computation Time

Execution time \(\approx 1.5\) secs for every 100 data points in the earthquake record.
SUBROUTINE FOR PARABOLIC BASE LINE CORRECTION

DIMENSION T(I), GA(I), VEL(I)

BE1 = 0.0
BE2 = 0.0
BE3 = 0.0
T(I) = 0.0
VEL(I) = 0.0
M = NDATA - 1
DEL2 = DELT * DELT
DO 1 I = 1, M
   IF (IFEQ I EQ 1) GO TO 2
   DELT = (T(I+1) - T(I))
   DEL2 = DELT * DELT
   GO TO 6
2 T(I+1) = T(I) + DELT
6 T2 = T(I) * T(I)
   T3 = T2 * T(I)
   T4 = T(I) * T(I+1)
   T5 = T(I+1) * T(I+1)
   T6 = T(I+1) * T5
   VEL(I+1) = VEL(I) + 0.5 * DELT * (GA(I) + GA(I+1))
   BE1 = BE1 + 0.5 * VEL(I) * DELT * (T(I) + T(I+1)) + 0.0416667 * (GA(I) * (3 * T(I) + 1))
   1 + 5.0 * (T(I+1) + GA(I+1) * T(I+1) + 3 * T(I+1)) * DEL2
   BE2 = BE2 + 0.3333333 * VEL(I) * DELT * (T2 + T4 + T5) + 0.0166667 * DEL2 * (GA(I) * T6)
   1 + 4.0 * T2 + 7.0 * T4 + 9.0 * T5 + 11.0 * T6 + 6.0 * T(I+1) * T(I+1)
   2 + GA(I+1) * (T3 + 3.0 * T2 + T(I+1) + 6.0 * T(I) * T5 + 10.0 * T6)
1 CONTINUE
XO = T(NDATA)
BETA1 = BE1 / XO
BETA2 = BE2 / XO
BETA3 = BE3 / XO
CA1 = 300.0 * BETA1 - 900.0 * BETA2 + 630.0 * BETA3
CA2 = -1800.0 * BETA1 + 5760.0 * BETA2 - 4200.0 * BETA3
CA3 = 1890.0 * BETA1 - 6300.0 * BETA2 + 4725.0 * BETA3
C0 = CA1
C1 = CA2 / XO
C2 = CA3 / XO
DO 5 I = 1, NDATA
5 GA(I) = GA(I) - C0 - C1 * T(I) - C2 * T(I) * T(I)
RETURN
END
APPENDIX III
SUBROUTINE FOR COMPUTING
GROUND VELOCITY AND DISPLACEMENT

Identification
PCN02 - Fortran IV Subroutine

Purpose
To compute the ground velocity and displacement for a component of an earthquake by integrating the digitized ground acceleration record.

Method
Starting with given initial conditions, ground velocity, \( v(t) \) and displacement, \( u(t) \) are computed by successive applications of the following formulae due to Berg and Housner\(^{(13)}\).

\[
\begin{align*}
    u(t_{i+1}) &= u_{i+1} = u_i + v_i \Delta t_i + \frac{1}{6} (\Delta t_i)^2 (2a_i+a_{i+1}) \\
    v(t_{i+1}) &= v_{i+1} = v_i + \frac{1}{2} \Delta t_i (a_i+a_{i+1})
\end{align*}
\] (AIII. 1)

in which \( u(t) \) = the ground displacement at time \( t \), \( v(t) \) = the ground velocity at time \( t \), \( a(t) \) = the ground acceleration at time \( t \), and \( \Delta t_i = t_{i+1} - t_i \).

Usage
Call PCN02 (NDATA, T, GA, VEL, DISP, VELO, DISPO, DELT, IFEQ)
where

- \( \text{NDATA} \) = number of data points in the acceleration record;
- \( \text{T} \) = an array containing time coordinates of the original accelerogram;
- \( \text{GA} \) = an array containing acceleration coordinates;
- \( \text{VEL} \) = an array containing the ground velocity coordinates computed by the subroutine;
- \( \text{DISP} \) = an array containing the ground displacement.
coordinates computed by the subroutine;

\[ \text{VELO} = \text{the initial velocity to be set by the user (usually } \text{VELO is set equal to zero}); \]

\[ \text{DISPO} = \text{the initial displacement to be set by the user} \]

(usually \( \text{DISPO} \) is set equal to zero);

\[ \text{DELT} = \text{the interval of digitization, if constant;} \]

\[ \text{IFEQ} = \text{a control variable: if set to 1, it implies} \]

that input data is sampled at equal time intervals.

In this case, only \( \text{GA} \) needs to be specified

and the array \( \text{T} \) is generated by the subroutine.

If \( \text{IFEQ} \neq 1 \) it implies that input data is sampled

at unequal time intervals and in this case both

\( \text{GA} \) and \( \text{T} \) must be specified.

**Input/Output**

Input and Output are controlled by the calling program.

**Computation Time**

Execution time \( \approx 3.0 \text{ secs for every 100 data points in the earthquake record.} \)
***********************

SUBROUTINE FOR GROUND DISPLACEMENT AND VELOCITY

PCN02

***********************

SUBROUTINE PCN02(NDATA,T,GA,VEL,DISP,VELO,DISPO,DELT,IFEQ)
DIMENSION T(1),GA(1),VEL(1),DISP(1)
DISP(1)=DISPO
VEL(1)=VELO
T(1)=0.0
M=NDATA-1
DO 2 I=1,M
1 IF(IFEQ.EQ.1) GO TO 1
2 POT=T(I+1)-T(I)
GO TO 3
1 T(I+1)=T(I)+DELT
3 DISP(I+1)=DISP(I)+VEL(I)*DELT+0.16666667*DELT*DELT*(2.*GA(I)+
1*GA(I+1))
2 VEL(I+1)=VEL(I)+0.5*DELT*(GA(I)+GA(I+1))
RETURN
END
APPENDIX IV

SUBROUTINES FOR COMPUTING SPECTRA FROM EARTHQUAKE RECORDS DIGITIZED AT EQUAL TIME INTERVALS

Identification

PCN03
PCN04

Fortran IV Subroutines

Purpose

To compute response spectra from earthquake records digitized at equal time intervals.

Method

The response of a damped, simple oscillator is calculated by successive application of Eqs. 2.7 and 2.8 and spectrum values are obtained by monitoring the maximum values of the response.

Usage

The computer program consists of the following:

1. Main program for Input/Output
2. Subroutine PCN03 for computation of spectra
3. Subroutine PCN04 for computation of matrices A and B

Subroutine PCN03 is called in the main program by the statement

Call PCN03 (ID, IP, N, DEL, DMP, PD, GA, SF, SD, SV, SA, FS)

where

ID = the number of damping values;
IP = the number of periods;
N = the number of sample points in the earthquake record
DEL = the interval of digitization;
DMP (I) = an array containing damping values as fractions of critical damping;
PD (I) = an array containing values of the periods;
GA (I) = an array containing acceleration coordinates;
SF = the scale factor for acceleration;
SD (I, J) = the displacement spectrum values for different
generated by the subroutine;
SV (I, J) = the velocity spectrum values for different
generated by the subroutine
SA (I, J) = the acceleration spectrum values for different
generated by the subroutine;
FS (I) = an array containing velocity at the end of the
excitation, generated by the subroutine. For the undamped oscillator, this array gives the
Fourier spectrum of the acceleration record.

Subroutine PCN04 is called within subroutine PCN03.

Input/Output

The input and output are controlled by the main program. A sample of such a program, including both printed and plotted output, is given below.

Remarks

The computer programs, in Fortran IV, for subroutines PCN03 and PCN04 are based on the following criteria:
1. Interval of integration, $\Delta t \leq \frac{\text{Period}}{20}$ or interval of digitization, whichever is less.

2. The response of an undamped oscillator tends to build up with time during the excitation so that spectrum values often occur toward the end of the record. For this reason the record is assumed to continue with zero acceleration after the last data point until such time as the relative velocity had changed sign three times. This procedure insures that true maxima (for the record given) are found.

3. The user must prescribe the dimension statements in the main program (I/O) and the subroutines, depending upon the data points in the earthquake record and the number of damping and period values for which spectra are to be obtained. The dimension of the array GA, in the main program and subroutine PCN03 must be $> N + 50$ or $N + 2*\text{PMAX}/\text{DEL}$, whichever is greater, where PMAX = the maximum value of the period for which spectra are computed.

4. Average execution time = 1.0 sec per spectrum point for records of 30 second duration.
SAMPLE PROGRAM FOR INPUT/OUTPUT

SPECTRA CALCULATION FOR EARTHQUAKE RECORDS DIGITIZED
AT EQUAL TIME INTERVALS

DIMENSION DMP(5),PD(50),GA(3000),SD(5,50),SV(5,50),SA(5,50),
1VM(50),FS(50)

LOGICAL CL

*************** DATA READ-IN ******************

CL=.FALSE.
READ(5,100) ID,IP,N,DEL,DF,PM
READ(5,101)(DMP(D),J=1,ID)
READ(5,102)(PD(J),J=1,IP)
READ(5,103) (GA(J),J=1,N)
CALL PCNS1(ID,IP,N,DEL,UMP,PD,GA,FS,SD,SV,SA,FS)

*************** OUTPUT-PRINT ******************

DO 3 J=1,ID
3 D=DMP(J)
   IF(D.GT.1.E-03) GO TO 24
   WRITE(6,104) D
   GO TO 23
24   WRITE(6,105) D
23   DO 4 I=1,IP
4   P=PD(I)
   DMAX=SD(J,I)
   VMAX=SV(J,I)
   AMAX=SA(J,I)
   IF(P.GE.1.E-03) GO TO 25
   VEND=FS(I)
   GO TO 26
25   IF(D.GE.1.E-03) GO TO 26
   WRITE(6,106) P,DMAX,VMAX,VEND,AMAX
   GO TO 20
26   WRITE(6,107) P,DMAX,VMAX,AMAX
20   VM(I)=ABS(VMAX)
   CONTINUE

*************** OUTPUT-PLT ******************

IF(CL) GO TO 21
   CL=.TRUE.
   VM=ABS(SV(1,1))
   DO 5 IX=2,IP
      IF(ABS(SV(1,IX)).LE.VM) GO TO 5
      VM=ABS(SV(1,IX))
5   CONTINUE
   IF(VMS.GE.100.) GO TO 51
   IF(VMS.GT.50.) GO TO 52
   VMS=50.
   GO TO 21
51   VMS=200.
   GO TO 21
52   VMS=100.
   CALL XYPLOT(IP,PD,VM,0.0,PM,0.0,VMS,DD,0)
CONTINUE

FORMAT (3I4, 3F10.4)

FORMAT (4F5.3)

FORMAT (12F5.2)

FORMAT (6F11.7)

FORMAT (5X, 8HDAMPING=, F4.2, 5X, 6HPERIOD, 5X, 4HMAX, 8X, 4HMAX, 8X, 14HVEND, 10X, 4HAMAX/6X, 4HSEC, 6X, 4HINS, 7X, 8HINS, SEC, 4X, 4HINS, SEC/SEC, 44X12HINS, SEC/SEC)

FORMAT (5X, 8HDAMPING=, F4.2, 5X, 6HPERIOD, 5X, 4HMAX, 8X, 4HMAX, 10X, 14HAMAX/6X, 4HSEC, 6X, 4HINS, 7X, 8HINS, SEC, 4X, 12HINS, SEC/SEC)

FORMAT (6X, F4.2, 4X, F7.4, 5X, F8.4, 4X, F7.4, 7X, F8.4)

STOP

END

********** EXPLANATION **********

PMS = MAX. VALUE OF PERIOD (ABSCISSA) FOR PLOT

VMS = MAX. VALUE OF VELOCITY (ORDINATE) FOR PLOT
SUBROUTINE FOR COMPUTATION OF SPECTRA FROM EARTHQUAKE RECORD
DIGITIZED AT EQUAL TIME INTERVALS

PCN03

SUBROUTINE PCN03(D,IP,N,DEL,DMP,P,D,G,A,S,F,S,F,S,F,S,F)
DIMENSION X(3),G(2),DMP(5),P(50),A(2,2),B(2,2),GA(3000),TY(3),
SD(5,1),SV(5,1),SA(5,1),FS(5)
DO4,J=1,ID
D=DMP(J)
DO4,K=1,ID
P=P*DMP(K)
W=2.*3.*141592654/P

***CHOICE OF INTERVAL OF INTEGRATION*****
DELP=P/20.
L=DELP/DELP+1.*1.E-05
DELT=DELP/FLOAT(L)

***COMPUTATION OF MATRICES A AND B*****
CALL PCN04(D,W,DELT,A,B)

********INITIATION***********
X(1)=0.0
X(3)=0.0
X(2)=0.0
DMAX=0.0
VMAX=0.0
AMAX=0.0
I=1
DW=2.*W#D
W2=W**2
IA=2.*P/DELT+1.*E-05

*****COMPUTATION OF RESPONSE*****
SL=(G(A(I+1)-G(A(I)))/FLOAT(L)
DO6,M=1*L
G(1)=(G(A(I)+SL*FLOAT(M-1)))/SF
G(2)=(G(A(I)+SL*FLOAT(M)))/SF
TY(1)=A(1,1)*X(1)+A(1,2)*X(2)-B(1,1)*G(1)-B(1,2)*G(2)
TY(2)=A(2,1)*X(1)+A(2,2)*X(2)-B(2,1)*G(1)-B(2,2)*G(2)
TY(3)=-(DW*TY(2)+W2*TY(1))

******MONITORING THE MAX. VALUES*****
IF(ABS(TY(1)).LE.ABS(DMAX)) GO TO 14
DMAX=TY(1)

14 X(1)=TY(1)
IF(ABS(TY(2)).LE.ABS(VMAX)) GO TO 15
VMAX=TY(2)

15 X(2)=TY(2)
IF(ABS(TY(3)).LE.ABS(AMAX)) GO TO 16
AMAX=TY(3)

16 X(3)=TY(3)
CONTINUE

****TEST FOR END OF INTEGRATION****

I = I + 1
IF (I .EQ. N) GO TO 18
GO TO 19
VEND = SQRT (X(2)**2 + W2*(X(1)**2))
IF (I .EQ. (N + IA)) GO TO 8
IF (I .GE. N) GO TO 17
GO TO 7
GA(I + 1) = 0.0
GO TO 7
IF (D .GE. 1.0E-03) GO TO 26
FS(K) = VEND
SD(J*K) = DMAX
SV(J*K) = VMAX
SA(J*K) = AMAX
CONTINUE
CONTINUE
RETURN
END
SUBROUTINE PCN04(D,W,DELT,A,B)
DIMENSION A(2,2),B(2,2)
D=W*D
D2=D**2
A0=EXP(-D*DELT)
A1=W*SQR(1.0-D2)
A2=SIN(A1)
A3=COS(A1)
W2=W**2
A4=(2.0*D2-1.0)/W2
A5=D/W
A6=2.0*A5/W2
A7=1.0/W2
A8=(A1*A3-DW*A2)*A0
A9=-(A1*A2+DW*A3)*A0
A10=A8/A1
A11=A0/A1
A12=A11*A2
A13=A0*A3
A14=A10*A4
A15=A12*A4
A16=A6*A13
A17=A9*A6
A(1,1)=A0*(DW*A2/A1+A3)
A(1,2)=A12
A(2,1)=A10*DW+A9
A(2,2)=A10
B(1,1)=(-A15-A16+A6)/DELT-A12*A5-A7*A13
B(1,2)=(A15+A16-A6)/DELT+A7
B(2,1)=(-A14-A17-A7)/DELT-A10*A5-A9*A7
B(2,2)=(A14+A17+A7)/DELT
RETURN
END
APPENDIX V

SUBROUTINES FOR COMPUTING SPECTRA FROM EARTHQUAKE RECORDS DIGITIZED AT UNEQUAL TIME INTERVALS

Identification

PCN05
PCN06
PCN07

Fortran IV Subroutines

Purpose

To compute response spectra from earthquake records digitized at unequal time intervals.

Method

The response of a damped, simple oscillator is calculated by successive application of Eqs. 2.7 and 2.8 and spectrum values are obtained by monitoring the maximum values of the response.

Usage

The computer program consists of the following:

1. Main program for input/output
2. Subroutine PCN05 for computation of spectra
3. Subroutine PCN06 for subdivision and indexing of time coordinates
4. Subroutine PCN07 for computation of matrices A and B.

Subroutine PCN05 is called in the main program by the statement

Call PCN05 (ID, IP, N, DELM, DMP, PD, TY, RT, SFT, SFA, SD, SV, SA, FS)

where
ID = the number of damping values;
IP = the number of periods;
N = the number of sample points in the earthquake record;
DELM = the upper limit on the size of the maximum interval of integration (suggested value of DELM = 0.05 sec);
DMP (I) = an array containing damping values as fractions of critical damping;
PD (I) = an array containing values of periods;
TY (I) = an array containing acceleration coordinates;
RT (I) = an array containing time coordinates;
SFT = the scale factor for time;
SFA = the scale factor for acceleration;
SD (I, J) = the displacement spectrum values for different values at damping (I) and periods (J), generated by the subroutine;
SV (I, J) = the velocity spectrum values for different values of damping (I) and periods (J), generated by the subroutine;
SA (I, J) = the acceleration spectrum values for different values of damping (I) and periods (J), generated by the subroutine;
FS (I) = an array containing velocity at the end of the excitation, generated by the subroutine. For the undamped oscillator, this array gives the Fourier spectrum of the acceleration.

Subroutines PCN06 and PCN07 are called within subroutine PCN05.
Input/Output

The input and output are controlled by the main program. A sample of such a program is given below.

Remarks

The computer programs, in Fortran IV, for subroutines PCN05, PCN06 and PCN07 are based on the following criteria.

1. Maximum interval of integration. \((\Delta \tau)_m \leq \frac{Period}{20}\), the interval of digitization or 0.05 secs, whichever of the three is least.

2. The time coordinates of the accelerogram are rounded by the programs to the nearest 0.005 sec.

3. The record is assumed to continue with zero acceleration after the last data point until such time as the relative velocity had changed sign three times.

4. The user must prescribe the dimension statements in the main program (I/O) and the subroutines, depending upon the data points in the earthquake record and the number of damping and period values for which spectra are to be obtained. The dimensions of various arrays should be fixed as follows:

<table>
<thead>
<tr>
<th>Array</th>
<th>Dimension</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT,TY,TU</td>
<td>( \geq N )</td>
<td></td>
</tr>
<tr>
<td>GA,IND</td>
<td>( \geq (200*D+100) ) or ( (N+50) )</td>
<td>For periods less than 0.2 secs.</td>
</tr>
<tr>
<td></td>
<td>( \geq \max \left( \frac{20*D}{P} + 200 \right) )</td>
<td>For periods greater than or equal to 0.2 secs.</td>
</tr>
<tr>
<td></td>
<td>( \left( \frac{D}{DELM} + \frac{2*P\text{MAX}}{DELM} + 100 \right) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{or} (N+50) ) whichever is greater</td>
<td></td>
</tr>
</tbody>
</table>

where \( D \) = duration of the earthquake record in seconds, \( P \) = value of
the periods in seconds and PMAX = the maximum value of the periods.

5. Average execution time = 0.8 sec for spectrum point for a record of 30 sec duration.
Start

Read Data In

Round-Off Time Coordinates to 0.005 sec

Call PCN05

Set Period (T)

Choose Maximum Interval of Integration

Subdivide and Index the Time Coordinates and Compute Accln. Coordinates at New Data Points

Call PCN06

Set Damping (β)

Compute

A(ω, β, Δτ₁)
B(ω, β, Δτ₂)

Call PCN07

Initial Conditions

Compute and Monitor Max. Response

Write and Plot

End
SAMPLE PROGRAM FOR INPUT/OUTPUT

SPECTRA CALCULATION FOR EARTHQUAKE RECORDS DIGITIZED
AT UNEQUAL TIME INTERVALS

DIMENSION DMP(5), PD(50), RT(1000), TY(1000), SD(5), SV(5), SA(5), FS(5), VMP(50)
LOGICAL CL

DATA READ-IN

CL=.FALSE.
READ(5,101)ID, IP, N, DELM, SFT, SFA, PMS
READ(5,102) (DMP(J), J=1, ID)
READ(5,103) (PD(J), J=1, IP)
READ(5,104) (RT(J), TY(J), J=1, N)
CALL PCN05(ID, IP, N, DELM, DMP, PD, RT, SFT, SFA, SD, SV, SA, FS)

OUTPUT - PRINT

DO 10 J=1, ID
D=DMP(J)
IF(D.GT.1.E-05) GO TO 12
WRITE(6,108) D
GO TO 15
12 WRITE(6,105) D
15 DO 14 K=1, IP
DMAX=SD(J,K)
VMAX=SV(J,K)
AMAX=SA(J,K)
P=PD(K)
VMP(K)=ABS(VMAX)
IF(D.GT.1.E-05) GO TO 13
VEND=FS(K)
WRITE(6,106) P, DMAX, VMAX, VEND, AMAX
GO TO 14
13 WRITE(6,107) P, DMAX, VMAX, AMAX
14 CONTINUE

OUTPUT - PLOT

IF(CL) GO TO 34
CL=.TRUE.
VMS=ABS(SV(1,1))
DO 5 IX=2, IP
IF(ABS(SV(1,IX)).LE.VMS) GO TO 5
VMS=ABS(SV(1,IX))
5 CONTINUE
IF(VMS.LE.50) GO TO 18
IF(VMS.LE.100) GO TO 57
VMS=200.
GO TO 34
18 VMS=50.
GO TO 34
57 VMS=100.
34 CALL XYPL0T(IP, PD, VMP, 0.0, PMS, 0.0, VMS, DD, 0)
10 CONTINUE
**Explanation**

PMS = MAX. VALUE OF PERIOD (ABSCISSA) FOR PLOT
VMS = MAX. VALUE OF VELOCITY (ORDINATE) FOR PLOT
*****SCALING AND ROUNDIING-OFF THE TIME COORDINATES TO 0.005 SECS*****

DO 20 J=1,N
RT(J)=RT(J)*SFT
IND(J)=RT(J)*100.+0.5
TU(J)=FLOAT(IND(J))/100.
DIF=RT(J)-TU(J)
IF(ABS(DIF).GE.0.0025) GO TO 91
GO TO 20

91 TU(J)=TU(J)+SIGN(1.*DIF)*0.005

TY(J)=TY(J)*SFA
DO1K=1,IP
P=PD(K)
W=6.28318/P

*****CHOICE OF INTERVAL OF INTEGRATION AND SUBDIVISION WITH INDEXING*****

TY1=20.*DELM+1.*E-05
IF(FL) GO TO 26
IF(P.GT.TY1) GO TO 24
IF(P.GT.(0.2-1.*E-03)) GO TO 22
IF(CL) GO TO 60
DELT=0.005
CL=.TRUE.
L=1
CALL PCN06(N,TU,TY,GA,IND,DELT,M)

60 IF(P.LT.(0.1-1.*E-04)) GO TO 25
DELT=0.005
IC=1
GO TO 26

25 IF(P.LT.(0.05-1.*E-04)) GO TO 61
DELT=0.0025
IC=2
GO TO 26

61 DELT=0.001
IC=5
GO TO 26

22 IR=P*5.+6.*E-03
DELT=0.01*FLOAT(IB)
CL=.FALSE.
IF(FL.EQ.(IB+1)) GO TO 26
L=IND+1
GO TO 23

24 CL=.FALSE.
FL = TRUE
DELT = DELM
L = 100.0 * DELM + 1.0 * E-03 + 1
CALL PCNO6(NMC, NM, NTY, GA, IND, DELT, M)
IA = 2.0 * P / DELT + 1.0 * E-05
DO 3 J = 1, I1
D(J) = DMP(J)
3 CONTINUE

***** COMPUTATION OF MATRICES A AND B *****
CALL PCNO7(D, W, L, A, B, DELT)

******** INITIATION *********

DO 4 IB = 1, 3
X(IB) = 0.0
DMAX = 0.0
VMAX = 0.0
AMAX = 0.0
DW = 2.0 * D*W
W2 = W**2
I = 1
4 IB = IND(I)

***** COMPUTATION OF RESPONSE FOR PERIOD LT 0.2 SEC. *****
IF(CL) GO TO 56
GO TO 27
56 SL = (A(I) + GA(I) - DELT / 0.005)
DO 28 IX = 1, IC
G(I) = GA(I) + FLOAT(IX-1) * SL
G(2) = GA(I) + FLOAT(IX) * SL
TX(1) = A(IB, 1) * X(1) + A(IB, 1, 2) * X(2) - B(IB, 1, 1) * G(1) -
1B(IB, 1, 2) * G(2)
TX(2) = A(IB, 2) * X(1) + A(IB, 2, 2) * X(2) - B(IB, 2, 1) * G(1) -
1B(IB, 2, 2) * G(2)
TX(3) = -(DW*TX(2) + W2*TX(1))
IF(ABS(TX(1)) .LE. ABS(DMAX)) GO TO 34

***** MONITORING THE MAX. VALUES *****
DMAX = TX(1)
X(1) = TX(1)
IF(ABS(TX(2)) .LE. ABS(VMAX)) GO TO 35
VMAX = TX(2)
X(2) = TX(2)
IF(ABS(TX(3)) .LE. ABS(AMAX)) GO TO 36
AMAX = TX(3)
X(3) = TX(3)
28 CONTINUE
GO TO 29

***** COMPUTATION OF RESPONSE FOR PERIOD GE 0.2 SEC. *****
G(I) = GA(I)
G(2) = GA(I+1)
TX(1) = A(IB, 1, 1) * X(1) + A(IB, 1, 2) * X(2) - B(IB, 1, 1) * G(1) -
1B(IB, 1, 2) * G(2)
TX(2) = A(IB, 2, 1) * X(1) + A(IB, 2, 2) * X(2) - B(IB, 2, 1) * G(1) -
1B(IB, 2, 2) * G(2)
TX(3) = -(DW*TX(2) + W2*TX(1))
C

*****MONITORING THE MAX. VALUES*****

IF (ABS(TX(1)) .GE. ABS(DMAX)) GO TO 44
DMAX=TX(1)

44 X(1)=TX(1)

IF (ABS(TX(2)) .LE. ABS(VMAX)) GO TO 45
VMAX=TX(2)

45 X(2)=TX(2)

IF (ABS(TX(3)) .LE. ABS(AMAX)) GO TO 46
AMAX=TX(3)

46 X(3)=TX(3)

C

*****TEST FOR END OF INTEGRATION*****

29 I=I+1

IF (I .GE. (M+1)) GO TO 31
IF (I .LT. M) GO TO 7
IF (I .GT. M) GO TO 9

VEND = SQRT (X(2)**2 + W2*(X(1)**2)
GA(I+1)=0.0
IND(I)=L
GO TO 7

0 GA(I+1)=0.0
IND(I)=L
GO TO 7

8 SD(J*K)=DMAX
SV(J*K)=VMAX
SA(J*K)=AMAX
IF (DGE1.03) GO TO 5
FS(K)=VEND
CONTINUE
5 CONTINUE
3 CONTINUE
1 CONTINUE
RETURN
END
SUBROUTINE PCNX6(N, RT, TY, GA, IND, DELP, J)
DIMENSION RT(1), TY(1), GA(1), IND(1)
M=N-1
J=1
GA(1)=TY(1)
DO2I=1*M
DELM=DELP
DEL=RT(I+1)-RT(I)
SL=(TY(I+1)-TY(I))/DEL
IF(DEL.LT.5.*1E-03) GO TO 6
IF(DEL.GE.(DELM-2.*E-04)) GO TO 1
DELM=DELM-0.01
GO TO 3
1 GA(J+1)=GA(J)+SL*DELM
IND(J)=DELM*100*+1.E-03+1.
J=J+1
DFL=DFL-DELM
IF(DEL.GT.5.*1E-03) GO TO 3
6 IF(DEL.GT.1.E-03) GO TO 4
GA(J)=TY(I+1)
GO TO 2
4 GA(J+1)=TY(I+1)
IND(J)=1
J=J+1
2 CONTINUE
RETURN
END
SUBROUTINE FOR COMPUTATION OF MATRICES A AND B
PCN07

SURROUNDD PCN07(D,W,L,A,B,DELT)
DIMENSION A(10,2,2),B(10,2,2),TY(10)
IF(DELT.GT.(0.005+1.E-04)) GO TO 3

TY(1)=DELT
GO TO 4
DELT=0.*01
TY(1)=0.*005
DO 1 J=2*L
   TY(J)=DELT
1   DEL=DEL+0.*01
   DO2 J=1*L
      DEL=TY(J)
      DW=D**2
      D2=D**2
      A0=EXP(-DW*DEL)
      A1=W*SORT(1.-D2)
      A0-A1*DELT
      A2=SIN(A11)
      A3=COS(A11)
      W2=W**2
      A4=(2.*D2-1.)/W2
      A5=D/W
      A6=2.*A5/W2
      A7=1./W2
      A8=(A1*A3-DW*A2)*A0
      A9=-(A1*A2+DW*A3)*A0
      A10=A8/A1
      A11=A0/A1
      A12=A11*A2
      A13=A0*A3
      A14=A10*A4
      A15=A12*A4
      A16=A6*A13
      A17=A9*A6
      A(J+1,1)=A0*(DW*A2/A1+A3)
      A(J+1,2)=A12
      A(J+2,1)=A10*DW+A9
      A(J+2,2)=A10
      B(J+1,1)=(-A15-A16+A6)/DEL-A12*A5-A7*A13
      B(J+1,2)=A15+DEL-A16/A16*DEL
      B(J+2,1)=(-A14-A17-A7)/DEL-A10*A5-A9*A7
      B(J+2,2)=(A14*A17+A7)/DEL

CONTINUE
RETURN
END