CALIFORNIA INSTITUTE OF TECHNOLOGY

EARTHQUAKE RESEARCH LABORATORY

An Electric Analog Type Response Spectrum Analyzer for Earthquake Excitation Studies

by

T. K. Caughey and D. E. Hudson

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Abstract

The design of a response spectrum analyzer for earthquake excitation studies is described. Electric analog techniques are used, with a series inductance, capacitance, and resistance circuit forming a direct analog to the mechanical structure. The circuit arrangement permits a determination of system response for a sequence of periods at constant damping. Provision is made for obtaining zero damping in the circuit. An arbitrary function generator of the variable width film - photoelectric cell type is described. The results obtained with the function generator - spectrum analyzer system for a half-sine wave pulse are compared with the mathematically obtained exact answers for the zero damping case, and the accuracy of the system is shown to be satisfactory.

Introduction

The present report has the following objects:

(1) To describe briefly the general design considerations behind the development of the response spectrum analyzer,

(2) To serve as operating instructions for persons using the device, and

(3) To provide details on circuits and instrument arrangement to serve as a guide for checking and maintenance of the analyzer.

In the report these three aspects are treated more-or-less simultaneously, since it is believed that an understanding of the basic design is essential for proper use of the instrument.

A previous report [1] discussed the application of electric analog techniques to the determination of the response spectra of strong motion earthquake acceleration records and presented a large number of such response spectra for typical earthquakes. For that work the general purpose electric analog computer of the California Institute of Technology's Analysis Laboratory was used. Although this general purpose instrument was satisfactory, it was felt that the amount of future computations of this type that would be required might justify the design and construction of a small special purpose instrument for this particular work. Such a unit, it was thought, might be designed so that the particular measurements required for the response spectra could be made more quickly or more conveniently than on the general purpose computer, and, in any event the continual availability of a special machine would be an advantage. The present report describes the design evolved for this special purpose instrument, and the results which have been obtained with the completed analyzer which is now available for a continuing program of earthquake investigations.

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1) Alford, J. L., Housner, G. W., and Martel, R. R., Spectrum Analysis of Strong-Motion Earthquakes. 1st Technical Report, onr-081-095, Pasadena, California, Aug. 1951.

Basic Design Principles

The problem which is to be solved may be stated as follows: Given a single-degree-of-freedom system of mass m, spring constant k, and viscous damping c (Fig. 1). For a prescribed base acceleration $\ddot{y}(t)$, find the maximum relative motion $(y - x)_{max}$ between the mass and the base. This corresponds to a determination of the strains set up in a simple building by earthquake ground motions. In particular, a plot of this maximum relative motion versus the undamped natural period, $\tau = 2\pi\sqrt{\frac{m}{k}}$ of the system is desired, for various values of damping expressed as a fraction of critical damping ($n = \frac{c}{c_c}$, where $c_c = 2\sqrt{km}$).

As is discussed in detail in the previous report, for the particular type of transient exciting forces involved in earthquakes, it is more convenient to plot the relative velocity spectrum $(\dot{y} - \dot{x})_{max}$ rather than the relative displacement spectrum, since the velocity spectrum exhibits the essential features of the shape of the spectrum curve in the clearest form. The desired relative displacements for such earthquake excitations can then be obtained to a satisfactory approximation from the relative velocity spectrum by the relation $(y - x)_{max} \approx \frac{\tau}{2\pi}(\dot{y} - \dot{x})_{max}$.

The differential equation describing Fig. 1 is:

$$(\ddot{y} - \ddot{x}) + \frac{c}{m}(\dot{y} - \dot{x}) + \frac{k}{m}(y - x) = \ddot{y}(t) \qquad (1)$$

we are given $\ddot{y}(t)$ and wish to find $(\dot{y} - \dot{x})_{max}$ for various values of the parameters $\frac{c}{m}$ and $\frac{k}{m}$. Considering the complexity of the earthquake exciting forces, and the ranges of the basic parameters involved in typical building structures, something like 300-400 points representing solutions of the differential equation must be obtained to satisfactorily define the response spectrum curves. This large number of points and the very complex nature of the exciting functions, suggests that some kind of machine calculation technique should be used. The desired speed of operation and the wide range of system parameters involved indicates an electrical computing system, and the fact that only moderate accuracy is required (2 - 5%) suggests an analog type of computer rather than a digital type.

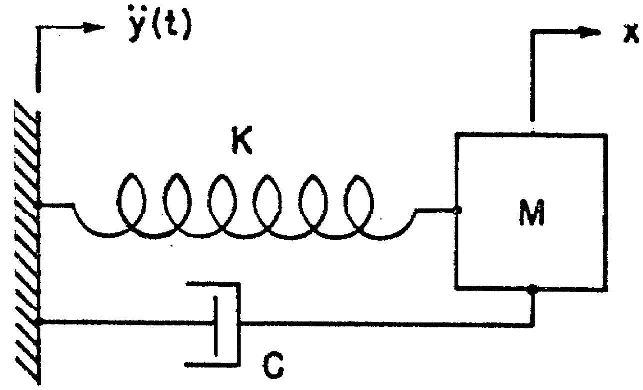


FIG. 1. SINGLE-DEGREE-OF-FREEDOM MECHANICAL SYSTEM

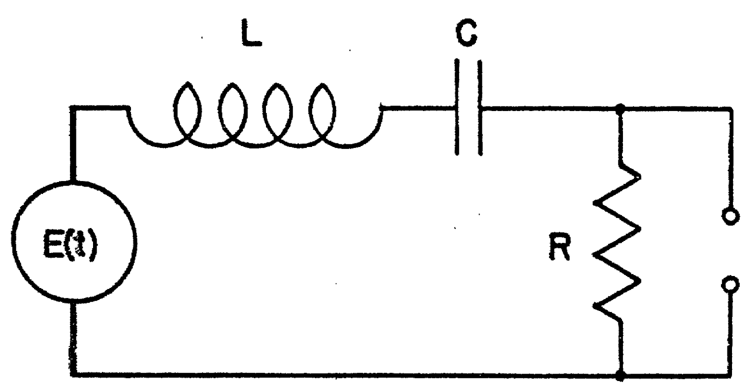


FIG. 2. BASIC ELECTRIC ANALOG CIRCUIT

Two basic kinds of electric analog computers would be suitable for the present calculations. The first type sets up a direct mechanical - electrical analogy by constructing a passive electrical circuit whose behavior is described by the same differential equations as the mechanical system. In the second type the electrical elements are used to carry out the operations indicated by the differential equation. Either method would seem to be satisfactory for the present problem. Since the work already done had used the first method with complete success, it was decided to continue with this passive system.

The advantage of the passive system lies in the fact that for this particular mechanical system the equivalent electric system is a very simple one, and hence a simpler computer results. In particular, the operational type of analog would require a larger number of electronic amplifiers, and would thus be a more complicated device to construct and to maintain.

The basic computer circuit is shown in Fig. 2. The input function is applied as a voltage $E(t)$ which varies in time in the same way as the base acceleration $\ddot{y}(t)$ of Fig. 1. The differential equation describing the electrical circuit of Fig. 2 is:

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C} Q = E(t) \tag{2}$$

where L = inductance, R = resistance, C = capacitance, Q = charge, and $E(t)$ is the applied voltage. A direct comparison of equations (1) and (2) will show the formal analogy between the systems, and it will be seen that the voltage across the resistance in the electrical circuit is analogous to the relative velocity in the mechanical system.

Since rapid operation of the whole computing device was required it was felt that no recording system, either photographic or pen-writing, should be employed. The most desirable arrangement would result in a display of the response on a cathode-ray tube, which would enable one not only to note the maximum responses rapidly, but would also permit an examination of the whole time response picture for interesting features. This cathode-ray tube display also lends itself directly to a rapid study of system parameter changes. In order to examine transient responses by eye on a cathode-ray tube it is necessary to repeat the cycle of events at a sufficiently high rate that a stationary pattern is obtained. For the cathode-ray tubes ordinarily used in standard oscilloscopes a repetition rate of 10 cycles per second is satisfactory, and this rate plus the detail which must be reproduced during each cycle fixes the general frequency limits involved for the computer elements.

Although the simple circuit of Fig. 2 contains the essential elements of the spectrum analyzer, there are a number of additional features that are required in a practical device. These additional features will first be listed, and then the means of attaining them will be discussed.

(1) It must be possible to adjust the circuit parameters in such a way that the natural period goes through a sequence of values while the damping remains constant.

(2) Various values of damping, including zero damping, must be attainable without otherwise changing the circuit characteristics.

(3) Measurements of the response must be possible without an undue disturbance of the circuit characteristics.

(4) General requirements of ease of operation and maintenance, stability of operation and calibration, etc., must be met.

Time Period Adjustment

The undamped natural period of the electrical circuit of Fig. 2, $\tau = 2\pi\sqrt{LC}$, can be varied by changing either the inductance L or the capacitance C. If, however, either L or C is changed alone, the per cent of critical damping of the circuit would be altered, since the critical damping resistance in such a circuit is $R_c = 2\sqrt{\frac{L}{C}}$. In using the general purpose analog of the Analysis Laboratory, it was necessary to compute for each period the required circuit resistance to give the desired damping ratio, and then to adjust the circuit to this value. It was primarily this time-consuming feature that suggested the desirability of a special purpose spectrum analyzer.

In the present instrument this adjustment is accomplished by changing L and C simultaneously in such a way that the quantity $\sqrt{\frac{L}{C}}$ remains a constant. This not only maintains the damping constant, but has the additional feature that the period becomes directly proportional to the inductance, thus giving a linear period scale with standard decade inductors.

The single series inductor and capacitor of Fig. 2 are thus replaced by the system of Fig. 3.

The variable interpolation inductance shown permits a continuous variation of period between the discrete steps available in the main switching unit. For most work the 100 steps in the switching unit, which cover a range of actual time periods of from 0.0001 to 0.011 seconds, are sufficiently close together to make interpolation unnecessary. For some exciting functions, however, it is desirable to be able to study the effect of very small changes of period on the system response. For quantitative work this interpolation system should also include a simultaneously continuously variable capacitance, in order to maintain the constant $\sqrt{\frac{L}{C}}$ in the circuit. In practice, $\sqrt{\frac{L}{C}}$ can be kept sufficiently constant for the required accuracy by correcting the interpolation system at discrete steps. An interval on the interpolation inductance dial of 10 - 20 (10 millihenries) can be made equivalent to one complete small time period duration (one unit) by adding capacity to the circuit as the interpolation inductance dial setting is changed. An interpolation capacitance is provided which enables the interpolation range of 10 - 20 to be divided linearly into fifths, so that a time period of 0.00002 sec. can be read. For each one fifth of the interval (2 small divisions on the interpolation inductance dial) the auxiliary capacitance should be advanced one step to maintain the linearity of the circuit.

For most work the use of the interpolation inductance dial and the interpolation capacitance will be unnecessary, and the interpolation inductance and capacitance dials can be left at the standard setting 10.

For the particular circuit elements used, the constant factor $\sqrt{\frac{L}{C}} = 2\pi(100)$, with L in henries and C in farads. Putting this constant

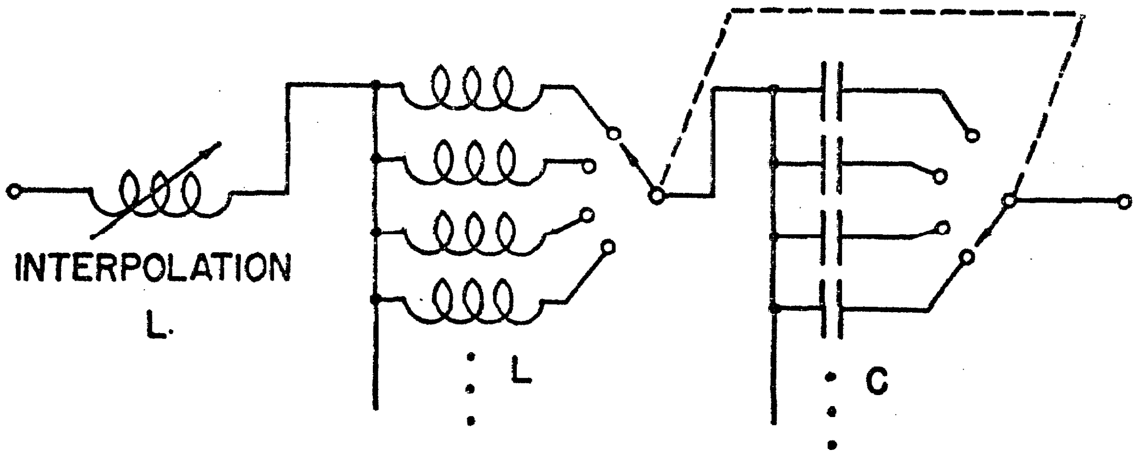


FIG. 3. BASIC L-C SWITCHING ARRANGEMENT

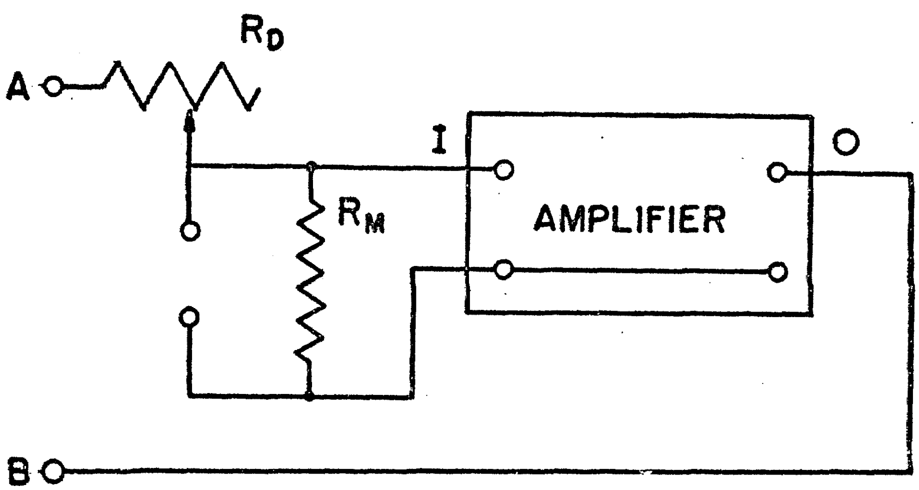


FIG. 4. ZERO-DAMPING AMPLIFIER AND MEASURING RESISTOR

in the expression for the undamped natural period of the system $\tau = 2\pi\sqrt{LC}$, we obtain $\tau = \frac{L}{100}$ seconds.

Zero Damping System

The damping in the simple circuit of Fig. 2 can of course never be reduced to zero, since there is a small resistance associated with the inductance. In addition to this difficulty, it is found that even for moderate values of damping, which could be attained in the passive circuit, the resistance R becomes so small that the voltage drop across the resistor becomes difficult to measure. To solve this measurement problem an auxiliary measuring resistor, R_m, is introduced, and the circuit is arranged in such a way that a relatively large value (1000 Ω) can be used. To reduce the damping to zero an amplifier is employed in such a way that it acts effectively as a negative resistance to cancel out the resistance of the rest of the circuit.

The general arrangement of this zero-damping amplifier and of the measuring resistor are shown in Fig. 4. The resistor R of Fig. 2 is replaced by the circuit AB of Fig. 4. If the gain G of the amplifier is just unity, then the voltage at the amplifier output O, which is the same as at B, would be the same as the voltage at the amplifier input I. Thus the total voltage drop between A and B would be just that due to the damping resistor R_D. By increasing the gain of the amplifier above unity, the voltage at B can be increased, and the voltage drop from A to B can be decreased. The amplifier thus evidently acts like a negative resistance, and can be adjusted to balance out the damping resistor R_D and the resistance of the inductance, thus attaining zero damping in the whole circuit.

The system is tested for zero damping by applying a step function to the circuit, thus setting up free oscillations. By repeating this pulse periodically a stationary pattern of these free oscillations can be seen on the cathode-ray tube, as shown in Fig. 5. The gain of the amplifier is then adjusted until the oscillations maintain a constant amplitude, as in Fig. 5(c).

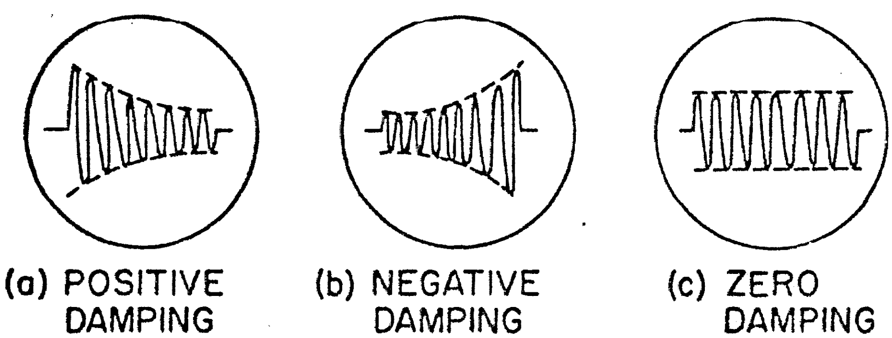


FIG. 5. FREE OSCILLATIONS AT VARIOUS AMPLIFIER GAIN SETTINGS

Various amounts of positive damping can be introduced into the system either by adjusting the damping resistance or by decreasing the amplifier gain.

Amplifier Characteristics

The characteristics which are desired in the zero damping amplifier are:

(1) Positive gain, adjustable from 1-1.6.

(2) Negligible phase shift from 10-10,000 cps.

(3) Flat response (5%) from 10-10,000 cps.

(4) Input impedance - 1 megohm.

(5) Output impedance - 10 ohms.

It is the combination of low output impedance with negligible low frequency phase shift that makes the amplifier design critical. For earthquake analysis work a lower frequency limit of 100 cps would be satisfactory; the 10 cps limit, however, increases the usefulness of the instrument for single pulse work. It should be noted that the power amplifier used in the function generator has appreciable phase shift below 20 cps, and hence is the limiting factor in the complete system.

The requirement of flat amplifier response is not a critical one unless the spectrum to be analyzed contains a number of strong components at widely spaced frequencies. Otherwise gain adjustments at various frequencies near the ends of the frequency bands could be made if necessary.

A simplified schematic diagram of the amplifier, laid out to show the essential features of the design, is shown in Fig. 6. It will be seen that the amplifier consists of a negative gain stage capacitively coupled into a D. C. system which has a cathode-follower output stage. To reduce the output impedance of the cathode-follower stage, two tubes are used in parallel, thus doubling the mutual conductance of the tube system. The 6L6 tube is used as a cathode impedance. A large amount of negative feedback is used to stabilize the system and to provide a low output impedance.

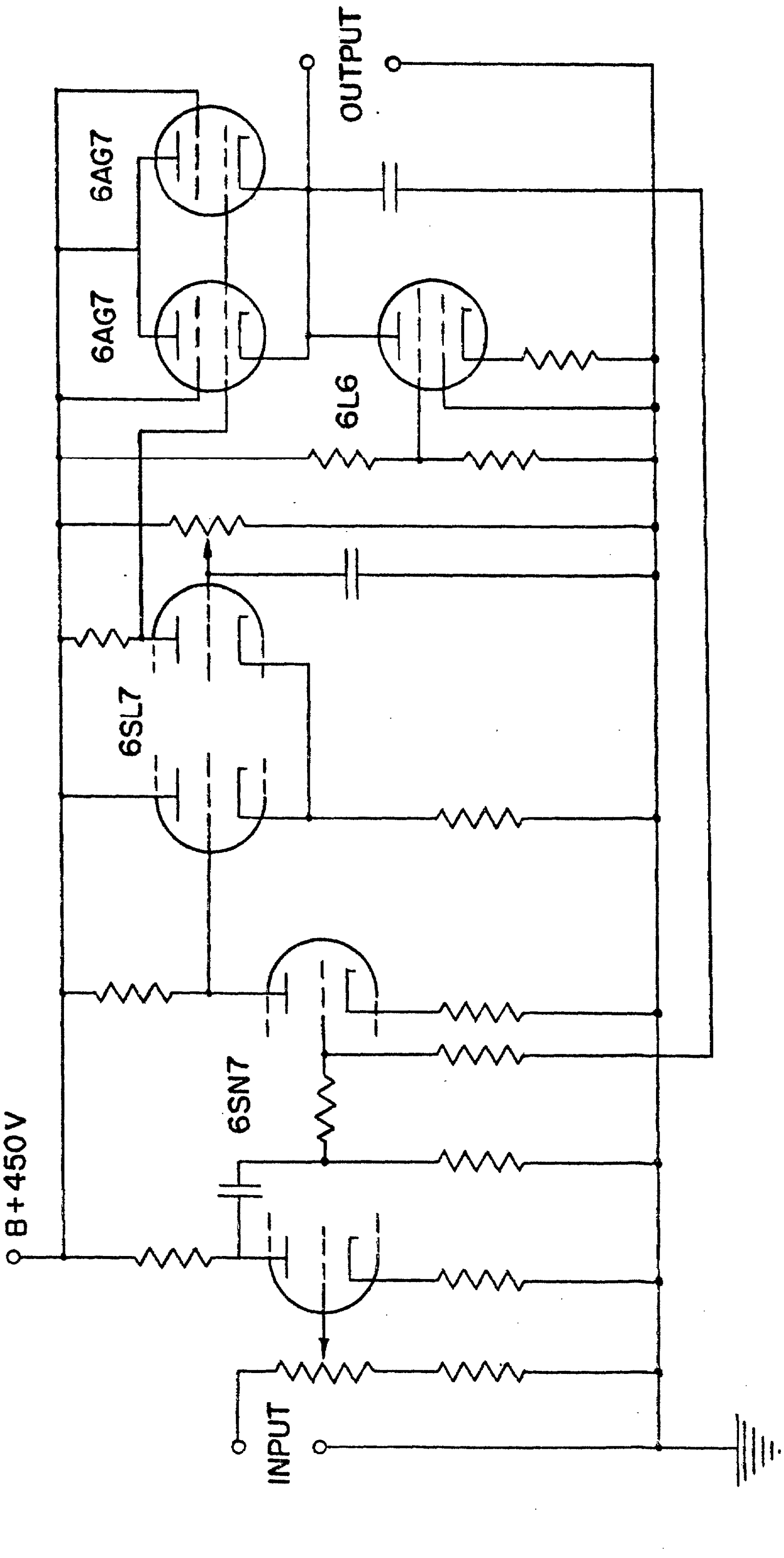


FIG. 6. SIMPLIFIED SCHEMATIC DIAGRAM OF AMPLIFIER

Damping Arrangements

For convenient operation of the analyzer it is necessary to have a means of periodically checking the zero damping setting of the amplifier, and of setting into the system any desired amount of damping. The first requirement is met by arranging the function generator so that by operating a switch on the analyzer panel the function generator output is replaced by a step function which excites the free oscillations discussed in connection with Fig. 5. For values of damping other than zero, the circuit is first adjusted to zero damping, and then series resistances of known values are switched into the circuit. For the basic circuit of Fig. 2 the critical damping resistance is $R_c = 2\sqrt{\frac{L}{C}}$, since $\sqrt{\frac{L}{C}} = 2\pi(100)$, we have $R_c = 1255$ ohms, and hence resistances to give any desired fraction of critical damping can easily be computed. Built into the analyzer are resistances giving 2.5, 5, 10, 15, and 20 per cent of critical damping.

One other type of damping circuit is required for the system. Since the input function is applied to the analyzer circuit periodically, the circuit must be brought periodically to rest before the next cycle occurs. This is accomplished by replacing the input function for a portion of each cycle by a resistor which gives somewhat greater than critical damping in the circuit, thus damping out the oscillations and bringing the system to rest ready for the next cycle. A slip ring and commutator assembly driven from the film record motor drive shaft is designed so that this damping resistance is in the circuit for about 1/3 of each cycle. The useful portion of the input function film record must thus be limited to about 2/3 of the circumference of the film record. A mark is placed on the film mounting plate to insure that the film record is properly phased with the damping portion of the cycle. This same commutator system is used to apply the step function for the zero damping test.

Incorporated in the same commutator system mentioned above is the pulse generating system for oscilloscope synchronization. This pulse signal is available on the analyzer panel at the terminal marked "sync". Since the synchronous motor driving the function generator record disk is driven from the same 60 cps supply that powers the oscilloscope, line voltage synchronization will also ordinarily be satisfactory. On the complete diagram of the function generator (Fig. 16) it will be seen that this synchronizing pulse is generated by an isolating transformer-diode system. The direct pulse from the commutator would be suitable, but the construction of the particular commutator used made the above arrangement necessary.

General Layout and Arrangement

The photographs of Figs. 7, 8, and 9 will give a general idea of the layout and construction of the analyzer. A complete circuit diagram of the whole instrument is given in Fig. 10.

Time Scale Factor Determinations

The relations between the record spacing on the film record, the rotational speed of the film record in the function generator, and the actual natural period of the electric system in the spectrum analyzer fix the time factors for the device. The time factor N is defined as the ratio between actual or prototype time, and the equivalent analog time.

For example, if a transient acceleration having an actual time duration of 15 sec. was reproduced on 180° of the film record disk, then, since the disk rotates at 10 cyc/sec, the time factor would be:

$$N = \frac{15 \text{ sec}}{\left(\frac{1}{10}\ \frac{\text{sec}}{\text{rev}}\right)\left(\frac{180^{\circ}}{360^{\circ}}\right) \text{rev.}} = 300.$$

The numbers on the time period switching unit scales should be divided by 10,000 to give the actual analog time in seconds. Unless the interpolation system is to be used, all readings should be taken with the time period interpolation inductance dial set at "10" (the minimum value), the interpolation capacity switch set at "10", and one unit (1) should be added to the marked readings of the time period switches. If, for example, the left time period switch is set at 70, the right at 5, the interpolation inductance dial at 10, and the interpolation capacitance switch at 10, the time period of the electric circuit would be:

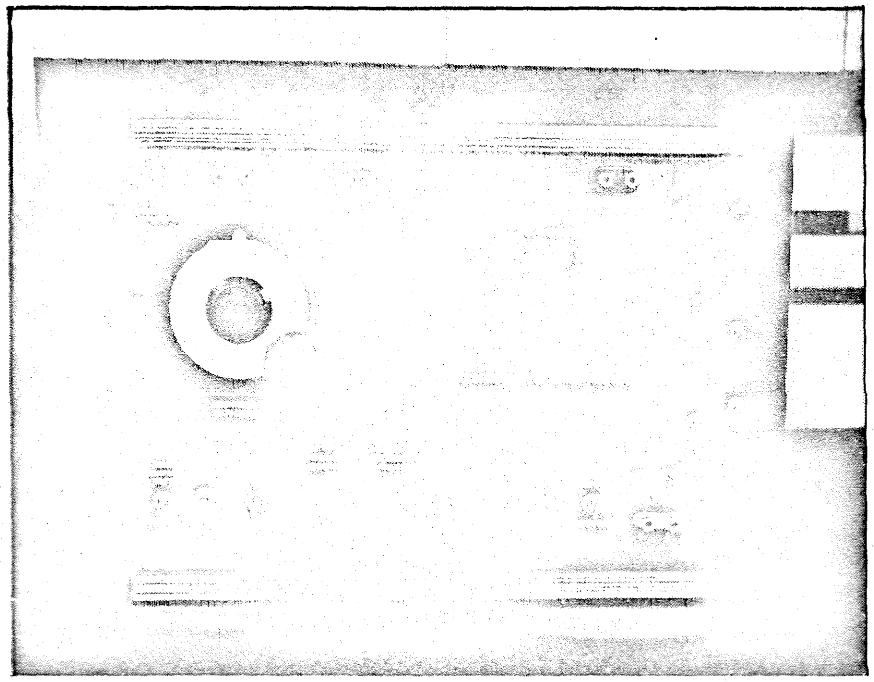


FIGURE 7. Arrangement of Panel Controls of Response Spectrum Analyzer

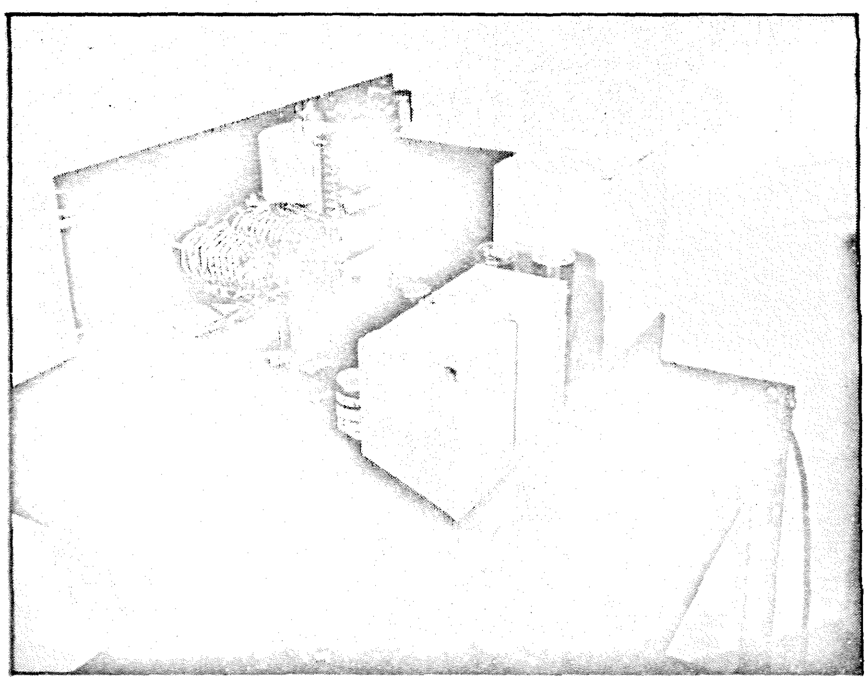


FIGURE 8. Details of Construction of Spectrum Analyzer

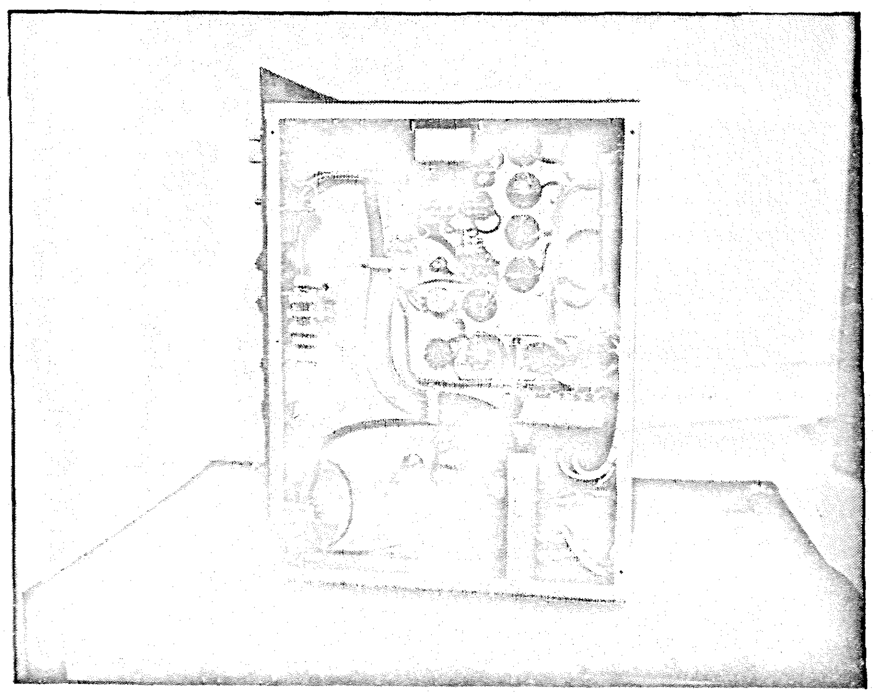
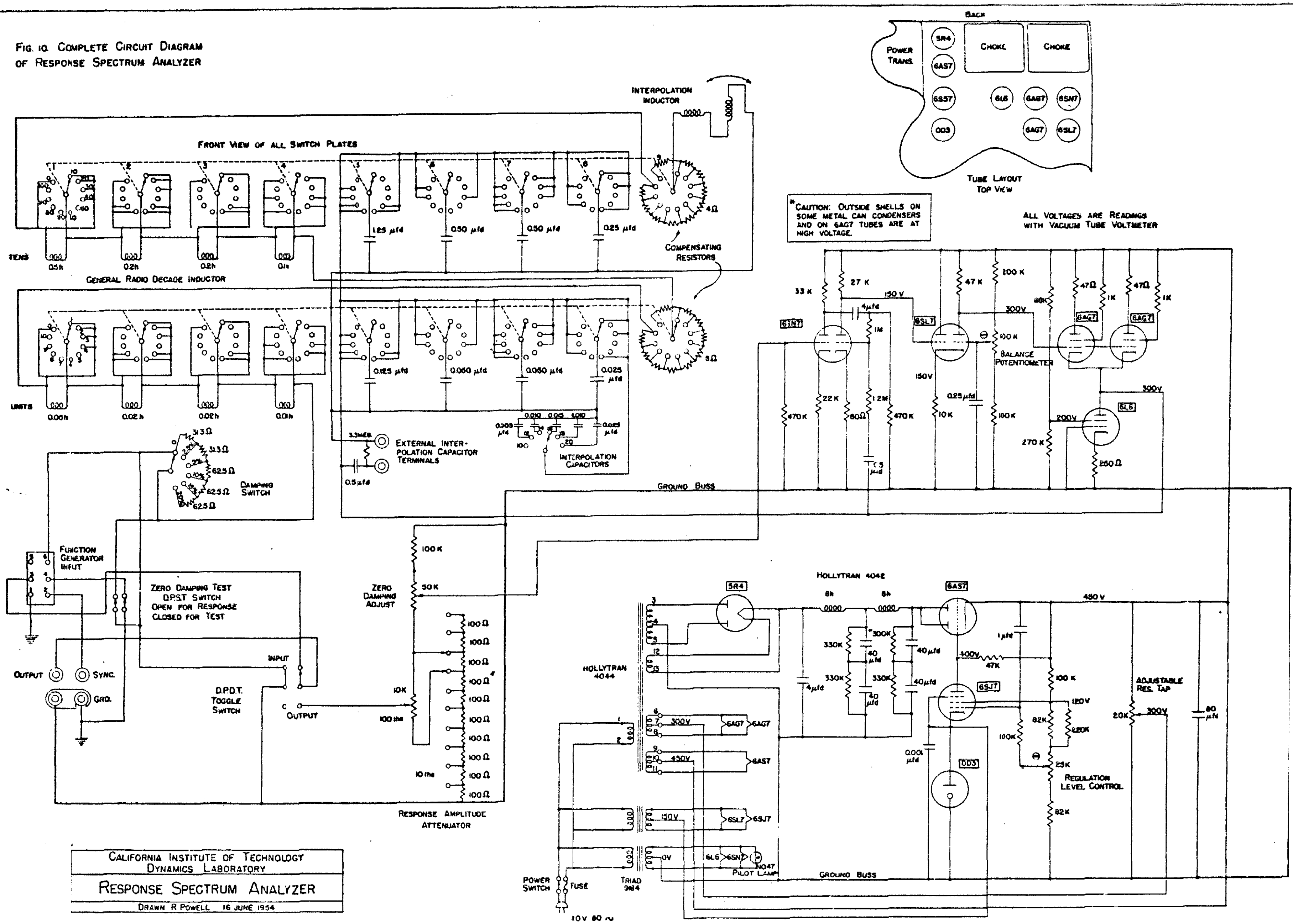


FIGURE 9. Details of Wiring and Construction of Spectrum Analyzer

Fig. 10 Complete Circuit Diagram of Response Spectrum Analyzer



$$(70 + 5 + 1.0) \times 10^{-4} = 0.0076 \text{ sec.}$$

For interpolation, for example, the interpolation inductance dial could be advanced to "12", and the interpolation capacitance switch set to "12", then with the main time period switch set as before, the time period would be:

$$(70 + 5 + 1.2) \times 10^{-4} = 0.00762 \text{ sec.}$$

If the time factor N for the particular record being used for the above settings was N = 300, then the actual time to be plotted on the final spectrum curve would be:

$$(300)(0.00762) \text{ sec.} = 2.286 \text{ sec.}$$

Response Scale Factor Determinations

The relations between the response factors for the mechanical and the electrical systems may be established in the following way.

For the mechanical system of Fig. 1, we put $\ddot{y}(t) = A_o f(t)$ and rewrite Eq. (1) as follows:

$$m(\ddot{y} - \ddot{x}) + c(\dot{y} - \dot{x}) + k(y - x) = m A_o f(t) \qquad (3)$$

The solution of this equation, supposing that the system starts from rest at $t = 0$, is:

$$(y - x)_T = \frac{A_o}{\omega_p} \int_o^T f(t)\, h\left[\omega_p (T - t)\right] dt \qquad (4)$$

where ω_p = natural frequency of mechanical system

and $$h\left[\omega_p (T - t)\right] = e^{-\frac{c}{2\sqrt{km}}(T - t)} \sin \omega_p (T - t)$$

We wish the relative velocity, so differentiating (4) we obtain:

$$(\dot{y} - \dot{x})_T = A_o \int_o^T f(t)\, h'\left[\omega_p (T - t)\right] dt \qquad (5)$$

For Eq. (2), describing the electrical circuit of Fig. 2 we obtain, in a similar way:

$$\dot{Q}_T = \frac{E_o}{L} \int_o^T f(t)\, h'\left[\omega_a (T - t)\right] dt \tag{6}$$

where $E(t) = E_o f(t)$ and ω_a = natural frequency of electrical system.

To investigate the effect of time scale changes on these quantities, we make the integrals in Eqs. (5) and (6) dimensionless by introducing in (5) the new variable ($\omega_p t$) and in (6) the new variable ($\omega_a t$). We then obtain

$$\frac{\dot{\omega}_p (\dot{y} - \dot{x})_T}{A_o} = \int_o^{\omega_p T} f(t)\, h'\left[\omega_p (T - t)\right] \omega_p\, dt$$

$$\frac{\omega_a L \dot{Q}_T}{E_o} = \int_o^{\omega_a T} f(t)\, h'\left[\omega_a (T - t)\right] \omega_a\, dt$$

These two integrals now have the same value (note that any particular $f(t)$ would also be expressed in terms of some dimensionless quantity, e.g., (ωt), where ω would be subject to the same time scale changes that apply to ω_p and ω_a , and we may put

$$\frac{\omega_p (\dot{y} - \dot{x})_T}{A_o} = \frac{\omega_a L \dot{Q}_T}{E_o}$$

By the definition of the time factor N, we have:

$$\omega_a = N \omega_p$$

so

$$\left(\frac{\dot{y} - \dot{x}}{A_o}\right)_T = N L \left(\frac{\dot{Q}}{E_o}\right)_T \tag{7}$$

In the electric curcuit $\dot{Q}$ is determined by measuring the voltage drop across a measuring resistor, R_m , which, by the arrangement of Fig. 4 is made independent of the damping resistor. Introducing R_m into Eq. (7), we have the desired relation between the mechanical quantities and the measured electrical quantities

$$\left(\frac{\dot{y} - \dot{x}}{A_o}\right)_T = \frac{N L}{R_m} \left(\frac{R_m \dot{Q}}{E_o}\right)_T \tag{8}$$

Expression (8) holds for any time T. It is usually convenient to take as a reference value in finding the ratio $\left(\frac{R_m \dot{Q}}{E_o}\right)$ the peak or maximum occurring during the cycle, and this peak value ratio can be related directly to the peak value of acceleration on the original accelerogram.

For the particular circuit constants used in the spectrum analyzer, the factor $\left(\frac{N L}{R_m}\right)$ can be evaluated as follows:

$$\frac{N L}{R_m} = \frac{N L \left(\frac{1}{\sqrt{LC}}\right)}{R_m \; \omega_a} = \frac{N \sqrt{\frac{L}{C}}}{\omega_a \; R_m}$$

$$\sqrt{\frac{L}{C}} = 2\pi\,(100) \quad \text{and} \quad R_m = 1000 \text{ ohms}$$

so

$$\frac{N L}{R_m} = \frac{N\,(2\pi)\,(100)}{\omega_a\,(1000)}$$

also $\frac{2\pi}{\omega_a}$ = natural period of analog circuit

= τ_a seconds

and thus $\frac{NL}{R_m} = 0.1\, N\, \tau_a$

Thus, for this instrument:

$$(\dot{y} - \dot{x})_{peak} = 0.1\, N\, \tau_a \left(\frac{R_m \dot{Q}}{E_o}\right)_{peak} \cdot (A_o)_{peak} \qquad (9)$$

The ratio $\left(\frac{R_m \dot{Q}}{E_o}\right)_{peak}$ can be read from the analyzer in two different ways. The peak input and response voltages can be directly compared on the scale of the cathode-ray tube, or for the usual case when the response voltage is greater than the input voltage, the response voltage $(R_m \dot{Q})$ can be reduced by the calibrated attenuator on the analyzer until the two voltages show the same amplitude on the oscilloscope. The voltage ratio can then be read directly from the attenuator scales.

Example:

Suppose that an earthquake record having a peak acceleration of 0.28 g is to be analyzed. Given the time factor $N = 378$.

With the time period switch set at 22, interpolation inductance dial set at 10, and the interpolation capacitance switch set at 10, we would have for the actual prototype period to be plotted on the spectrum:

$$(22 + 1)\, 10^{-4} \times 378 = 0.892 \text{ seconds.}$$

Suppose that at this setting the voltage ratio between the response and the input, as obtained from the attenuator setting required to make the input and output voltages equal on the cathode-ray tube scale is 0.72, then

$$\begin{aligned}(\dot{y} - \dot{x})_{peak} &= (0.1)(0.892)(0.72)(0.28)(32.2) \\ &= 0.58 \text{ ft/sec.}\end{aligned}$$

and this would be the plotted ordinate on the relative velocity response spectrum.

The Arbitrary Function Generator

The arbitrary function generator which is used with the response spectrum analyzer is a modification of a type developed by the Analysis Laboratory and used in the work described in the previous report.

Various method have been used to generate arbitrary input functions for electric analog computing work. One method which has been widely used, the "photoformer" method, employs an opaque template having the form of the desired function. This template is placed in front of a cathode-ray tube and the beam is driven to follow the pattern. A second method makes use of a variable width film in conjunction with a light source and photocell. This second method was selected for the work because of the complexity of the input function and the accuracy desired. By the use of a 10 in. diameter circular track, a record length of over two feet is available, which enables one to reproduce in considerable detail even very complex functions such as the earthquake acceleration record of Fig. 19. It was thought that it would not be possible to reproduce such records in the small size required for photoformer generation and retain all small details that might have an effect on system response.

The general features of the function generator are shown in the block diagram of Fig. 11.

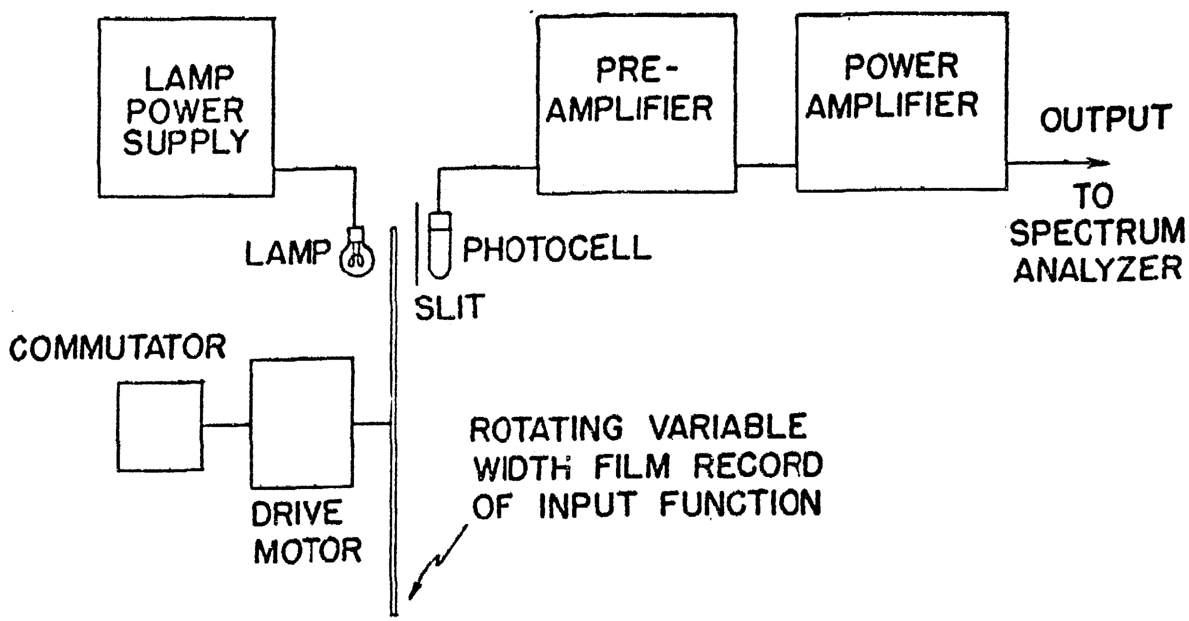


FIG. 11. BLOCK DIAGRAM OF ARBITRARY FUNCTION GENERATOR

In the previous report on Earthquake Spectrum Analysis will be found details of the method of making the film record, and Fig. 12, reproduced from that report, shows typical film records and the device used in transferring to the film the graph of the input function. The radius of the film record center line is 10 in., and the record is rotated at a speed of 10 rev/sec by the drive motor.

A rotary slip ring and commutator assembly is also driven from the motor shaft. One segment provides a synchronizing pulse, and another segment provides that for about 120° of rotation the analog circuit is shorted through a damping resistor to return the analog circuit to its initial condition at the beginning of each cycle. The rectangular wave from this damping segment is also used as the step function to excite free oscillations for the zero damping test.

The light source is an automobile headlight bulb which is ordinarily operated at a voltage of 6 - 8 volts. The lamp power supply must be very well filtered so that the ripple voltage is less than 1%. A 0.025 in. wide slit is placed between the film and the phototube as shown. The phototube is a type 929 with a maximum output voltage of about 0.1 volts. It is of particular importance that the light intensity over the width of the film be uniform, otherwise non-linear distortion of the record may result. The phototube preamplifier has a gain of approximately 100, a frequency response which is essentially flat from 2 - 70,000 cyc/sec, and an output impedance of 10 ohms. This preamplifier could perhaps be eliminated by using a type 931a photomultiplier tube, but it is believed that the overall noise level would be higher than with the present system.

The power amplifier has a frequency response curve which is essentially flat from 2 - 7,000 cyc/sec, and has negligible phase shift above about 20 cyc/sec. The output impedance is about 1/2 ohm. The voltage output level from the power amplifier is of the order of 10 volts, and the overall noise level through the whole system is about 5 millivolts. The spectrum analyzer is usually operated at a voltage level of about 2 volts.

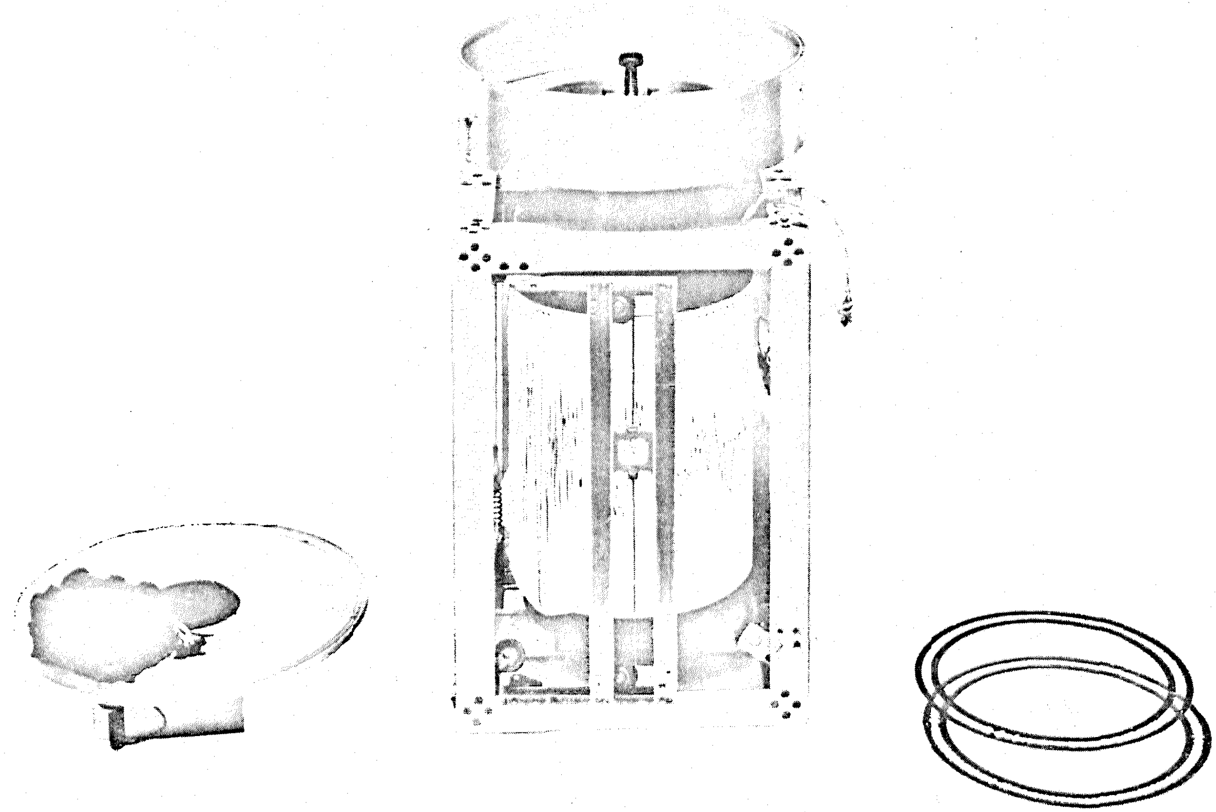


FIGURE 12. Plotting Table for Film Records

The photograph of Fig. 13 shows the film record disk in place in the drive motor system. Fig. 14 shows a back view of the function generator unit, and indicates general layout and method of construction. The upper portion of the cabinet contains the motor drive unit and the lamp and photocell system. The middle unit is the lamp power supply and the photocell preamplifier, and the lower unit is the power amplifier. A general view of the entire spectrum analysis system consisting of the arbitrary function generator, the response spectrum analyzer, and the measuring oscilloscope is shown in Fig. 15. In Fig. 16 is given a complete wiring diagram of the arbitrary function generator.

Operation and Tests of the Spectrum Analyzer

To give an overall idea of the performance and accuracy of the spectrum analyzer, an analysis was made of a single half-sine pulse, for which a mathematical solution can be easily obtained.

In Fig. 17 is shown a photograph of the half-sine pulse as reproduced on the cathode-ray tube screen by the function generator. Fig. 18 gives the response spectrum for this pulse, as computed from the following expressions:

T = pulse duration

a = pulse acceleration

τ = period

t = time

The velocity during the pulse is:

$$\dot{x}(t) = \frac{\frac{a}{\pi} T}{1 - \frac{T^2}{\pi^2 \tau^2}} \left(\cos \frac{\pi t}{T} - \cos \frac{t}{\tau}\right)$$

The velocity after the pulse is:

$$\dot{x}(t)_{max} = \frac{\frac{2a}{\pi} T}{1 - \frac{T^2}{\pi^2 \tau^2}} \cos \frac{T}{2\tau}$$

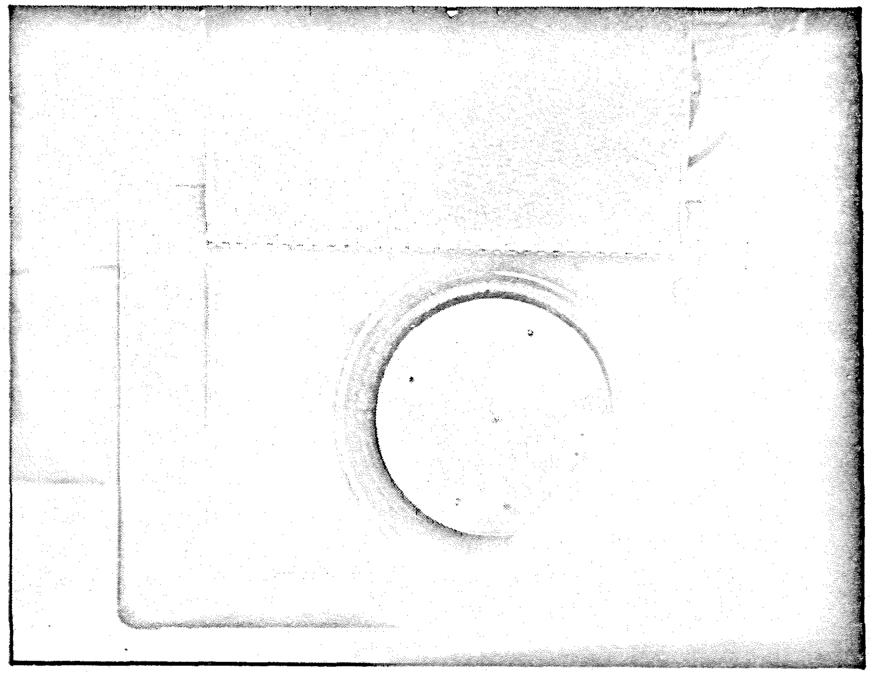


FIGURE 13. Film Record in Place in Drive Motor System

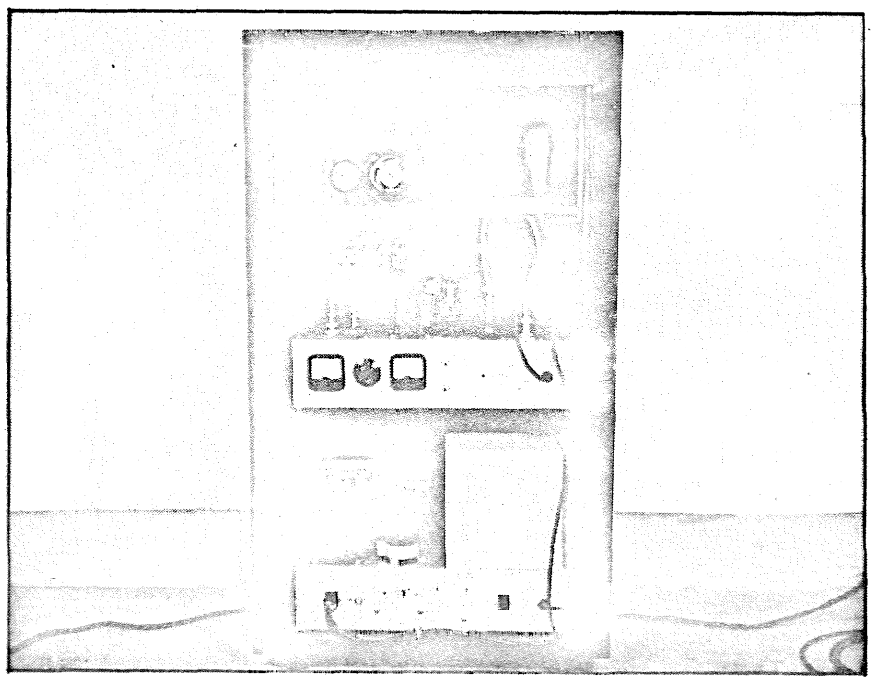


FIGURE 14. Construction Details of the Arbitrary Function Generator

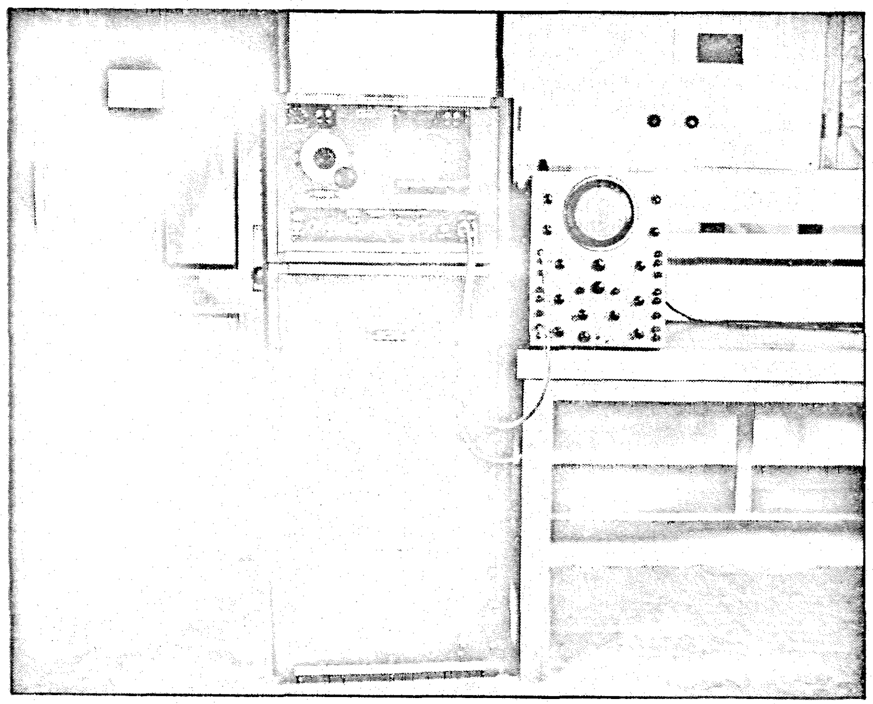


FIGURE 15. General View of the Whole Spectrum Analyzer System

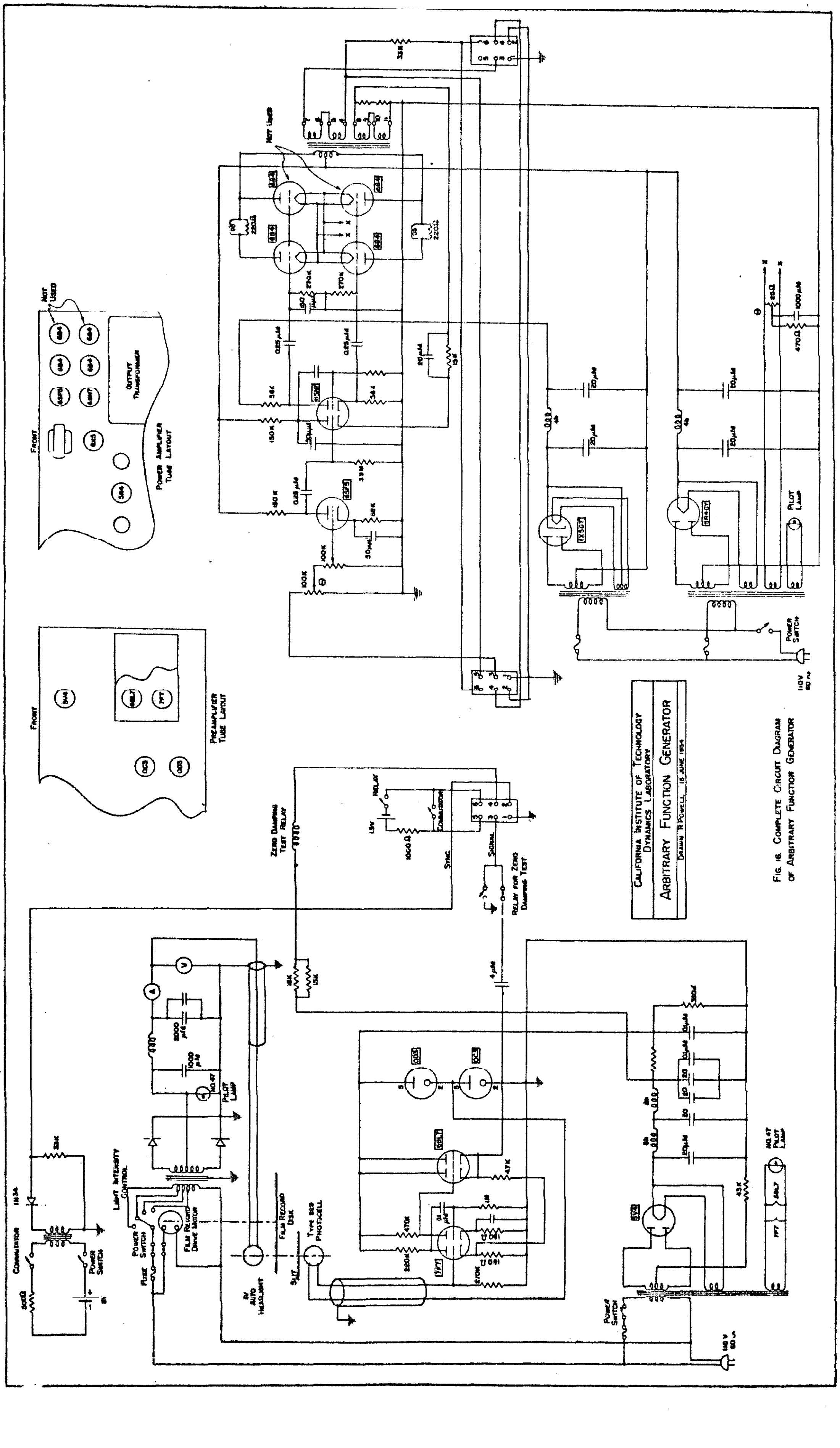


Fig. 16. Complete Circuit Diagram of Arbitrary Function Generator

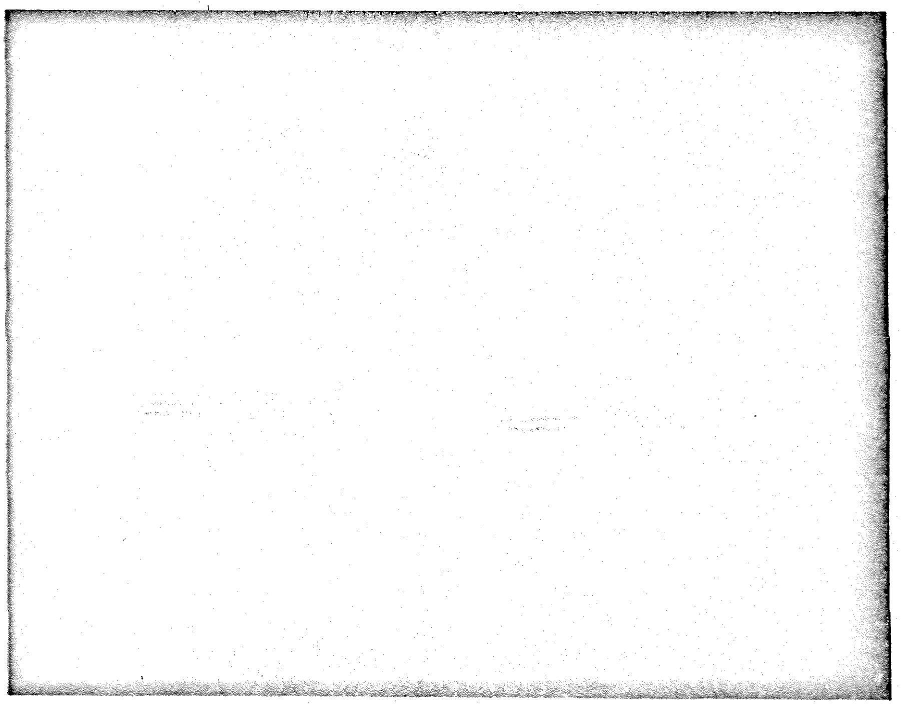


FIGURE 17. Half-Sine Pulse as Reproduced by the Arbitrary Function Generator

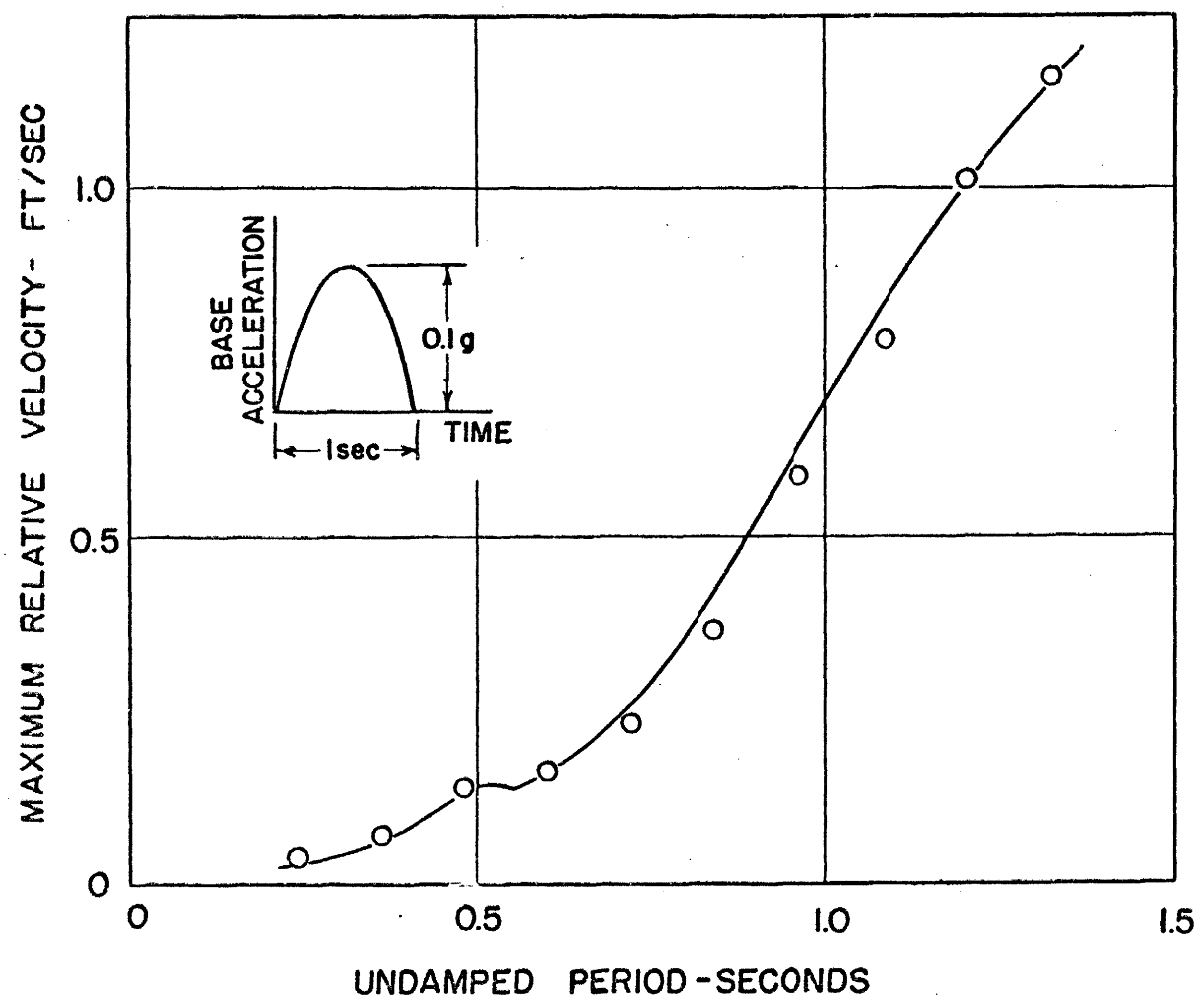


FIG. 18. RELATIVE VELOCITY SPECTRUM FOR A HALF-SINE PULSE (ZERO DAMPING)

The curve of Fig. 18 gives in each case the maximum velocity for each period, which sometimes occurs during the pulse, and sometimes after the pulse. The curve is computed for the specific case of a pulse for which $T = 1$ sec; $a = 0.1$ g, and the film record time factor is $N = 120$.

The points shown on Fig. 18 are obtained from the spectrum analyzer. The deviation of the points from the curve can be accounted for almost entirely by inaccuracies in reading amplitudes on the cathode-ray tube screen. One possible source of error in the period scale lies in the fact that the values of the inductances change slightly with current, so that at widely different analyzer levels, corrections to the period readings should be made. Even for the wide output level ranges encountered for the pulse, it is seen that this correction is a small one. For the ranges covered in earthquake work it should not be necessary to make any correction of this type.

In Fig. 19 is shown a typical earthquake acceleration record, and Fig. 20 gives the velocity spectra for this earthquake, taken from the previous report. The points on Fig. 20 are check points taken with the new spectrum analyzer using the same function generator film record used for the previous determinations. It will be seen that the agreement is in general well within the required limits of accuracy.

Acknowledgement

We wish to express our appreciation to Sheldon Rubin and Robert Powell for their many contributions to the development and construction of the response spectrum analyzer and the Arbitrary Function Generator.

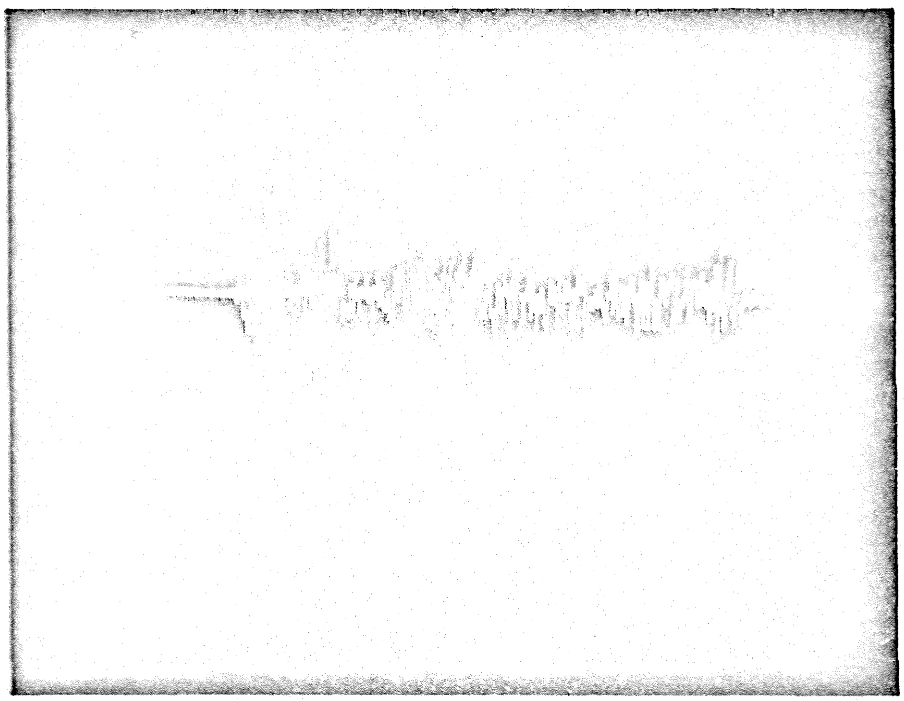


FIGURE 19. Typical Earthquake Accelerogram as Reproduced by the Arbitrary Function Generator

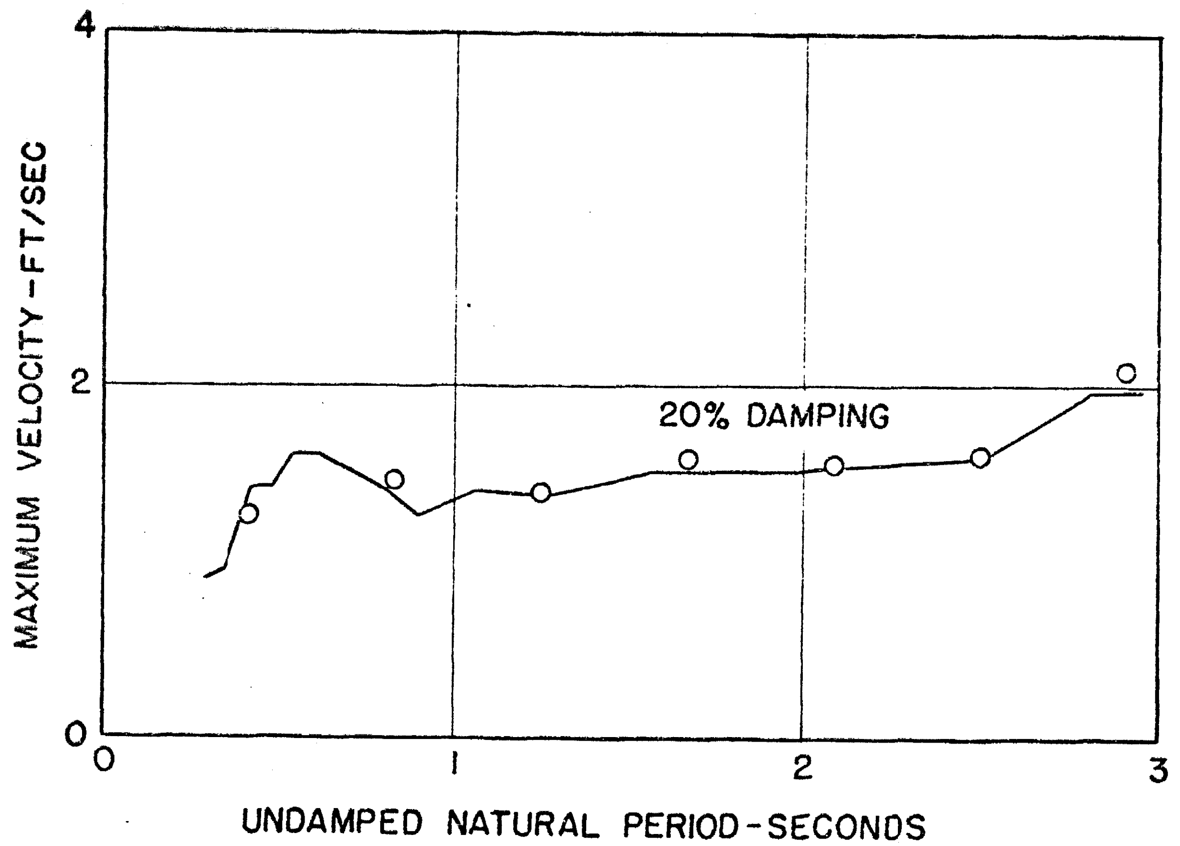


FIG. 20. RELATIVE VELOCITY SPECTRUM FOR EARTHQUAKE (EL CENTRO 18 MAY 1940, NS)