Ph 103c: The Physics of LIGO

LECTURE 12.
Seismic Isolation
Lecture by Lisa Sievers

Assigned Reading:
II. Leonard Meirovitch, *Elements of Vibration Analysis* (McGraw-Hill, 1986), pp. 48–58. [This reference develops the basic concepts of using mass-spring-damper systems for vibration isolation; and it discusses the measurement of vibrations, and two types of damping that can occur in mechanical systems: viscous damping and structural damping. These two types of damping will play an important role in the lectures on thermal noise next week.]

JJ. R. del Fabbro, A. di Virgilio, A. Giazotto, H. Kautzky, V. Montelatici, and D. Pas- suello, “Three-dimensional seismic super-attenuator for low frequency gravitational wave detection,” *Physics Letters A*, 124, 253–257 (1987). [This reference describes and analyzes an early version of the ambitious mass-spring-damper vibration-isolation stack that is being developed by the Pisa, Italy group as their prime contribution to the VIRGO Project. The analysis of the LIGO isolation stacks is similar, though their initial design is less ambitious.]

Suggested Supplementary Reading:
II. Leonard Meirovitch, *Elements of Vibration Analysis* (McGraw-Hill, 1986), pp. 39–48. [This is largely foundational material underlying the assigned reading (item 1. above); you may find it helpful.]

KK. C. A. Cantley, J. Hough, and N. A. Robertson, “Vibration isolation stacks for gravitational wave detectors—Finite element analysis,” *Rev. Sci. Instrum.*, 63, 2210–2219 (1992). [This paper, by the Glasgow gravity-wave group, illustrates an isolation-stack analysis that is more sophisticated than the simple models used in class and in reference 2, and that reveals pitfalls in the design of a stack.]


A Few Suggested Problems: See the next page.
1. You have the 2 stage spring/mass stack shown in Figure 1 and want to decide the best mass ratio $m_1/m_2$ in the 2 stages so that you achieve maximum isolation at frequencies well above $w_0^2=k_1/m_1$. A good designer would assume that the springs are compressed to their maximum limit in order to get the most bang for their buck, therefore the strain energy in each spring should be assumed equal ($k_1/m_1 = k_2/(m_1+m_2)$). Show that the transmissibility $X_1(f)/X_0(f)$ is maximized as the mass ratio $m_1/m_2$ goes to zero but that a point of diminishing returns is reached when the ratio is about 1.

2. Work out the equations of motion for the 1 and 2 stage pendula shown in Figure 2. Compare the amount of isolation achieved at two different frequencies: $\omega = 2\sqrt{2}$ and $\omega = 10\sqrt{2}$.

3. A method for mechanically damping a high Q mechanical resonance is to use a "proof mass damper" as shown in Figure 3. The proof mass damper is a damped oscillator whose mass is much smaller than the mass to be damped and whose resonant frequency and damping coefficient is tuned specifically to damp the system in the most effective way. Assume $m_1=20m_2$, $f_1=4$Hz, and $f_2=(1+m_2/m_1)f_1$. Plot the transmissibility function $X_1(f)/X_0(f)$ for 3 different damping coefficients, c. [Definition of damping coefficient, c: If a particle of mass m moves under the combined influence of a linear restoring force $-kx$ and a resisting force $-c\dot{x}$, the differential equation which describes the motion is $mx'' + cx' + kx = 0$ ....... c is inversely proportional to Q]

   1. $c=0$
   2. $c=\text{infinity}$
   3. $c=2m_2(2\pi f_1) \sqrt{\frac{3m_2/m_1}{8(1+m_2/m_1)^2}}$

The third case is the case where you get the maximum attenuation possible (i.e. $X_1(f_1)/X_0(f_1)=\sqrt{1 \pm 2m_1/m_2}$).

[A proof mass damper has been experimentally implemented in Mark I. One of the stacks (i.e. optics plate mounted on rubber), had a high Q horizontal resonance at $f_1=4$Hz. In a compact vacuum sealed vessel, we built a pendulum whose bob was 1/20 the mass of the offending optics plate. The pendulum was partially submerged in motor oil whose damping coefficient was given in (3). The length of the pendulum bob was tuned to the resonant frequency of $f_2$. The stack resonance was damped without compromising the isolation at higher frequencies.]
Figure 1

Figure 2

\[ \omega_1 \gamma = \sqrt{\frac{k_1}{m_1}} \]
\[ \omega_2 \gamma = \sqrt{\frac{k_2}{m_2}} \]

Figure 3
Lecture 12
Seismic Isolation
by Lisa Sievers, 6 May 1994

Sievers lectured from the following transparencies.
LIGO DISPLACEMENT NOISE

Frequency (Hz)

$\frac{\text{x}(f)}{\text{m/Hz}^{1/2}}$

Caltech Seismic Background

LIGO Site Seismic Background

Target Displacement Noise
Seismic Isolation of test mass is composed of 2 components:

- Stack Isolation
- Pendulum Suspension

The ground noise at the sites drives the requirement on the amount of isolation necessary.
CONCEPTS FOR DESIGNING SPRING/MASS
PASSIVE ISOLATION SYSTEMS

MODEL OF
1 LAYER STACK

\[ \frac{X_1}{X_G} = \frac{K}{M} \]

\[ \omega_0^2 = \frac{K}{M} \]

\[ \text{As } \omega \to \infty \text{ slope is } \omega^{-2} \]
Effect of Spring Stiffness on Vibration Isolation of a Simple Harmonic Oscillator

- $K = 0.1$
- $K = 1.0$
- $K = 10.$

Gain

Radians/sec
• **EFFECT OF MULTIPLE LAYERS:**

\[ \left| \frac{X_1}{X_G} \right| \text{ ROLLS-OFF AS } \omega^{-2N} \text{ WHERE } N \text{ IS THE NUMBER OF LAYERS.} \]

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Simple Model of Mark 2 Stack Isolation (vertical)

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• **WHY NOT USE A STACK WITH MANY MANY LAYERS TO MAXIMIZE ROLL-OFF?**
Constant Static Deflection

4 and 2 Layer Stacks with silicone springs

qp

40  20  0  -20  -40  -60  -80  -100
EFFECTS OF DAMPING ON ISOLATION

\[ \frac{X_1}{X_G} = \frac{K/M + C/M}{(j\omega)^2 + C/M(j\omega) + K/M} = \frac{\omega_0^2 + \frac{\omega}{Q}}{(j\omega)^2 + \frac{\omega_0}{Q}(j\omega) + \omega_0^2} \]

\[ \omega_0^2 = \frac{K}{M} \]
\[ Q = \frac{\omega_0}{C/M} \]

As \( \omega \to \infty \) slope is \( \omega \)

WHY DO WE NEED DAMPING IN STACK?

- In prototypes, damping is essential; seismic noise at Caltech and MIT is high enough that cavities would be much more difficult to lock and stay in lock
- May also be nonlinear coupling into gravity wave signal due to seismic peak motion
- Have no verification for minimum damping required in LIGO but believe that \( Q < 10 \) is more than adequate
Effect of Damping on Vibration Isolation of a Simple Harmonic Oscillator

![Graph showing the effect of damping on vibration isolation. The graph plots gain against radians per second for different values of Q, with critical damping denoted as a dotted line.](image-url)
CONSIDERATIONS FOR CHOOSING SPRING/DAMPER MATERIAL

1. VACUUM COMPATIBILITY?
2. DAMPING?
3. STIFFNESS?
4. LOAD (TOTAL LOAD IN MARK II IS OVER A TON)

- METAL SPRINGS ARE VACUUM COMPATIBLE BUT DON'T PROVIDE MUCH DAMPING
- ELASTOMER SPRINGS PROVIDE DAMPING BUT NEED TO BE SPECIALLY PROCESSED BEFORE THEY ARE VACUUM COMPATIBLE
- ELASTOMERS HAVE NICE PROPERTY THAT DAMPING IS FREQUENCY DEPENDENT; LOTS OF DAMPING AT LOW FREQUENCIES AND LITTLE DAMPING AT HIGHER FREQUENCIES
  - RTV: FLEXIBLE (ABOUT 40% DEFLECTION) BUT LITTLE DAMPING
  - VITON: STIFFER (ABOUT 20% DEFLECTION) BUT MORE DAMPING
- PICKED A SIZE FOR THE SPRINGS SO COULD LOAD WITH 55 KG PER SPRING
REAL SPRING/MASS ISOLATORS MAY BE MORE DIFFICULT TO MODEL

- REAL SYSTEMS ARE 6 DIMENSIONAL, NOT 1; TILTS AND TRANSLATIONS COUPLE INTO TEST MASS MOTION

- Why be concerned with vertical isolation?

- SPRINGS AND DASHPOTS ARE NOT NECESSARILY "BEST MODEL" FOR REAL SPRING AND DAMPING ELEMENTS

- MASSES ARE NOT RIGID AT ALL FREQUENCIES

curvature of earth is .5 mrad for 4 Km
Measuring Stack Transfer Functions

- Drive base of stack with shaker and measure ratio between $a_1$ and $a_2$
- Above about 40 Hz, signal in $a_2$ is mainly acoustic pickup so must do measurements in vacuum
- Above about 100 Hz ($10^{-5}$ attenuation) signal is mainly sensor electronics noise
- To get higher frequency points use mechanical amplifier
PENDULA AS VIBRATION ISOLATORS

TRANSMISSIBILITY FUNCTION:

\[ \frac{X_1}{X_G} = \frac{g/l}{(j\omega)^2 + g/l} \]

Effects of:

- shortening pendulum ⇒ same as stiffening spring
- adding damping (e.g. air) ⇒ same as in spring/mass case; roll-off varies between \( \omega^{-1} \) and \( \omega^{-2} \) depending on level of damping
- pendulum in series ⇒ same as adding more layers (get \( \omega^{-2 \times N} \) roll-off where N is the number of pendulum stages)
CALCULATING DIRECT TRANSMISSION OF GROUND MOTION TO TEST MASS MOTION

- MEASURE POWER SPECTRAL DENSITY OF GROUND MOTION:
  \[ X_G(f) \]

- MEASURE STACK TRANSFER FUNCTION:
  \[ \frac{X_{MP}(f)}{X_G(f)} \]

- MEASURE PENDULUM TRANSFER FUNCTION:
  \[ \frac{X_{TM}(f)}{X_{MP}(f)} \]

\[ X_{TM}(f) = \frac{X_{TM}(f)}{X_{MP}(f)} \frac{X_{MP}(f)}{X_G(f)} X_G(f) \]
Comparison of Vibration Isolation Stacks

Horizontal Transmission

Present Design (oil whirl)

New Design (Viton and RMY)

Frequency (Hz)

10^{-8}

10^{-6}

10^{-4}

10^{-2}

10^{0}

10^{2}

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LIGO DISPLACEMENT NOISE

\[ x(f) \text{ (m/Hz}^{1/2}) \]

- Vion Stacks
- Vion/RTV Stacks
- Target Displacement Noise

Frequency (Hz)
OTHER APPLICATIONS FOR VIBRATION ISOLATION

1. CAR SUSPENSION

2. MACHINERY RAFTS

3. DENTAL DRILLS

4. Many many others
METHODS FOR FUTURE VIBRATION ISOLATION SYSTEMS

WHERE SHOULD ACTIVE CONTROL BE APPLIED?

1. **ON TEST MASS DIRECTLY**
   NO NO NO!!!! Have to reduce seismic noise before get to test mass or can't detect gravity wave signal

2. **AT SUSPENSION POINT**
   Need a very sensitive sensor. Have to worry about tampering with Q of suspension. No work in progress

3. **ACTIVE STACKS**
   Need a very good 6-D model of stack to design controller (think you are driving translations but really driving tilts). Work in progress at JILA

4. **ISOLATION OUTSIDE VACUUM AT SUPPORT POINTS**
   Don't have to worry about vacuum compatibility issues. Actuators need to support loads in the tons with 10 micron stroke. Work in progress at MIT
System Configuration

Bedrock

FOOT
ELECTRONIC
MODULE

FOOT

USER INTERFACE / CONTROLLER
ACTIVE ISOLATION OUTSIDE VACUUM AT SUPPORT POINTS (BARRY MOUNTS)

- Sensor and PZT actuator pair for each of 3 translational degrees of freedom
- Provides about a factor of 30–100 isolation between 3 Hz and 100 Hz
PASSIVE METHODS (POSSIBLY COMBINED ACTIVE) FOR FUTURE VIBRATION ISOLATION SYSTEMS

- 5-STAGE PENDULUM HORIZONTAL ISOLATION (VIRGO PROJECT)
- 5-STAGE BLADE SPRING ISOLATION FOR VERTICAL (AUSTRALIAN PROJECT)

![Diagram of isolator elements]

Figure 2. Configuration of 1 isolator element.
Figure 3. Four element stack.
Horizontal isolation performance of multi-stage pendulum (not include test mass stage) 
\[ m_1 = 300 \text{kg}, \quad m_2 = m_3 = 100 \text{kg}, \quad m_4 = 200 \text{kg}, \quad Q_{\text{all}} = 100 \]

Figure 5. Comparison of horizontal isolation performance for a 4 and 5 element stack design.