LECTURE 12.

Seismic Isolation

Lecture by Lisa Sievers

Assigned Reading:

II. Leonard Meirovitch, *Elements of Vibration Analysis* (McGraw-Hill, 1986), pp. 48–58. [This reference develops the basic concepts of using mass-spring-damper systems for vibration isolation; and it discusses the measurement of vibrations, and two types of damping that can occur in mechanical systems: viscous damping and structural damping. These two types of damping will play an important role in the lectures on thermal noise next week.]

JJ. R. del Fabbro, A. di Virgilio, A. Giazotto, H. Kautzky, V. Montelatici, and D. Passuello, "Three-dimensional seismic super-attenuator for low frequency gravitational wave detection," *Physics Letters A*, **124**, 253–257 (1987). [This reference describes and analyzes an early version of the ambitious mass-spring-damper vibration-isolation stack that is being developed by the Pisa, Italy group as their prime contribution to the VIRGO Project. The analysis of the LIGO isolation stacks is similar, though their initial design is less ambitious.]

Suggested Supplementary Reading:

II. Leonard Meirovitch, *Elements of Vibration Analysis* (McGraw-Hill, 1986), pp. 39–48. [This is largely foundational material underlying the assigned reading (item 1. above); you may find it helpful.

KK. C. A. Cantley, J. Hough, and N. A. Robertson, "Vibration isolation stacks for gravitational wave detectors—Finite element analysis," *Rev. Sci. Instrum.*, **63**, 2210–2219 (1992). [This paper, by the Glasgow gravity-wave group, illustrates an isolation-stack analysis that is more sophisticated than the simple models used in class and in reference 2, and that reveals pitfalls in the design of a stack.]

LL. M. Stephens, P. Saulson, and J. Kovalik, "A double pendulum vibration isolation system for a laser interferometric gravitational wave antenna," *Rev. Sci. Instrum.*, **62**, 924–932 (1991). [This paper, passed out for other reasons in Lecture 10, analyses the use of compound pendula for vibration isolation.]

MM. L. Ju, D. G. Blair, H. Peng, and F. van Kann, "High dynamic range measurements of an all metal isolator using a sapphire transducer," *Mass. Sci. Technol.*, **3**, 463–470 (1992). [This paper, by the Perth resonant-bar gravitational-wave-detector group, describes a type of all-metal isolator which might be a precursor to an isolation stack for advanced LIGO detectors; see transparencies 20, 21, and 22 of Sievers' lecture.]

A Few Suggested Problems: See the next page.

1. You have the 2 stage spring/mass stack shown in Figure 1 and want to decide the best mass ratio m_1/m_2 in the 2 stages so that you achieve maximum isolation at frequencies well above $w_o^2=k_1/m_1$. A good designer would assume that the springs are compressed to their maximum limit in order to get the most bang for their buck, therefore the strain energy in each spring should be assumed equal ($k_1/m_1 = k_2/(m_1+m_2)$). Show that the transmissibility $X_1(f)/X_g(f)$ is maximized as the mass ratio m_1/m_2 goes to zero but that a point of diminishing returns is reached when the ratio is about 1.

2. Work out the equations of motion for the 1 and 2 stage pendula shown in Figure 2. Compare the amount of isolation achieved at two different frequencies: $\omega = 2\sqrt{\frac{g}{l}}$ and $\omega = 10\sqrt{\frac{g}{l}}$

3. A method for mechanically damping a high Q mechanical resonance is to use a "proof mass damper" as shown in Figure 3. The proof mass damper is a damped oscillator whose mass is much smaller than the mass to be damped and whose resonant frequency and damping coefficient is tuned specifically to damp the system in the most effective way. Assume $m_1=20m_2$, $f_1=4$Hz, and $f_2=(\frac{1}{1+m_2/m_1})f_1$. Plot the transmissibility function $X_1(f)/X_g(f)$ for 3 different damping coefficients, c. [Definition of damping coefficient, c: If a particle of mass m moves under the combined influence of a linear restoring force -kx and a resisting force $-c\dot{x}$, the differential equation which describes the motion is $m\ddot{x} + c\dot{x} + kx = 0$ ..... c is inversely proportional to Q]

   1. c=0
   2. c=infinity
   3. $c=2m_2(2\pi f_1)\sqrt{\frac{3m_2/m_1}{8(1+m_2/m_1)^3}}$

   The third case is the case where you get the maximum attenuation possible (i.e. $X_1(f_1)/X_g(f_1)=\sqrt{1+2m_1/m_2}$)

   [A proof mass damper has been experimentally implemented in Mark I. One of the stacks (i.e. optics plate mounted on rubber), had a high Q horizontal resonance at $f_1=4$Hz. In a compact vacuum sealed vessel, we built a pendulum whose bob was 1/20 the mass of the offending optics plate. The pendulum was partially submerged in motor oil whose damping coefficient was given in (3). The length of the pendulum bob was tuned to the resonant frequency of f_2. The stack resonance was damped without compromising the isolation at higher frequencies.]

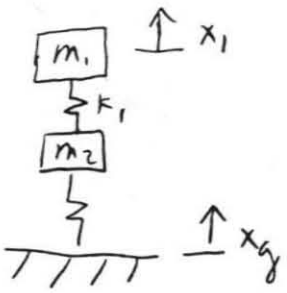


Figure 1

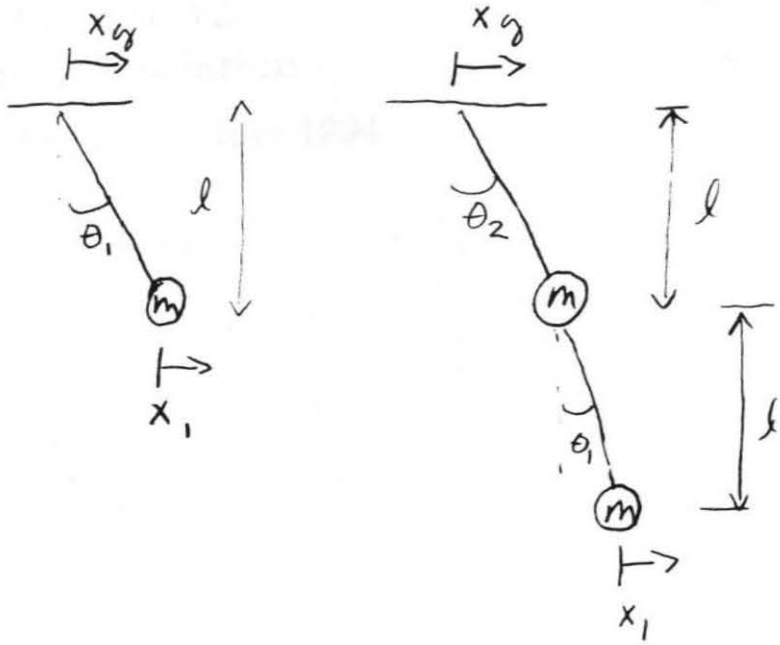
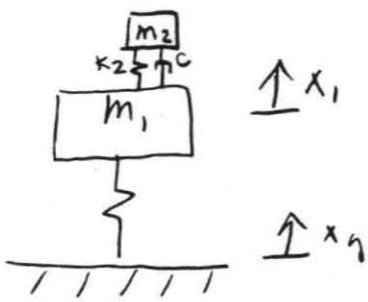


Figure 2



$$2\pi f_1 = \sqrt{\frac{k_1}{m_1}}$$

$$2\pi f_2 = \sqrt{\frac{k_2}{m_2}}$$

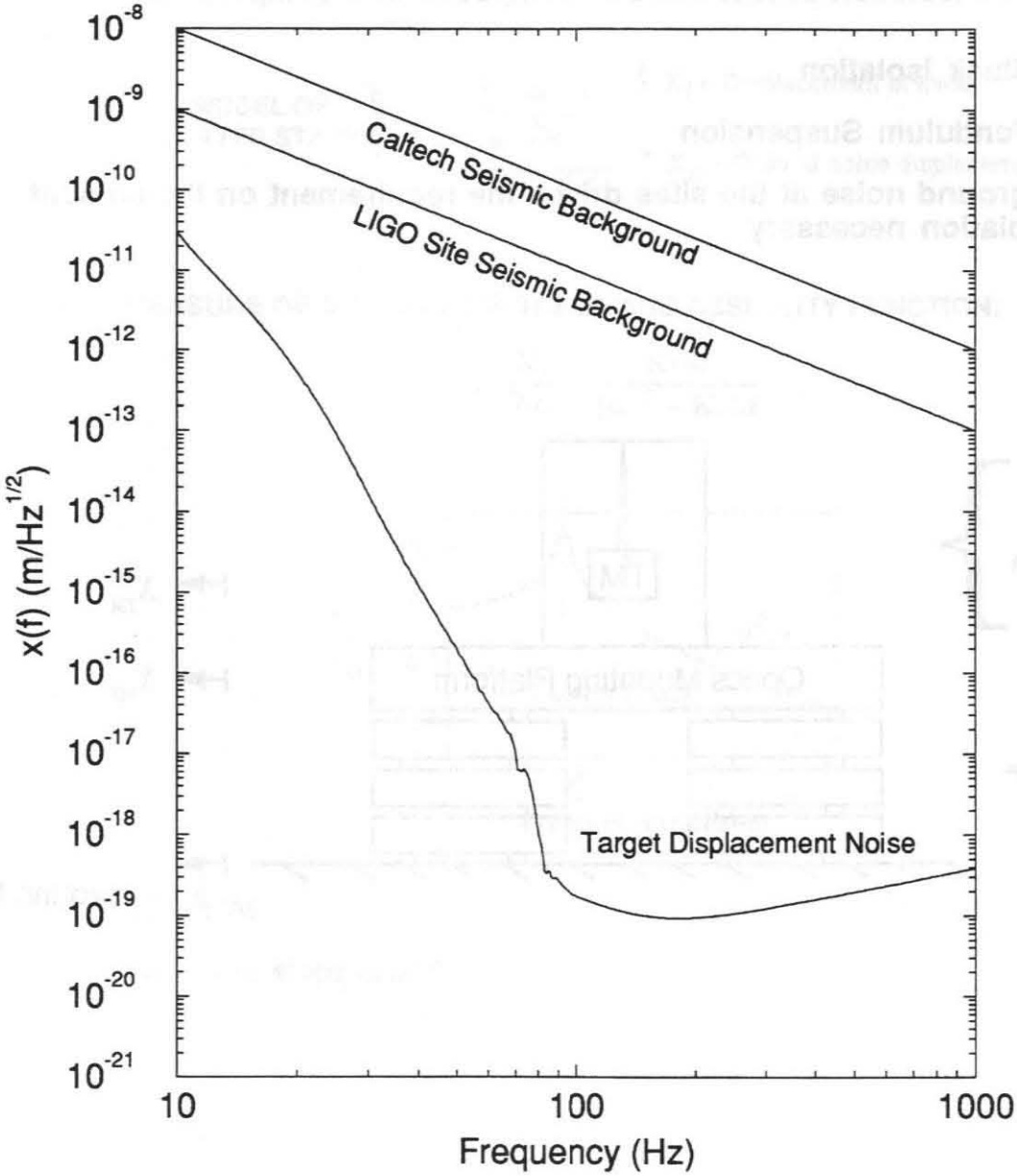
Figure 3

Lecture 12
Seismic Isolation

by Lisa Sievers, 6 May 1994

Sievers lectured from the following transparencies.

LIGO DISPLACEMENT NOISE

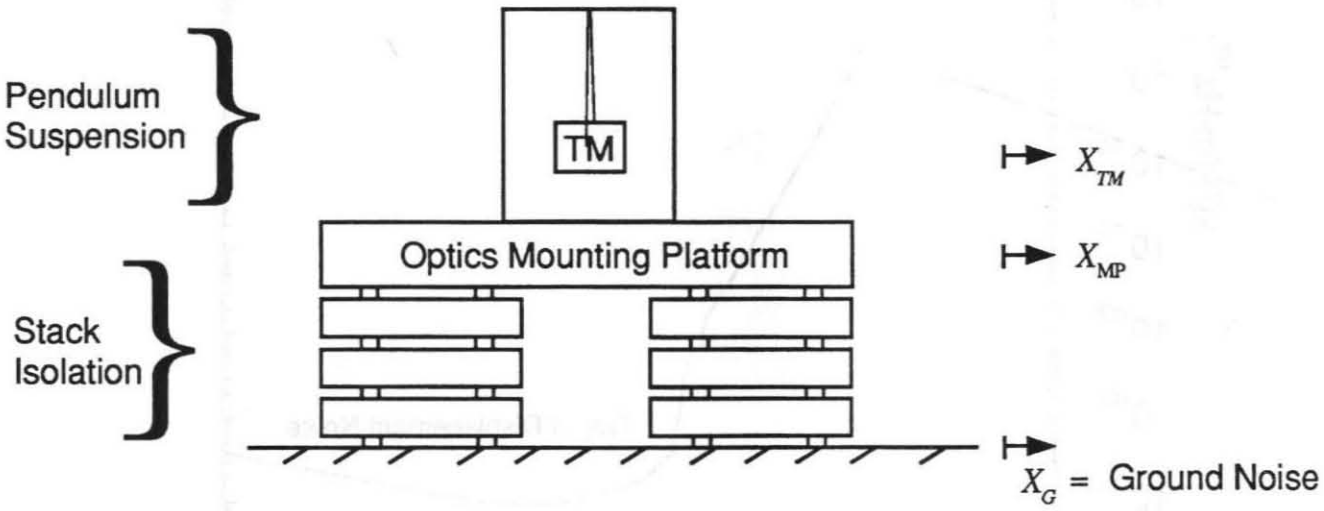


SEISMIC ISOLATION OF TEST MASS

Seismic Isolation of test mass is composed of 2 components:

- **Stack Isolation**
- **Pendulum Suspension**

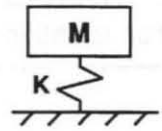
The ground noise at the sites drives the requirement on the amount of isolation necessary



LIGO

CONCEPTS FOR DESIGNING SPRING/MASS PASSIVE ISOLATION SYSTEMS

MODEL OF 1 LAYER STACK

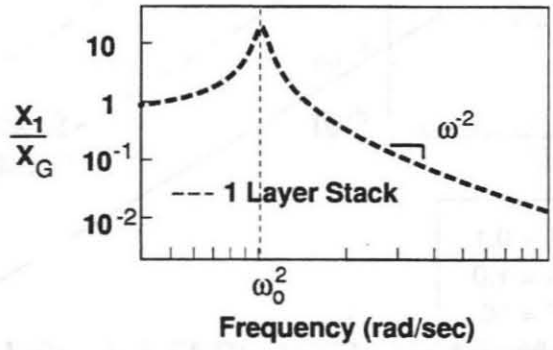


X_1 = Displacement of mass

X_G = Ground noise displacement

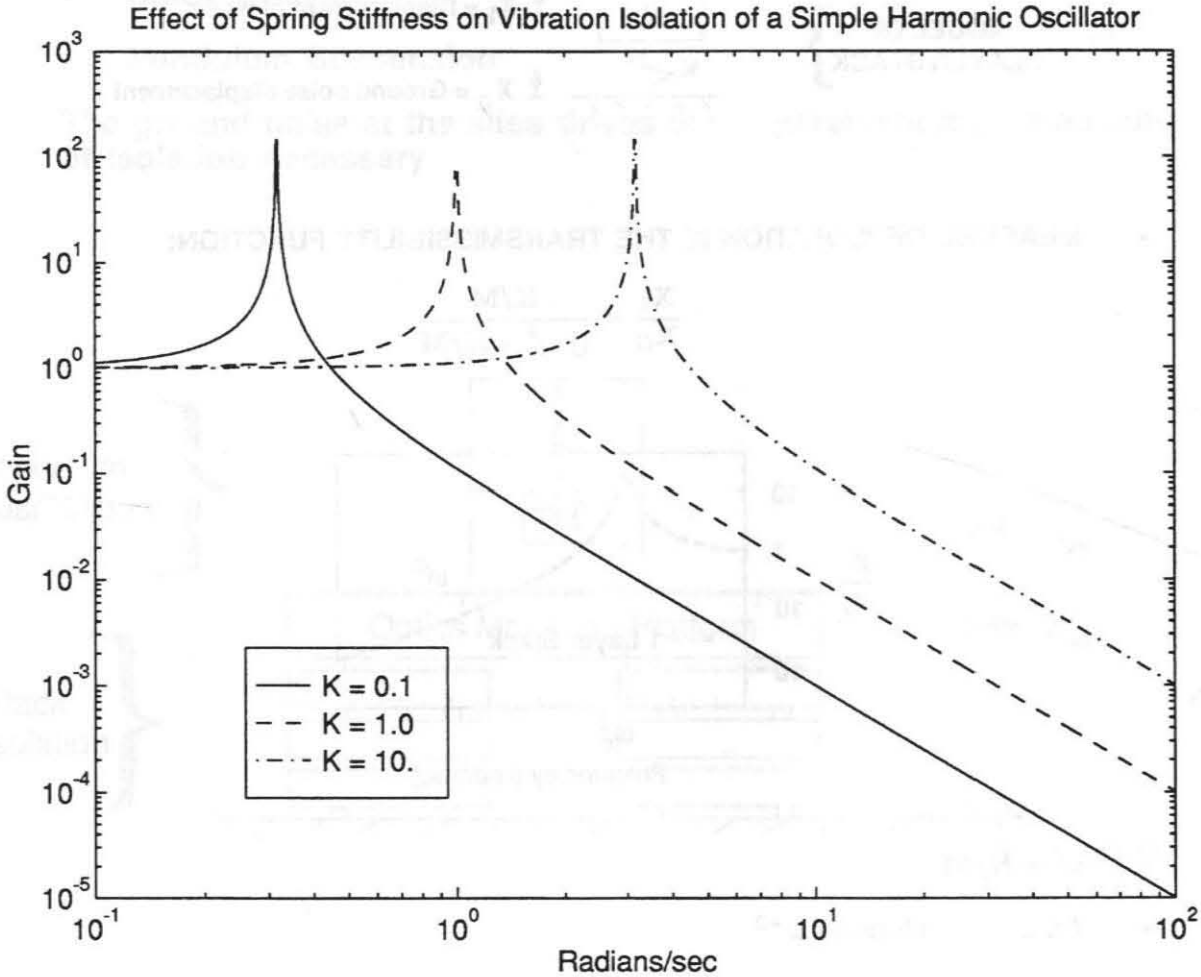
- **MEASURE OF ISOLATION IS THE TRANSMISSIBILITY FUNCTION:**

$$\frac{X_1}{X_G} = \frac{K/M}{(j\omega)^2 + K/M}$$



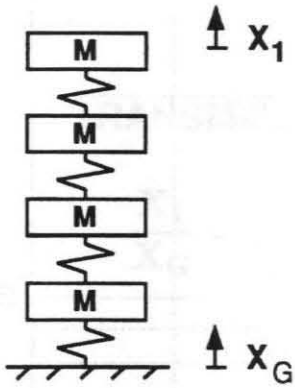
- $\omega_o^2 = K/M$
- **As** $\omega \to \infty$ **slope is** ω^{-2}

LIGO

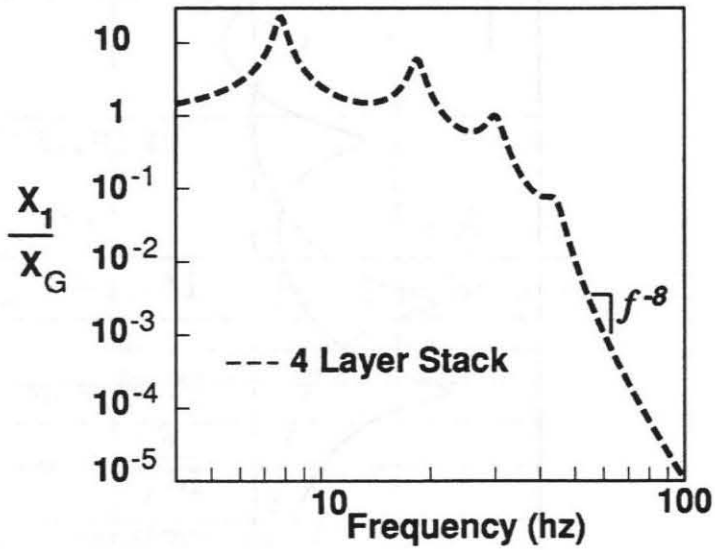

Effect of Spring Stiffness on Vibration Isolation of a Simple Harmonic Oscillator
Gain
Radians/sec
K = 0.1
K = 1.0
K = 10.

- **EFFECT OF MULTIPLE LAYERS:**

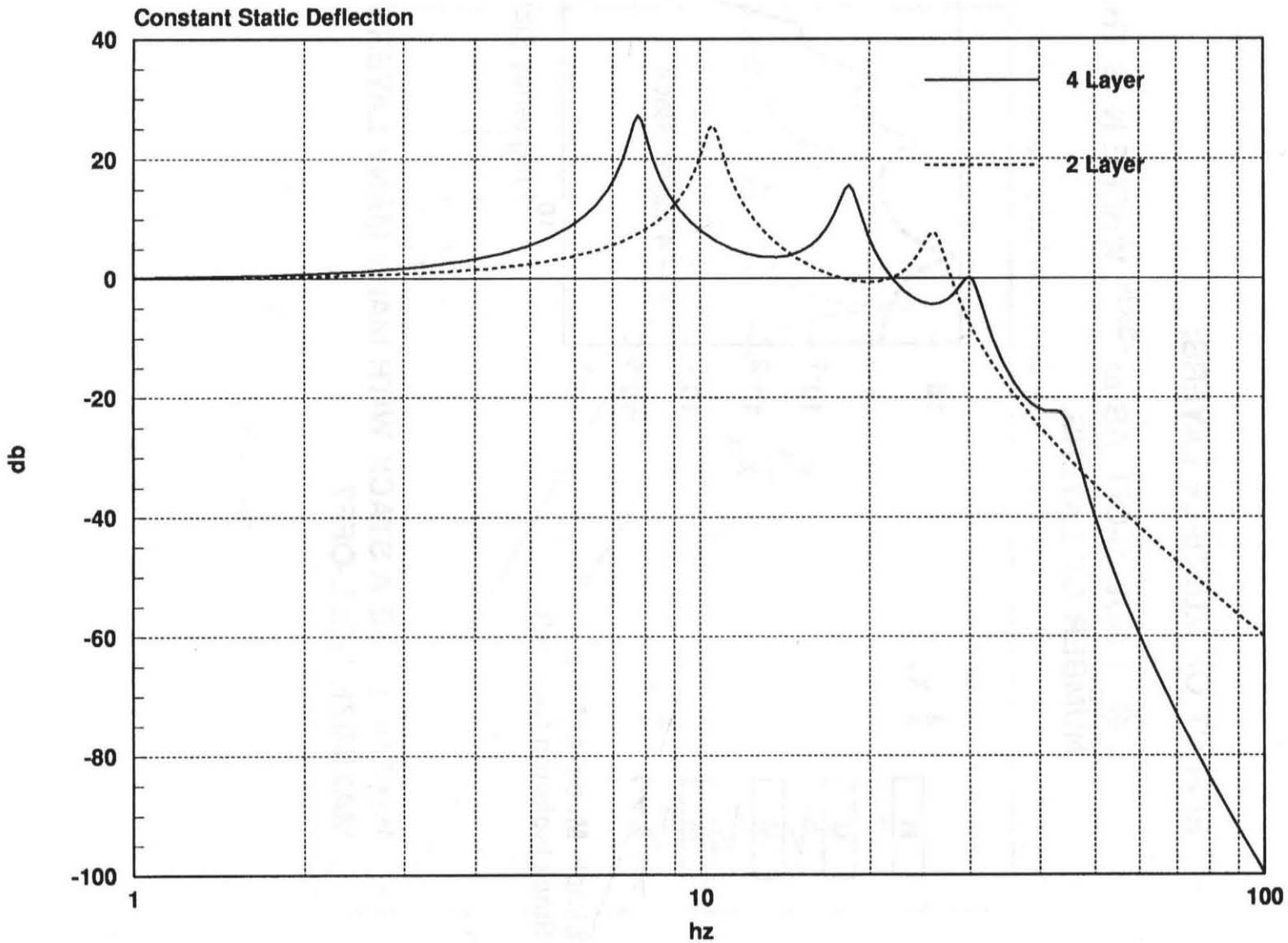
  $\left| \frac{X_1}{X_G} \right|$ **ROLLS-OFF AS** $\omega^{-2\times N}$ **WHERE N IS THE NUMBER OF LAYERS.**



Simple Model of Mark 2 Stack Isolation (vertical)

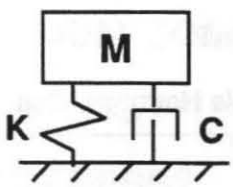


- **WHY NOT USE A STACK WITH MANY MANY LAYERS TO MAXIMIZE ROLL-OFF?**



4 and 2 Layer Stacks with silicone springs

EFFECTS OF DAMPING ON ISOLATION



X_1 = Displacement of mass

X_G = Ground noise displacement

- **TRANSMISSIBILITY FUNCTION:**

$$\frac{X_1}{X_G} = \frac{K/M + C/M}{(j\omega)^2 + C/M(j\omega) + K/M} = \frac{\omega_o^2 + \frac{\omega}{Q}}{(j\omega)^2 + \frac{\omega_o}{Q}(j\omega) + \omega_o^2}$$

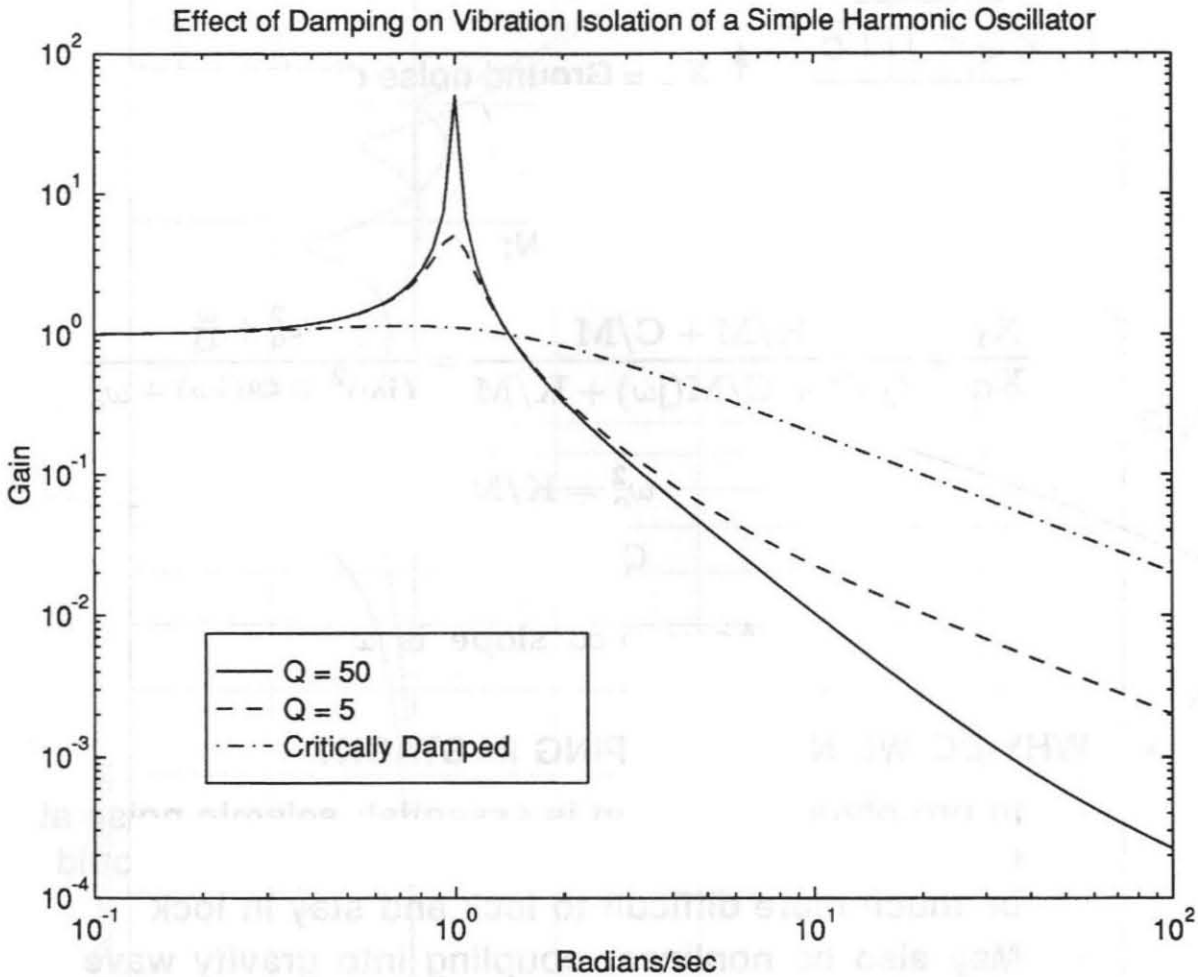
$$\omega_o^2 = K/M$$

$$Q = \frac{\omega_o}{C/M}$$

$$\text{As } \omega \to \infty \text{ slope is } \omega$$

- **WHY DO WE NEED DAMPING IN STACK?**
  - In prototypes, damping is essential; seismic noise at Caltech and MIT is high enough that cavities would be much more difficult to lock and stay in lock
  - May also be nonlinear coupling into gravity wave signal due to seismic peak motion
  - Have no verification for minimum damping required in LIGO but believe that Q < 10 is more than adequate

LIGO


Effect of Damping on Vibration Isolation of a Simple Harmonic Oscillator
Gain
Radians/sec
Q = 50
Q = 5
Critically Damped
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^{0}
10^{1}
10^{2}

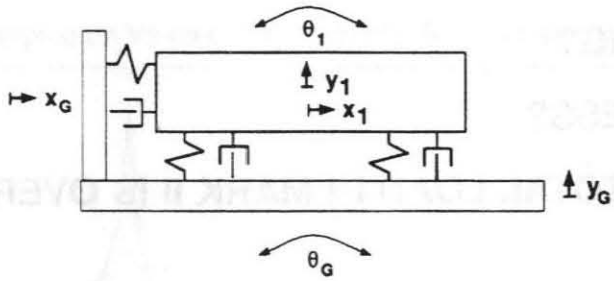
CONSIDERATIONS FOR CHOOSING SPRING/DAMPER MATERIAL

1. VACUUM COMPATIBILITY?
2. DAMPING?
3. STIFFNESS?
4. LOAD (TOTAL LOAD IN MARK II IS OVER A TON)

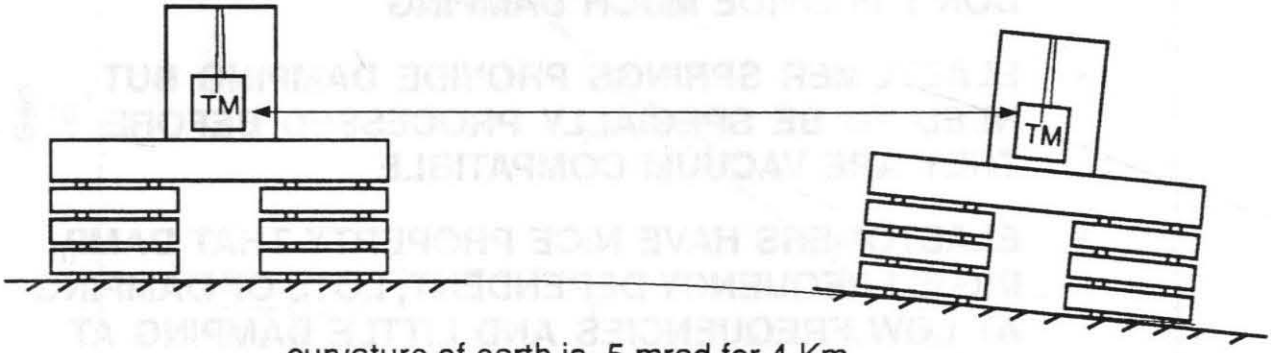
- METAL SPRINGS ARE VACUUM COMPATIBLE BUT DON'T PROVIDE MUCH DAMPING
- ELASTOMER SPRINGS PROVIDE DAMPING BUT NEED TO BE SPECIALLY PROCESSED BEFORE THEY ARE VACUUM COMPATIBLE
- ELASTOMERS HAVE NICE PROPERTY THAT DAMPING IS FREQUENCY DEPENDENT; LOTS OF DAMPING AT LOW FREQUENCIES AND LITTLE DAMPING AT HIGHER FREQUENCIES
  - RTV: FLEXIBLE (ABOUT 40% DEFLECTION) BUT LITTLE DAMPING
  - VITON: STIFFER (ABOUT 20 % DEFLECTION) BUT MORE DAMPING
- PICKED A SIZE FOR THE SPRINGS SO COULD LOAD WITH 55 KG PER SPRING

REAL SPRING/MASS ISOLATORS MAY BE MORE DIFFICULT TO MODEL

- **REAL SYSTEMS ARE 6 DIMENSIONAL, NOT 1; TILTS AND TRANSLATIONS COUPLE INTO TEST MASS MOTION**

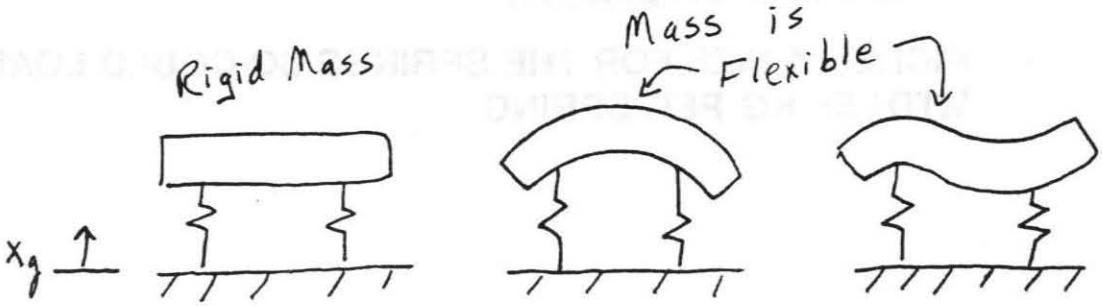


- **Why be concerned with vertical isolation?**



curvature of earth is .5 mrad for 4 Km

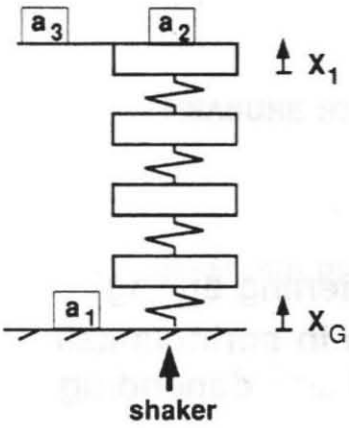
- **SPRINGS AND DASHPOTS ARE NOT NECESSARILY "BEST MODEL" FOR REAL SPRING AND DAMPING ELEMENTS**
- **MASSES ARE NOT RIGID AT ALL FREQUENCIES**

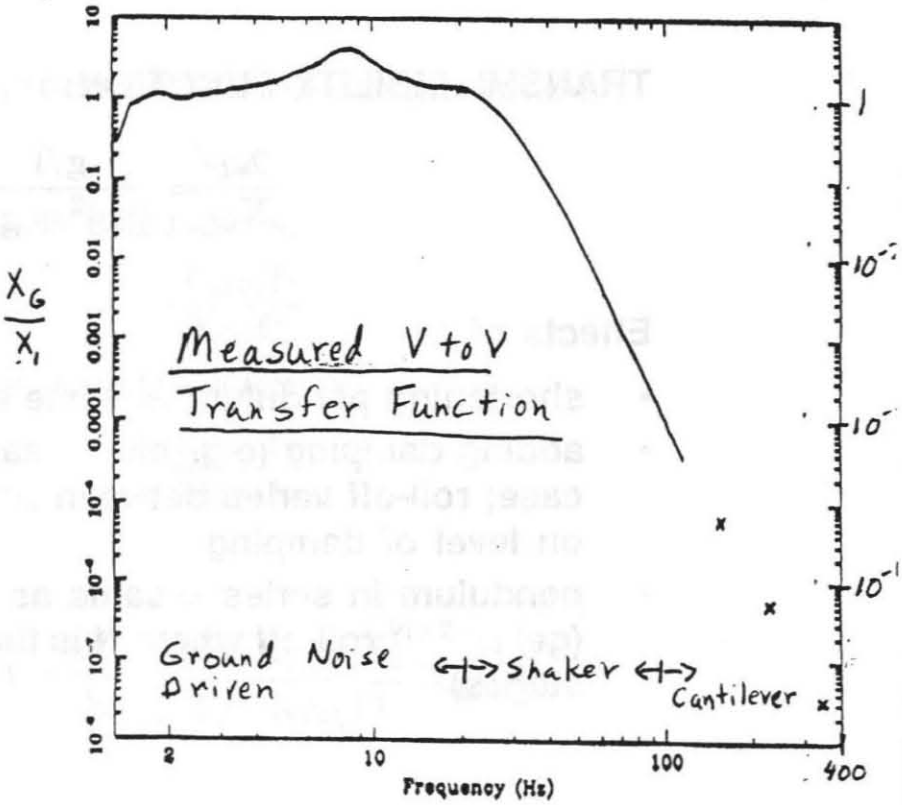


LIGO

Measuring Stack Transfer Functions

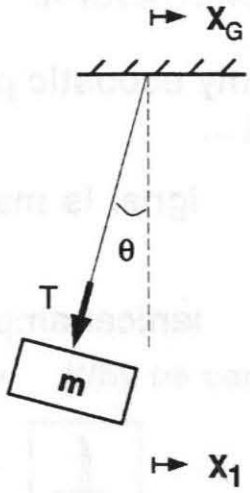
- Drive base of stack with shaker and measure ratio between a1 and a2
- Above about 40 Hz, signal in a2 is mainly acoustic pickup so must do measurements in vacuum
- Above about 100 Hz (10^{-5} attenuation) signal is mainly sensor electronics noise
- To get higher frequency points use mechanical amplifier

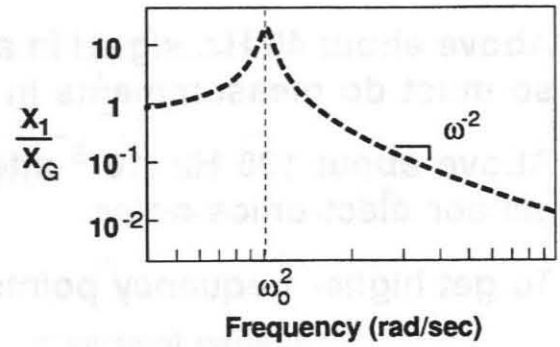




LIGO

PENDULA AS VIBRATION ISOLATORS





$\omega_0^2 = g / l$

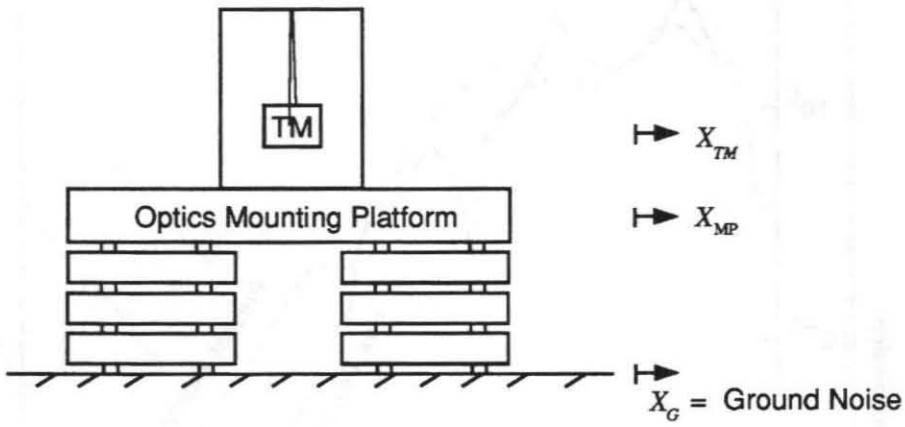
TRANSMISSIBILITY FUNCTION:

$$\frac{X_1}{X_G} = \frac{g/l}{(j\omega)^2 + g/l}$$

Effects of :

- **shortening pendulum ⇒ same as stiffening spring**
- **adding damping (e.g. air) ⇒ same as in spring/mass case; roll-off varies between ω^{-1} and ω^{-2} depending on level of damping**
- **pendulum in series ⇒same as adding more layers (get $\omega^{-2\times N}$ roll-off where N is the number of pendulum stages)**

CALCULATING DIRECT TRANSMISSION OF GROUND MOTION TO TEST MASS MOTION



- **MEASURE POWER SPECTRAL DENSITY OF GROUND MOTION:**

$$\mathbf{X_G(f)}$$

- **MEASURE STACK TRANSFER FUNCTION:**

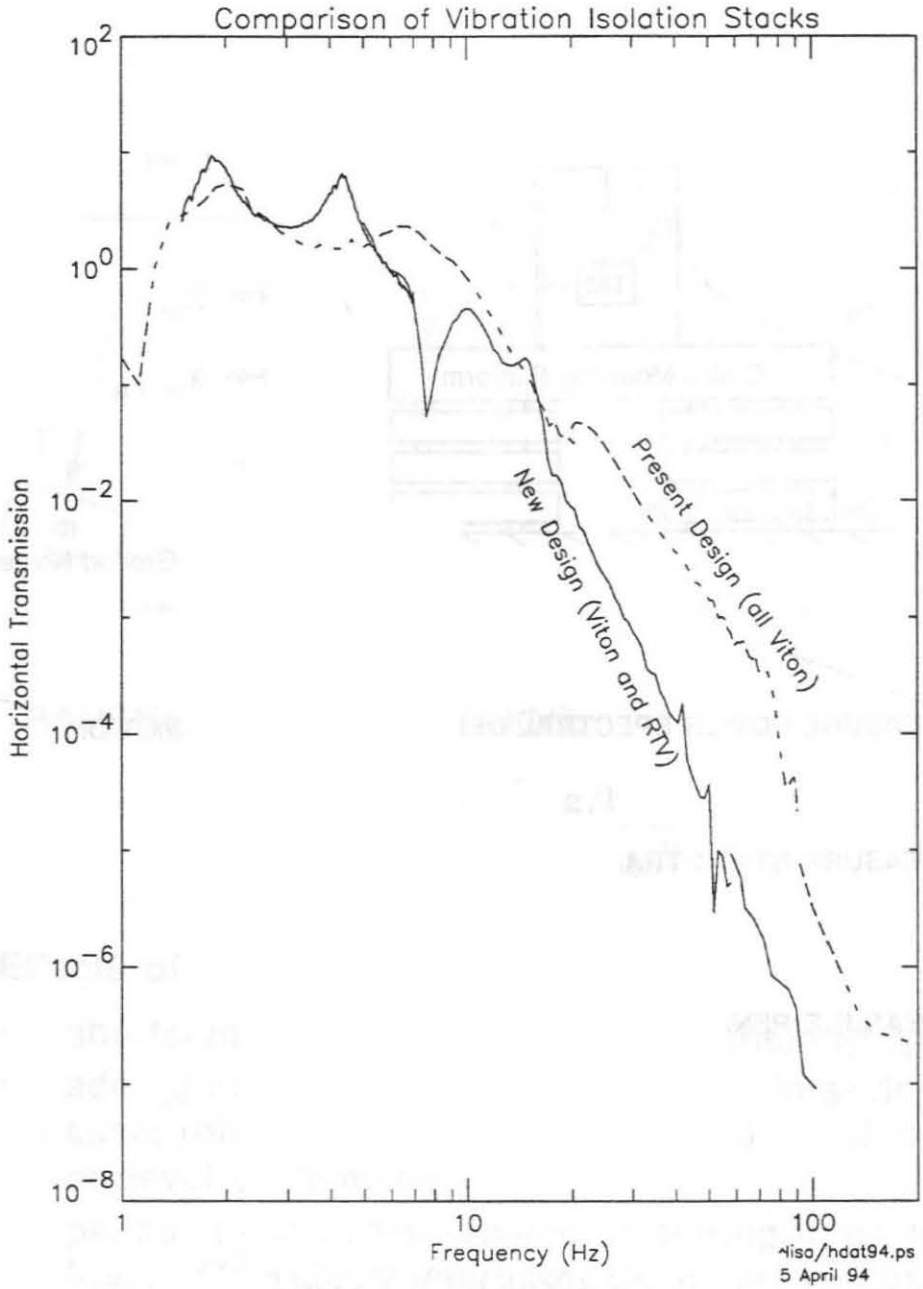
$$\frac{\mathbf{X_{MP}(f)}}{\mathbf{X_G(f)}}$$

- **MEASURE PENDULUM TRANSFER FUNCTION:**

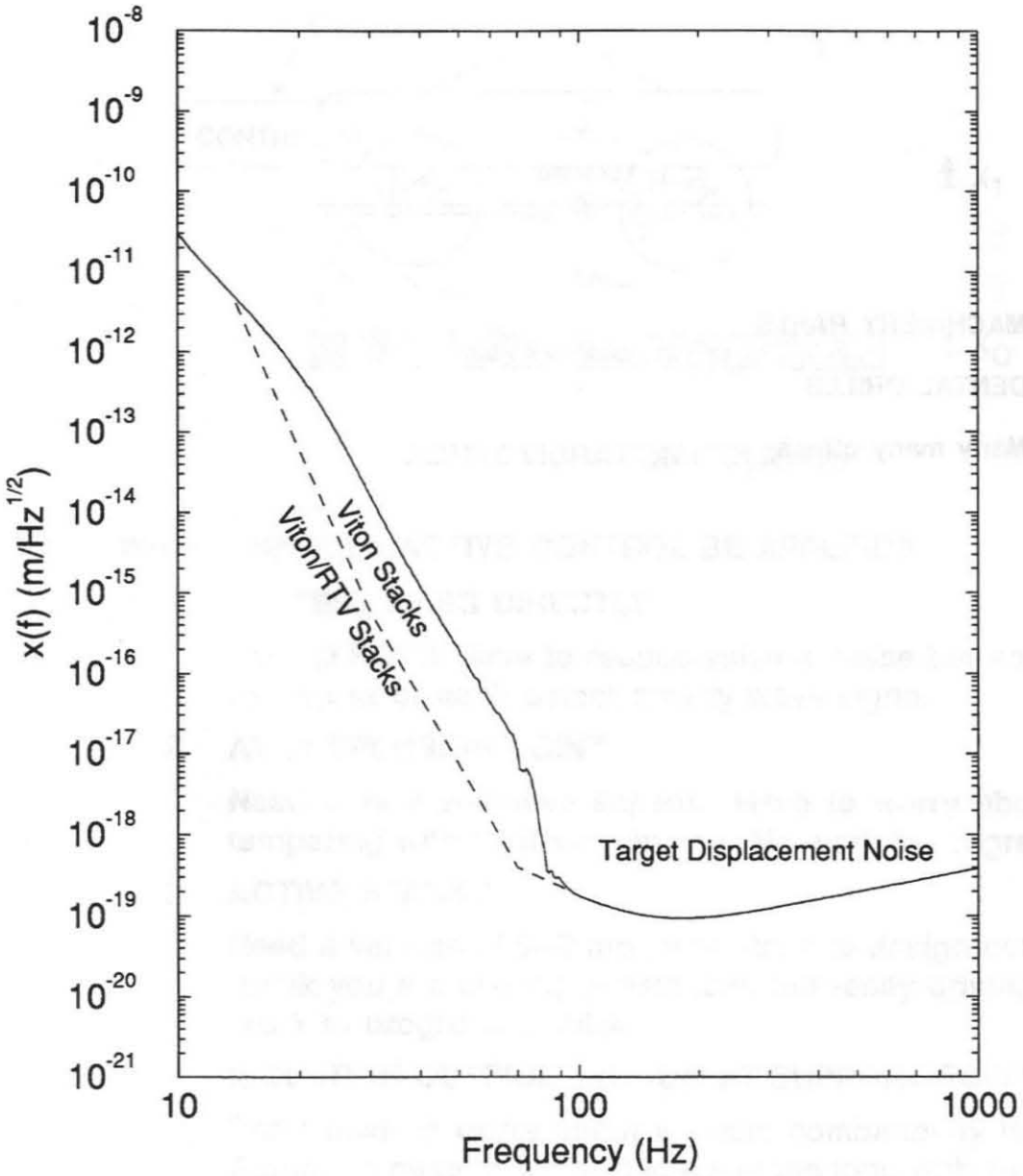
$$\frac{\mathbf{X_{TM}(f)}}{\mathbf{X_{MP}(f)}}$$

$$\mathbf{X_{TM}(f)} = \frac{\mathbf{X_{TM}(f)}}{\mathbf{X_{MP}(f)}} \frac{\mathbf{X_{MP}(f)}}{\mathbf{X_G(f)}} \mathbf{X_G(f)}$$

LIGO

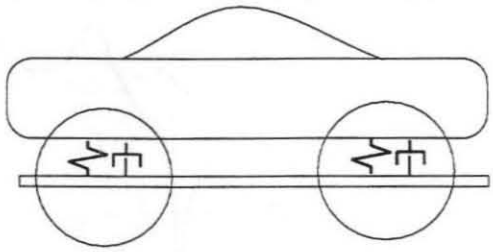

Comparison of Vibration Isolation Stacks
Horizontal Transmission
Frequency (Hz)
Present Design (all Viton)
New Design (Viton and RTV)
Alisa/hdat94.ps
5 April 94

LIGO DISPLACEMENT NOISE



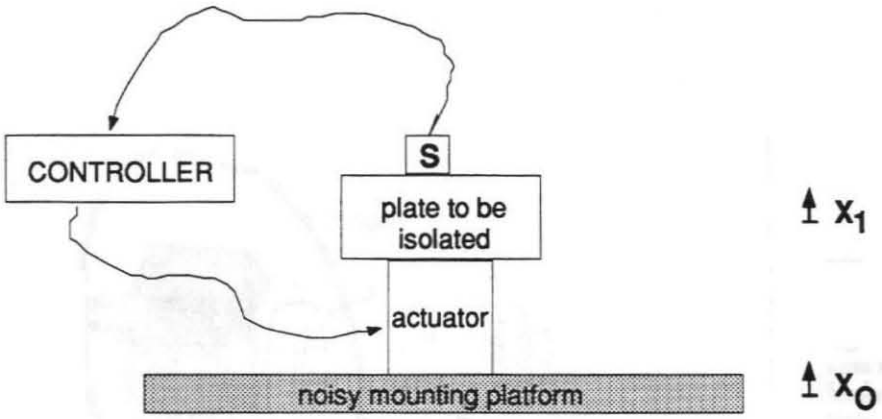
OTHER APPLICATIONS FOR VIBRATION ISOLATION

1. **CAR SUSPENSION**



2. **MACHINERY RAFTS**
3. **DENTAL DRILLS**
4. **Many many others**

METHODS FOR FUTURE VIBRATION ISOLATION SYSTEMS



ACTIVE VIBRATION ISOLATION

WHERE SHOULD ACTIVE CONTROL BE APPLIED?

1. **ON TEST MASS DIRECTLY**

   NO NO NO!!!! Have to reduce seismic noise before get to test mass or can't detect gravity wave signal

2. **AT SUSPENSION POINT**

   Need a very sensitive sensor. Have to worry about tampering with Q of suspension. No work in progress

3. **ACTIVE STACKS**

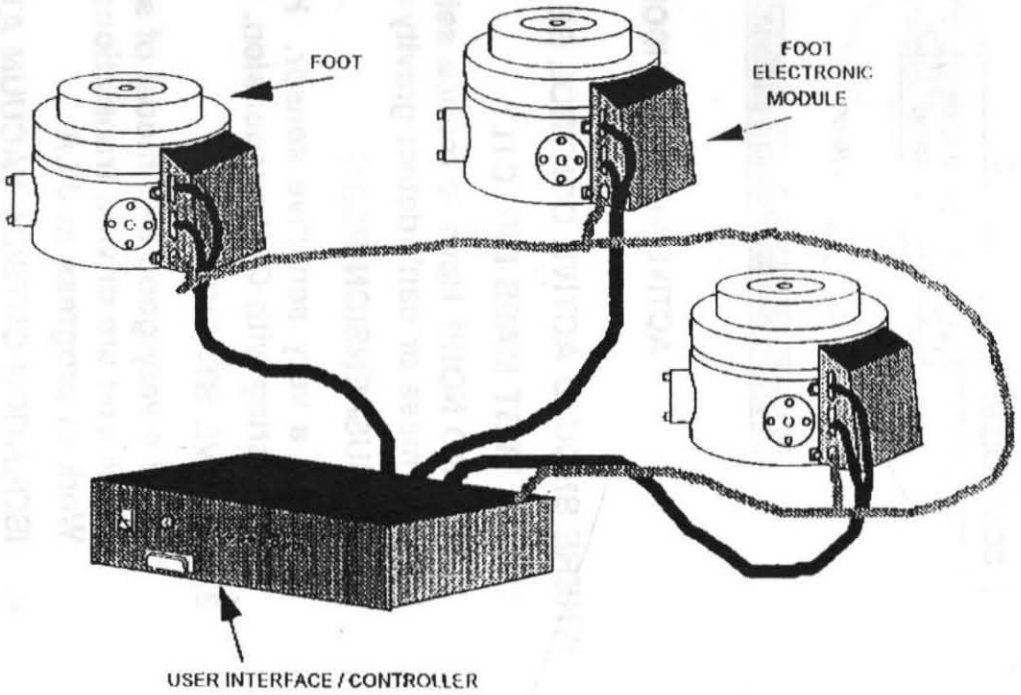
   Need a very good 6–D model of stack to design controller (think you are driving translations but really driving tilts). Work in progress at JILA

4. **ISOLATION OUTSIDE VACUUM AT SUPPORT POINTS**

   Don't have to worry about vacuum compatibility issues. Actuators need to support loads in the tons with 10 micron stroke. Work in progress at MIT

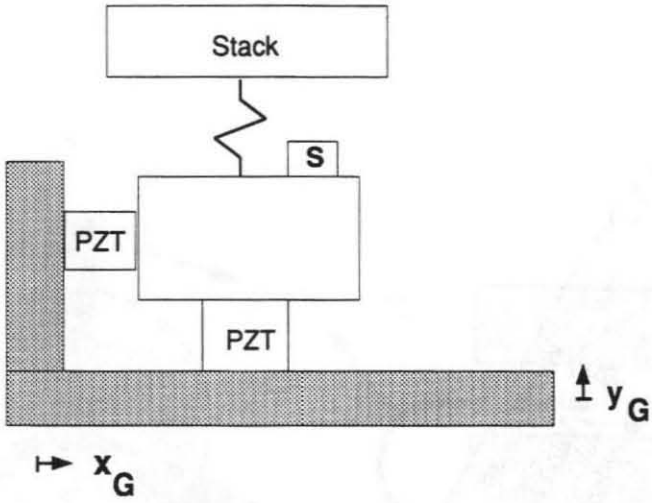
LIGO

System Configuration



BARRY CONTROLS
A UNIT OF APPLIED POWER INC.

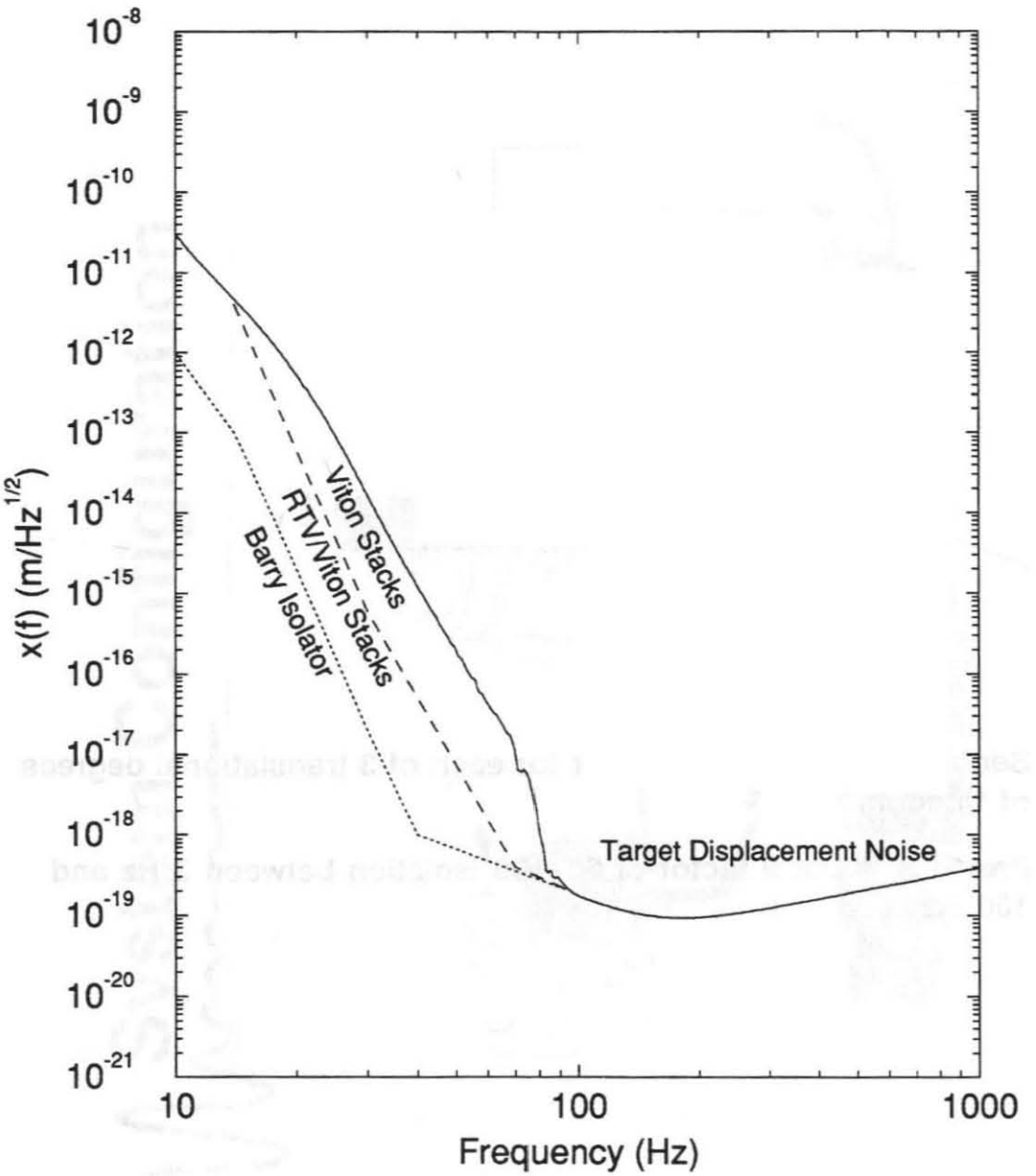
ACTIVE ISOLATION OUTSIDE VACUUM AT SUPPORT POINTS (BARRY MOUNTS)



- **Sensor and PZT actuator pair for each of 3 translational degrees of freedom**
- **Provides about a factor of 30–100 isolation between 3 Hz and 100 Hz**

LIGO

LIGO DISPLACEMENT NOISE



PASSIVE METHODS (POSSIBLY COMBINED ACTIVE) FOR FUTURE VIBRATION ISOLATION SYSTEMS

- **5–STAGE PENDULUM HORIZONTAL ISOLATION (VIRGO PROJECT)**
- **5–STAGE BLADE SPRING ISOLATION FOR VERTICAL (AUSTRALIAN PROJECT)**

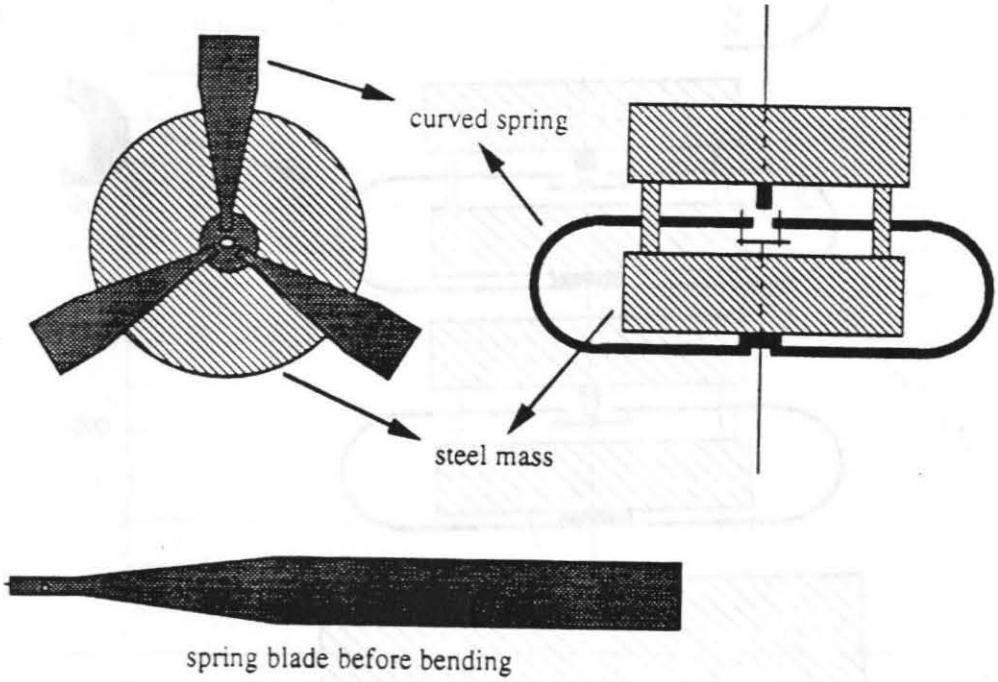


Figure 2. Configuration of 1 isolator element.

LIGO

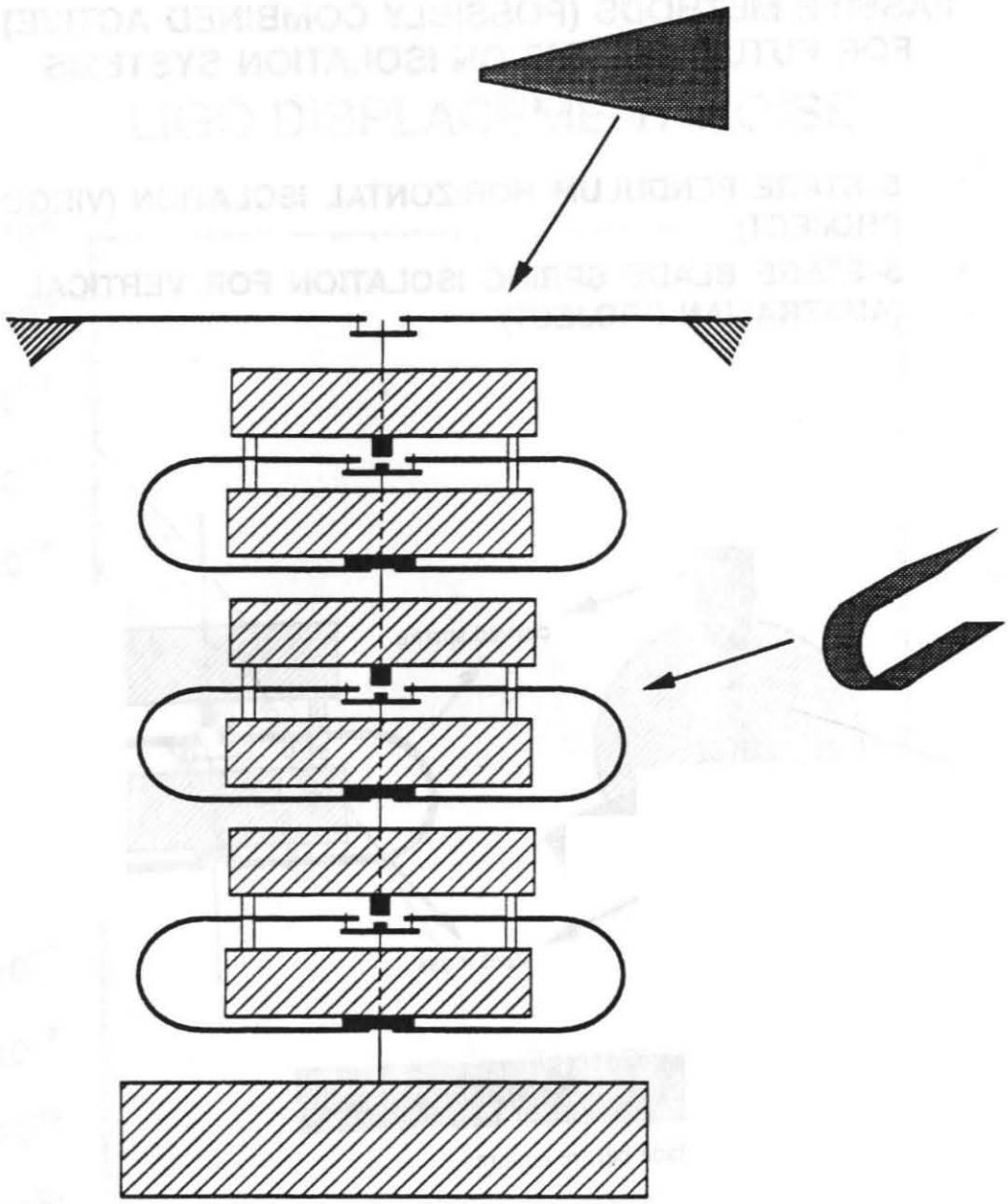


Figure 3. Four element stack.

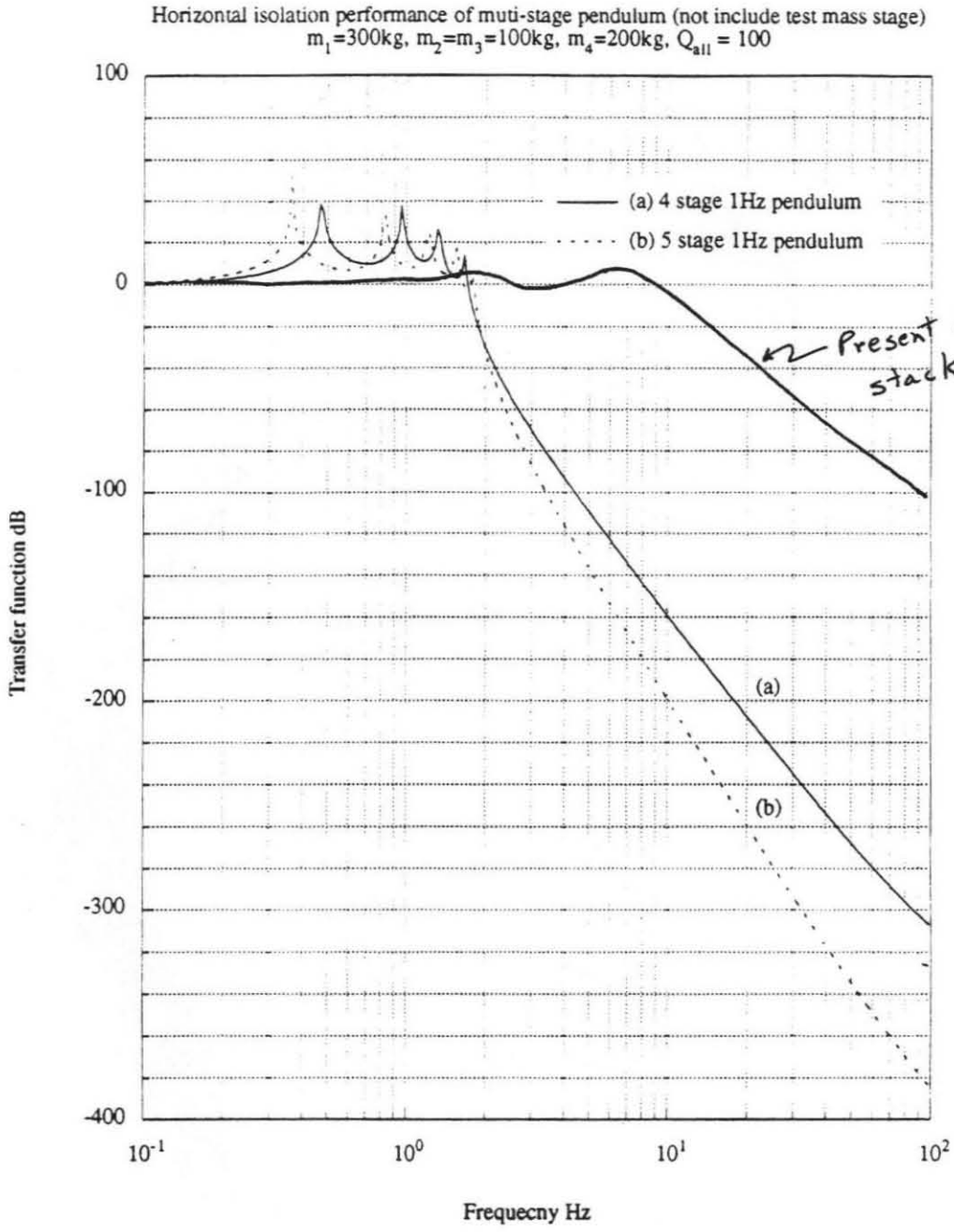


Figure 5. Comparison of horizontal isolation performance for a 4 and 5 element stack design.