Lecture 18

The 40 Meter prototype Interferometer as an Example of Many of the Issues Studied in this Course

Lecture by Robert Spero

Assigned Reading:


YY. Robert L. Forward, "Wideband laser-interferometer gravitational-radiation experiment," *Phys. Rev. D* 17(2), 379–390 (1977). [A description of the first interferometric gravitational-wave detector to be built (a 2 m Michelson interferometer) and the first search for gravitational waves using such a detector (a coincidence run conducted in 1972 between the interferometer and several bar detectors).]

A Few Suggested Problems: Note: Your homework for Lectures 17 and 18 is to be turned in to Shirley Hampton in room 151 Bridge Annex before 1:00PM Friday June 3

1. In this course you have encountered all the significant noise sources for interferometric detectors that the LIGO team is now aware of. Which of these were unanticipated by Weiss in Ref. 1 above; and are they “fundamental” or are they “technical”?

2. The interferometer described in Weiss’s paper has a different optical configuration from the 40 m interferometer, but the shot noise calculated by Weiss is similar to that achieved in the 40 m. Why? Compare the shot noise limited sensitivity calculated by Weiss with the sensitivity achieved by Forward, and account for the difference.

3. The thermal noise calculated by Weiss is based on viscous damping $|\phi(\omega)\propto\omega|$. How does the thermal noise prediction change when if the damping is structural $|\phi(\omega)$ independent of $\omega|$? cf. Lectures 13 and 14.

4. Weiss calculated noise from laser intensity fluctuations acting on the test masses via radiation pressure. Intensity fluctuations also result in noise (in a manner discussed in Spero’s lecture) if the interferometer’s operating point is offset from a “dark fringe” at the photodiode. Under what circumstances will this noise be larger than that due to radiation-pressure fluctuations.
Lecture 18
The 40 Meter Prototype Interferometer as an Example of Many of the Issues Studied in this Course
by Robert Spero, 27 May 1994

Spero lectured from the following transparencies. Kip has annotated them a bit.
LECTURE 18

THE 40 m INTERFEROMETER

AS EXAMPLE OF ISSUES STUDIED IN COURSE

R. SPERO

27 MAY 1994
Time Domain Data, Pulse Height Distribution, and Pulse Calibration [Lecture 3]

View interferometer output in time domain,

\[ x(t) \]

and analyze as a train of pulses of varying height \( x(t) \)

Gaussian distribution: \( N(x) = e^{-\frac{1}{2} \left( \frac{x}{\sigma} \right)^2} \)

where \( N(x) \) = "density" of pulses with amplitude \( x \)

\( \sigma \) = average (rms) pulse height

Then

signal-to-noise ratio (SNR) of a pulse = \( \frac{x}{\sigma} \)

Histogram the time series

\[ \log N(x) \]

Slope = \( -\frac{1}{2\sigma^2} \)
Calibration requires converting from voltage to $\Delta L$ or $h$: inject large ($x > 100$) displacement pulse of known amplitude.

Q: How can one calibrate the calibrator; i.e., convert from $V(t)$ to $\Delta L(t)$?

Calibration pulse is separate from noise:

$\sigma = \text{Pulse sensitivity of interferometer}$
Histogram of Pulse Heights

Non-Gaussian Pulse Rates:
- Mk 1 ~ 1/sec to 1/min
- Mk 2 ~ 1/hr

Tape 3/2/94, Section 1
(46 minutes)

Impressed Calibration Peaks
Gaussian Noise
Non-Gaussian Pulse
Number

0
1
10
100
1000
10000
Voltage
MATCHED FILTER TO INCREASE SNR

(A SIMULATION)

FILTER INPUT

FILTER OUTPUT

NO EVIDENCE OF SIGNAL

PULSE HEIGHT DISTRIBUTIONS

SIGNAL

TIME SERIES
IMPERFECTIONS IN PHASE MODULATION USING Pockels CELLS

[LECTURES 7, 8]

1. IDEAL Pockels CELL PHASE MODULATOR

Polarizer is aligned with crystal axis, so Pockels cell does not alter polarization.

Only effect is small modulation of index of refraction $n$, resulting in a change in optical path length or $V(t)$.

Misalignment of axes w.r.t. beam results in intensity modulation (maximum when $x'$, $y'$ rotated by 45°).
THREE POCKELS CELLS

Calculated effect of small misalignments

Angles in degrees: theta1 = 0.01, theta2 = 0.03, theta3 = 0.01

Dashed line: no drive to 2nd Pockels cell

Alpha = -0.03

P = 12 mW (audio)

Demodulated output short noise (1Hz BW)
SERVO MODEL OF POCKET C CELC MISALIGNMENT

\[
\begin{align*}
\text{A} & \quad \text{UNINTENDED FEEDBACK PATH} \\
\begin{array}{c}
\Delta f_1' \quad \Delta f_1' \\
\Delta f_2 \quad \Delta f_2 \\
\Delta f_3 \quad \Delta f_3 \\
\end{array}
\end{align*}
\]

- \( f_{MC} \): Frequency of the light out of the mode cleaner
- \( f_1 \): Resonance frequency of the primary cavity
- \( f_0 \): Stabilized frequency
- \( \Delta f_1 \): True frequency deviation (between \( f_0 \) and \( f_1 \))
- \( \Delta f_1' \): Measurable frequency deviation (between \( f_0 \) and \( f_1 \))
- \( A \): Open loop transfer function of the primary servo
- \( f_{PC} \): Equivalent correction frequency to the PC
- \( B_1 \): Transfer function from PC correction frequency to deviation frequency (in the primary) due to PC misalignment (through intensity noise around 12MHz)
- \( f_{B1} \): False frequency deviation due to PC misalignment
- \( f_2 \): Resonance frequency of the secondary cavity
- \( \Delta f_2 \): True frequency deviation (between \( f_0 \) and \( f_2 \))
- \( \Delta f_2' \): Measurable frequency deviation (between \( f_0 \) and \( f_2 \))
- \( B_2 \): Transfer function from PC correction frequency to deviation frequency (in the secondary) due to PC misalignment
- \( f_{B2} \): False frequency deviation due to PC misalignment
Fig. 1  Frequency suppression of the primary cavity servo.

A: Measured
B: Calculated (with spurious paths)
C: Calculated (without spurious paths)
MIRROR HEATING AS LIMIT TO OPTICAL POWER

[LECTURE 9]

Two HEATING EFFECTS

1) MECHANICAL DISTORTION VIA THERMAL EXPANSION ($\Delta L$)

2) OPTICAL DISTORTION VIA TEMPERATURE DEPENDENCE OF INDEX OF REFRACTION ($\Delta n/\Delta T$)

Both result in lensing and beam distortions that are difficult to control if the absorbed power is $P_{\text{abs}} \gtrsim 1$ W.
HEATING EFFECTS MINIMIZED BY

1) REDUCING ABSORPTION IN COATING
\[ \frac{P_{\text{absorbed}}}{P_{\text{incident}}} < 10^{-5} \] coatings are available

2) SELECTING SUBSTRATE MATERIAL
   - SMALL \( d \), SMALL \( \frac{\Delta n}{\Delta T} \) LARGE THERMAL CONDUCTIVITY \( \lambda \) (thermal expansion)

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>( \alpha ) (10^{-6}/\text{K})</th>
<th>( \frac{\Delta n}{\Delta T} ) (10^{-6}/\text{K})</th>
<th>( \lambda ) (W/m-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fused Silica</td>
<td>0.59</td>
<td>12</td>
<td>1.3</td>
</tr>
<tr>
<td>BK7</td>
<td>7.1</td>
<td>0.6</td>
<td>1.1</td>
</tr>
<tr>
<td>Sapphire</td>
<td>8.4</td>
<td>13</td>
<td>35</td>
</tr>
</tbody>
</table>

It appears that coating heating will always dominate over substrate heating.

3) PIECES IN THE SKY
   a) ADVANCED OPTICAL ARRANGEMENTS, SUCH AS DUAL RECYCLING
   b) ADAPTIVE OPTICS
ANTI-NOISE FILTERS AND SERVO BANDWIDTH

Lecture 10

Uncontrolled test mass is quiet in tilts in signal band $f \geq 100 \text{ Hz}$, but has large motion near $f = 1 \text{ Hz}$.

Optical lever sensing system is adequately stable near $1 \text{ Hz}$, but noisy for $f \geq 100 \text{ Hz}$ (vibration of laser, connection currents deflecting beam, ...)

(Q: How can one make a quiet optical lever?)

Low pass filter is intended to pass the low frequency control signals, and block the sensing noise before it can disturb the test mass.
PRACTICAL LIMITATIONS TO LOW PASS FILTER

1) PHASE SHIFT AFFECTS SERVO PERFORMANCE
2) FILTER OUTPUT NOISE

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = \left(1 + \frac{\omega}{\omega_0}\right)^n \]

\( \omega \gg \omega_0 \Rightarrow \left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \left(\frac{\omega}{\omega_0}\right)^{-n} \) ("STOPBAND ATTENUATION")

PHASE SHIFT IN "PASSBAND" - \( \omega \ll \omega_0 \):
\( \omega \ll \omega_0 \Rightarrow \frac{V_{\text{out}}}{V_{\text{in}}} \approx 1 - \frac{n}{\omega_0} \frac{\omega}{\omega_0} \); PHASE \( \approx -\frac{n}{\omega_0} \).

EXAMPLE: \( n = 12 \), \( f_0 = 10 \text{ Hz} \) attenuates noise at 100 Hz by \( \approx 10^{12} \).
But causes \( \approx 70^\circ \) phase shift at \( f = 1 \text{ Hz} \),
and may make servo with gain at 1 Hz unstable.
(In this case, lowering the bandwidth is an adequate solution; roll off the gain at \( f \approx 1 \text{ Hz} \) and thereby get a lower phase shift at unity gain)
JOHNNON NOISE OF RESISTOR

\[ V_{\text{noise}}^2 = 4A_T R \]

\[ \sqrt{I} = \sqrt{4A_T R} \]

\[ \sqrt{30} = 9 \times 10^{-10} \text{ V/} \sqrt{\text{Hz}} \]

**Practical Consequence:** Noise at filter output can be significant, especially at frequencies where loop gain is small.

**Johnson Noise in Longitudinal Control Signal**

\[ V(f) = 0.10^{-10} \text{ V/} \sqrt{\text{Hz}} \]

\[ x = \frac{\beta V}{f^2} \quad \beta = 5 \times 10^{-5} \frac{\text{m}}{\text{sec}^2 \cdot \text{Volt}} \]

\[ \Rightarrow \quad x(f) = 10^{-18} \frac{\text{m}}{\text{VHz}} \cdot \alpha + 100 \text{ Hz} \]
STARTING UP THE CONTROL SYSTEM:
LOCK ACQUISITION

[LECTURE 11]

PROBLEM: Most (> 99.9%) cavity is out of resonance and there is no signal for the control system to use.

\[ \lambda / 2 \]

\[ \rightarrow 1 \] \[ \lambda / 2 \] \[ \rightarrow 1 \]

\[ \lambda / 2 \] \[ \Omega = 5000 \]

DESIGN STRATEGY:

1) Slow down fringes by seismic isolation, active damping
2) Construct servo to have high gain and stable operation when in resonance
3) Attend to electronic saturation properties
4) Maximize dynamic range

\[ \Delta x_{\text{BERNE LOCK}} \approx 10^{-6} \text{ m} \]
\[ \Delta x_{\text{IN LOCK}} \approx 10^{-12} \text{ m} \]
Requirements for lock acquisition compete with requirement of low noise after resonance is established. Trick: switch from "Acquire" mode to "Run" mode. Switch must be gentle; otherwise lock is disrupted.

**Acquire Mode:** Input to amplifier G is attenuated by D₁, to reduce overloading.

**Run Mode:** Attenuate output of G for less noise (and less dynamic range). Attenuation same as D₁.

Also: low frequency boost inserted. Increases gain below 100 Hz, does not affect servo stability.
DEPARTURE FROM IDEAL \( \left( \frac{1}{f^2} \right) \)

SUSPENSION: RESONANT NORMAL MODES
OF COUPLED MECHANICAL OSCILLATORS

[LECTURE 12]
To test the suspension's performance (measure transfer function from ground motion to interferometer output)
TOP PLATE SHAKER vs. INTERFEROMETER OUTPUT

In-air accelerometer calibration of shaker used

- 21 June 91
- 25 July (uncertain calibration)

Suspension solution (dB)

f (Hz)

Ideal simple pendulum

POOR COHERENCE
Suspension Resonances Evident in Interferometer Output

Displacement Sensitivity of Caltech 40 m Interferometer

- Measured upper limit to seismic transmission at East End
- Calculated shot noise, 3/94

Frequency (Hz)

$x (\text{m/Hz}^{1/2})$
(Some) Reduction of Suspension Resonances by (Slight) Simplification of Suspension

Constraint wires removed from H-mass control block

Interferometer response to top-plate shaker (arbitrary units)

--- Before modification
--- After

Frequency (Hz)

8 Sep 92; wccmp,pro wcJ, htphf
Seismic Isolation Stack Imperfection:
Interferometer is more responsive to vertical excitation!

Seismic Feedthrough at East End, Compared to Total Noise

seis-pred2.xmgr; 13 Dec 93

Predictions based on measured ground motion and measured transfer function

Total Noise
3/29/94
Design Exercise:

SEISMIC ISOLATION DESIGN FOR MARK II:
OLD SUSPENSION, NEW STACK

ISOLATION STACK PERFORMANCE

Predicted Mk II Seismic $x(f)$
(includes only horizontal floor motion, not vertical)

NEXT: NEW SUSPENSIONS, SOFTER STACKS
Two Types of Intensity Noise

[LECTURE 7, 16-17]

1) Imbalanced radiation pressure fluctuations

\[ F(t) \propto \text{Power} \tilde{P}(t) \]

Fluctuation is "fundamental" if power is stabilized so well that dominant effect is shot noise; the fluctuating force is

\[ F(t) = \sqrt{\frac{h \tilde{P}}{\lambda_0}} \]

Coupling to offset from center of resonance

\[ \Delta x = \frac{x_0}{\lambda} \frac{\Delta I}{I} \]

- Operating point: offset from dark fringe

Output of photodiode after the mixer

- higher intensity \( \Rightarrow \) steeper response curve
Measured effect of intensity noise is small.

Laser intensity noise
(linear extrapolation)

Displacement x(f) (m/√Hz)

10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}

f (Hz)

100
500
1000
5000

Note: This is an old noise spectrum.
UPCONVERSION BY SCATTERING

[LECTURE 15]

Mechanical vibrations at low frequency & large amplitude produce high-frequency noise

Unintended resonator

aligned optical component

Test mass

$X_p > \lambda$

Maximum fringes/sec = $f_p$

$\frac{V_p}{\lambda/2} = \frac{2\pi X_p f_0}{\lambda/2}$

e.g. $X_p = 10\lambda$, $f_0 = 1$ Hz $\Rightarrow f_p = 100$ Hz

("upconversion")

Reduce effect by damping resonances (especially effective if $X_p < \frac{\lambda}{2}$ - no upconversion)

And by optical isolation