LECTURE 2: RANDOM PROCESSES

Lecture by Kip S. Thorne

Assigned Reading:
D. Pages 5-1 through 5-24 of “Chapter 5. Random Processes” from the textbook manuscript Applications of Classical Physics by Roger Blandford and Kip Thorne.

Suggested Supplementary Reading:
a. L. A. Wainstein and V. D. Zubakov, Extraction of Signals from Noise (Prentice Hall, London, 1962; Dover, New York, 1970). [This wonderful book—a sort of biblical primer on the subject—is long since out of print. Kip will put his personal xerox copy on reserve in Millikan Library for a few weeks, along with the library’s only copy.]

Two Suggested Problems from Blandford and Thorne’s “Chapter 5, Random Processes”:
5.1 Bandwidths of a finite-Fourier-transform filter and an averaging filter [page 5-21]
5.2 Wiener’s Optimal Filter [page 5-22]. This is an especially important exercise, since the optimal filter underlies much of the data analysis to be done in LIGO.
Lecture 2
Random Processes
by Kip S. Thorne, 1 April 1994

This lecture actually consumed only half of the 90 minutes on 1 April; the completion of Lecture 1 consumed the other half.

This lecture was largely just a blackboard presentation of the key issues in Reference D [pages 5-1 through 5-24 of “Chapter 5. Random Processes” from the textbook manuscript Applications of Classical Physics by Roger Blandford and Kip Thorne]. Since that reference is included in Volume II, we here present, as a record of Lecture 2, only the scrawled notes from which Kip lectured.
Lecture 2: Random Processes

1. Expanded SP $I(t)$: $h(t) = C \cdot I_p(t)$; $X(t)$ stationary

2. Sp Noise Spectrum

   $\text{like F.T.}$: $h(t) = \int_{-\infty}^{\infty} h(t) e^{i2\pi ft} dt$

   $\Rightarrow h(f) = \sqrt{S_h(f)}$

   Try:
   $$\lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} h(t) e^{i2\pi ft} dt \right|^2$$

   $\Rightarrow \mathcal{G}_h(f) \equiv S_h(f) \equiv \int h(f) \mathcal{G}^2 = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} h(t) e^{i2\pi ft} dt \right|^2$

3. Correlation function

   $C_h(t) = \int_{0}^{\infty} C_h(t) \cos 2\pi ft dt$

   $C_h(t) = 4 \int_{0}^{\infty} C_h(t) \cos 2\pi ft dt$

4. Wiener-Khintchine Theorem

   $C_h(t) = \int_{0}^{\infty} G_h(f) \cos 2\pi ft dt$

   $G_h(f) = 4 \int_{0}^{\infty} C_h(t) \cos 2\pi ft dt$

5. Variance:

   $C_h(0) = \int_{0}^{\infty} G_h(f) df = C_h^2 = ^2$ but might not converge @ low $f$

6. Filtering

   a. $H(t) \equiv \int_{-\infty}^{\infty} K(t-t') h(t') dt'$

   b. If $w(t)$ uncorrelated $\Rightarrow$ $H(f) = k'(f) h(f)$

   c. From RP $G_h(f) = |k(f)|^2 G_h(f)$
7. Band Pass Filter

\[ G_h(t) = \int \left| \frac{G_h(t)}{K_0^2} \right|^2 \, dt = G_h(t_0) \int \left| \frac{K(t)}{K_0^2} \right|^2 \, dt \]

\[ G_h(t) = [G_h(t_0) \cdot \Delta f] \cdot K_0^2 \]

Rms fluctuations of \( h \)
in bandwidth \( \Delta f \)

a. Single example of Bilinear filter:

\[ H(t) = t \int \cos \left( 2\pi f_0 (t - t) \right) y(t) \, dt \]

\[ t - \Delta t \]

\[ \Delta f = 1/\Delta t \]

8. Example: A line spike in spectrum... large rms fluctuations
   with \( f \).

9. Wiener Optimal Filter:

a. \( h(t) = A \delta(t) + n(t) \) ... want to find \( h(t) \) and \( s(t) \)='dual
   unknown
   ... so how strong, \( A \)
   
   But very shall be it cross correlates: \( h(t) \cdot s(t) \) dt
   
   
   Better! Suppose common frequency, when detect it in
   noisy... \( S_F^2(f) = S(f) G_h(f) \)
   
   then cross correlates:

\[ W = \int h(t) s_p(t) \, dt = \int \frac{h(t) s(t) \, dt}{G_h(f)} \]
b. Signal contributes to \( W = S + N \) where \( N \) is random. Thus \( \bar{N} : W = S + N \).

\[
\frac{S}{\bar{N}} = 4 \sum_{0}^{15} \frac{15'(\epsilon)^{12}}{C_{h}(+1)}
\]

11. Known spectrum does not belong \( \equiv P(N) \), the probability distribution of \( N \). But if we know it is Gaussian, etc.

\[
P(N) = \frac{1}{\sqrt{2\pi N}} \exp \left( \frac{N - N_{0}}{2N} \right)
\]

\[
P(N) = \frac{1}{\sqrt{2\pi N^{2}}} \exp \left[ - \frac{N^{2}}{2N^{2}} \right]
\]

12. If not enough, then \( \equiv \text{not likely} \), noise is superposition of many different things; Central limit theorem \( \equiv \text{Gaussian} \).

13. Central issue will it be Gaussian or not?