LECTURE 4.
IDEALIZED THEORY OF INTERFEROMETRIC DETECTORS — I.

Lecture by Kip S. Thorne

Assigned Reading:
A. “Gravitational Radiation” by Kip S. Thorne, in 300 Years of Gravitation, eds. S. W. Hawking and W. Israel (Cambridge University Press, 1987), pages 414–425; ending at beginning of first full paragraph on 425. [This material uses the phrase beam detector for an interferometric gravitational-wave detector. The principal results quoted in this lecture are derived in the exercises below.]

G. The following portions of “Chapter 7. Diffraction” from the textbook manuscript Applications of Classical Physics by Roger Blandford and Kip Thorne: Section 7.2 (pages 7-2 to 7-7), and Section 7.5 (pages 7-20 to 7-27). [This material develops the foundations of the theory of diffraction (Green’s theorem and the Helmholtz-Kirchoff formula), explores semi-quantitatively the spreading of a transversely collimated beam of light, develops the formalism of paraxial Fourier optics for analyzing quantitatively the propagation of collimated light beams, and uses that formalism to derive the evolution of the cross sectional shape of a Gaussian beam, of the sort used in LIGO.]

Suggested Supplementary Reading:

A Few Suggested Problems

1. Shot Noise. Reread the discussion of shot noise on pages 5-20 and 5-21 of Blandford and Thorne, Random Processes (which was passed out last week). In that discussion let the random process $y(t)$ be the intensity $I(t) = d(energy)/dt$ of a laser beam, and let $F(t)$ be the intensity carried by an individual photon, which has frequency $\omega$.
   (a) Explain why $\hat{F}(0)$, the Fourier transform of $F$ at zero frequency, is the photon energy $\hbar\omega$.
   (b) Show that the spectral density of $I$ (the “shot-noise spectrum”) is
   \[ G_I(f) = 2\bar{I}\hbar\omega, \]
   where $\bar{I}$ is the beam’s mean intensity.
   (b) Let $N(t)$ be the number of photons that the beam carries into a photodiode between time $t$ and time $t + \hat{t}$ (so $\hat{t}$ is the averaging time): $N(t) = \int_t^{t+\hat{t}} I(t')dt'$. 


1
This $N(t)$ is a linear functional of $I(t')$. Use the theory of linear signal processing to derive the spectral density $G_N(f)$ of $N(t)$, and then compute the mean square fluctuations of $N$: $(\sigma_N)^2 = \int_0^\infty G_N(f)df$. Your result should be $\sigma_N = \sqrt{\bar{N}}$, where $\bar{N}$ is the mean number of photons that arrive in the averaging time $\bar{t}$. This is the standard "square-root-of-$N$" fluctuation in photon arrival for a laser beam.

2. Reciprocity Relations for a Mirror and a Beam Splitter. Modern mirrors, beam splitters, and other optical devices are generally made of glass or fused silica (quartz), with dielectric coatings on their surfaces. The coatings consist of alternating layers of materials with different dielectric constants, so the index of refraction $n$ varies periodically. If, for example, the period of $n$'s variations is half a wavelength of the radiation that impinges on the device, then waves reflected from successive dielectric layers build up coherently, producing a large net reflection coefficient. In this exercise we shall derive the reciprocity relations for a mirror of this type, with normally incident radiation. The generalization to radiation incident from other directions, and to other dielectric optical devices is straightforward.

The foundation for the analysis is the wave equation,

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{c^2}{n^2(x)} \nabla^2\right) \psi = 0$$

satisfied by any Cartesian component $\psi$ of the electric field, and the assumption that $\psi$ is precisely monochromatic with angular frequency $\omega$. These imply that the spatial dependence of $\psi$ is governed by the Helmholtz equation with spatially variable wave number $k(x) = n(x)\omega/c$: $\nabla^2 \psi + k^2 \psi = 0$.

Let waves $\psi_i e^{ikz}$ impinging perpendicularly (z direction) on the mirror from the “unprimed” side produce reflected and transmitted waves $\psi_r e^{ikz}$ and $\psi_t e^{ikz}$; these waves and their corresponding $\psi$ inside the mirror are one solution $\psi_1$ of the Helmholtz equation. The complex amplitudes of this solution are related by reflection and transmission coefficients, $\psi_r = r \psi_i$, $\psi_t = t' \psi_i$. Another solution, $\psi_2$, consists of incident waves from the opposite, “primed” side, $\psi_\nu e^{-ikz}$ and reflected and transmitted waves $\psi_r e^{ikz}$, $\psi_t e^{-ikz}$, and the corresponding $\psi$ inside the mirror; and this solution’s complex amplitudes are related by $\psi_{r'} = r' \psi_\nu$, $\psi_t = t\psi_\nu$.

(a) Show that $\psi$ obeys Green’s theorem [Equation (7.3) of Blandford and Thorne] throughout the mirror. Apply Green’s theorem, with $\psi$ and $\psi_0$ chosen to be various pairs of $\psi_1$, $\psi_2$, $\psi_1^*$, $\psi_2^*$ (where the star denotes complex conjugation). Thereby obtain four relationships between $r$, $r'$, $t$, and $t'$.

(b) Show that these relationships can be written in the form

$$r = \sqrt{R} e^{2i\beta}, \quad r' = -\sqrt{R} e^{2i\beta'}, \quad t = t' = \sqrt{T} e^{i(\beta+\beta')}$$

2
where $\beta$ and $\beta'$ are unconstrained phases, and $R$ and $T$, the power reflection and transmission coefficients are related by

$$R + T = 1,$$

(2)

which is just energy conservation.

(c) Show that, if one moves the origin of coordinates as seen from the unprimed side by $\delta z = -k\beta$, and moves the origin as seen from the primed side by $\delta z = +k\beta'$, one thereby will make all the reflection and transmission coefficients real:

$$t = t' = \sqrt{T}, \quad r = -r' = \sqrt{R}.$$

(3)

Thus, with an appropriate choice of origin on each side of the mirror, the coefficients can always be made real.

The same is true for the reflection and transmission coefficients of any other optical device made of a lossless, spatially variable dielectric. In particular, for a perfect, 50/50 beam splitter, the transmission coefficient becomes, with appropriate choice of origins, $1/\sqrt{2}$ from each and every one of the four input ports, and the reflection coefficient becomes $+1/\sqrt{2}$ from the input ports on one side of the beam splitter and $-1/\sqrt{2}$ from the input ports on the other side of the beam splitter. These results are summarized by the following figures:

2. Transfer Function and Photon Shot Noise for a Delay-Line Interferometer In class, Kip derived the “transfer function” for a delay-line interferometer in the limiting regime where the waveform $h(t)$ is nearly constant during the time $2BL/c$ that the light is stored in the interferometer arms (during $B$ round trips in an arm whose length is $L$). His result was

$$I_{PD}(t) = I_1(t) + 2\sqrt{I_1 I_0} B k L h(t)$$

(4)

where $I_0$ is the mean laser input power entering the beamsplitter, $I_1(t)$ is the (slightly fluctuating because of shot noise) intensity of the light falling onto the photodiode in
the absence of a gravitational-wave signal, \( \bar{I}_1 \) is the mean intensity onto the photodiode, \( B \) is the number of round trips in the arms of the interferometer, \( k = \omega / c = 2\pi / \lambda_e \) is the light's wave number, \( L \) is the arm length, and \( h(t) \) is the gravitational waveform. Kip used this and the shot-noise spectral density [Eq. (1) above] to derive the following expression for the shot-noise contribution to the interferometer's gravitational-wave noise output:

\[
G_h(f) = \frac{\hbar \omega}{2I_0(BkL)^2}.
\]  

(a) Use the same method of analysis as Kip did in class to derive the transfer function when the gravitational wave is sinusoidal in time with angular frequency \( \Omega = 2\pi f \), i.e. when \( h(t) = h_o \cos(\Omega t) = h_o \text{Re}(e^{-i\Omega t}) \), with a frequency \( f \) high enough (gravitational wavelength short enough) that the waveform can vary significantly while the light is stored in the arms. Your result should be the same as Eq. (4), with \( B \) replaced by

\[
B_{\text{eff}} = B \sin(f/f_0) \quad f_0 = \frac{c}{2\pi BL} = \frac{119\text{ Hz}}{(B/100)(L/4\text{ km})}.
\]  

(b) Show that the shot-noise contribution to \( G_h(f) \) has the form (5) with \( B \) replaced by \( B_{\text{eff}} \).

3. Transfer Function and Photon Shot Noise for a Fabry-Perot Interferometer. In class, Kip showed that for a Fabry-Perot interferometer in the regime of slow variations of \( h(t) \) the transfer function and photon shot noise have the forms (4) and (5), with \( B \) replaced by

\[
B_{\text{eff}} = \frac{4}{(1 - R)}
\]  

where \( R \) is the power reflectivity of the interferometer's corner mirrors and where it is assumed that the end mirrors are perfectly reflecting. Show that, if the variations of \( h(t) \) are not assumed to be slow, then the transfer function (for monochromatic gravitational waves) has the form (4) and the shot noise contribution to \( G_h(f) \) has the form (5), with \( B \) replaced by

\[
B_{\text{eff}} = \frac{B}{\sqrt{1 + (f/f_0)^2}},
\]  

where \( f_0 \) is as in Eq. (6) above.
Thorne lectured at the blackboard. The following are the notes from which he lectured, cleaned up a bit to make them more understandable.
1. Overview of [known works & orders of magnitude]

- **Goal**: $h \approx 10^{-22}$; $\frac{\Delta L}{L} = h_z$; $L = 4\text{km} = 10^6\text{cm}$
  \[ \Rightarrow \Delta L \approx 10^{-16}\text{cm}; \quad \frac{2e}{c} \approx 0.5\text{mm} = 10^{-4}\text{cm} \]
  \[ \Rightarrow \text{measure} \quad 10^{-12} \text{of} \ 2e \quad \text{seems outrageous} \]

- **GW** $f \approx 100\text{Hz}$

  $2 \approx 3000\text{km}$

  \[ \text{Light} \ \text{period} \ \text{length} = \ \text{light straight through}\]

  \[ \Rightarrow B = (2\text{GHz}) \]

  \[ \Rightarrow \frac{2e}{2L} = \frac{1500\text{km}}{8\text{km}} = 200 \]

  (So light is struck in each arm

  for, on average, $B \approx 200$

  round trips)

2. **Phase shift**

   $\Delta \Phi = \frac{2m}{2e} \Rightarrow 2\Delta L = 200 \times \frac{2\pi}{8\times 10^{-5}} = 2\times 10^{-16}\text{cm}$

   $\approx 10^{-9}$

3. **How accurately can this phase shift be measured?**

   \[ \text{If} \quad \Delta \Phi \approx \frac{1}{\sqrt{N}} \quad \Rightarrow \quad \text{[ Photon shot noise]} \]

   \[ \text{need} \quad 10^{18} \text{photons in} \ 0.01\text{sec} \quad [\text{from laser} = \text{time} \times \frac{1}{f} = 0.01\text{sec}] \]

   \[ \Rightarrow \quad I = 10^{18} \times \frac{e}{h} = 5 	imes 10^{15} \times 27 + 18 + 2 \approx 5 \times 10^{-2}\text{sec} \]

   $\approx 50\text{Watts}$

   [Can be achieved with 5 Watt laser and a 10-fold recycling of used light]
e. Won't vibrations of atoms in mirror prevent measurement of such tiny motions? No—

ii. Individual atoms vibrate at $f \approx 10^{13}$ Hz, far above interferometers' gravity-wave band

iii. Only concern is lowest frequency normal mode which have thermal amplitudes

$$\sqrt{\frac{kT}{m \omega^2}} = \sqrt{\frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{10^4 \text{ g} \cdot (10^{15} \text{ Hz})^2}} = \sqrt{4 \times 10^{-12} \text{ cm}}$$

$= 2 \times 10^{-14} \text{ cm}$.

The interferometer averages over many periods and sees only changes of amplitude—which are made small by giving mirrors high mechanical $Q$'s.

This thermal noise will be discussed in later lectures.
2. Ways to Produce Multiple Beams in Interferometric Arms
   a. Delay line [many distinct spots on each mirror]
   b. Fabry-Perot [one spot on each mirror]

3. Delay Line
   a. Describe light by \( P = \frac{E_x}{V_{rf}} \) ... so \( P = 14 \text{dB} \)

   \[ P = 4 e^{-\alpha x}; \quad P = e^{ikx} \text{ propagates in } x \text{ direction}; \quad k = \frac{632.8}{2}\lambda \]

   b. Field at various points:

   \[ P_D = \frac{1}{2} (P_2 - P_1) \]

   c. A bit of algebra:

   \[ P_L = \frac{1}{2} P_2 (e^{i\phi_2} + e^{i\phi_1}) \quad \text{[is field going back toward laser from interferometer]} \]

   \[ \Rightarrow |P_L|^2 = \frac{14P^2}{1} \cos^2 \left( \frac{\phi_2 - \phi_1}{2} \right) \]

   \[ P_D = \frac{1}{2} P_2 (e^{i\phi_2} - e^{i\phi_1}) \quad \text{[field going toward photodiode]} \]

   \[ \Rightarrow |P_D|^2 = \frac{14P^2}{1} \sin^2 \left( \frac{\phi_2 - \phi_1}{2} \right) \]

   d. Operate PD on "dark fringe" \[ P_D = \frac{I_0}{P_L} \text{ before wave mixing} \]
c. Effect of wave: 
\[ \Delta \Phi_2 = -\Delta \Phi_1 = k \cdot \frac{h}{2L} \cdot 2B \]
\[ \Delta \Phi = \frac{1}{2} (\Delta \Phi_2 - \Delta \Phi_1) \]
\[ \Delta \Phi = B \cdot 2k \delta L = B \cdot k L h_0 \]

f. \[ I_{PD}(t) = I_0 \left( \Phi_0^2 + 2\Phi_0 B k L h(t) \right) \] for \( 2k \delta L \ll 2k \omega L \)

9. **Why dark port toward PD?**
- Keep power on PD low
- Send light toward laser, so it can be recycled back into interferometer with new laser light

4. Fabry-Perot:

a. Lock on a single arm \([\text{cavity}]\)

b. How it gets excited:

i. Turn on light suddenly, on resonator \(2kL = m \pi + \frac{1}{2} \pi \)

ii. First pass of light down arm
\[ \Psi(t) = \frac{t_4}{1} \]
\[ L = \text{amplitude transmission} \]
\[ r = \text{reflectance} \]

returns in phase \( \implies \text{next pass} \]
\[ \Psi(t) = \frac{t_4}{1} \]

returns in phase again \( \implies \text{next:} \]
\[ \Psi_{\text{inside}} = \frac{t_4}{1} (1+r+r^2+...) = \frac{t_4}{1-r} = \frac{1}{1-r} \]
\[ \Psi_{\text{inside}} = \frac{2t_4}{1} \]
\[ \text{Input:} \frac{4}{1-r} I_0 \]
5. Shot Noise [Cf. Random Processes, Chapter Ref. D]
   a. The beam $I_{pp} = I_0 q_0^2$ in absence of GWs consists of photons which arrive randomly at photodiode. Each photon carries energy $h\nu$, so average rate of arrival is $R = \frac{I_{pp}}{h\nu} \approx (10^{18} \text{ sec}^{-1}) \frac{I}{1W}$
   Since duration of each pulse is $\approx 10^{-15} \text{ sec}$, $R\tau_p \gg 1$

   b. This random arrival means $I_{pp}$ fluctuates randomly,
      $I_{pp}(t)$, with some spectral density
      $G_{I_{pp}}(f)$.

5.1. Frequency of interest is $f \approx 0.01 \text{ sec}$, and $\ll \frac{1}{\tau_p}$

At these frequencies, the shape of the pulse is small. Hence $G_{I_{pp}}(f)$ is small.
d. Exercise: From requirements that \( N = \frac{I}{\hbar \omega} \) is mean that arrive during time \( T \), the
\[
\hat{N} = \sqrt{N}, \text{ we get}
\]
\[
\hat{G}_{N} = \frac{I^2}{\hbar \omega} \quad \text{ - classical: } \frac{I^2}{\hbar \omega}
\]

6. Shot noise \( \hat{h}(t) \): (Translate this photon shot noise into an equivalent noise in, C, wave
signal)

a. \( I_{pp}(t) = I_{o} \left[ \varphi_{o}^2 + 2 \varphi_{o} B \kappa \lambda h(t) \right] \)

b. Rewrite \( \frac{I_{pp}(t)}{I_{o} \varphi_{o}^2} = \hat{I}_1(t) \); \( I_{o} \varphi_{o} = \sqrt{\hat{I}_1} \)

\[
I_{pp} = \hat{I}_1(t) + 2 \sqrt{\hat{I}_1 I_{o} B \kappa \lambda h(t)}
\]

Then \( \hat{G}_{I_1}(t) = 4 \hat{I}_1 I_{o} (B \kappa \lambda)^2 \hat{G}_{h}(t) \)

\[
\Rightarrow \hat{G}_{h}(t) = \frac{\hat{G}_{I_1}(t)}{4 \hat{I}_1 I_{o} (B \kappa \lambda)^2}
\]

c. \( \hat{G}_{h}(t) = \frac{\hbar \omega}{2 I_{o} (B \kappa \lambda)^2} \quad \text{ - white noise spectrum}
\]

d. \( \text{rms} = \sqrt{\hat{G}_{h}(t)} = \frac{1}{2 B \kappa \lambda} \sqrt{I_{o} / \hbar \omega} \quad \frac{\Delta f}{2 B L}
\]
\[
\text{# of photons available in } \frac{1}{2} \text{ GW period}
\]

These are the shot noise limits on our interferometer - valid both for Delay line and Fabry Perot.
7. What if \( f_{ew} \geq 2B \lambda L \)?

a. Delay Line:

\[
\Rightarrow B_{\text{eff}} = B \cdot \frac{\sin(B \cdot (2\pi f L/c))}{B \cdot 2\pi f L/c} = B \frac{\sin(f/\omega)}{f/\omega}
\]

\[
\frac{f_0}{c} = \frac{2\pi \lambda L}{c} = \frac{119\text{ Hz}}{(B/100)(L/4\text{ km})}
\]

\[
\frac{B_{\text{eff}}}{f} = \frac{c}{2\pi \lambda B} = \frac{f_{ew}}{L}
\]

b. Fabry-Perot:

- photons stand for a statistically varying length of time \( \Rightarrow \) smooth die out \( \Rightarrow B_{\text{eff}} \):

\[
B_{\text{eff}} = \frac{B}{\sqrt{1 + (f/f_0)^2}} = \begin{cases} B \quad & f < f_0, \\ \frac{B_{\text{eff}}}{f} = \frac{2\pi L}{c} & \frac{f_{ew}}{f_0} \leq f < f_{ew}. \end{cases}
\]
8. Bottom Line

\[ h_{\text{rms}} = \sqrt{f \cdot G_h(f)} = \frac{2e}{2BL} \frac{1}{\sqrt{(I_0 M_h)(1/2)}} \cdot \sqrt{1 + (f/f_0)^2} \]

Fewer bounces
(1/2 vs. B)

"knee" is at \( f = f_0 \)
9. Gaussian Beams & Paraxial Optics [§7.5 - Ref. 6]

a. Basic idea of wave spreading:

\[ \Delta k y \approx R_0 \approx 1 \]

\[ \Delta k y \approx \frac{1}{R_0} \quad \frac{\Delta k y}{k} \approx \frac{\Delta \theta}{R_0} = \text{angular spread} \]

b. This fixes size of beam in interferometer arms:

\[ R_0 \approx \sqrt{\frac{\lambda L}{2}} \approx \sqrt{(4x10^{-5} \text{ cm}) (4x10^{-5} \text{ cm})} \approx 4 \text{ cm} \]

c. Transverse profile is Gaussian:

\[ y \approx \exp \left( -\frac{r^2}{R_0^2 \left[ 1 + \frac{r^2}{R_0^2} \right]} \right) \]

\[ B_0 = \frac{R_0^2}{\Delta \theta} \]
A Beam is matched into cavity using lenses that manipulates its radius and its radius of curvature of phase fronts.
LECTURE 5.
IDEALIZED THEORY OF INTERFEROMETRIC DETECTORS—II.
Lecture by Ronald W. P. Drever

Assigned Reading:
I. R. W. P. Drever, “Fabry-Perot cavity gravity-wave detectors” by R. W. P. Drever, in The Detection of Gravitational Waves, edited by D. G. Blair (Cambridge University Press, 1991), pages 306–317. [This is a qualitative overview of Fabry-Perot gravitational-wave detectors, with emphasis on recycling in the later part (pages 312—317).]

J. B. J. Meers, “Recycling in laser-interferometric gravitational-wave detectors,” Phys. Rev. D, 38, 2317–2326. [This is the paper in which Meers introduced his idea of dual recycling and sketched out its features. You are not expected to master all the equations in this paper—which Meers just gives without derivation—but you might try deriving some of the equations as a homework exercise.]

Suggested Supplementary Reading:
L. B. J. Meers and R. W. P. Drever, “Doubly-resonant signal recycling for interferometric gravitational-wave detectors.” (preprint) [This paper introduces a new recycling configuration, not considered in previous papers.]
N. R. W. P. Drever, “Interferometric Detectors of Gravitational Radiation,” in Gravitational Radiation, N. Deruelle and T. Piran, eds. (North Holland, 1983); section 8 (pages 331-335). [This is the article in which Drever first presented in detail his ideas of power recycling and resonant recycling.]
A Few Suggested Problems

Note: Of all configurations for a recycled interferometer, the only one that is reasonably easy to analyze is power recycling. For this reason, and because this is the type of recycling planned for the first LIGO interferometers, I have chosen to focus solely on power recycling in the following exercises. — Kip.

1. Simplified Configuration of Nested Cavities that Illustrates Power Recycling: Consider the configuration of two nested optical cavities shown below:

   ![Diagram of nested cavities](image)

   All three mirrors are assumed ideal in the sense that they do not scatter or absorb any light; therefore each of them satisfies the reciprocity relations of Assignment 4, Eq. (3). Assume that the power reflectivities of the subcavity are fixed: $R_e$ is the highest reflectivity the experimenter has available; $R_c$ is a much lower reflectivity, carefully designed to store the light in the subcavity for a chosen length of time. What reflectivity $R_r$ should the recycling mirror have in order to maximize the light intensity in the subcavity, when both cavities are operating on resonance? Use physical reasoning to guess the answer before doing the calculation.

2. Optimization of a Power Recycled Interferometer. Consider the power-recycled interferometer shown below:

   ![Diagram of power-recycled interferometer](image)

   a. Suppose the interferometer is operated with the photodiode very near a dark fringe, so the light power $I_2$ is many orders of magnitude less than $I_1$. As in exercise 1, let $R_e$ and $R_c$ be fixed. How should $R_r$ be chosen to maximize the power in the interferometers' two arms? Guess the answer on physical grounds before doing the calculation.

   b. Again, suppose that $I_2$ is many orders of magnitude less than $I_1$. Let a low-
frequency gravitational wave (one with $2\pi fBLC/c \ll 1$ where $B = 4/(1 - R_c)$ is the effective number of round trips in the arms) impinge on the interferometer. How should $R_r$ be chosen so as to maximize the gravitational-wave signal to noise ratio in the interferometer? Guess the answer on physical grounds before doing the calculation.

c. Suppose that the mirrors in the two arms are slightly imperfect, and their imperfections cause a mismatching of the phase fronts of the light from the two arms at the beam splitter. As a result, the ratio $I_2/I_1 \equiv \alpha$ has some modest value (e.g. 0.01) instead of being arbitrarily small. In this case, how should $R_r$ be chosen so as to maximize the signal to noise ratio? Guess the answer on physical grounds before doing the calculation.

3. Scaling of Photon Shot Noise with Arm Length. We saw in Kip’s lecture that, if one has mirrors of sufficiently high reflectivity and one uses a simple (nonrecycled) interferometer, then the photon shot noise $h_{\text{rms}} = \sqrt{fG_h(f)}$ is independent of the interferometer’s arm length.

Suppose, instead, that (i) the highest achievable power reflectivity is $R = 1 - 10^{-5}$, (ii) one can do as good a job of phase-front matching at the interferometer as one wishes, so in the above drawing $I_2/I_1 = \alpha$ can be made as small as one wishes, (iii) one has a fixed laser power $I_0$ (say, 10 Watts) available, (iv) one operates the interferometer in a power-recycled mode, as in the above figure. Show that in this case the photon shot noise $h_{\text{rms}}$ scales as $1/\sqrt{L}$ in the full LIGO frequency band (a result quoted on page 314 of Ref. I).

Note: Another example of arm-length scaling is described on page 316 of Ref. I: A resonant-recycled or dual-recycled interferometer looking for periodic gravitational waves, e.g. from a pulsar, has photon shot noise $h_{\text{rms}} \propto 1/L$. 
Lecture 5
Idealized Theory of Interferometers — II.

by Ronald W. P. Drever, 13 April 1994

Drever lectured from the attached transparencies. His lecture focussed on optical configurations for interferometers that are more complex than the simple interferometers of Thorne’s lecture:

- A \textit{recycling interferometer} (more normally called \textit{power recycling interferometer} these days), in which the light going back from the interferometer toward the laser gets recycled back into the interferometer together with and in phase with new laser light. Such power recycling (invented in the early 1980s by Drever) will be used in LIGO’s first interferometers.

- A \textit{dual recycling interferometer} (also called \textit{signal recycling}) in which light power is recycled at the laser’s port of the interferometer, and the gravity-wave signal is recycled (and thereby enhanced) at the photodetector’s port. Such signal recycling can be used to enhance the interferometer’s performance in the vicinity of (most) any desired gravity-wave frequency and over (most) any desired bandwidth around that frequency. It is likely to find application, for example, in deep searches for the gravitational waves from pulsars. A dual recycled interferometers has the added benefit of less sensitivity to irregularities in the mirrors and beam splitter than an ordinary interferometer.

- A \textit{resonant recycling interferometer}, a configuration that accomplishes the same thing as signal recycling but in a more complicated and less practical way. (This configuration was invented by Drever in the early 1980s; signal recycling is an improvement on it, devised by Brian Meers in the late 1980s.)

- A \textit{doubly resonant signal recycling interferometer}. This configuration, invented by Drever and Meers, recycles (and enhances) both signal sidebands of the light’s carrier frequency; ordinary signal recycling recycles and enhances only one signal sideband.

- A \textit{resonant sideband extraction interferometer}, whose configuration looks just like that of a dual recycling interferometer but performs quite differently because of a different fine tuning of the location and reflectivity of the recycling mirrors. This configuration, invented recently by M.J. Mizuno (a Japanese graduate student working with the Garching, Germany group) stores the carrier-frequency light in the arms for a long time, while resonantly extracting the signal sideband from the arms after about a half cycle of the gravitational wave. It thereby achieves a broad-band sensitivity comparable to that of a power recycled interferometer, but with far less light power passing through the beam splitter and hence with less problem from high-power heating of the splitter.
DUAL RECYCLING INTERFEROMETER
(Also Resonant Side Band Extraction Interferometer)
Combined" Photodiode measures phase difference between C1 and C2 (F1 and F2 made to cancel)

Alternate pair of Photodiodes measure phase differences between C1 and F1, C2 and F2

Resonant cavity 1

Resonant cavity 2

\[ \Delta x \]

Laser
RESONANT RECYCLING INTERFEROMETER
DOUBLE-RESONANT SIGNAL RECYCLING INTERFEROMETER

LASER

Input Recycling Mirror

Output Recycling Mirror

Detector