R-modes in Accreting and Young Neutron Stars

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Abstract. Recent work has raised the exciting possibility that r-mode pulsations (Rossby waves) in rotating neutron star cores may be strong gravitational wave sources. Rapidly rotating young neutron stars born in supernovae enter the r-mode instability region within the first minutes of their lives and may spin down by substantial amounts due to gravitational radiation from r-modes. Accreting neutron stars in low-mass X-ray binaries (LMXBs) are spun up by accretion to such short rotation periods that they may be unstable to r-mode pulsations as well. Gravitational waves from these neutron stars are strong enough to be detectable by second-generation, "enhanced" gravitational wave interferometers. I review the recent progress in understanding the r-mode instability in young and accreting neutron stars, with the focus on the issues of the coupling of the pulsations to the crust and nonlinear saturation amplitudes.

I INTRODUCTION

Since the work of Chandrasekhar [1], it has been known that the loss of energy and angular momentum due to emission of gravitational waves (GW) can make certain pulsation modes of rotating stars grow, rather than decay. This counter-intuitive situation is easy to understand. Imagine a star that rotates with angular frequency Ω and undergoes a pulsation of frequency $\omega_r > 0$ (in the rotating frame), so that the azimuthal dependence of the perturbation is $e^{im\phi+i\omega_r t}$. Such a wave propagates in the direction opposite to the rotation (retrograde), and carries negative angular momentum. As seen from the inertial frame, this wave has a frequency $\omega_i = \omega_r - m\Omega$. The spin of the star or the m of the mode may be large enough that $\omega_i < 0$, i.e., the mode appears prograde in the inertial frame. Gravitational radiation, since it lives in the inertial frame, removes positive angular momentum from such mode. This means that the mode's *negative* angular momentum (in the rotating frame) must become more negative, i.e., the mode amplitude must grow. This instability is generic: in any rotating perfect-fluid star one can find a mode with sufficiently high m that it is unstable to gravitational radiation reaction [2].

Gravitational radiation therefore offers an exciting possibility to set an upper limit on the spin frequencies of rotating neutrons stars (NS), both young pulsars

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born in supernovae and old millisecond pulsars thought to be spun up by accretion in close binaries. However, the generic nature of the CFS (Chandrasekhar-Friedman-Schutz) instability does not guarantee that it is applicable to real NSs. Indeed, up until a few years ago, research in this field concentrated on stability of modes that couple to gravitational radiation via mass multipoles, mainly f-modes ("fundamental" modes, or surface waves). Frequencies of such modes are comparable to the breakup frequency[1], Ω_b, and do not change significantly with rotation. In order to satisfy the CFS instability criterion, $\omega_r(\omega_r - m\Omega) < 0$, the NS has to be rotating close to Ω_b. Moreover, real NSs have viscosity, which damps oscillations. The net result [4] (see also Lai's contribution in this volume) is that the instability in normal-fluid NSs is completely suppressed unless the temperature is between 10^7 and 10^{10} K. Even then, the critical frequency Ω_c (the frequency such that NSs with $\Omega < \Omega_c$ are stable and ones with $\Omega > \Omega_c$ are unstable) is $\gtrsim 0.9\Omega_b$. Thus, the parameter space for the CFS instability of f-modes is rather small.

This field underwent a renaissance in 1998, when Andersson [5] and Friedman & Morsink [6] realized that a special class of fluid modes, called r-modes, is CFS-unstable at *any* Ω. R-modes are global equivalents of what geophysicists call Rossby waves, which have been studied extensively since 1930s. They are predominantly horizontal fluid motions for which the restoring force is the Coriolis force. For example, sinusoidal distortions of the jet stream are caused by a long-wavelength Rossby wave propagating through the atmosphere. Rossby waves have also been detected on the Sun [7]. In the astrophysical context, they were first considered by Papaloizou & Pringle [8], who coined the term "r-modes".

For slow rotation, the r-mode velocity field is mostly transverse:

$$\delta\vec{v} = \alpha\Omega R \left(\frac{r}{R}\right)^m e^{i\omega_r t}\vec{r} \times \vec{\nabla} Y_{mm}(\theta, \phi) + \mathcal{O}(\Omega^3), \tag{1}$$

where $l = m$. A snapshot of this velocity field is shown by arrows in Figure 1a. Individual fluid elements oscillate around their equilibrium positions, while the entire pattern propagates counter-clockwise in the corotating frame. Radial motion in an r-mode is generally negligible (radial displacements at the surface are $\sim 0.1\alpha R(\Omega/\Omega_b)^2$), but is quite important for pulsations in NSs with crusts (Sec. II).

The dispersion relation for r-modes (for small Ω) is

$$\omega_r = \frac{2m\Omega}{l(l+1)} + \mathcal{O}(\Omega^3), \tag{2}$$

so they indeed satisfy the CFS instability criterion, $\omega_r(\omega_r - m\Omega) < 0$ at any spin. However, unlike the case with f-modes, various damping processes are not nearly as effective in suppressing the instability of r-modes. Remarkably, even when viscous damping is included [9,10], the critical frequency for the onset of the r-mode instability, Ω_c, can be as small as $0.1\Omega_b$! The solid line in Figure 1b shows Ω_c

[1] Almost independently of the equation of state, NS breakup frequency is $\Omega_b \approx \frac{2}{3}\Omega_o$, where $\Omega_o = \sqrt{\pi G \bar{\rho}}$ [3].

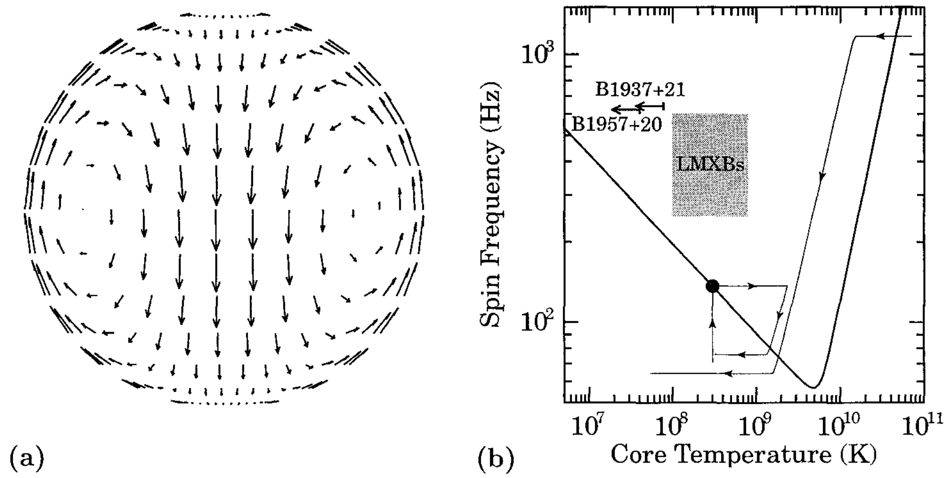

FIGURE 1. (a) Velocity field of an $l = m = 2$ r-mode. (b) Critical stability curve (thick solid line) for $l = m = 2$ r-modes in fluid NSs. The curve and the loop marked with arrows denote the evolution scenarios for newborn NSs and NSs in LMXBs, respectively.

for $l = m = 2$ r-mode of a particular NS model [9–11]. The low-temperature ($T \lesssim 5 \times 10^9$ K) part of the stability curve is determined by the competition between GW growth, with e-folding time $\tau_{\rm gw} \propto \Omega^{-6}$, and damping due to shear viscosity, with $\tau_{\rm v} \propto T^2$. At high temperatures ($T \gtrsim 5 \times 10^9$ K) bulk viscosity (neutrino emission which arises because compression and rarefaction of matter during oscillations drive the fluid out of β equilibrium) is the dominant dissipation mechanism, with $\tau_{\rm b} \propto \Omega^{-2} T^{-6}$.

Clearly, the parameter space for the r-mode instability to operate is quite large (for comparison, Ω_c for f-modes would be off the vertical axis in Figure 1b). A purely fluid NS can stably exist only if its Ω and T place it below the instability line. In any NS that somehow found itself above this instability line an r-mode would quickly grow, and angular momentum loss due to GW emission would quickly spin the star down and out of the instability region.

This discovery dramatically widened the applicability of the CFS instability in astrophysical situations. Two major scenarios have so far been proposed. Lindblom et al. [9], Owen et al. [12], and Andersson et al. [10] considered the effect of this instability on newborn NSs, which may rotate near breakup at birth. These NSs rapidly cool from $T_i \sim 10^{11}$ K by neutrino emission, and reach $T \sim 10^9$ K within a year from birth. However, seconds after the start of cooling they enter the r-mode instability region (follow the curve marked with arrows in Figure 1b). Any initial velocity perturbations get rapidly amplified by the CFS instability. If the amplitude of the unstable r-mode saturates near $\alpha \sim 1$, the GW torque is so large that it can spin down the NS from near-breakup to roughly $0.1\Omega_b$. In this scenario, a NS would lose as much as 99% of its rotational energy to GWs, which would be detectable by LIGO-II from as far as the Virgo cluster, where the event rate could be several per year [12]. The major uncertainty in this scenario is the nonlinear saturation of r-modes, discussed in Sec. III.

Bildsten [13] and Andersson et al. [14] pointed out that the r-mode instability may also explain the spin frequencies of NSs in LMXBs. There is now a mounting body of evidence (see contributions by Strohmayer and Bildsten in this volume) that accreting NSs in LMXBs have a rather narrow range of spin frequencies, $\approx 300-600$ Hz. Remarkably, despite the fact that the spinup time for a NS in an LMXB is much smaller than the lifetime of such a system, none of the observed NSs are spinning anywhere near the breakup. Bildsten [13] conjectured that accretional spinup in these systems is halted by GW emission. The steep frequency dependence of the GW torque can then account for the small range of observed frequencies, in spite of the large range of accretion rates in LMXBs. GWs could be due to either "mountains" supported by NS crusts [13,15] (reviewed in Bildsten's article in this volume) or r-modes in their cores.

Accretion of high angular momentum gas spins up the NS in an LMXB (follow the loop marked with arrows in Figure 1b) at roughly a constant temperature until it reaches the r-mode instability line. Since the r-mode amplitude needed to balance the accretion torque is rather small (corresponding to fluid displacements of tens of centimeters), it was originally believed [16,13,14] that the NS can just hover at the instability line. At this equilibrium point (thick dot in Figure 1b) the accretion torque would be balanced by GW emission from an unstable r-mode. The narrow range of spin frequencies stems from the similarity between core temperatures of accreting NSs. If such equilibrium obtains, then Sco X-1, with $h_c \approx 2 \times 10^{-26}$, would be detectable by LIGO-II, as would a handful of other bright LMXBs [13].

However, this cannot be the whole story. First, Levin [17] and Spruit [18] showed that steady-state equilibrium between accretion and r-modes is thermally unstable. Dissipation from shear viscosity in the r-mode quickly heats the star. Shear viscosity of a normal fluid scales as T^{-2}, so the increase in the core temperature decreases the viscosity, thereby increasing the r-mode's growth rate. The ensuing runaway cycle [17] is depicted by a loop in Figure 1b. Instead of just hovering near the instability line, the r-mode rapidly grows to saturation, heats up the star, and spins it down and out of the instability region in less than 1 yr. The implications of this scenario for detectability of GWs from r-modes in LMXBs is discussed in Sec. II. Second, at $T = \text{few} \times 10^8$ K, typical of LMXBs (shaded box in Figure 1b), the equilibrium spin frequency would be ≈ 150 Hz, rather than $300-600$ Hz. Moreover, the existence of two 1.6 ms recycled radio pulsars (spins and upper limits on temperatures of which are shown by arrows in Figure 1b) means that rapidly rotating NSs are formed in spite of the r-mode instability. While it is not clear whether their *current* core temperatures place them inside the r-mode instability region, normal-fluid r-mode theory says that they were certainly unstable during spinup. It was speculated that including superfluid dissipation ("mutual friction") would move the instability curve to higher frequencies, and perhaps into agreement with the observed spins [13,14]. But tour-de-force calculations of Lindblom & Mendell [19] showed that, except for $\approx 2\%$ of the allowed parameter space, mutual friction gives only a modest increase in the damping rate, insufficient to account for the observed high spins of NSs. It now appears likely that the presence of a solid crust plays a crucial role in this mechanism, and, as argued by Bildsten & Ushomirsky [20], dissipation in the boundary layer between the solid crust and the core is indeed sufficiently strong to reconcile this discrepancy.

In the remainder of this contribution, I will describe recent work on the role of the crust in the r-mode instability (Sec. II), as well as the current understanding of nonlinear saturation of r-modes in young neutron stars. Several in-depth reviews have recently been published [21,22], and I refer the reader to them for the many issues not covered here.

II CRUSTS AND R-MODES IN LMXBS

The instability line in Figure 1b is computed [9–11] using a very simple NS model: a fluid ball with a polytropic equation of state. As discussed in Sec. I, the low-temperature part of the instability curve is determined by energy loss due to the shearing motions of the r-mode, and the damping timescale is $\tau_{\rm v} = 3\times10^6$ s T_8^2 for a particular NS model [9]. For comparison, the gravitational wave growth timescale for the same model is $\tau_{\rm gw} = 20$ s $(1\ {\rm kHz}/f_{\rm spin})^6$. Clearly, shear viscosity in the bulk of the star is quite feeble compared to radiation reaction. This is because the energy loss rate due to shear viscosity depends on the strength of the velocity shear, i.e., the length scale for the velocity gradient, and, for an r-mode in a fluid star, the length scale for the shear is of order the stellar radius R.

However, all but the hottest NSs have crusts that occupy the outer $\sim 1-2$ km of the star. The fluid motions of the r-mode are mostly transverse, indicated schematically by arrows in Figure 2a. This means that the fluid rubs against the solid crust. Neglecting viscosity is an excellent approximation in the bulk of the star, away from the crust-core boundary, and, in the absence of viscosity, there is no extra dissipation due to the rubbing as well. However, for a viscous fluid, there can be no relative motion at the crust-core boundary, leading to a boundary layer where the relative transverse velocity drops from a large value to zero [20], as indicated schematically in Figure 2a.

The width d of this boundary layer is set by the balance between the viscous term in Navier-Stokes equations, $\sim \nu\delta v/d^2$ and either the acceleration term, $\sim \omega\delta v$, or the Coriolis force term, $\sim \Omega\delta v$. The former is referred to as a viscous boundary layer, while the latter is called an Ekman layer. For r-modes in rotating stars, Ekman layer treatment is applicable [23,24], however, the widths of the layers and the damping rates are comparable since $\omega \sim \Omega$ (i.e., Coriolis force is responsible for the acceleration of the fluid elements). It turns out that the boundary layer is very thin,

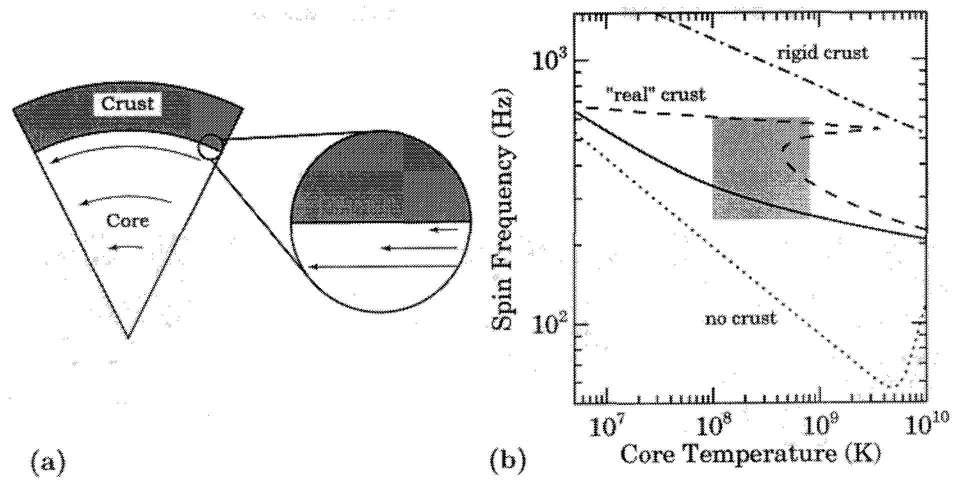

FIGURE 2. **(a)** Schematic depiction of r-mode fluid motion in the bulk of the star and in the boundary layer at the crust-core interface. **(b)** R-mode instability curves for fluid NSs (dotted line), NSs with perfectly rigid crusts (dot-dashed line), and realistic, elastic crusts (solid and dashed lines).

$$d = \left(\frac{\nu}{2\Omega}\right)^{1/2} = 1 \text{ cm } \frac{1}{T_8}\left(\frac{1 \text{ kHz}}{f_{\text{spin}}}\right)^{1/2} \tag{3}$$

Thus, the rate of viscous dissipation in the boundary layer exceeds the damping rate in the interior of the star by a factor $\approx R/d = 10^6$. Lindblom et al. [24] performed an exhaustive survey for a variety of equations of state, and found that the dissipation timescale is in the range

$$\tau_{\text{rub}} \approx 30 - 60 \text{ s } T_8 \left(\frac{1\text{kHz}}{f_{\text{spin}}}\right)^{1/2}. \tag{4}$$

The new instability line for NSs with crusts is shown by the dot-dashed line in Figure 2b. For comparison, $\Omega_c(T)$ for fluid NSs is shown by the dotted line (same as the corresponding line in Figure 1b). The critical frequency for the r-mode instability is a factor of $\sim (R/d)^{2/11}$ higher for neutron stars with crusts. Clearly, boundary layer damping plays an important role in the r-mode instability.

The dissipation rate, Eq. (4), depends on the square of the relative velocity between the bulk of the liquid and the crust. In the above discussion, it was implicitly assumed that the crust is stationary in the rotating frame, and does not participate in the oscillations. However, Levin & Ushomirsky [25] and Yoshida & Lee [26] demonstrated that this picture is not accurate for crusts of real NSs. The crust does participate in r-mode oscillations to some degree, so the amplitude of the relative velocity $\Delta v/v$ ("slippage") between the crust and the core is not 1, as is implicit in Eq. (4). The damping rate, taking into account the motion of the crust, is then [25]

$$\tau_{\rm bl} = \tau_{\rm rub} \left(\frac{v}{\Delta v}\right)^2 . \tag{5}$$

The crux of the matter is that the NS crust is not very rigid. NS matter crystallizes when the ratio of the Coulomb energy of the ion lattice, Z^2e^2/a to the thermal energy, k_BT, exceeds ≈ 172 [27], i.e., at densities $\gtrsim 10^8$ g cm^{-3}. The ratio of the shear modulus to the bulk modulus (i.e., pressure) is rather small, $\mu/p \approx 10^{-2} - 10^{-3}$, throughout the crust. At best, the crust is more like Jell-o rather than a rigid solid. The small $(\mathcal{O}(\Omega/\Omega_b)^2)$ radial motions of the r-mode push on the crust, and are quite effective in making the crust move almost in unison with the core. Another way to look at this situation is as follows. The $l = m = 2$ torsional mode in a non-rotating crust, which has the same angular displacement pattern as the corresponding r-mode, has $f_{\rm cr} \simeq 50$ Hz [28]. This frequency is a few times lower than the r-mode frequency in rapidly rotating stars, indicating that the elastic restoring force is quite weak compared to the Coriolis force. Therefore, at a sufficiently high spin, one would expect the crust to oscillate more or less like a liquid, with elasticity only slightly modifying the mode's structure. If the shear modulus μ of the crust were exactly zero, there would be no slippage Δv between the crust and the core. Since μ is non-zero but small, one would expect the slippage to be proportional to the ratio of the elastic restoring force to the Coriolis force, $\Delta v/v \sim (f_{\rm cr}/f_{\rm rmode})^2$.

Quantitatively [25], the behavior of $\Delta v/v$ with frequency is somewhat more complicated. The crust possesses a spectrum of torsional and spheroidal modes, so the whole (core+crust) system has many modes that look like simple r-modes in the core, but have non-trivial behavior in the crust. The restoring force in the crust then depends on which particular crustal mode is closest in frequency to the core's preferred frequency. At some Ω the crust may be able to effectively expel r-mode pulsations, while at other, nearby frequencies, pulsations may easily penetrate the crust. This phenomenon of avoided crossings (see Figure 1 of [25]) is well-known in asteroseismology. The net result is that the slippage $\Delta v/v$ is not a monotonically decreasing function of frequency. Instead, while typically $\Delta v/v \approx 10^{-2} - 10^{-1}$, it rises sharply to ≈ 1 at the frequencies of avoided crossings. The resulting r-mode instability lines are shown by the solid and dashed lines in Figure 2b, computed for a rather simple NS model, in which the crust is elastic, but has a constant density [25]. The solid line corresponds to the crust occupying the outer $0.1R$, while the dashed line is for a thicker crust, $0.2R$ (this is approximately the range of crustal thicknesses in realistic NS models). The instability line for a thick crust model has a peculiar double-valued shape due to an avoided crossing between the r-mode and a crustal mode at $f_{\rm spin} \approx 550$ Hz. Qualitative behavior of the instability lines for more realistic NSs is expected to remain the same.

The new stability curves cut right through the observed range of spins of both NSs in LMXBs and of millisecond pulsars, for temperatures of interest for accreting NSs (shaded box in Figure 2). An accreting NS can continue its spinup so long as it is located to the left of the stability curve in Figure 2. However, once it crosses the critical stability curve and is located to the right of it, the r-mode will grow and halt the spinup, probably forcing the NS into a thermal runaway cycle [17,18]. The details of what happens during this runaway and how violent it is are a topic of current research. For example, the displacements induced in the crust may cause it break when the strain (which is of order the r-mode amplitude α) exceeds a critical value ($\lesssim 10^{-2}$, see § 6.1 of [15] and references therein for a summary) or melt if the heating due in the boundary layer raises its temperature above $\approx 10^{10}$ K (see [24] and Sec. III). The evolution of the spin frequency and the temperature depends on the saturation of r-mode's growth (see below). Despite these uncertainties, it is clear that, if the r-mode instability operates in accreting NSs, the non-trivial shape of the stability curve and its sensitivity to the crustal structure will be reflected in a peculiar, non-uniform spin distribution of accreting NSs and millisecond pulsars (cf. Strohmayer; Bildsten; this volume). At present, the role of r-modes in setting the spins of NSs in LMXBs cannot be ruled out, and appears to be as plausible as the crustal quadrupole GW equilibrium [13] or magnetospheric equilibrium [29] scenarios. However, many issues still need to be explored to ascertain the location of the r-mode instability curve, such as (1) the presence of the magnetic field that could couple the crust to the core, reducing the effectiveness of boundary layer damping (2) possible pinning of core superfluid to the crust (3) presence of superfluid neutrons in the crust, etc.

The presence of the crust also leads to saturation of r-mode growth. As noted by Wu et al. [30], the Reynolds number in the boundary layer, defined appropriately for oscillatory flow as $Re = \delta v^2/\omega_r \nu$ exceeds the (experimentally measured) critical value of 2×10^5 at a rather small α. Turbulent energy loss rate scales as α^3, while energy input rate from radiation reaction is $\propto \alpha^2$. Turbulence in the boundary layer can halt r-mode growth at $\alpha_{\rm sat} = 3.5 \times 10^{-3} (v/\Delta v)^3 (f_{\rm spin}/1 \text{ kHz})^5$ [30], where the dependence of the prefactor on the temperature is only logarithmic. It therefore seems certain that, for most of the parameter space, the saturation amplitude in the presence of the crust is expected to be less than 1, but is not exceedingly small. This gives us the first indication that the r-mode instability may indeed be directly relevant in astrophysical situations. However, the actual value of $\alpha_{\rm sat}$ is still quite uncertain. The drag coefficient used in deriving it [30] is extrapolated from experimental data for flow of water over solid surfaces, and it is not clear how well they would apply to (possibly superfluid) flow at the crust-core boundary. Convective motions in the turbulent boundary layer may lead to powerful energy losses due to bulk viscosity (neutrino emission), which would lower $\alpha_{\rm sat}$. It is also not clear what role an equipartition magnetic field that may be set up by the turbulence in the boundary layer may play. Despite the uncertainties, the work by Wu et al. [30] is clearly a step in the right direction.

Are GWs from r-modes in LMXBs detectable by ground-based interferometers?

If $\alpha_{\rm sat} \lesssim 3.5 \times 10^{-5}(\dot{M}/\dot{M}_{\rm Edd})^{1/2}(300 \text{ Hz}/f_{\rm spin})^{7/2}$ (i.e., the value needed to balance the accretion torque by GW emission), and is not temperature-sensitive, then accretion-GW equilibrium [16,13,14] is still possible and stable. In this case, GW emission is persistent, and the signal strength h_c is roughly the same as in the crustal quadrupole emission scenario (see Sec. I), but would be distinguishable from it since the radiation would be emitted at $4f_{\rm spin}/3$, rather than $2f_{\rm spin}$. Such stable equilibrium can be ruled out on the basis of existing X-ray observations of low-luminosity, transiently accreting LMXBs [31], since the heat deposited by viscosity into the NS core would make these systems (Aql X-1 in particular) appear about 5-10 times brighter between accretion outbursts than is actually observed. However, current observations cannot rule out this possibility in bright X-ray sources, such as Sco X-1, where core neutrino emission can easily get rid of the extra heat.

If $\alpha_{\rm sat}$ exceeds the above value, then LMXBs are caught in a runaway cycle [17,18]. For $\alpha_{\rm sat} \approx 1$, the spindown phase, during which GWs are emitted, lasts only a fraction of a year. GW signal in this case is similar to that from spindown of newborn NSs, so such LMXBs should be detectable by LIGO-II from distances of tens of Mpc. However, even for a star accreting at $\dot{M}_{\rm Edd}$, the "inactive" spinup phase lasts $\sim 10^7$ yr. The duty cycle for $\alpha_{\rm sat} \approx 1$ is therefore quite low, and Levin [17] concluded that, in order to observe one event per year, the detector must reach a volume encompassing $\gtrsim 10^6$ galaxies (assuming $10 - 100$ LMXBs per galaxy), beyond the capabilities of LIGO-II.

Two things that have changed since the original analysis [17] may bring some renewed optimism for detecting GWs from LMXBs. First, X-ray timing observations (see Swank's article in this volume) have revealed that, in addition to 50 or so persistent LMXBs, there are perhaps as many as several hundred transient LMXBs in the Galaxy. Moreover, recent *Chandra* observations of nearby galaxies (see [32,33] and many others) have resolved most of their diffuse X-ray backgrounds into similar numbers of individual LMXBs. It is now possible to obtain arcsecond-accurate positions of these extragalactic accreting NSs. Second, since the spindown time during the runaway cycle scales as $1/\alpha_{\rm sat}^2$, a small saturation amplitude would lead to a much more favorable duty cycle for GW emission. Using the results [30] for the turbulent saturation amplitude, the spindown phase lasts ≈ 1500 yr $(\Delta v/v)^{0.4}$, leading to a duty cycle of $\approx 2\times10^{-4}(\dot{M}/\dot{M}_{\rm Edd})(\Delta v/v)^{0.4}$. Therefore, the volume one needs to sample is much smaller. Moreover, since the spindown time much exceeds the typical observation time, GW signal is nearly monochromatic. Therefore, a viable strategy is to search for GWs from extragalactic LMXBs, using existing GW pulsar search techniques and codes, with precise positions obtained by *Chandra*.

III SPINDOWN OF YOUNG NEUTRON STARS

About a minute after its birth, a young NS can cool to $T_m \approx 10^{10}$ K, the temperature at which a crust begins to form at $\rho \approx 1.5 \times 10^{14}$ g cm^{-3}. The r-mode instability curves applicable to this situation are then those shown in Figure 2b, with Ω_c a factor of 2−3 higher than the pure-fluid value. Therefore, if a solid crust does form, the final frequency to which a young NS can spin down would be a factor of several higher than if the crust did not form. Since most of the GW signal-to-noise (S/N) during spindown is accumulated at the lowest spin frequencies, where the source spends most of the time, raising the frequency cutoff by a factor of few would correspondingly reduce the S/N.

However, the crust has to form in the presence of the shearing motion of the r-mode, which can substantially delay this process.[2] If a solid crust manages to form, viscous dissipation in the boundary layer will heat the fluid at the crust-core boundary. An r-mode with amplitude exceeding $\alpha_c = 4.7 \times 10^{-3}(T_m/10^{10}\ \mathrm{K})^{5/2}(\Omega_b/\Omega)^{5/4}$ (obtained simply by balancing boundary-layer heating with local neutrino emission and heat conduction into the core, [24]) will heat the crust-core boundary to T_m and melt the crust. The would-be crust is constantly losing energy to neutrino emission and wants to solidify. However, as soon as it does, it can be re-melted by the r-mode if its amplitude exceeds α_c. Clearly, neither a completely solid, nor a completely liquid state is possible in this case. Lindblom et al. [24] argued that the outer layers of a nascent NS will be in a mixed liquid-solid state: a NS ice flow. The viscosity in such granular flow is set by the sizes of individual ice chunks and the distances between them, and much exceeds ordinary molecular viscosity. This flow is self-regulating: chunk sizes adjust (typical size ~ 1 cm) to keep the dissipation at the level needed to keep the temperature at the melting point. The net result is that the star can spin down to a frequency much lower than if a solid crust were present. The spindown can continue as long as the r-mode has enough energy to melt the crust. Calculations of Lindblom et al. [24] show that the final frequency is $\Omega \approx 0.14\Omega_b\alpha_{\mathrm{sat}}^{-1/4}$ (where α_{sat} is the *pure-fluid* saturation amplitude), i.e., almost the same as if the crust did not form at all.

This brings us to the central question for r-modes in young NSs: what is the saturation amplitude in a fluid star? So far, we have assumed that the r-mode would be able to grow to a "large" amplitude ($\alpha = 0.01 - 1$). Do the full equations of hydrodynamics allow this, or does the nonlinearity present in them limit the growth at small values of α? For example, radial pulsations in Cepheids easily reach amplitudes ~ 1, while g-modes in white dwarfs saturate at rather small amplitudes, so either possibility is not excluded a priori. One can approach this problem either by direct numerical simulations (i.e., studying the fully nonlinear regime), or by retaining, in addition to the linear terms, the terms of $\mathcal{O}(\alpha^2)$ or higher in the equations of motion (i.e., weakly nonlinear regime).

What magnitude of saturation amplitude may one expect? In the linearized equations of motion, employed to compute r-mode frequencies and damping times, all modes are independent and do not couple to each other. If the mode amplitude is small but finite, we expect that the effect of the nonlinear terms is to couple the linear modes and allow energy transfer among them. Suppose that lowest order, quadratic coupling to daughter modes is responsible for saturation of the

[2] Differential rotation present in the nascent NS may also delay crust formation.

parent r-mode. This means that the rate at which gravitational radiation reaction is depositing energy into the parent r-mode, $\sim \alpha E_{\rm mode}/\tau_{\rm gw}$, is the same as the rate at which quadratic coupling drains the energy from it, $\sim \alpha^2 C E_{\rm mode}\omega_r$, where C is a dimensionless number signifying the efficiency of coupling (Phinney, private communication). If the modes are well-coupled ($C \approx 1$), then $\alpha_{\rm sat} \sim 10^{-3}$ for stars rotating near breakup, and even smaller at slower Ω. However, there is no easy way to estimate C. It has to be computed, and there are no compelling theoretical arguments to prefer large or small values of C. The main problem becomes to identify the modes that couple most strongly to the parent r-mode, and to compute the coupling coefficients C for them. Very preliminary indications (Morsink, private communication) are that, if only coupling to other r-modes is considered, no saturation is observed until the parent mode amplitude grows to unrealistically large values. Coupling to other classes of modes (e.g., g-modes) may turn out to play a significant role.

So far, direct numerical simulations are hinting at roughly the same conclusion. Stergioulas & Font [34] imposed a large-amplitude ($\alpha \approx 1$) r-mode on a rotating star, and evolved the system for 26 rotation periods without much change in the mode amplitude. Lindblom et al. [35] evolved a small-amplitude r-mode under the influence of radiation reaction (artificially increased by a factor of 4500). They observed exponential growth, as predicted by perturbation theory, until $\alpha \approx 2$, while at $\alpha \approx 3.4$ shocks developed and quickly damped out the mode. These results seem to imply that even an r-mode with $\alpha \approx 1$ is dynamically stable, and that nonlinear hydrodynamical coupling, if it occurs, happens on timescales much longer than the dynamical time. However one needs to exercise caution in interpreting the results: these pioneering simulations were carried out for barotropic stars that do not have g-modes, and their resolution was likely insufficient to resolve the length scales ($l \gtrsim 50$) on which microscopic viscosity is able to dissipate the energy input due to gravitational radiation reaction. It is also uncertain what role the differential rotation, observed in simulations of Lindblom et al. [35] (see also [36,37]), or the magnetic field [38] will play.

If shock formation does turn out to be the mechanism that limits the growth of r-modes, then it has some interesting implications for the young NS spindown scenario [35]. Since the spindown timescale is $\propto 1/\alpha^2$, the spindown will occur in $\approx 1/10$ the time originally expected [12]. GWs will then be emitted in a much narrower band, $\Delta f \approx 0.05f$, which somewhat simplifies the data analysis problem. Finally, after the shocks dissipate the r-mode, the star is still left rather rapidly rotating, so subsequent bursts of r-mode spindown are possible.

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