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Variability associated with α accretion disc theory for standard and advection-dominated discs

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ABSTRACT

The α turbulent viscosity formalism for accretion discs must be interpreted as a mean field theory, modelling a steady state only on spatial or time-scales greater than those of the turbulence. The extent of the scale separation determines the relative precision error (RPE) of the predicted luminosity L_ν. Turbulence and the use of α implies that (1) field line stretching gives a magnetic pressure $\gtrsim\alpha^2/6$ of the total pressure generally, and a one-to-one relation between α and the pressure ratio for thin discs, and (2) large turbulent scales in advection-dominated accretion flows (ADAFs) predict a lower L_ν precision than thin discs for a given observation duration and central mass. The allowed variability (or RPE) at frequency ν increases with the size of the contributing region. For X-ray binary ADAFs, the RPE $\sim$5 per cent at $R \leq 1000$ Schwarzchild radii (R_s) for averages over $\gtrsim$1000 s. However, current data for galaxies like NGC 4258 and M87 give RPEs in L_ν of 50–100 per cent even at $R \leq 100R_S$. More data are required, but systematic deviations from ADAF predictions are more significant than random deviations, and may constrain properties of the turbulence, the accretion mode, the assumption of a steady state or the accretion rate.

Key words: accretion, accretion discs – binaries: general – Galaxy: centre –galaxies: active – galaxies: nuclei.

1 INTRODUCTION

Accretion discs (e.g Pringle 1981; Papaloizou & Lin 1995) can explain a variety of phenomena such as active galactic nuclei (AGN), X-ray binary systems, cataclysmic variables (CVs), and dwarf novae. As accreting gas orbits a central massive source, internal energy dissipation drains the rotational energy, allowing material to move in and angular momentum to move out. The dissipation sustains steady accretion and some fraction of the dissipated energy accounts for the observed luminosity. Microphysical viscosities are too small to explain observed luminosities, so an enhanced transport mechanism, probably involving turbulence, is essential. Since astrophysical discs are surely magnetized to some non-zero level, the 'Balbus-Hawley' shearing instability (cf. Balbus & Hawley 1998), which produces self-sustaining turbulence, is a natural and likely ubiquitous driver of angular momentum transport for at least thin discs and possibly thick discs as well. Significant dissipation may also occur above a thin disc, in a corona (e.g Haardt & Maraschi 1993; Field & Rogers 1993; DiMatteo, Blackman & Fabian 1997). For low enough accretion rates, the dissipated energy may be primarily advected rather than radiated (Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1995a,b), forming an advection-dominated accretion flow (ADAF) thick disc. ADAFs have been effective in modelling quiescent phases of accretion in a variety of X-ray binaries and galactic nuclei (see Narayan, Mahadevan & Quataert 1998a).

While non-linear instabilities in thin discs have been simulated locally (e.g. Brandenburg et al. 1995; Stone et al. 1996; Balbus, Hawley & Stone 1996), a useful approach to global models swipes the details of the stress tensor into a turbulent viscosity of the form (Shakura 1973; Shakura & Sunyaev 1973)

$$\nu_{\rm tb} = \alpha c_{\rm s} H \simeq v_{\rm tb} l_{\rm tb}, \tag{1}$$

where H is the disc height, $c_{\rm s}$ is the sound speed, $l_{\rm tb}$ is the dominant turbulent eddy scale, $v_{\rm tb}$ is the eddy speed at that scale, and $\alpha < 1$ is taken to be a constant. Sometimes a 1/3 appears on the right side. This formalism *requires* a mean field theory. Viscous coupling of differentially rotating fluid elements is a local paradigm, so assumptions of azimuthal symmetry and steady inflow (e.g. Pringle 1981; Narayan & Yi 1995a,b) require smoothing of turbulence over the time and/or spatial scales on which mean quantities vary.

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The required mean field approach is similar to that employed in mean field magnetic dynamo theory (Parker 1979), where the induction equation for the magnetic field is averaged and solved. In the kinematic (and therefore incomplete) dynamo theory, the velocity is imposed and the momentum equation is ignored. For the simplest global α accretion disc approach, the focus is reversed (but also incomplete); the momentum, energy and continuity equations are solved, with the inclusion of the magnetic field as a pressure rather than employing the magnetic induction equation (e.g. Narayan & Yi 1995a,b). However, the usual disc equations are not usually derived from the formal averaging approach. Balbus & Hawley (1998) have addressed some of these points, but the conditions for which the standard equations result when $H \sim R$ have not been extensively studied.

Despite being incomplete, the α formalism provides a useful framework for thin and ADAF discs. Here I show that $l_{\rm tb} \leq H$ and field line stretching place a non-trivial relation between α and the magnetic pressure. I also estimate the precision of the α turbulent disc formalism for thin and thick ADAF discs. For ADAFs the precision is lower, and the allowed variability higher, than for thin discs with a given central mass and fixed observation duration. A discussion of the implications for stellar versus galactic nuclei presumed to be ADAF systems is given. Balbus, Gammie & Hawley (1994) give a discussion of fluctuations in a thin disc and a relation to luminosity, but a different approach and different questions are addressed here. Although quantities like velocity can be formally separated into mean and fluctuating parts, the turbulence gives a non-negligible rms error to the mean only when the disc radius is much larger than the scale of the turbulence. Here this rms error is estimated as a precision measure of the α formalism, as a function of the averaging time.

2 PRECISION OF α-ACCRETION DISC THEORY

The usual slim disc equations are derived (e.g. Abramowicz et al. 1988; Narayan & Yi 1995a) by vertically averaging the continuity, Navier–Stokes, and energy equations and with the magnetic field incorporated only as an additional pressure. Without presenting the formalism, here I assume that the equations of (e.g.) Narayan & Yi (1995a) hold, but emphasize that the standard simple replacement of the microphysical viscosity with a turbulent viscosity hides the requirement of radially and/or temporally averaging (in addition to the usual azimuthal and vertical smoothing). For the radial average, a scale ξ must be chosen such that $l_{\rm tb} < \xi < R$, where R is the disc radius. The spatial average of a quantity like velocity $\boldsymbol{V}(R)$ is then $\boldsymbol{V}_0(R) = \langle \boldsymbol{V}(R)\rangle_{\rm s} \simeq \langle \boldsymbol{V}(R,t')\rangle_\xi = \int_{\lambda\leq\xi} \boldsymbol{V}(R+\lambda,t')\mathrm{d}\lambda$, where the similarity follows from the assumption that the time dependence is only due to turbulent fluctuations which are intended to be smoothed for mean quantities. The subscript 0 indicates the mean quantity to be used in standard α disc theory. For a temporal average, taken over a duration $t_{\rm obs}$, we have $\boldsymbol{V}_0(R) = \langle \boldsymbol{V}(R)\rangle_{t_{\rm obs}} = (1/t_{\rm obs})\int_t^{t+t_{\rm obs}} \boldsymbol{V}(R,t')\mathrm{d}t'$. The temporal average is meaningful only over times such that $t_{\rm obs} > t_{\rm tb}$ where $t_{\rm tb}$ is the dominant energy containing eddy turnover time-scale.

How precise is the assumption that the mean speed behaves as a steady monotonic function of R in the presence of turbulence? Note that 'precision' is taken to mean that defined in Bevington & Robinson (1992) with the α disc theory as the measuring device. A relative precision error (RPE) measures how effective the theory is at predicting, not how accurate the predictions are. The RPE error around the total mean speed V_0 is given by $\Delta V_0/V_0 = [(\Delta V_0)_{\rm fl}/V_0 + \partial_R V_0(\Delta R)_{\rm rs}/V_0]$, and the two terms on the right measure two rms contributions to the RPE. The first is

$$(\Delta V_0)_{\rm fl} = v_{\rm tb}/N_{\rm fl}^{1/2} \simeq v_{\rm tb}(\xi/l_{\rm tb} + t_{\rm obs}/t_{\rm tb})^{-1/2}$$

where $N_{\rm fl}$ measures the 'effective' number of eddies per radial averaging length. With increasing $t_{\rm obs}$, $N_{\rm fl}$ can well exceed the 'snapshot' $t_{\rm obs} = 0$ value $\xi/l_{\rm tb}$. The second rms contribution to the RPE above results from the fact that

$$R - (\xi/2)N_{\rm rs}^{-1/2} \leq R \leq R + (\xi/2)N_{\rm rs}^{-1/2}$$

is indistinguishable once ξ is chosen. Here

$$N_{\rm rs}^{-1/2} \simeq (1 + t_{\rm obs}/t_{\rm tb})^{-1/2}$$

and measures the 'effective' number of averaging scales per ξ which increases from its snapshot value of 1 for long $t_{\rm obs}$. The fluctuation and resolution numbers ($N_{\rm fl}$ and $N_{\rm rs}$) increase with $t_{\rm obs}$ because the turbulence does not generate eddies in the same location over time, and this smoothing reduces the 'effective' averaging and eddy scales.

Since the total speed for both thin and thick discs is dominated by a contribution $\propto R^{-1/2}$, we can then estimate a total RPE for R as

$$\Delta R/R \simeq [2(\Delta V_0)_{\rm fl}/V_0 + (\Delta R)_{\rm rs}/R] \simeq [2(v_{\rm tb}/V_0)N_{\rm fl}^{-1/2} + (\xi/2)N_{\rm rs}^{-1/2}]. \tag{2}$$

Assuming a constant accretion rate, (2) translates into an RPE in the luminosity given by

$$\Delta L_\nu/(L_\nu) \simeq |\psi|(\Delta R/R) \tag{3}$$

where ν is the frequency of emission and $|\psi| \equiv |R\partial_R[\ln(L_\nu)]|$. The RPE can be used to estimate the variability allowed for a given $t_{\rm obs}$.

Though phenomenologically derived, the RPE formulae have properties which show that they capture the limiting cases correctly. First, for $t_{\rm obs} >> t_{\rm tb}$, they are reduced as expected. Secondly, for $t_{\rm obs} = 0$, there is an optimal scale of

$$\xi/R = \xi_{\rm opt}/R = (v_{\rm tb}/V_0)^{2/3}(l_{\rm tb}/R)^{1/3} \tag{4}$$

for which the error is minimized: a larger ξ reduces the rms effect of the turbulent velocity, but one pays the price with a coarser spatial resolution. When $\xi_{\rm opt} < l_{\rm tb}$, the RPE is dominated by the resolution term but then the minimum $\xi = l_{\rm tb}$ must be used.

3 ENERGY CONSTRAINTS AND RELATIONS BETWEEN CHARACTERISTIC SPEEDS

In a highly conducting turbulent plasma, the magnetic field is naturally amplified to the extent that $v_{\rm tb} \simeq B/(4\pi\rho)^{1/2} \equiv v_A$, the Alfvén speed (e.g. Parker 1979). Shearing box simulations, in which turbulence is driven by a seed magnetic field (Stone et al. 1996; Brandenburg et al. 1995; Balbus & Hawley 1998), show $v_A \gtrsim v_{\rm tb}$. Because of field line stretching, equipartition of turbulent and Alfvén speeds is generally a more applicable rule of thumb than any relation between the particle and magnetic pressures.

When the magnetic field is tangled on scales much smaller than those on which mean quantities vary (which for ADAFs probably requires temporal averaging, as seen below) averaging the Lorentz force gives an effective magnetic pressure

$$P_{\rm mag}/\rho \simeq B^2/(24\pi\rho) = v_A^2/6 = (1-\beta_a)c_{\rm s}^2, \tag{5}$$

where β_a is a parameter. Using (5) in (1) and $v_{\rm tb} \simeq v_A$ we have

$$l_{\rm tb} = \alpha H/K_1^{1/2}, \tag{6}$$

where $K_1 \equiv 6(1-\beta_a)$. Because $l_{\rm tb} \leq H$, we have the constraint

$$0 \leq \beta \leq (1-\alpha^2/6), \tag{7}$$

or $\alpha \leq 6^{1/2}(1-\beta_a)$. Sometimes (1) is written with an isotropizing factor of 1/3 on the right, in which case we would have $\alpha \leq (6^{1/2}/3)(1-\beta_a)$.

When the instability driving the turbulence is a magneto-shearing instability, we have in the steady state $t_{\rm tb}\sim$ the instability growth rate, so $t_{\rm tb} \simeq R/v_{0,\phi}$, where $v_{0,\phi}$ is the mean azimuthal speed. Since, $v_A \sim v_{\rm tb}$ from field line stretching, $\nu = \alpha c_{\rm s} H \simeq v_{\rm tb} l_{\rm tb}/3 \sim v_A^2(R/3v_{0,\phi})$ we have $\alpha \sim 2(1-\beta_a)$ for thin discs. For thick ADAF discs, this relation does not apply. This is because the rotation speed is not Keplerian. If one applies the same argument about shearing instability for ADAFs directly, the result violates the upper limit discussed below equation (7). An alternative approach for ADAFs is to note that if a shearing instability drives the turbulence, the time-scale for its growth (= $t_{\rm tb}$ in the steady state) must be less than the radial infall time. Using the ADAF solution, this gives a limit comparable to (7). The above relations also imply

$$v_{\rm tb} = l_{\rm tb} t_{\rm tb} = K_1^{1/2} c_{\rm s}. \tag{8}$$

4 RPE OF THIN DISC MODELS

For thin discs, $H << R$ and $V_0 \simeq V_{\phi,0} \sim V_{\rm ff} \equiv (GM/R)^{1/2}$. Also, $c_{\rm s} \simeq V_{\rm ff} H/R$ from vertical hydrostatic equilibrium. Using these, along with (3), (6), (8) and the definitions of $N_{\rm fl}$ and $N_{\rm rs}$, we have

$$\begin{aligned}\Delta L_\nu/L_\nu &= |\psi|\Delta R/R \simeq 2|\psi|(v_{\rm tb}/V_{0,\phi})N_{\rm fl}^{-1/2} + 0.5|\psi|(\xi/R)N_{\rm rs}^{-1/2}\\ &\simeq \frac{2|\psi|(H/R)(K_1/1)^{1/2}}{\{{\rm Max}[1, 22(K_1/1)^{2/3}(\alpha/0.01)^{-2/3}] + 2.3\times10^5 t_{\rm obs}(K_1/1)(\alpha/0.01)^{-1}(M/{\rm M}_\odot)^{-1}(R/10R_{\rm s})^{-3/2}\}^{1/2}}\\ &+\frac{0.005|\psi|(\alpha/0.01)(K_1/1)^{-1/2}(H/R){\rm Max}[1,22(K_1/1)^{2/3}(\alpha/0.01)^{-2/3}]}{[1+2.3\times10^5 t_{\rm obs}(K_1/1)(\alpha/0.01)^{-1}(M/{\rm M}_\odot)^{-1}(R/20R_{\rm s})^{-3/2}]^{1/2}},\end{aligned} \tag{9}$$

where ξ has been replaced by the Max[,] as per the discussion below (4). The temperature in an optically thick thin disc goes as $T_{\rm e} \propto R^{-3/4}$ (e.g. Frank, King & Raine 1992). Then, for example, in the Rayleigh–Jeans regime $[h\nu << kT(R)]$ where $L_\nu \propto \nu^2 \int_{R_{\rm min}}^{R_{\rm max}} T_{\rm e}(r) r {\rm d}r$, the luminosity within a radius R at a given frequency goes as $L_\nu \propto \nu^2 R^{5/4}$. Thus $|\psi| = 5/4$.

The RPE of (9) is small compared to what will be found for ADAFs. A careful check, keeping equation (7) and the discussion below it in mind, ensures that for all allowed β_a the RPE $\lesssim|\psi|H/R$. The RPE is further reduced for large $t_{\rm obs}$. The low RPE results because $H/R << 1$ and $v_{\rm tb} << V_0 \sim V_{0,\phi} \sim v_{\rm ff}$ for thin discs.

5 IMPLICATIONS AND RPE FOR THICK ADAF DISCS

For thick ADAF discs things are more subtle. From (7) and the standard ADAF choice of $\alpha = 0.3$ (Narayan et al. 1998a) we have $0 \leq \beta_a \leq 0.985$. Defining $K_2 \equiv 2/(7-2\beta_a)$ and using $c_{\rm s} = K_2^{1/2} V_{\rm ff} \sim (H/R)V_{\rm ff}$ (Narayan et al. 1998a), we have $v_{\rm tb} = (K_1K_2)^{1/2}V_{\rm ff}$, and thus

$$l_{\rm tb}/v_{\rm tb} = t_{\rm tb} = \alpha H/(K_1 c_{\rm s}) = \alpha H/(K_1K_2)^{1/2}V_{\rm ff} = \alpha R/(K_1 V_{\rm ff}), \tag{10}$$

where (6) and (8) were used. Furthermore, defining $K_3 \equiv 12(1-\beta_a)/(7-2\beta_a)$, the total mean speed is (Narayan 1998a) $V_0 \sim [(9\alpha^2/4)K_2^2 + K_3]^{1/2}V_{\rm ff}$. Define

$$K_4 \equiv v_{\rm tb}/V_0 = (K_1K_2)^{1/2}/[(9\alpha^2/4)K_2^2+K_3]^{1/2} = [12(1-\beta)(7-2\beta)]^{1/2}/[9\alpha^2 + 12(1-\beta)(7-2\beta)]^{1/2}. \tag{11}$$

This $K_4 \sim 1$ over the allowed range of $0 \leq \beta_a \leq 0.985$. Using (3) (4), (10) and (11) gives

$$\begin{aligned}\Delta L_\nu/L_\nu &= |\psi|\Delta R/R \sim 2|\psi|(v_{\rm tb}/V_0)N_{\rm fl}^{-1/2} + |\psi|(\xi/2)N_{\rm rs}^{-1/2} \\ &\simeq \frac{1.22|\psi|(K_4/1)(\alpha/0.3)^{1/2}}{\{\mathrm{Max}[1, 3.9(\alpha/0.3)^{-\frac{2}{3}}(K_4/1)^{\frac{2}{3}}(K_1/3)^{\frac{1}{3}}(K_2/0.58)^{-\frac{1}{3}}] + 2.4\times10^3 t_{\rm obs}(K_1/3)(M/\mathrm{M}_\odot)^{-1}(R/20R_{\rm s})^{-\frac{3}{2}}\}^{-\frac{1}{2}}} \\ &+ \frac{0.06|\psi|(\alpha/0.3)(K_1/3)^{-\frac{1}{2}}(K_2/0.58)^{\frac{1}{2}}\mathrm{Max}[1, 3.9(\alpha/0.3)^{-\frac{2}{3}}(K_4/1)^{\frac{2}{3}}(K_1/3)^{\frac{1}{3}}(K_2/0.58)^{-\frac{1}{3}}]}{[1+8\times10^3 t_{\rm obs}(\alpha/0.3)^{-1/2}(K_1/0.3)(M/\mathrm{M}_\odot)^{-1}(R/20R_{\rm s})^{-3/2}]^{-1/2}}. \end{aligned} \tag{12}$$

Recall that $K_{1,2,3}$ all depend only on β_a. The rigorous form for $\xi_{\rm opt}$ from (4) has also been employed. Over the allowed range $0 \leq \beta_a \leq 1 - \alpha^2/6$, the RPE is relatively insensitive to β_a (except through the α and β_a relation of Section 3.) This is because decreasing β_a increases $l_{\rm tb}$ while lowering $v_{\rm tb}$, and vice versa. The RPE is sensitive to *both* $v_{\rm tb}$ and $l_{\rm tb}$.

I now estimate $|\psi|$ for various emission regimes based on ADAF scaling relations (Mahadevan 1997). Consider the Rayleigh–Jeans radio regime. Here $L_\nu \sim L_{\nu_{\rm c}}$ where $\nu_{\rm c}$ is the peak frequency at each $R < R_{\rm max}$ and is determined by synchrotron absorption. In the ADAF, $\nu_{\rm c} \propto BT_{\rm e}^2 \propto T_{\rm e}^2 R^{-5/4}$, and is therefore a function of R. The spectrum traces the envelope of peak frequencies, with each frequency corresponding to a particular R. For moderate accretion rates by ADAF standards (but below the critical value required for an ADAF solution) compressive electron heating is unimportant (Narayan et al. 1998a). Then, $\partial_R T_{\rm e} = 0$. Using $\nu = \nu_{\rm c}$ we then have $L_\nu \propto T_{\rm e}^5 R^{-1/2}$ and $|\psi| = 1/2$. However, when compressive heating is important, $\partial_R T_{\rm e}(R) \neq 0$. Fitting the $T_{\rm e}(R)$ curve of Narayan et al. (1998b), I obtain $\log T_{\rm e}(R) \sim 9.8 - 0.3\log(R/R_{\rm s}) - 0.06[\log(R/R_{\rm s})]^2$, so then

$$R\partial_R[\ln T_{\rm e}(R)] = -0.7 - 0.3\log(R/R_{\rm S}), \tag{13}$$

and $|\psi| = 1/2 + 3.5 + 1.5\log(R/R_{\rm s})$.

For ν below $\nu_{\rm c} = \nu_{c,\rm min}$, the critical frequency corresponding to the maximum disc radius $R_{\rm max}$, the spectrum is simply $\propto\nu^2$ at fixed $R_{\rm max}$, and in the constant $T_{\rm e}$ regime, $|\psi| = 2$. For the $\partial_R T_{\rm e}(R) \neq 0$ regime using (13), $|\psi| = 1.3 + 0.3\log(R/R_{\rm s})$. In the Compton-dominated submillimetre/X-ray regime,

$$L_\nu \propto \nu_{\rm c}^{\alpha_{\rm c}} T_{\rm e}^5 R^{-1/2}\nu^{-\alpha_{\rm c}} \propto T_{\rm e}^{5+2\alpha_{\rm c}} R^{-(2+5\alpha_{\rm c})/4}\nu^{-\alpha_{\rm c}},$$

which is sensitive to the Comptonization parameter $\alpha_{\rm c}$ and $|\psi| = 0.5 + 5\alpha_{\rm c}/4$ for constant $T_{\rm e}(R)$. In the regime where (13) is applicable, $|\psi| = 0.5 + 5\alpha_{\rm c}/4 + (5+2\alpha_{\rm c})[0.7 + 0.3\log(R/R_{\rm s})]$. In the Bremsstrahlung-dominated submillimetre X-ray regime,

$$L_\nu \propto \ln(R_{\rm max}/R_{\rm min})F(T_{\rm e})T_{\rm e}^{-1}\exp[-h\nu/kT_{\rm e}],$$

where $F(T_{\rm e})$ is dominated by a term $\propto T_{\rm e}$ when $kT_{\rm e} > m_{\rm e}c^2$ and dominated by a term $\propto T_{\rm e}^{1/2}$ when $kT_{\rm e} < m_{\rm e}c^2$. If $\partial_R T_{\rm e} = 0$, $|\psi| \sim 1/\ln(R_{\rm max}/R_{\rm min}) \sim 0.4$, for $R_{\rm max}/R_{\rm min} = 1000$, but this is sensitive to radial dependences of $T_{\rm e}$ since the latter appears in the exponential for the Bremsstrahlung regime. In the limit that $T_{\rm e} > mc^2$, $|\psi| \sim 2.2(\nu/10^{20}\ \mathrm{Hz})(T_{\rm e}/3\times10^9\ \mathrm{K})^{-1}[0.7 + 0.3\log(R/R_{\rm s})]$. For $T_{\rm e}(R) < mc^2$,

$|\psi| \sim |[0.5 - 2.2(\nu/10^{20}\ \mathrm{Hz})(T_{\rm e}/3\times10^9\ \mathrm{K})^{-1}][0.7 + 0.3\log(R/R_{\rm s})]|$. Generally, for reasonable $(R/R_{\rm s})$,$1/2 \leq |\psi| \leq 10$.

6 IMPLICATIONS FOR OBSERVATIONS OF PRESUMED ADAFS

We can see from (12) that for $M \sim 10\,\mathrm{M}_\odot$ (e.g. X-ray binary type systems) and $|\psi| \leq 10$, predictions probing the inner $20R_{\rm s}$ and averages over $t_{\rm obs} \gtrsim 10^3$ s are quite precise, that is $\Delta L_\nu/L_\nu \leq 0.05$. At $R = 1000R_{\rm s}$, and $|\psi| = 1/2$, $\Delta L_\nu/L_\nu \sim 0.05$. The allowed variability decreases as $t_{\rm obs}^{-1/2}$.

Now consider the galactic nucleus of NGC 4258 with central mass $M \simeq 3.5\times10^7\mathrm{M}_\odot$. For the radio-emitting regime of this source near 22 GHz, Herrnstein et al. (1998) found no detection of 22 GHz emission in NGC 4258 with a 3σ upper limit of 220 μJy. This frequency is safely in the Rayleigh–Jeans regime and the best-fitting models of ADAFs to NGC 4258 have $T_{\rm e}(R)$ approximately constant in this regime. Herrnstein et al. interpret this non-detection to mean that any ADAF proposed for this source (Lasota, Narayan & Yi 1996) cannot extend outside a radius defined by $\nu_{\rm c} = 22\,\mathrm{GHz}$, namely $R \sim 100R_{\rm s}$. For the observations, $t_{\rm obs} \sim 10^5$. However, at $100R_{\rm s}$ the $t_{\rm obs}$ term does not contribute significantly to (12). Since in this regime $|\psi| \sim 1/2$, we have from (12) $\Delta L_\nu/L_\nu \sim 0.4$, so this would reduce the significance of non-detection at 22 GHz to $\sim1\sigma$.

For the Galactic Centre, the presumed central mass is $\simeq 2.5\times10^6\ \mathrm{M}_\odot$. Then from (12), at $R = 20R_{\rm s}$, $t_{\rm obs}$ must be $> 10^3$ s to contribute to significantly reducing the RPE. The X-ray observations and many radio observations above 10 GHz, when taken together, provide enough total $t_{\rm obs}$ for low RPE in this range (Narayan et al. 1998b and references therein) However, for frequencies $\lesssim$1 GHz, $R \geq 1000R_{\rm s}$, and the total $t_{\rm obs}$ must be $\gtrsim 10^6$ s, for which there is insufficient data. Using $|\psi| = 1.65$ in (12), $\Delta L_\nu/L_\nu \gtrsim 1$.

Application of ADAFs to larger galactic nuclei (Fabian & Rees 1995) require longer observation times and more data for precise predictions. For M87, $M \sim 3\times10^9\ \mathrm{M}_\odot$ so at $20R_{\rm s}$, the required $t_{\rm obs}$ time would be $\gtrsim 10^6$ s for the $t_{\rm obs}$ term in (12) to reduce the RPE well below 1, while at $1000R_{\rm s}$ the limit would be $\gtrsim 10^8$ s of total time. X-ray observations have been made for 1.4×10^4 s (Reynolds et al. 1996) and radio observations have been made for only of order hours at particular frequencies, e.g. 2×10^4 s at 1.7 GHz (Reid et al. 1989). and 7.2×10^3 s at 22 GHz (Spencer & Junor 1986) Recent observations of several large systems such as M60 (DiMatteo et al. 1998) seem to indicate a radio peak reduced well below that of ADAFs. While there may be a trend, the interpretation should still be taken with the RPE in mind.

7 CONCLUSIONS

The presumption that accretion discs are turbulent implies that the standard steady disc equations represent mean field equations. Predictions of steady state turbulent accretion disc theory would not be expected to match observations taken over a period less than $t_{\rm tb}$ since the system would not be in a steady-state on that time scale. This leads to an allowed variability or RPE in the predicted luminosity. The large turbulent scales and speeds for ADAFs lead to an RPE significantly larger than for thin discs. The RPEs of (9) and (12) can be used to roughly predict allowed deviations or variabilities in the predicted L_ν for a given $t_{\rm obs}$, and indicate when longer or additional observations are needed to properly compare with disc models. The RPE is reduced over large $t_{\rm obs}$ because such averaging amounts to smoothing over an ensemble of many turbulent realizations.

Conclusions about any ADAF transition radius in NGC 4258 (Herrnstein et al. 1998) based on a 22 GHz non-detection must be interpreted with the RPE of equation (12) in mind. For the Galactic centre below 1 GHz, and for larger mass systems, more data than expected would be needed for robust comparisons. Systematic deficits from ADAF predictions in several large elliptical galaxies (DiMatteo et al. 1998) and/or the absence of predicted variability, would be stronger evidence against ADAFs than random deviations when the data is sparse. However, if the energy dissipation occurs in rapid flares, the probability of seeing the system in quiescence during a snapshot might be greater than seeing the system active. When the amount of data is sufficient, the absence of such variability in presumed large α systems could also be regarded as a diagnostic of whether such systems even have ‘canonical’ turbulence in the sense of (1). The accretion could be episodic or produce an outflow. Finally, note that the total and radio peak luminosities depend on the accretion rate to powers of 1 and 3/2, respectively (e.g. Mahadevan 1997). Since the accretion rate in ellipticals is estimated far away from the central engine (DiMatteo et al. 1998; Peres 1998), this provides another source of RPE.

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