# Two-Dimensional Interleaving Schemes with Repetitions

**Mario Blaum**
IBM Almaden Research Center
650 Harry Rd.
San Jose, CA 95120
blaum@almaden.ibm.com

**Jehoshua Bruck***
California Institute of Technology
MS 136-93
Pasadena, CA 91125
bruck@paradise.caltech.edu

**Patrick G. Farrell**
Electrical Engineering Laboratories
The University, Manchester
M13 9PL, England
farrell@man.ac.uk

*Abstract* — **We present 2-dimensional interleaving schemes, with repetition, for correcting 2-dimensional bursts (or clusters) of errors, where a cluster of errors is characterized by its area. Known interleaving schemes are based on arrays of integers with the property that every connected component of area $t$ consists of distinct integers. Namely, they are based on the use of 1-error-correcting codes. We extend this concept by allowing repetitions within the arrays, hence, providing a trade-off between the error-correcting capability of the codes and the degree of the interleaving schemes.**

Most 2-dimensional burst error-correcting codes that have been studied in the literature so far consider burst errors of a given shape, including rectangular and circular shapes [1, 4, 5, 6]. Here we present 2-dimensional interleaving schemes for correcting 2-dimensional bursts (or clusters) of errors, where a cluster of errors is characterized by its area. A recent application of correction of 2-dimensional clusters appeared in the context of holographic storage [7]. In [2] we presented interleaving schemes that are based on arrays of integers with the property that every connected component of area $t$ consists of distinct integers. These arrays are called $t$-interleaved arrays. Here, we extend the concept of $t$-interleaved arrays by allowing repetitions within the arrays. Namely, a $t$-interleaved array with repetition $r$ is an array of integers with the property that in every connected component of area $t$, every integer is repeated at most $r$ times.

**Definition 1** We say that a set of $t$ elements in a 2-dimensional array is a *cluster* of size $t$, if any two elements in the cluster belong in a path contained in the set.

**Definition 2** Let $t \geq 1$ and $r \geq 1$ be integers. Let $A(t,r)$ be a 2-dimensional array of integers, namely, the elements of the array are labeled by integers. We say that $A(t,r)$ is *$t$-interleaved* with repetition $r$ if every cluster of size $t$ in $A(t,r)$ consists of integers that repeat at most $r$ times. The *degree of interleaving* of the array is the number of distinct integers it contains.

For example, the following is an $A(5,2)$ array with degree of interleaving 5. In fact, this is the optimal degree:

$A(5,2) =$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 |
| 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 |
| 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 |
| 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 |

* Supported in part by the NSF Young Investigator Award CCR-9457811, by the Sloan Research Fellowship and by DARPA and BMDO through an agreement with NASA/OSAT.

| $t$ | Lower bound | Upper bound |
|---|---|---|
| 3 | 2 | 2 |
| 4 | 3 | 3 |
| 5 | 5 | 5 |
| 6 | 7 | 7 |
| 7 | 8 | 10 |
| 8 | 10 | 12 |
| 9 | 13 | 16 |
| 10 | 15 | 19 |
| 11 | 18 | 22 |
| 12 | 21 | 27 |

Table 1: Lower and upper bounds on the degree of interleaving of $A(t,2)$.

Notice that, if the integers represent different codes (like in the one-dimensional case), then $r$-error-correcting codes distributed in a $t$-interleaved array with repetition $r$ can correct any cluster of size up to $t$. In the above example, we need only five 2-error correcting codes to correct any cluster of size 5.

Our goal is to construct $A(t,r)$ arrays with minimal degree. In [2] we presented optimal degree constructions of $A(t,1)$ arrays for arbitrary $t$. Here we focus on the case $r = 2$ and present lower bounds and constructions. Table 1 summarizes, for specific numbers, the lower bounds and the upper bounds on the degree of interleaving of $A(t,2)$. For more details we refer the reader to [3].

## References

[1] K. A. S. Abdel-Ghaffar, R. J. McEliece and H. C. A. van Tilborg, *Two-Dimensional Burst Identification Codes and their Use in Burst Correction,* IEEE Trans. on Information Theory, IT-34, pp. 494–504, 1988.

[2] M. Blaum and J. Bruck, *Interleaving Schemes for Multidimensional Cluster Errors,* IBM Research Report, RJ 9984, Oct. 1995.

[3] M. Blaum, J. Bruck and P. G. Farrell, *Two-Dimensional Interleaving Schemes with Repetitions,* IBM Research Report, RJ 10047, Oct. 1996.
See also http://paradise.caltech.edu/ETR.html.

[4] M. Blaum and P. G. Farrell, *Array Codes for Cluster-Error Correction,* Electronic Letters, Vol. 30, No. 21, pp. 1752-3, Oct. 1994.

[5] E. M. Gabidulin and V. V. Zanin, *Codes Correcting Array Bursts,* The Workshop on Information Protection, Proc., pp. 24-25, Moscow, 1993.

[6] H. Imai, *A Theory of Two Dimensional Cyclic Codes,* Information and Control, Vol 34, pp. 1–21, 1977.

[7] S. A. Lis and P. D. Henshaw, *Ultra-Dense Optical Mass Storage,* Report prepared for USAF, AFSC, AD-A232 767, 1991.