Supporting Information

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SI Text

Relationship Between ([Fe/H]) and M. for Dwarf Spheroidal Galaxies (dSphs).

The data on [Fe/H] and M. for the eight dSphs shown in Fig. 2 plus another six from literature are given in Table S1 and shown in Fig. 3. All values of [Fe/H] are taken from ref. 1. The values of M. for Fornax, Leo I, Leo II, Sculptor, Draco, Sextans, and Ursa Minor (filled diamonds in Fig. 3) are taken from ref. 2, whereas those for Canes Venatici I, Hercules, Ursa Major I, Leo IV, Canes Venatici II, Ursa Major II, and Coma Berenices (open diamonds) are taken from ref. 3. These values of M. are inferred from observations* and have some considerable uncertainties. For example, Draco is common to the datasets of refs. 2 and 3 in view of this uncertainty, we treat log(M. / M⊙) as a function of the better determined ([Fe/H]) and obtain a least-square linear fit

\[
\log(M. / M⊙) = 10.08 \pm 0.59 - (2.41 \pm 0.29) \langle [\text{Fe/H}] \rangle. \quad \text{(S1)}
\]

The solid line in Fig. 3 corresponds to the above fit. We estimate that the 1σ error in the inferred values of log(M. / M⊙) is 0.47 dex. This error is indicated by the two dashed lines in Fig. 3.

The relationship in Eq. S1 is essentially the same as that found in ref. 2, which used a set of more massive dwarf galaxies (4.0 \times 10^5 \leq M. / M⊙ \leq 4.6 \times 10^5, including both irregular and spheroidal galaxies) and earlier and less precise data on [Fe/H]. That work also used a different parameter log Z = log ([Fe/H]) - log Z⊙, where Z⊙ = 0.019 is the total mass fraction of metals in the sun (4). In the representation used here, the result obtained by ref. 2 becomes

\[
\log(M. / M⊙) = 10.37 + 2.5 \langle [\text{Fe/H}] \rangle. \quad \text{(S2)}
\]

Our result in Eq. S1 for a different mass range (4.8 \times 10^5 \leq M. / M⊙ \leq 1.9 \times 10^7) using only dSphs is in excellent agreement with Eq. S2. The dot-dashed line in Fig. 3 has a slope of 2.5 and passes through the point defined by the average values of the data. It corresponds to

\[
\log(M. / M⊙) = 10.26 + 2.5 \langle [\text{Fe/H}] \rangle. \quad \text{(S3)}
\]

and is nearly the same as the solid line. The mass ranges covered by Eqs. S2 and S3 overlap and both include the seven dSphs shown as filled diamonds in Fig. 3. Together these two results show that log(M. / M⊙) increases with ([Fe/H]) with a slope of 2.5 over the wide range of 4.8 \times 10^5 \leq M. / M⊙ \leq 4.6 \times 10^5 for dwarf galaxies.

Parameters of the Model. We can estimate the parameters of our model based on properties of the dark matter halo hosting a dSph and characteristics of core-collapse supernovae (CCSNe) and type Ia supernovae (SNe Ia). The rate of CCSNe is

\[
R_{\text{CC}}(t) = \int_0^\infty \frac{d^2N}{dt dm} dm = 7.42 \times 10^{-3} \lambda_{\text{CC}}(t) \frac{M_h(t)}{M_h}. \quad \text{(S4)}
\]

For simplicity, we assume that the rate of SNe Ia is \( R_{\text{Ia}}(t) = k R_{\text{CC}}(t) \), where k is a constant. The average Fe yield of each CCSN is

\[
\langle Y_{\text{Fe}}^{\text{CC}} \rangle = \frac{\int_0^{100} Y_{\text{Fe}}^{\text{CC}}(m) m^{-2.35} dm}{\int_0^{100} m^{-2.35} dm} = 4.58 \times 10^{-2} M_h. \quad \text{(S5)}
\]

where \( Y_{\text{Fe}}^{\text{CC}}(m) \) is the Fe yield of a CCSN from a progenitor of mass m and we have taken \( Y_{\text{Fe}}^{\text{CC}}(m) = 3 \times 10^{-3} M_h \) for \( 8 \leq m \leq 11 \) (5) and \( 7 \times 10^{-2} M_h \) for \( 11 \leq m \leq 100 \) (e.g., ref. 6) in the second step. We take the average Fe yield of each SN Ia to be \( \langle Y_{\text{Fe}}^{\text{Ia}} \rangle = 0.7 M_h \) (e.g., ref. 7).

The net Fe production rate is

\[
P_{\text{Fe}}(t) = \lambda_{\text{Fe}} X_{\text{Fe}}^\odot M_h(t) = \langle Y_{\text{Fe}}^{\text{CC}} \rangle R_{\text{CC}}(t) + \langle Y_{\text{Fe}}^{\text{Ia}} \rangle R_{\text{Ia}}(t). \quad \text{(S6)}
\]

Using the above results and \( X_{\text{Fe}}^\odot = 1.3 \times 10^{-3} \) (4), we obtain

\[
\lambda_{\text{Fe}} = 0.261(1 + 15.3k) \lambda_{\odot}. \quad \text{(S7)}
\]

Because both CCSNe and SNe Ia produce Fe, the assumption that \( R_{\text{Ia}}(t)/R_{\text{CC}}(t) \) is a constant k does not critically affect the model presented here for Fe in consideration of the approximations made. However, other elements such as O, Mg, and Si are produced by CCSNe but not by SNe Ia. For these elements, the detailed distribution of the delay between the birth and death of the progenitors for SNe Ia must be taken into account.

The rate of SN-driven outflows is \( F_{\text{out}}(t) = \eta \lambda_{\text{Fe}} X_{\text{Fe}}^\odot M_h(t) \) and can be estimated by

\[
\frac{GM_h}{r_h} F_{\text{out}}(t) = \langle E_{\text{kin}}^{\text{CC}} \rangle R_{\text{CC}}(t) + \langle E_{\text{kin}}^{\text{Ia}} \rangle R_{\text{Ia}}(t). \quad \text{(S8)}
\]

where G is the gravitational constant, \( \langle E_{\text{kin}}^{\text{CC}} \rangle \) and \( \langle E_{\text{kin}}^{\text{Ia}} \rangle \) are the average kinetic energy imparted to the surrounding gas by each CCSN and SN Ia, respectively. Specifically,

\[
\langle E_{\text{kin}}^{\text{CC}} \rangle = \frac{\int_0^{100} E_{\text{kin}}^{\text{CC}}(m) m^{-2.35} dm}{\int_0^{100} m^{-2.35} dm} = 6.75 \times 10^{49} \text{erg}. \quad \text{(S9)}
\]

where \( E_{\text{kin}}^{\text{CC}}(m) \) is the kinetic energy imparted to the surrounding gas by a CCSN from a progenitor of mass m and we have taken \( E_{\text{kin}}^{\text{CC}}(m) = 10^{49} \) erg for \( 8 \leq m \leq 11 \) and \( 10^{50} \) erg for \( 11 \leq m \leq 100 \) in the second step. This estimate assumes that 10% of the explosion energy of each SN (e.g., refs. 6 and 8) is used to drive the bulk motion of the surrounding gas. We take \( \langle E_{\text{kin}}^{\text{Ia}} \rangle = 10^{50} \) erg. Using the above results, we obtain

\[
\eta = 58.5(1 + 1.48k) \left( \frac{r_h}{\text{kpc}} \right) \left( \frac{10^8 M_h}{M_h} \right). \quad \text{(S10)}
\]

According to the theory of hierarchical structure formation in cold dark matter cosmology, a halo with a total mass \( M_h \) collapsing at redshift \( z > 1 \) has a radius

\[
r_h = 1.54 \left( \frac{M_h}{10^8 M_h} \right)^{1/3} \left( \frac{10}{1+z} \right) \text{kpc}. \quad \text{(S11)}
\]

which can be obtained from equation 24 of ref. 9 for the current cosmological parameters (Ω_m = 0.27 and a Hubble constant of 70 km s^{-1} Mpc^{-1}). As was recognized in ref. 10, this purely
theoretical result is in excellent agreement with what is required (i.e., \( r_h \propto M_h^{1/3} \)) to account for the slope of 2.5 for the observed relationship between \( \log(M_h/M_\odot) \) and \( \langle {\text{Fe}}/{\text{H}} \rangle \) for dSphs. We note that there is no a priori basis for choosing an explicit time (i.e., \( z \)) for the formation of dSphs. It is most plausible that they formed early in cold dark matter cosmology. Taking \( 1+z=10 \) and substituting \( r_h \) from Eq. S11 into Eq. S10, we obtain

\[
\eta = 90.1(1 + 1.48k) \left( \frac{10^{8} M_\odot}{M_h} \right)^{2/3}.
\]  

[S12]

In our model, \( \langle {\text{Fe}}/{\text{H}} \rangle \approx \log(\lambda_{\text{Fe}}/(1+\eta)\lambda_{\odot}) \approx \log(0.261(1+15.3k)/\eta) \) (see Eq. S7) and \( M_g \approx 9.65 \times 10^{-2} M_\odot/\eta \) (see Eq. 14 in

the main text). Because \( \eta \) is determined by \( M_h \) and \( k \) through Eq. S12, \( \langle {\text{Fe}}/{\text{H}} \rangle \) and \( M_g \) are functions of \( M_h \) and \( k \). We estimate \( M_h \) and \( k \) from the data on \( \langle {\text{Fe}}/{\text{H}} \rangle \) and \( M_g \) for each of the dSphs shown in Fig. 3 and give the results in Table S1. As expected, these dSphs have very strong SN-driven outflows with \( \eta \approx 10-300 \). The estimated values of \( M_h \) range from \( 1.8 \times 10^{7} M_\odot \) to \( 2.8 \times 10^{9} M_\odot \) and are approximately 3-70 times larger than the total mass within the half-light radius \( M_{1/2} \) inferred from observations (11). The estimated values of \( k \) range from 0.02 to 0.48 and are typically approximately 0.1. In particular, the relationship between \( \log(M_g/M_\odot) \) and \( \langle {\text{Fe}}/{\text{H}} \rangle \) in Eq. S3 corresponds to \( k = 0.14 \).


Table S1. Characteristics of dSPhs

<table>
<thead>
<tr>
<th>Galaxies*</th>
<th>( \langle {\text{Fe}}/{\text{H}} \rangle )</th>
<th>( M_g )</th>
<th>( M_{1/2} )</th>
<th>( M_{0.2} )</th>
<th>( M_{0.6} )</th>
<th>( k )</th>
<th>( \eta )**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fornax</td>
<td>-0.99</td>
<td>1.9 \times 10^7</td>
<td>7.4 \times 10^7</td>
<td>2.8 \times 10^9</td>
<td>0.30</td>
<td>14</td>
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</tr>
<tr>
<td>Leo I</td>
<td>$-1.43$</td>
<td>4.6 \times 10^6</td>
<td>2.2 \times 10^7</td>
<td>1.1 \times 10^9</td>
<td>0.14</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Leo II</td>
<td>$-1.62$</td>
<td>7.3 \times 10^6</td>
<td>5.4 \times 10^8</td>
<td>0.15</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sculptor</td>
<td>$-1.68$</td>
<td>1.2 \times 10^6</td>
<td>2.3 \times 10^7</td>
<td>4.8 \times 10^8</td>
<td>0.13</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Draco</td>
<td>$-1.93$</td>
<td>9.1 \times 10^5</td>
<td>2.1 \times 10^7</td>
<td>3.8 \times 10^8</td>
<td>0.052</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Sextans</td>
<td>$-1.93$</td>
<td>8.5 \times 10^5</td>
<td>3.5 \times 10^7</td>
<td>3.6 \times 10^8</td>
<td>0.056</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>Ursa Minor</td>
<td>$-2.13$</td>
<td>5.6 \times 10^5</td>
<td>5.6 \times 10^7</td>
<td>2.8 \times 10^8</td>
<td>0.023</td>
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<td></td>
</tr>
<tr>
<td>Canes Venatici I</td>
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<td>3.0 \times 10^5</td>
<td>2.8 \times 10^7</td>
<td>2.0 \times 10^8</td>
<td>0.10</td>
<td>65</td>
<td></td>
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<tr>
<td>Hercules</td>
<td>$-2.41$</td>
<td>3.7 \times 10^5</td>
<td>7.5 \times 10^7</td>
<td>5.7 \times 10^8</td>
<td>0.078</td>
<td>147</td>
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<tr>
<td>Ursa Major I</td>
<td>$-2.18$</td>
<td>1.9 \times 10^4</td>
<td>1.3 \times 10^7</td>
<td>4.4 \times 10^7</td>
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<td>Leo IV</td>
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<td>Canes Venatici II</td>
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<td>Ursa Major II</td>
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<td>7.9 \times 10^7</td>
<td>2.0 \times 10^7</td>
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<td></td>
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<td>Coma Berenices</td>
<td>$-2.60$</td>
<td>4.8 \times 10^5</td>
<td>2.0 \times 10^7</td>
<td>1.8 \times 10^7</td>
<td>0.16</td>
<td>354</td>
<td></td>
</tr>
</tbody>
</table>

* Dwarf spheroidal galaxies in descending order of \( M_g \).
  \* Mean metallicities taken from ref. 1.
  \* Stellar masses taken from ref. 2 for the seven most massive dSphs and from ref. 3 for the rest.
  \* Total dynamic masses within half-light radii taken from ref. 11.
  \* Estimated total masses of dark matter halos.
  \* Estimated relative rates of SNe Ia to CCSNe.
  ** Estimated efficiencies of SN-driven outflows.