Limitation on holographic storage in photorefractive waveguides

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We show that photorefractive waveguide devices are subject to a limitation on their holographic storage capability owing to transversely nonuniform nonlinear losses to evanescent and radiation modes.

Recent developments in the growth of photorefractive (PR) thin films,[1] as well as in optical fibers,[2,3] offer practical uses of the PR waveguides for applications such as multimode-to-single-mode conversion,[4–7] low-power all-optical switching,[8] and high-capacity holographic storage.[9]

In this Letter we draw attention to the fact that there exists a basic limitation on the performance of PR waveguide devices, which is due to the irreversible coupling of power from the guided modes to evanescent and radiation modes. The resulting nonlinear loss reduces the storage capability and the resolution of the stored images in PR slab waveguides and fibers, especially in the proposed PR fiber bundles.[9] The limitation arises from transversely nonuniform losses, which cannot be compensated for, even by an ideal phase-conjugate reconstruction[10] of the stored hologram. This translates to reductions in the resolution of the stored images, in the angular selectivity, and in the multiple-image storage capability. We analyze the sources for this loss and its dynamics and demonstrate its influence in two applications: as a nonlinear mode coupler (a funneling device[4]) and as a holographic memory with an ideal phase-conjugate reconstruction.[10] As an example we demonstrate the loss of pictorial information in a crude image that consists of two spatial guided modes stored in a two-dimensional (slab) waveguide.

Consider a simplified model of a slab dielectric waveguide, in which the guided modes are represented by their propagation constant β_i, where i is the guided-mode serial number ($i = 1$ is the lowest-order mode), and form a discrete set. The propagation constant is $\beta_i = k \cos \theta_i$, where θ_i is the angle of propagation with respect to the waveguide axis z and $k = \omega n/c$ (n is the refractive index of the slab). The spectrum of guided modes is restricted to the range $|\theta_i| \leq |\theta_c|$, where θ_c is the critical angle for propagation in the waveguide. For angles larger than θ_c (and smaller than $\pi/2$), the light propagates in a continuum of radiation modes. Both guided and radiation modes are characterized by a real propagation constant β. The complete spectrum of waveguide modes also includes a continuum of evanescent modes.[11] These propagate in the waveguide at angles occupying the same range as the guided modes but that consists of the complementary plane-wave basis, i.e., at all angles that do not correspond to guided modes. The evanescent modes have complex propagation constants, since they do not satisfy the waveguide boundary condition for real β. They represent plane waves with equal-amplitude planes perpendicular to their phase fronts (equal-phase planes), and hence they are sometimes called inhomogeneous waves.[11] As a result of their complex β, the intensity of the evanescent modes decays exponentially with z, and they all but disappear within a distance of a few wavelengths. Their influence is significant only as spatial transients in the vicinity of inhomogeneities and discontinuities in the waveguide. We represent these modes by their complex propagation constant $\beta_q + i\alpha_q$, where $\beta_q = k \cos \theta_q$ and $q = k \sin \theta_q$, with θ_q their angle of propagation.

The holographic recording in a PR waveguide is made through the interaction between the pairs of guided modes. The role of a reference wave can be played by each of the guided modes or any subgroup of them. For simplicity, we restrict ourselves to transmission gratings only. This recording process results in a dynamic volume hologram, which is, in principle, identical to the one that caused the nonlinear mode-coupling effects.[4–8,10] Note that our analysis remains valid for reflection gratings, as demonstrated in the storage scheme of Ref. 9. Considering the interaction between pairs of guided modes only, and assuming that the nonlinear interaction (the dynamic volume hologram) is solely a power exchange between pairs of modes,[4–8] one can write a dynamic equation for the intensity I_i of each individual guided mode i:

$$\frac{\mathrm{d}I_i}{\mathrm{d}z} = \frac{1}{I_0} \sum_{j=1}^{N} I_i I_j \Gamma_{ij}, \tag{1}$$

where $I_0 = \Sigma_N I_i$ and is constant with z (we neglect here the normalization to $\widetilde{I}_i$ given in Refs. 1 and 2).

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The PR intensity coupling coefficient, $\Gamma_{ij} = \Gamma_{ij}(q_i, q_j)$, between each pair of plane waves q_i and q_j is calculated given the material parameters and the polarization of the waves,[4] and its values are real. In Eq. (1) we neglect all the uniform losses to the guided modes, including material absorption and light scattering owing to inhomogeneities. The essential reason for this is that in an ideal hologram reconstruction, transversely uniform losses can be tolerated, since they affect only the efficiency, whereas a nonuniform loss degrades the reconstruction quality.

Inclusion of the interaction with the continua of both the ordinary radiation and the evanescent modes [represented by their intensities $I_q(z)$] yields

$$\frac{\mathrm{d}I_i}{\mathrm{d}z} = \frac{I_i(z)}{I_0(z)}\left[\sum_{j=1}^{N} I_j(z)\Gamma_{ij}(q_i, q_j) - \int_{-k}^{k} I_q(z)\Gamma(q_i, q)\mathrm{d}q\right] \tag{2}$$

for each individual guided mode i. Note that the absolute intensity guided in the waveguide $[I_0(z)]$ may vary with z since light power is constantly escaping from the waveguide core owing to coupling to the radiative modes, which do not confine their power to the vicinity of the core. Equation (2) can be simplified by recalling that in any waveguide the intensity of both the radiation and the evanescent modes decays much faster than can be replenished by the PR gain. Nevertheless, we assume that there is always a small amount of guided energy in these modes, owing to scattering from the guided modes. Assuming homogeneous distribution of scattering centers, we may take the ratio of the intensity within these modes $I_q(z)$ to the total intensity $I_0(z)$ to be a constant for a given system: $\eta = I_q(z)/I_0(z)$. This constant can be calculated if the scatterers are assumed to be known[12,13] (this scattered noise was used to explain and analyze the Fanning effect[12] and the PR backscattering[13]). Under this assumption Eq. (2) simplifies to

$$\frac{\mathrm{d}I_i}{\mathrm{d}z} = I_i(z)\sum_{j=1}^{N}\left[\frac{I_j(z)}{I_0(z)}\Gamma_{ij}(q_i, q_j) - \eta G_i\right], \tag{3}$$

where $G_i = \int_{-k}^{k}\Gamma(q_i, q)\mathrm{d}q$.

In principle, a radiative mode can either "milk" energy from a guided mode or transfer power to it (the direction of the energy transfer is determined by the sign of Γ_i). However, while energy scattered out of a coherent guided mode is almost entirely lost, electromagnetic energy scattered randomly into that mode is not phase matched to the propagating mode (the phase of the light in the radiative mode is random), and thus its contribution is negligible. As a result, this nonlinear process is not reciprocal, i.e., energy that escaped from the guided modes cannot be recovered [for the same reason that we neglected the nonlinear interaction between pairs of nonguided modes in Eq. (2)]. The effective nonlinear loss is calculated by accounting for $\Gamma_i = \Gamma_i(q_i, q) > 0$ only, and G_i is defined as an integral over the loss region only,

$$G_i = \int_{\nabla q_i}\Gamma(q_i, q)\mathrm{d}q, \tag{4}$$

where ∇q_i is the modal region for which $\Gamma_i > 0$, for an individual guided mode i. The total nonlinear loss as a result of the coupling to the nonguided modes is given by the change in the abolute light intensity,

$$\frac{\mathrm{d}I_0(z)}{\mathrm{d}z} = \sum_{i=1}^{N}\frac{\mathrm{d}I_i(z)}{\mathrm{d}z} = -\eta\sum_{i=1}^{N} I_i(z)G_i. \tag{5}$$

Since in general G_i depends on the mode number i, the nonlinear loss is transversely nonuniform. Each individual guided mode experiences its own loss, which is not necessarily identical to the losses of the other guided modes, and this loss does not depend on z only but depends on the transverse coordinate as well.

Up to this point we have formulated the mode-coupling process in a PR waveguide. For the given boundary conditions at $z = 0$, one can refer to it as a recording process of the PR volume hologram in the waveguide. The ideal reconstruction process consists of a propagation in the opposite direction, starting with a mode distribution that is the output of the recording process, i.e., a phase-conjugate reconstruction.[10] When the modes propagate in the negative z direction, the mode-coupling dynamics in Eq. (3) is changed. The coupling between pairs of guided modes simply reverses the sign ($\Gamma_{ij} \to -\Gamma_{ij}$), but the nonlinear effective loss term, $-\eta G_i$, remains negative ($G_i > 0$), yet it may differ from the one for propagation in the positive z direction. In any case, the nonlinear loss remains nonuniform and does not turn into effective gain for the reason described above. In general, a nonuniform loss deteriorates the quality of the reconstructed hologram in any reconstruction scheme, including an ideal phase-conjugate reconstruction.[14]

As an example, we demonstrate the degradation of a simple stored image, as a result of the nonuniform nonlinear loss, for an ideal phase-conjugate reconstruction. Consider a $BaTiO_3$ PR slab waveguide of

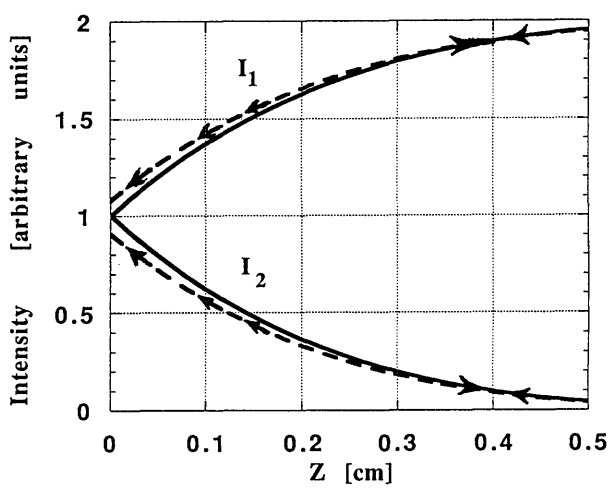

Fig. 1. Dynamics of the guided modes' intensity along the PR slab waveguide. The solid curves show the evolution of the funneling and antifunneling effects for the noise-free case, where the original intensities are perfectly reconstructed at each point along the propagation direction z; the dashed curves show the reconstruction for the case of a background noise ratio of $\eta = 10^{-4}$.

the material parameters of Ref. 4 and of a thickness that permits only two guided modes. We launch a crude image that consists of two guided modes of identical intensity from the facet $z = 0$ and observe its dynamics in a 5-cm-long waveguide, where the z axis coincides with the $+c$ crystalline axis. The two guided modes, I_1 and I_2, satisfy Eq. (3), with the boundary conditions $I_1(0) = I_2(0) = 1$. The reconstruction of this volume hologram is made by feeding back the mode outputs, by a phase-conjugate mirror with a reflectivity of unity, and examining the reconstruction at the front facet $z = 0$. The solid curves in Fig. 1 show the calculated mode-coupling dynamics I_1 and I_2 versus z (only the range $0 \leq z \leq 0.5$ cm is shown) for the case of $\eta = 0$ (no energy in the nonguided modes). The geometry of this proposed experiment is of a funneling device,[4,8] and all the energy is transfered from the higher-order mode I_2 to the lower-order one I_1. The reconstruction, as expected, restores the original inputs, and the evolution of the mode intensities retraces itself until full reconstruction $I_1^*(0) = I_2^*(0) = 1$ (where the asterisk represents the phase-conjugate nature of the reconstruction) is obtained. A real-life experiment is demonstrated by the dashed curves in Fig. 1, where we assumed that the background scattered noise results in a value of $\eta = 10^{-4}$ (see Refs. 12 and 13). This time the mode evolution did not retrace itself, and the reconstructed values were $I_1^*(0) = 1.0757$ and $I_2^*(0) = 0.9057$ (in the forward positive z propagation the dashed and the solid curves almost coincide and are practically indistinguishable). The overall power loss in this process is 0.0188, which corresponds to less than 1% of the input power, but the modes show intensity deviations of $+7.57\%$ and -9.43% from the optimal reconstructed values. The nonuniform loss affects the mode-coupling process between the guided modes and thus gives rise to a nonreciprocity in this process.

Examination of the above results for the two applications of interest (nonlinear mode couplers and holographic memories) finds the former to be less affected by coupling to evanescent modes. The mode conversion efficiency, from the higher mode (2) to the lower one (1), is almost unaffected by the nonlinear loss, but the distortion of the reconstructed image is large. The interaction with the radiative modes can be somewhat reduced by a reduction in the density of the scattering centers, which thus reduces the seeding of these modes. Nevertheless a finite amount of light will always exist in these evanescent modes, and it will always interact with, and lead to a milking of, the guided modes. The lowest limit on the density of the scattering centers in the PR crystals is the dopants' density (which also determines the storage capacity), but in practice the actual scatterer density is much higher. The nonlinear loss affects thin waveguides more than thick ones, and it disappears completely in bulk media. Therefore the proposed storage scheme,[9] which uses bundles of thin waveguides, is affected by it more strongly than is the multimode fiber scheme. In both cases, one should reconsider the figure of merit obtained by using PR waveguides instead of bulk crystals as storage media. It is not obvious that the benefits of uniformity in the material,[9] which are improved in waveguides, are worth the limitations on the resolution and the storage capacity that are introduced by the nonlinear loss.

In conclusion, we described a basic limitation on the performance of photorefractive waveguide devices and its influence on their storage capability and their operation as nonlinear mode couplers. We suggest that the storage capacity and the resolution of stored images in photorefractive waveguide devices are limited by effective nonlinear loss, induced by coupling to radiative and evanescent modes, rather than strictly by the material volume.

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