

The Hagedorn transition in noncommutative open string theory

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The Hagedorn transition in noncommutative open string theory (NCOS) is relatively simple because gravity decouples. For NCOS theories in no more than five space-time dimensions, the Hagedorn transition is second order, and the high temperature phase involves long, nearly straight fundamental strings separating from the D-brane on which the NCOS theory is defined. Above five spacetime dimensions interaction effects become important below the Hagedorn temperature. Although this complicates studies of the transition, we believe that the high temperature phase again involves long strings liberated from the bound state. © 2001 American Institute of Physics. [DOI: 10.1063/1.1372176]

I. INTRODUCTION

Just as noncommutative field theories (i.e., quantum field theories on noncommutative spaces) can be obtained as certain limits of D-branes with a background magnetic field,¹ noncommutative open string (NCOS) theories are defined as a special limit of Type II D-branes with a uniform electric field.^{2,3} The bosonic part of the world volume action has the standard form

$$S = \int_{\Sigma} d^2\sigma \left[\frac{1}{4\pi\alpha'} ((\partial_a X^0)^2 - (\partial_a X^1)^2) - \frac{1}{4\pi\alpha'} \sum_{i=2}^9 (\partial_a X^i)^2 \right] + E \oint_{\partial\Sigma} X^0 \frac{\partial}{\partial\sigma} X^1. \quad (1)$$

In the NCOS limit the electric field approaches its critical value, $E \rightarrow E_c = 1/2\pi\alpha'$, and $\alpha' \rightarrow 0$ in such a way that the effective tension of an open string stretched along the direction of the electric field remains finite:

$$\alpha'_{\text{eff}} = \alpha' \frac{E_c^2}{E_c^2 - E^2}. \quad (2)$$

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There is a similar rescaling of the interaction strength:⁴

$$G_o^2 = g_{\text{str}} \sqrt{\frac{E_c^2 - E^2}{E_c^2}}, \quad (3)$$

and the open string coupling G_o is held fixed in the NCOS limit. The inverse-tension parameter for the transverse directions, α'_i , is finite from the start and independent of E (it is convenient to set $\alpha'_i = \alpha'_{\text{eff}}$). A remarkable property of this limit is that, even though $g_{\text{str}} \rightarrow \infty$, the closed strings decouple from the open strings.^{2,3} Therefore, the NCOS is a nongravitational string theory.

This definition of NCOS theories leads naturally to a space–time where the space and time directions do not commute, i.e., $[X^0, X^i] = i\theta^{0i}$. Unlike the situation in noncommutative Yang–Mills theories, here the noncommutativity scale, $|\theta|$, is intrinsically tied to the string scale. This implies that in order to make sense of the notion of a noncommuting space/time manifold, we would have to first give precise meaning to the notion of Einsteinian spacetimes down at the string scale.

The relation (3) implies that $g_{\text{str}} \rightarrow \infty$ in the NCOS limit. Therefore, *S*-duality may be used to map NCOS to D-brane systems at weak string coupling.^{5,6} A particularly simple example of such duality is 1+1-dimensional NCOS, which is found to be dual to maximally supersymmetric $U(N)$ gauge theory with one unit of electric flux. The open string coupling is $G_o^2 = 1/N$; it becomes weak in the large N limit. Therefore, the 1+1-dimensional NCOS provides a new example of duality between large N gauge theory and strings. The fact that we find open strings rather than closed is related to the presence of the electric flux tube which binds the N D-strings. Massive open strings are dual to the excitations of this theory where locally $SU(N)$ is broken to $SU(N-1) \times U(1)$. The massless open strings are dual to the $U(1)$ part of the spectrum (the overall vibrations of the bound state), and the duality with the gauge theory predicts that the massless states decouple. In Ref. 6 this prediction was confirmed by explicit NCOS calculations. A further check on the duality performed in Ref. 6 involves the high-energy behavior of the massive amplitudes: it is found to exhibit the same power-law fall-off as expected from the gauge theory.

Another classic way of subjecting strings to extreme conditions is to heat them up to a high temperature. For conventional superstring theory this was extensively studied in the late 80's⁷⁻⁹ and afterwards (see for instance Refs. 10, 11, 12). A complicating factor in these papers is that it is difficult to study the thermodynamics of gravitating systems. Nevertheless, a coherent picture has emerged suggesting a first-order phase transition happening well below the Hagedorn temperature.⁹

In this paper we study the thermodynamics of NCOS in various dimensions. Just like any other superstring theory, NCOS theory exhibits a Hagedorn density of states:

$$\rho(m) \sim m^{-9/2} e^{m/T_H}, \quad (4)$$

with the scale for the Hagedorn temperature set by α'_{eff} :

$$T_H = \frac{1}{\sqrt{8\pi^2 \alpha'_{\text{eff}}}}. \quad (5)$$

In particular, the partition function of NCOS theory appears to diverge above the temperature $T = T_H$, where the Hagedorn transition is expected to take place. Our goal is to understand the physics of this transition and describe the thermodynamics of NCOS theory at $T > T_H$. Since the NCOS theories are decoupled from gravity, we will not face the usual difficulties associated with gravitational thermodynamics. Furthermore, at least in 1+1 dimensions the dual gauge theory provides an important guide to what happens at the transition. Here we find that the transition is

to a phase where some finite fraction of the strings are freed from the bound state, i.e., where the theory enters the Higgs branch $SU(N) \rightarrow SU(N-K) \times U(1)^K$. A calculation of the free energy below and above the transition shows that it is second order.

Guided by the intuition from the 1 + 1-dimensional case we proceed to $p + 1$ dimensions. For $p > 1$, S -duality works differently, but on the NCOS side we may still think of some density of F -strings bound to a Dp -brane. We will show that for all $p < 5$ the Hagedorn transition is again second order and is associated with the liberation of strings from the bound state. For $p \geq 5$ the entropy of noninteracting open strings (and also the string length) diverges as T approaches T_H from below.¹³ This suggests that string interaction effects become important already below T_H . Nevertheless, we will argue that the high-temperature phase again contains a finite fraction of long strings liberated from the bound state. In all these cases the theory slightly above the transition appears to be effectively 1 + 1-dimensional, with a preferred direction chosen by the electric field.

In previous work³ it was suggested that there is a change in the behavior of zero-temperature NCOS at $p = 7$, where non-planar amplitudes begin to diverge at $k^2 = 0$ (k is the momentum flowing in the closed string channel). We calculate a cross-section for graviton production and confirm that NCOS theories do not decouple from gravity for $p \geq 7$. Our work further shows that, at finite temperature, there is new physics appearing in the NCOS theory at a lower dimension, $p = 5$: interaction effects become important already below T_H . A special role of $p = 5$ in open string thermodynamics was noted earlier in Ref. 13.

Other authors have recently studied phases of NCOS theories and Hagedorn behavior of string theories decoupled from gravity.¹⁴⁻¹⁷ These works focused primarily on results derivable from supergravity. The current work takes the rather different approach of examining the free energy of bound states in a field theory approximation. The relevant temperatures for our analysis are so low that the near-extremal supergravity solutions are highly curved on the string scale and hence unreliable.

There is also an extensive literature on Hagedorn behavior in asymptotically free gauge theories. For important early contributions to the subject, see Refs. 18, 19, 20. In the current work we focus on perturbative string techniques rather than field theory. However, some information about strongly coupled gauge theories may be extracted from our results, particularly in the 1 + 1-dimensional case.

II. NCOS THERMODYNAMICS FOR $T < T_H$

In order to obtain a reliable picture of the thermodynamics of NCOS theory for $T < T_H$ directly from the free string spectrum, two conditions must pertain. First, the open strings should interact weakly with one another. Second, the cubic coupling $\langle \phi \phi \sigma \rangle$ between an incipient thermal tachyon ϕ and the radius σ of the Euclidean time direction, which played a crucial role in the analysis of Ref. 9, is absent. This is because σ represents a closed string (gravitational mode), which decouples according to the arguments of Refs. 2, 3. This is the essential difference between the Hagedorn transition for NCOS theory and for critical string theory.⁹ Whereas essentially gravitational effects drive the Hagedorn transition first order in critical string theory, we will see that in NCOS theory the transition remains second order.

The free string analysis proceeds in a similar way regardless of the spatial dimension p in which the open strings live. The calculation of free energy of noninteracting open strings on a Dp -brane, which is not affected by the noncommutativity, was carried out in Ref. 13. We will largely rederive their results and adapt them for our purposes. The principal result is that the free energy is analytic in T for $T < T_H$, and that the leading nonanalytic behavior in the expansion of F around $T = T_H$ is

$$F \sim \begin{cases} (\text{analytic in } t) + t^{(7-p)/2} + \dots, & \text{for } p \text{ even,} \\ (\text{analytic in } t) + t^{(7-p)/2} \log t + \dots, & \text{for } p \text{ odd,} \end{cases} \quad (6)$$

where $t = (T_H - T)/T_H$. There are two equivalent means of obtaining this result. First, one may

directly evaluate the annulus diagram in the Matsubara formalism where the Euclidean time direction is compact with circumference $\beta = T^{-1}$. The one loop free energy for a Dp -brane with an electric field of strength E turned on is ²¹

$$\begin{aligned} Z_{\text{single string}} &= -c_1 \int_0^\infty \frac{dt}{(p+3)/2} \vartheta_2 \left(0 \left| \frac{i\beta^2}{2\pi^2 \alpha'_{\text{eff}} t} \right. \right) \left[\frac{\vartheta_2(0|it)}{\vartheta_1'(0|it)} \right]^4 \\ &= -\frac{c_1}{(2\pi)^4} \int_0^\infty \frac{d\tau}{\tau^{9-p/2}} \vartheta_2 \left(0 \left| \frac{i\beta^2 \tau}{2\pi^2 \alpha'_{\text{eff}}} \right. \right) \left[\frac{\vartheta_2(0|i\tau)}{\vartheta_1'(0|i\tau)} \right]^4. \end{aligned} \tag{7}$$

In the first line we have used t as the modular parameter of the cylinder. In the second line we substitute $\tau = 1/t$, which is the usual closed string modular parameter. The expressions in (7) are exact even away from the NCOS limit, provided we use the definition (2) and neglect coupling to closed strings. They are identical to the partition function of ordinary open superstrings on a Dp -brane, only with α' replaced by α'_{eff} . The constant c_1 is given as $V\beta\pi^4/2(2\pi)^5(2\pi\alpha'_{\text{eff}})^5$. The reduced Hagedorn temperature for the partition function was noted in Ref. 22.

The nonanalytic behavior arises from a divergence in the modular integral at large τ (the long cylinder limit). Near T_H , we can use the large τ asymptotics of the ϑ functions to obtain

$$F_{\text{NCOS}}(T \approx T_H) \sim - \int \frac{d\tau}{\tau^{(g-p)/2}} e^{(1/T_H^2 - 1/T^2)\tau}, \tag{8}$$

from which the claimed analyticity for $T < T_H$ and the leading nonanalyticity quoted in (6) are evident.

An equivalent, ‘‘elementary’’ approach to obtain the same result is to plug (4) into the standard formula for a partition function:

$$\begin{aligned} Z_{\text{single string}} &= \sum_{\text{states}} e^{-E/T} \\ &= \sum_{i \in \mathcal{H}_o^\perp} \int \frac{d^p k}{(2\pi)^p} e^{-\sqrt{k^2 + m_i^2}/T} \\ &\sim \int_0^\infty dm \rho(m) (mT)^{p/2} e^{-m/T} \sim \int_0^\infty dm m^{(p-9)/2} \exp \left[m \left(\frac{1}{T_H} - \frac{1}{T} \right) \right], \end{aligned} \tag{9}$$

where in the third step we have made an approximation to the momentum integration which becomes exact in the limit of large masses m_i .¹³ Evaluating the last integral leads again to (6).

Note that the free energy is finite at $T = T_H$ for $p < 7$ and diverges logarithmically for $p = 7$. The entropy, $S = -\partial F / \partial T$, remains finite only for $p < 5$, and diverges logarithmically for $p = 5$. For a single long string, the entropy is proportional to the length of the string. Hence when $t = (T_H - T) / T_H$ is small, the total entropy is proportional to the rms length of the excited open strings, l_{NCOS} , times the average number of these strings per unit volume, ρ_{NCOS} .²³ As $T \rightarrow T_H$ from below, we have the scalings

$$\rho_{\text{NCOS}} l_{\text{NCOS}} \sim \begin{cases} (\text{finite}), & \text{for } p < 5, \\ -\log t, & \text{for } p = 5, \\ 1/\sqrt{t}, & \text{for } p = 6, \end{cases} \tag{10}$$

and so on.

The quantity $\rho_{\text{NCOS}} l_{\text{NCOS}}$ is the average density of string at any given point. As long as this quantity remains finite, the effects of interactions may be suppressed by taking G_o sufficiently small. Thus for $p < 5$ one can ensure that string interactions are never significant, but for $p \geq 5$

they eventually will be. A figure of merit to measure the strength of string interactions is $\eta = G_o \rho_{\text{NCOS}} l_{\text{NCOS}}$. We work in units where $\alpha'_{\text{eff}} = 1$ to make η dimensionless. The free energy for $T < T_H$, neglecting interactions, is order G_o^0 . Interactions make a contribution of order η^2 to the free energy. Thus interactions become important when $\eta \gtrsim 1$, which is to say $\rho_{\text{NCOS}} l_{\text{NCOS}} \gtrsim 1/G_o$. One can now use (10) to make a rough estimate of the temperature at which string interactions matter. For $p = 5$ this temperature is $t \sim e^{-\text{const}/G_o}$, while for $p = 6$ it is $t \sim G_o^2$.

In the next sections, we will propose that the physics above T_H involves the gradual emission of long strings. We will assume that the free string picture is valid up to $T = T_H$: thus the discussion seems to be limited to $p < 5$. Note however that for $p = 5, 6$ the entropy of the open string gas at the temperature where interactions become important is of order $1/G_o$. For weak string coupling, this is still much smaller than the entropy in the liberated string phase which, as we show in Sec. IV A, is of order $1/G_o^2$. So we speculate that long string liberation starts taking place near T_H for $p = 5, 6$ as well.

The calculations that we present for $p < 5$ are clean because we can work in a limit where free string theory applies. The string liberation transition may still occur away from zero coupling, although it is possible that the transition becomes first order. The additional complication for $p = 5, 6$ is that there is no limit in which free string theory applies uniformly.

III. TWO-DIMENSIONAL NCOS THEORY AT $T > T_H$

In this section we focus on the specific example of the two-dimensional NCOS theory, $p = 1$. In this case one can use Type IIB S -duality to describe a D1-brane with a near-critical electric field as a $(1, N)$ bound state^{24,25} where the number, N , of D1-branes is related to the open string coupling constant, $G_o^2 = 1/N$. At low energies this system behaves as $SU(N)$ two-dimensional super-Yang–Mills theory with one unit of electric flux and coupling constant,

$$g_{\text{YM}}^2 = \frac{N^2}{\alpha'_{\text{eff}}}. \tag{11}$$

In this dual picture noncommutative open strings can be identified with excitations corresponding to the Higgsing $SU(N) \rightarrow SU(N-1) \times U(1)$. Indeed, to create an island (of size L) of the Higgs phase costs an energy^{5,6} of

$$E = L \left(\frac{g_{\text{YM}}^2}{4\pi(N-1)} - \frac{g_{\text{YM}}^2}{4\pi N} \right) \approx \frac{L g_{\text{YM}}^2}{4\pi N^2} = \frac{L}{4\pi \alpha'_{\text{eff}}}. \tag{12}$$

In the last equality we used the relation (11) between the Yang–Mills coupling constant and the tension of open strings. In terms of the $(1, N)$ bound state, (12) represents the energy to have a D-string split off from the bound state and run parallel to it for a distance L before rejoining.

At finite temperature, there is a gain in entropy when a string splits off from the bound state, due to small fluctuations of the string. Since a long string in light-cone gauge is described by a free supermultiplet, this entropy is $S = 4\pi L T$ [in the dual gauge theory it comes from the $U(1)$ part of the Higgsed gauge group $SU(N-1) \times U(1)$]. The corresponding free energy of these light modes is $F = -2\pi L T^2$. Therefore, the total free energy of a string liberated from the bound state,

$$F_{\text{liberated string}} = L \left(\frac{1}{4\pi \alpha'_{\text{eff}}} - 2\pi T^2 \right), \tag{13}$$

vanishes precisely at the Hagedorn temperature:

$$T_H = \frac{1}{\sqrt{8\pi^2 \alpha'_{\text{eff}}}} = \frac{1}{\sqrt{8\pi}} \frac{g_{\text{YM}}}{N}. \tag{14}$$

We would like to interpret the Hagedorn transition as the liberation of fundamental strings parallel to the electric field from the $(N,1)$ bound state. This interpretation is satisfying in that the Hagedorn transition is generally associated with the temperature at which it is favorable to create long closed strings (see for instance Refs. 10, 11). If we compactify in the direction of the electric field, then the liberated strings are precisely those long closed strings. What is special about the NCOS limit is that in the near-critical electric field, the closed strings are allowed to wind only in one direction. Furthermore, since their tension away from the bound state is α' , they are nearly straight: the massless $U(1)$ degrees of freedom represent only slight fluctuations.

A crucial aspect of the analysis is that, once one string has been liberated, the Hagedorn temperature of the NCOS theory on the $(N-1,1)$ bound state is slightly higher: after freeing one string, we have

$$\alpha'_{\text{eff}}{}^{\text{new}} = \frac{(N-1)^2}{g_{\text{YM}}^2}, \quad T_{\text{H}}^{\text{new}} = \frac{g_{\text{YM}}}{\sqrt{8\pi}} \frac{1}{N-1}. \tag{15}$$

In order to free another fundamental string, we must increase T to $T_{\text{H}}^{\text{new}}$. The analysis in (12) and (13) carries over without change to this case, and the Hagedorn temperature of the bound state increases again. Thus we have good control over the physics above the original T_{H} : the liberated fundamental strings are only slightly fluctuating, and the NCOS strings attached to the bound state remain at or below their Hagedorn transition.

It is possible to summarize the analysis in a way that will generalize easily to other cases. Suppose k out of the N fundamental strings have been liberated, $k \gg 1$. The free energy per unit length of the total system, consisting of the $(N-k,1)$ bound state plus the k liberated strings, is

$$\begin{aligned} \frac{F_k}{L} &= \frac{1}{2\pi\alpha'} \sqrt{(N-k)^2 + \frac{1}{g_{\text{str}}^2}} + \frac{k}{2\pi\alpha'} - 2\pi k T^2 + O(1) \\ &\approx \frac{N-k}{2\pi\alpha'} \left(1 + \frac{1}{2g_{\text{str}}^2(N-k)^2} \right) + \frac{k}{2\pi\alpha'} - 2\pi k T^2 \\ &= \frac{N}{2\pi\alpha'} - 2\pi N T^2 + \frac{1}{4\pi\alpha' g_{\text{str}}^2(N-k)} + 2\pi(N-k)T^2 \\ &\geq \frac{N}{2\pi\alpha'} - 2\pi N T^2 + T \sqrt{\frac{2}{\alpha' g_{\text{str}}^2}}. \end{aligned} \tag{16}$$

In the first line of (16), we have summed up the total tension of the $(N-k,1)$ bound state, the total tension of the k liberated strings, the free energy of the fluctuations of those k strings, and the $O(1)$ free energy coming from fluctuating open strings attached to the bound state.²⁶ The symbol $O(1)$ means, more precisely, that this contribution to the free energy is a finite quantity of order 1 times LT^2 . In the second line of (16) we have expanded the square root for $g_{\text{str}}(N-k) \gg 1$, and in the last line we have used the arithmetic–geometric mean inequality. Equality holds in the last line iff

$$N-k = \frac{1}{\sqrt{8\pi^2\alpha' g_{\text{str}} T}}. \tag{17}$$

Transforming to rescaled NCOS variables, we find that the fraction of liberated strings is

$$\nu = \frac{k}{N} = \frac{T - T_{\text{H}}}{T}, \tag{18}$$

where T_H is the original Hagedorn temperature defined in (14).

From (16) we immediately read off the total free energy for $T > T_H$:

$$\frac{F}{L} = \inf_k \frac{F_k}{L} = -2\pi N(T_H)^2 + O(1). \tag{19}$$

Another way to arrive at this formula is to note that the entropy of the k liberated strings is

$$S = -\frac{dF}{dT} = 4\pi kLT = 4\pi NL(T - T_H). \tag{20}$$

Integrating this equation with the boundary condition that F/L is of order 1 at $T = T_H$ reproduces the result (19).

Actually, since k is a discrete variable, $F/L = -2\pi N(T - T_H)^2$ and $\nu = (T - T_H)/T$ only represent an approximation to a series of discrete transitions, from F_0 to F_1 to F_2 and so on. However, since we are operating at large N , the discrete transitions are very closely spaced, and can effectively be regarded as a single continuous transition. At some level, the approach we have taken is only meaningful in the large N limit: we have examined various competing minima of the free energy, corresponding to different numbers of liberated fundamental strings, at a completely classical level, ignoring the fact that in one spatial dimension strong infrared fluctuations smooth out any nonanalyticity in the free energy. What saves the day is large N : when a finite fraction of fundamental strings have been liberated, their fluctuations “average out” to an extent such that ν is a good order parameter for the transition. One should not take too seriously the literal picture of a single string peeling off the bound state at $T = T_H$, followed shortly thereafter by another, and then another; rather, the free energy starts as an $O(1)$ quantity for $T < T_H$ and rises to $O(N)$ through a transition in which fundamental strings are collectively liberated. There is *not*, after all, a series of closely spaced first-order transitions—this would be in violation of the general analyticity properties of the free energy in one spatial dimension—instead, the maximally smoothed free energy has the form (19), which indicates a discontinuity $\Delta C = 2\pi N$ at $T = T_H$. This is essentially the classic picture of a second-order phase transition, only with integer critical exponents that just barely avoid the typical singularity in the specific heat.

Note that, since the equilibrium condition reads as

$$\frac{g_{\text{YM}}^2}{4\pi(N-k)^2} - 2\pi T^2 = 0, \tag{21}$$

the open strings on the $(N-k, 1)$ bound state are always at their effective Hagedorn temperature (which depends on k provided that g_{YM} is held fixed). Therefore, their contribution to F/L is aT^2 , where a is a constant of order 1 which may be found by evaluating (7) directly at the Hagedorn temperature. It is quite possible that additional $O(1)$ contributions to F arise when one considers interactions of the liberated long strings. Also, the interactions could change the critical exponents by terms of order $G_o^2 = 1/N$. In principle, the effects of interactions can be studied starting from the maximally supersymmetric Yang–Mills description of the bound state. The liberated strings admit a matrix string description,^{27–29} while the bound state represents a confining non-Abelian sector of the theory. As far as we can tell, the total problem is quite formidable, but some progress might be made via a lattice or DLCQ approach.

It is clear that as we increase the temperature, one unit of electric flux in the dual super-Yang–Mills theory becomes unimportant. Already when the temperature is a finite multiple of $T_H \sim g_{\text{YM}}/N$ (say $T = 2T_H$), the free energy is dominated by the matrix string phase (recall that we are mainly interested in the large N limit). When $T \sim g_{\text{YM}}/\sqrt{N}$, the proper description of the system is no longer matrix string theory plus a D1-f1 bound state, but rather a single near-extremal black string solution in type IIB supergravity.^{30,31} The considerations of Refs. 30, 31 were applied only to multiple, identical, (nearly) coincident branes, but their conclusions should carry over to

the current circumstance, because at $T = g_{\text{YM}}/\sqrt{N}$, nearly all the D1-branes are in the matrix string phase: $\nu = 1 - O(N^{-1/2})$. The supergravity regime, then, is described by^{30,31}

$$F \sim LN^{3/2} \frac{T^3}{g_{\text{YM}}}, \quad \text{for } \frac{g_{\text{YM}}}{\sqrt{N}} < T < g_{\text{YM}}\sqrt{N}. \quad (22)$$

Finally, as we reach the 't Hooft scale $T \approx g_{\text{YM}}\sqrt{N}$, we end up with a gas of free photons, N^2 in number. This crossover is in the general class of correspondence points studied by Horowitz and Polchinski.³² At very high temperature the free energy looks like this:

$$F \sim LN^2 T^2, \quad \text{for } g_{\text{YM}}\sqrt{N} < T. \quad (23)$$

Historically, the Hagedorn transition was originally expected to be essentially a deconfinement transition. In the NCOS context, we see that there are actually two other phases, or regimes, in between the Hagedorn transition and the free gluon phase. In a superficial matching analysis, the transitions between the matrix string regime, the supergravity regime, and the free gluon regime appear to be first order. It could easily be, however, that there is only a second-order transition, or no sharp transition at all, between these phases. As yet, we know of no method of analysis powerful enough to distinguish among the possibilities. For the transition into the supergravity regime from below, one may hope that the perturbation of the matrix string CFT by the DVV twist operator²⁷ provides some hint of the formation of a horizon.

In summary, we find four different phases of 1+1-dimensional NCOS theory. They are illustrated in Fig. 1. In more detail, we have the following.

- (i) In the NCOS phase, F/LT^2 is an analytic function of order 1 (that is, no factors of N). String interactions are suppressed by $G_o^2 = 1/N$. Without an understanding of the gauge theory and the possibility of going to a Higgsed phase $SU(N-k) \times U(1)^k$, this is the only part of the phase diagram we would be able to understand.
- (ii) Above $T = T_H$, we gradually liberate more and more fundamental strings from the bound state, so that very soon the system becomes dominated by the matrix string phase. The open strings on the bound state stay at their effective Hagedorn temperature: this temperature adjusts as more strings are liberated. The continuous transition so described is the essential new physics of this paper.
- (iii) At $T \approx g_{\text{YM}}/\sqrt{N}$ significant departures from conformal invariance and nontrivial interactions drive us into the black string regime, where the thermodynamics is read off from a regular horizon.
- (iv) Above $T \approx g_{\text{YM}}\sqrt{N}$, the N^2 non-Abelian gluons (light D1-branes stretched between fundamental strings) are deconfined.

It is straightforward to extend our discussion to (N, M) bound states corresponding to $SU(N)$ theory with arbitrary number, M , of flux units. In that case, open string coupling constant is given by

$$G_o^2 = \frac{M}{N},$$

and effective open string tension reads as

$$\frac{1}{\alpha'_{\text{eff}}} = g_{\text{YM}}^2 \frac{M^2}{N^2}.$$

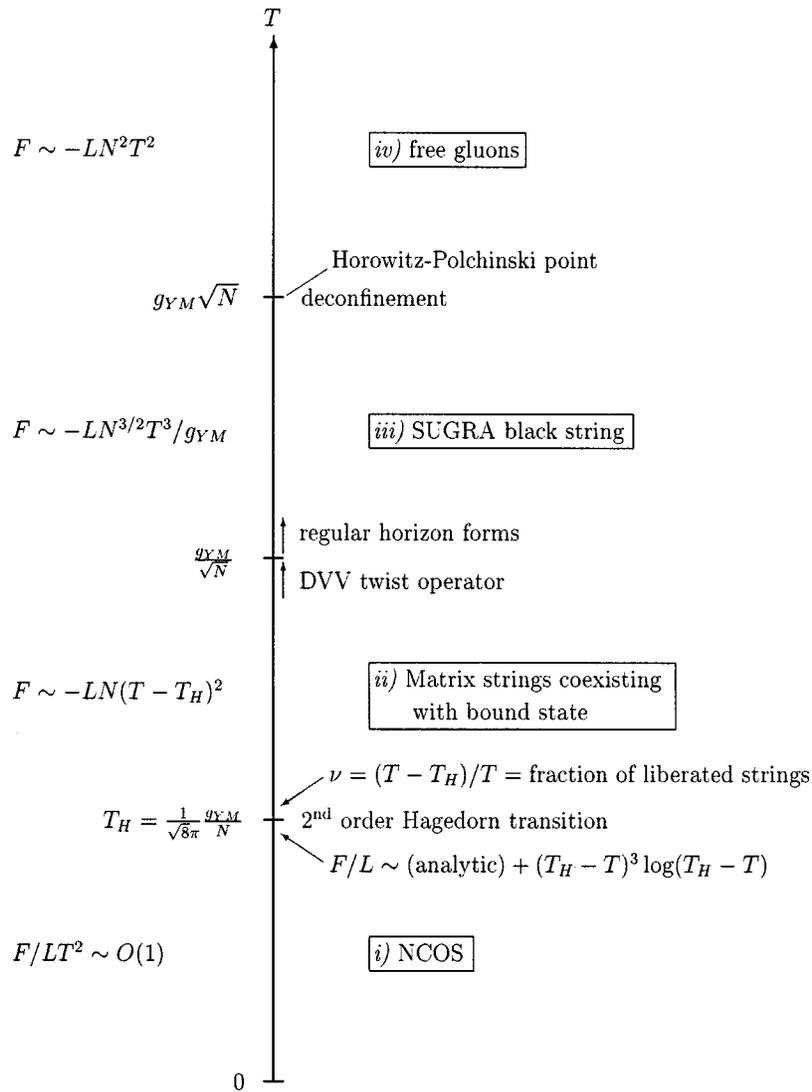


FIG. 1. The four phases of 1 + 1-dimensional NCOS theory.

The phase diagram of this system is similar to that of the $(N,1)$ bound state: in particular, the same four phases appear. The only difference is that the phase transitions between phases (i)–(ii)–(iii) occur at different temperatures, greater by a factor of M . For instance, the Hagedorn temperature of such a theory is given by

$$T_H = \frac{g_{YM}M}{\sqrt{8\pi^2}N}.$$

IV. HIGHER DIMENSIONAL EXAMPLES

Let us now elaborate on extensions of the ideas of the previous section to systems in which the strings are allowed to move in more than one spatial dimension (Sec. IV A), or where not strings but Dp -branes become light (Sec. IV B).

A. Thermodynamics of NCOS theories in higher dimensions

The Hagedorn transition in higher-dimensional NCOS theories (up to 4 + 1 dimensions) may be understood in a manner similar to the situation in 1 + 1 dimensions. The claim is that finitely above T_H , a finite fraction of long strings are liberated from the bound state. This process is gradual as in the previous case, and the fraction of long strings that decouple may be computed in a similar fashion. To see this we need the formula for the number of fundamental strings per unit transverse volume bound to a Dp -brane with electric field E :

$$\frac{N}{V_t} \sim (\alpha'_t)^{(1-p)/2} \frac{E}{g_{\text{str}} \sqrt{E_c^2 - E^2}}. \tag{24}$$

One way to get this formula is to consider the Dp -brane to be compactified on a circle of radius L in the direction of the electric field. Then the momentum conjugate to the gauge field is quantized: $P_1 = NL$. Equating this to the momentum calculated from the Born–Infeld action, as in Ref. 4, gives (24).

Note that (24) implies

$$G_o^2 \sim \frac{V_t (\alpha'_t)^{(1-p)/2}}{N}.$$

To fix the precise factor in this expression, consider the BPS formula for the mass of the bound state of N fundamental strings and a Dp -brane wrapped over a transverse torus of volume V_t :

$$\frac{L}{2\pi\alpha'} \sqrt{N^2 + \frac{V_t^2}{g_{\text{str}}^2 (2\pi)^{2p-2} (\alpha'_t)^{p-1}}} = \frac{L}{2\pi\alpha'} N \left(1 + \frac{V_t^2}{2N^2 g_{\text{str}}^2 (2\pi)^{2p-2} (\alpha'_t)^{p-1}} + \dots \right). \tag{25}$$

Calculating the energy required to free one fundamental string, we get

$$\frac{LV_t^2}{4\pi\alpha' g_{\text{str}}^2 (2\pi)^{2p-2} (\alpha'_t)^{p-1}} \left(\frac{1}{N-1} - \frac{1}{N} \right) \rightarrow \frac{LV_t^2}{4\pi\alpha'_{\text{eff}} G_o^4 N^2 (2\pi)^{2p-2} (\alpha'_t)^{p-1}}, \tag{26}$$

where we have used

$$\alpha' g_{\text{str}}^2 = \alpha'_{\text{eff}} G_o^4. \tag{27}$$

(26) should be equated to the energy of a closed string wound around the direction of the electric field, which is $L/(4\pi\alpha'_{\text{eff}})$.⁶ Thus, we find

$$G_o^2 = \frac{V_t (\alpha'_t)^{(1-p)/2}}{(2\pi)^{p-1} N}. \tag{28}$$

This formula shows that for $p > 1$, G_o^{-2} is not quantized, while for $p = 1$ it is.

Suppose we start with a Dp -brane with a near-critical electric field E , which loses a fraction of its long strings above T_H , such that the resulting system is a Dp -brane with a near-critical electric field E' and a bunch of free long strings. Assuming that the resulting brane configuration is right at its effective Hagedorn temperature as before, we find the ratio

$$\frac{T}{T_H} = \frac{\sqrt{E_c^2 - E'^2}}{\sqrt{E_c^2 - E^2}}.$$

Using (24) and the fact that both E and E' are near-critical, we find the relation between temperature and the fraction of strings remaining in the bound state:

$$\frac{N'}{N} = \frac{T_H}{T}. \tag{29}$$

This universal result is in accord with what we found in two-dimensional NCOS theory from its gauge theory dual; cf. (18).

Note that the transverse inverse-tension parameter, α'_t , remains fixed for $T > T_H$ because it does not depend on E . Thus, we may simply set $\alpha'_t = \alpha'_{\text{eff}}$. However, the effective parameter governing the 0 and 1 directions starts decreasing as in the 1 + 1-dimensional case. From (27) and (28) we find that

$$\alpha'_{\text{eff}}{}^{\text{new}} = \alpha'_{\text{eff}} \left(\frac{N'}{N} \right)^2.$$

As in 1 + 1 dimensions, the condition for equilibrium of long strings at temperature T is

$$\frac{1}{4\pi\alpha'_{\text{eff}}{}^{\text{new}}} - 2\pi T^2 = 0,$$

from which the relation (29) follows.

One may be concerned that for $T > T_H$ there are two different effective inverse-tension parameters: $\alpha'_{\text{eff}}{}^{\text{new}}$ for the 01 directions and α'_{eff} for the transverse directions. Which one sets the effective Hagedorn temperature? The answer is that it is $\alpha'_{\text{eff}}{}^{\text{new}}$, so that T is the effective Hagedorn temperature for $T > T_H$. The dispersion relation for open strings is indeed asymmetric:

$$(k_0^2 - k_1^2) - \frac{\alpha'_{\text{eff}}}{\alpha'_{\text{eff}}{}^{\text{new}}} \sum_{i=2}^p k_i^2 = \frac{\mathcal{N}}{\alpha'_{\text{eff}}{}^{\text{new}}},$$

where \mathcal{N} is the excitation level. We see that $\alpha'_{\text{eff}}{}^{\text{new}}$ determines the mass spectrum. Then following, for instance, the approach in (9) we find that the effective Hagedorn temperature is $(8\pi^2\alpha'_{\text{eff}}{}^{\text{new}})^{-1/2} = T$. Therefore, the free energy of the gas of open strings on the bound state is a finite (for $p < 7$) quantity of order 1, as far as the dependence on G_o is concerned.

For $T > T_H$ the free energy is dominated by that of the $N - N'$ free long strings:

$$F = -2\pi NL(T - T_H)^2.$$

Using (28) we observe that this expression is extensive:

$$F = -LV_t(2\pi)^{2-p}(\alpha'_{\text{eff}})^{(1-p)}G_o^{-2}(T - T_H)^2. \tag{30}$$

Just as for $p = 1$, the free energy is of order G_o^{-2} for $T > T_H$.

Now we are in a position to complete the phase diagram of the higher-dimensional NCOS theories. At very low temperatures, one has the open string phase of the NCOS. This description breaks down at the Hagedorn temperature (5), where one has a phase transition beyond which the temperature is two-dimensional although the free energy remains extensive. The details of the phase transition are dimension dependent. For all $p < 5$ the Hagedorn transition is second order. In $p \geq 3$ the specific heat diverges as $T \rightarrow T_H$ on the open string side.¹³

The free energy of noninteracting open strings becomes more singular with increasing p , and for $p \geq 5$ the entropy diverges at the transition. This implies that interaction effects become important already for $T < T_H$. Nevertheless, it is likely that the high-temperature phase again involves liberated long strings. We may argue for this as follows. The free energy of noninteracting open strings is of order G_o^0 , and interactions are unlikely to change this scaling. On the other hand, the free energy of liberated strings, (30), is of order G_o^{-2} . Therefore, for weak coupling and for T sufficiently above T_H , the system can lower its free energy by liberating long strings from

the bound state. It is not clear, however, whether the transition for $p > 4$ is second order; it may be a first-order transition for all values of G_o . Additional ideas on the Hagedorn transition for 5-branes have appeared in Refs. 33, 14, 15.

In fact, one may suspect that for large enough p the Dp -brane does not decouple from gravity in the NCOS limit. In Ref. 3 the nonplanar one-loop amplitude was calculated for 4 open strings, and it was shown that for $p < 7$ the amplitude is finite for $k^2 = 0$ (k is the momentum in the closed string channel). For $p \geq 7$ the amplitude blows up for $k^2 = 0$ which suggests that there is no decoupling from massless bulk modes. In order to check this, we have calculated the cross-section for two massless open strings of energy k_0 colliding along the electric field direction to produce an outgoing graviton. The term in the Born–Infeld action describing this process is

$$\frac{1}{2} \int d^{p+1}x (\partial_0 \Phi^i \partial_0 \Phi^j - \partial_1 \Phi^i \partial_1 \Phi^j) (\delta_{ij} + \sqrt{2} \kappa h_{ij}),$$

where we have rescaled the scalar fields so that they are canonically normalized. The cross-section we find,

$$\sigma \sim G_o^4 (\alpha'_e)^4 k_0^{9-p} (E_c^2 - E^2)^{(7-p)/2},$$

vanishes for $p < 7$, is finite for $p = 7$, and diverges for $p > 7$. This result is consistent with the annulus calculation in Ref. 3 and it indicates that nongravitational NCOS theories can exist only for $p < 7$. It is interesting to note that $p = 7$ is also special from the point of view of the thermodynamics: indeed here the free energy for the low energy phase, computed in Sec. II, diverges logarithmically at $T = T_H$.

To summarize this section, we can draw the general conclusion that the physics of the Hagedorn transition is similar for all NCOS theories with $p < 7$, in that above T_H the temperature dependence of F is effectively two-dimensional even though F is extensive in p dimensions. This is similar to the answer proposed in Ref. 9, although the justification there was different. In the NCOS case the two-dimensional behavior of the free energy above the Hagedorn temperature has to do with the presence of the electric field.

B. Extension to OD3 theory

In NCOS theory, a critical NS–NS 2-form field in presence of a Dp -brane leads to a decoupling limit in which fundamental strings are light. In certain variants of OM theory, a critical RR $(p+1)$ -form field applied to an NS5-brane is associated with a decoupling limit in which Dp -branes become light.^{5,16} No computational framework comparable to perturbative quantization of strings has emerged to study light Dp -branes for $p > 1$. Indeed, one may wonder if it is logically consistent for higher-dimensional branes ever to be the “fundamental” degrees of freedom of a theory.³⁴ However, it appears from the decoupling arguments of Refs. 5, 16 that there are decoupling limits of string theory where the lightest excitations are indeed open Dp -branes with $p > 1$.

It is tempting to adapt the reasoning used for NCOS theories to describe a possible phase transition for various OM-theories. In this section we will make an attempt in this direction, but our arguments will be much more heuristic than in previous sections. We will examine the relatively clean example of OD3-theory, which is the theory of open D3-branes on an NS5-brane in a decoupling limit with a critical 4-form potential turned on. Besides the obvious pitfall that the quantum states of fluctuating open D3-branes are hard to count, there is another interesting effect: fundamental strings living on the NS5-D3 bound state have a substantially reduced tension relative to their tension in flat space, and their Hagedorn behavior competes with the tendency to liberate D3-branes.

An NS5-brane with a near-critical RR 4-form potential can be described as a bound state of many D3-branes and a single NS5, such that the D3-branes make the dominant contribution to the tension. Let ρ be the number density of D3-branes in the two directions orthogonal to the D3-

branes but parallel to the NS5-brane. Then we require $\rho \tau_{D3} \gg \tau_{NS5}$. We will see below that this condition turns out to be trivially satisfied in the OD3 limit as defined in Ref. 5.

The tension of the NS5-D3 bound state is $\sqrt{\tau_{NS5}^2 + \rho^2 \tau_{D3}^2}$. The tension of an open D3-brane stuck to the NS5-brane is

$$\begin{aligned} \tau_{OD3} &= \frac{d}{d(\delta\rho)} (\delta\rho \tau_{D3} + \sqrt{\tau_{NS5}^2 + (\rho - \delta\rho)^2 \tau_{D3}^2})|_{\delta\rho=0} \\ &= \tau_{D3} \left[1 - \left(1 + \left(\frac{\tau_{NS5}}{\rho \tau_{D3}} \right)^2 \right)^{-1/2} \right] \approx \frac{1}{2} \left(\frac{\tau_{NS5}}{\rho \tau_{D3}} \right)^2 \tau_{D3}, \end{aligned} \tag{31}$$

where in the last line we have used $\rho \tau_{D3} \gg \tau_{NS5}$. The near-critical scaling limit is described by two parameters:⁵ a scale $\tilde{\alpha}'_{\text{eff}}$ and a coupling $G_{o(3)}^2$, which happens to be precisely the closed string coupling g_{str} (this last fact is special to OD3-theory). The precise scaling of the parameters is given as $\tilde{\alpha}' = \sqrt{\epsilon} \tilde{\alpha}'_{\text{eff}}$, the metric in directions transverse to the D3-branes scales as $g_{MN} = \epsilon \delta_{MN}$, and $g_s = G_{o(3)}^2$. The scaling of the metric implies that $\rho = \rho_0 / \epsilon$, where ρ_0 is of order unity. This implies $\rho \tau_{D3} \sim O(\epsilon^{-2})$, while $\tau_{NS5} \sim O(\epsilon^{-3/2})$, thereby satisfying the aforementioned condition that $\rho \tau_{D3} \gg \tau_{NS5}$.

Just as we found in NCOS theory that it is thermodynamically favorable to liberate strings from the bound state at a temperature $T_H \sim \sqrt{\tau_{\text{eff}}}$, so we will find here that it is favorable to liberate D3-branes at a temperature $T_{c,D3} \sim \tau_{OD3}^{1/4}$. The argument proceeds along similar lines. First we note that a free $U(1)$ gauge multiplet in a flat-space theory in $p+1$ dimensions and sixteen supercharges has

$$\frac{F}{L^p T^{p+1}} = -c_{D_p} \equiv -\frac{8 \text{Vol } S^{p-1}}{(2\pi)^p} \left(2 - \frac{1}{2^p} \right) \Gamma(p) \zeta(p+1). \tag{32}$$

Here F is the free energy, L^p is the spatial world-volume, and T is the temperature. The low-energy dynamics of the bound state is noncommutative super-Yang–Mills theory in $5+1$ dimensions. Clearly, then, the free energy at low temperatures is order 1 in the sense that it does not grow with a power of the number density ρ . Let us assume that this remains the case up through the temperature where liberating D3-branes becomes thermodynamically favorable. Then the same manipulations that we went through in (16) are justified at large ρ : the free energy after a number density $\delta\rho$ of D3-branes have been liberated is

$$\begin{aligned} \frac{F_{\delta\rho}}{L^5} &= \sqrt{\tau_{NS5}^2 + (\rho - \delta\rho)^2 \tau_{D3}^2} + \delta\rho \tau_{D3} - c_{D3} \delta\rho T^4 \\ &\approx \rho \tau_{D3} - c_{D3} \rho T^4 + \frac{\tau_{NS5}^2}{2(\rho - \delta\rho) \tau_{D3}} + c_{D3} (\rho - \delta\rho) T^4 \\ &\geq \rho \tau_{D3} + \tau_{NS5} \sqrt{2c_{D3} T^4 / \tau_{D3}} - c_{D3} \rho T^4, \end{aligned} \tag{33}$$

where we neglect terms which are subleading in ρ . Equality pertains in the last line of (33) if and only if the last two terms in the second line are equal. One reads off the fraction of liberated D3-branes, the free energy, and the critical temperature $T_{c,D3}$ as

$$\nu \equiv \frac{\delta\rho}{\rho} = \frac{T^2 - T_{c,D3}^2}{T^2},$$

$$\frac{F}{L^5} \approx \rho \tau_{D3} - c_{D3} \rho (T^2 - T_{c,D3}^2)^2, \tag{34}$$

$$T_{c,D3}^2 = \frac{\tau_{NS5}}{\sqrt{2c_{D3}\tau_{D3}\rho}} = \sqrt{\tau_{OD3}/c_{D3}} = \frac{1}{(2\pi)^{7/2}} \frac{1}{\sqrt{2c_{D3}}} \frac{1}{g_{str}^{3/2} \tilde{\alpha}'^2 \rho_0}.$$

[Formally, a similar analysis seems to be possible for many D5-branes bound to an NS5-brane. However, in this case, the absence of strong IR dynamics on N coincident D5-branes makes it likely that there are $O(N^2)$ massless degrees of freedom even at low energies. This would overwhelm the $O(N)$ effect due to liberated D5-branes, rendering the whole approach suspect.]

In the case of NCOS theory, it was essentially guaranteed that fundamental strings would start being liberated at the Hagedorn temperature of the noncommutative open strings, because the calculation of the free energy of liberated strings was equivalent to a computation in the light cone formalism of highly excited open strings. In OD3 theory, no analog of the latter computation exists as yet, so to be conservative we should regard $T_{c,D3}$ as an upper bound on the temperature where some transition must take place. In fact, as we will now show, when $G_{o(3)} \ll 1$, there is a Hagedorn transition for closed fundamental strings living on the NS5-D3 system at a substantially lower temperature than $T_{c,D3}$. These closed strings are excitations of the NS5-D3 bound state.

There is no net f_1 charge in the NS5-D3 bound state that we wish to analyze; however, in order to extract the tension of the closed strings which live on the NS5-D3 system, it is convenient to first consider a BPS arrangement where an NS5-brane is oriented in the 012345 directions, ρ D3-branes per unit 45-volume are oriented in the 0123 directions, and ρ_4 fundamental strings per unit 2345-volume are oriented in the 01 directions. The total tension is

$$\tau = \sqrt{(\tau_{NS5} + \rho_4 \tau_{f1})^2 + (\rho \tau_{D3})^2}. \tag{35}$$

The effective tension of a fundamental string bound to the NS5-D3 system is

$$\tau_{f1,eff} = \left. \frac{\partial \tau}{\partial \rho_4} \right|_{\rho_4=0} = \tau_{f1} \left. \frac{\tau_{NS5}}{\tau} \right|_{\rho_4=0} \approx \tau_{f1} \frac{\tau_{NS5}}{\rho \tau_{D3}} \ll \tau_{f1}, \tag{36}$$

where in the last two steps we have used the fact that most of the mass of the bound state is carried by the D3-branes. Fundamental strings which are orthogonal to the D3-branes but contained in the NS5-branes are much heavier.

Another way to derive the effective tension $\tau_{f1,eff}$ is to look at the supergravity solution for the NS5-D3 system. The string metric and dilaton are

$$ds_{str}^2 = \frac{1}{\sqrt{h_3}} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{h_3}(dx_4^2 + dx_5^2 + h_5(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2)),$$

$$e^{2(\phi - \phi_\infty)} = h_5, \quad h_3 = 1 + \frac{q_3}{r^2}, \quad h_5 = 1 + \frac{q_5}{r^2}, \tag{37}$$

$$r^2 = x_6^2 + x_7^2 + x_8^2 + x_9^2.$$

In the limit where $\rho \tau_{D3} \gg \tau_{NS5}$, we have $q_5/q_3 = \tau_{NS5}/(\rho \tau_{D3})$. The three-form field strengths need not concern us, except to note that $B_{\mu\nu}^{(NS)}$ may be chosen so that only B_{45} is nonzero. We are considering many D3-branes, but only a single NS5-brane, so the supergravity solution is trustworthy, in the sense that curvatures are sub-stringy, down to a radius $r_{match} = \sqrt{q_5}$. Following the philosophy of Ref. 32, we assert that the tension and coupling of fundamental strings bound to the NS5-D3 system can be read off, up to factors of order unity, from the properties of a test string located at the matching radius r_{match} . The tension so derived agrees with (36). The advantage of this more heuristic approach is that we can extract the string coupling for the strings bound to the NS5-D3 system: up to a factor of 2 it is just $g_{str} = G_{o(3)}^2$.

When $G_{o(3)} \ll 1$, we are entitled to use the free string spectrum to predict a Hagedorn temperature for the light fundamental strings whose orientation is within the D3-branes. It is

$$T_{c,f1} \sim \sqrt{\tau_{f1,eff}} = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{g_{str} \tilde{\alpha}'^2 \rho_0}}. \tag{38}$$

From

$$\frac{T_{c,f1}}{T_{c,D3}} \sim \frac{\sqrt{\tau_{f1,eff}}}{\tau_{OD3}^{1/4}} \sim g_{str}^{1/4} = \sqrt{G_{o(3)}}, \tag{39}$$

we learn that the fundamental string Hagedorn transition happens at a lower temperature when $G_{o(3)} \ll 1$. This supports the view that the most relevant degrees of freedom in OD3-theory with $G_{o(3)} \ll 1$ may be little strings. Before one can ask whether the D3-brane liberation transitions occurs, one must understand what contribution the fundamental strings make to the free energy above $T_{c,f1}$.

When $G_{o(3)} \gg 1$, one can obtain a more natural description of the theory by S -dualizing. The scaling limit leading to OD3-theory S -dualizes into the zero slope limit used in Ref. 1 to obtain noncommutative Yang–Mills theory.⁵ For $G_{o(3)} \gg 1$, this theory is weakly coupled in the sense that $g_{YM} \ll \sqrt{\theta}$. However, the interacting theory is nonrenormalizable, so it might be inappropriate to regard the quanta of the gauge field as the fundamental degrees of freedom. It was suggested in Ref. 5 that OD3-theory provides an ultraviolet completion of 5 + 1-dimensional noncommutative Yang–Mills theory. This is not a very effective description in the absence of a knowledge of how to quantize open D3-branes. It must be admitted that, for $G_{o(3)} \geq 1$, there is neither a natural little string theory description of OD3-theory, nor a renormalizable interacting quantum field theory description. Despite all this uncertainty, the analysis following (33) may still be valid: it only depends on the free energy of the bound state being $O(\rho^0)$ in a $\rho \rightarrow \infty$ limit with $G_{o(3)}$ fixed.

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