

Galley, Leibovich, and Rothstein Reply: The authors of [1] use the results of Nodvik [2] to argue that a term was missed in the worldline action that yields an order R correction to the Abraham-Lorentz-Dirac formula. We believe that the arguments made in the Comment are correct. The missed term linear in R in the worldline action is

$$C \int d\tau a^2 \quad (1)$$

where $a^2 = a^\mu a_\mu$ is the square of the four-acceleration and C is an undetermined coefficient that is found by matching onto a calculation from the full theory. For the case of a spherical shell of charge, the authors of [1] show that $C = -(2/9)e^2 R$. Importantly, the term in (1) vanishes for a neutral body.

It is interesting to note, however, that this term (1), while formally of order R , will be suppressed in any laboratory experiment. Thus its effects on proposals to measure finite size effects will be negligible. To see this we recall that the worldline of the point charge in the effective theory is not an observable. Hence, one may perform worldline shifts by amounts of order R without changing the predictions of the theory. In this regard, note that (1) can be eliminated by the following shift in the worldline

$$x^\mu \rightarrow x^\mu - \frac{C}{m} a^\mu \quad (2)$$

where C is the coefficient of the a^2 term in (1) and scales as $e^2 R$ [1]. The result of this shift is that the original action [3]

$$S[x^\mu, A_\mu] = -m \int d\tau + e \int d\tau u^\alpha A_\alpha(x) + C \int d\tau a^2 + C_d \int d\tau u^{[\alpha} a^{\beta]} F_{\alpha\beta}(x) + \dots \quad (3)$$

becomes

$$S[x^\mu, A_\mu] \rightarrow -m \int d\tau + e \int d\tau u^\alpha A_\alpha(x) + \left(\frac{eC}{m} + C_d\right) \int d\tau u^{[\alpha} a^{\beta]} F_{\alpha\beta}(x) + \dots,$$

which is obtained by expanding (3) in powers of R after applying the shift in (2). There are two key points to note. The first is that (1) is *cancelled* from the action. The second is that the acceleration-induced dipole moment term that we studied in [3] [i.e., the last term in (3)] receives a

correction to its matching coefficient. Hence, under this worldline shift, $C_d \rightarrow C_d + eC/m$.

Next, we ask under what conditions eC/m is much smaller than C_d . Since $C \sim e^2 R$ [1] and $C_d \sim eR^2$ [3], it follows that $(eC/m)/(C_d) \sim e^2/(mR)$, which is much less than unity if

$$m \gg \frac{e^2}{R}. \quad (4)$$

On the left side of (4) is the rest mass energy of the object and e^2/R is the potential energy of the extended charge. As the rest mass energy will almost assuredly be much larger than the charge's electrostatic energy in any real-world laboratory experiment then it follows that the contribution from (1) will not manifest itself. In fact, if one were in the situation where $m \sim e^2/R$ then the electrostatic energy is comparable to the object's rest mass energy. In this case, quantum processes will become very important since the spontaneous production of identical charged objects from the vacuum will occur. Of course, in such a scenario, the effective theory of [3], as well as any classical description of the extended charge, is no longer valid.

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