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THE ELECTROMOTIVE FORCE PRODUCED BY THE ACCELERATION OF METALS.

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THE modern theories of electricity have led to the belief that the passage of an electric current through a metal really consists in the progressive motion of "free" electrons contained in the body of the metal itself. If this be true we may now expect a number of effects arising from the *mass* of these electrons which were not predictable on the basis of older theories which thought of electricity as a sort of intangible massless fluid. As examples of such effects, we should expect the rear end of an *accelerated* rod of metal to become negatively charged owing to the lagging behind of the relatively mobile electrons which the metal contains, and should expect the periphery of a *rotating* disk to become negatively charged owing to the action of centrifugal force on the electrons in the disk. Such effects, however, would presumably be very small, owing to the exceedingly small mass probably associated with the electron.

PREVIOUS WORK WITH ELECTROLYTIC CONDUCTORS.

In the case of electrolytes effects such as these have long been known, their experimental determination being far less difficult than with metals owing to the relatively large masses associated with the carriers of electricity in such conductors.

An effect of this kind in electrolytes was first obtained by the *acceleration method* in 1882 by Colley[1] using a tube containing a solution of cadmium iodide and provided with electrodes at the two ends. The tube was given a sudden negative acceleration by dropping it and then bringing it to rest in a box of sand, the small pulse of electricity being noted which flowed through a galvanometer connected in circuit with the electrodes. More elaborate measurements of the effect of acceleration

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[1] Colley, Wied. Ann., *17*, p. 55, 1882.

have since been made by Tolman and Osgerby[1] using different electrolytes, potassium, sodium and lithium iodides, and varying both the acceleration and distance between the electrodes. Within the necessarily large limits of error these results were found to agree with those theoretically predicted.

In the case of electrolytes much larger effects can be obtained by the *centrifugal method* than by the acceleration method, and these were first detected by Des Coudres[2] in 1892. Since then one of the present authors,[3] using a very powerful centrifugal machine, has made a systematic study of the effect in four different electrolytes, potassium, sodium and lithium iodides and hydriodic acid, and has found an entirely satisfactory agreement between theory and experiment.

PREVIOUS WORK WITH METALLIC CONDUCTORS.

In the case of metallic conductors, Maxwell,[4] himself, was the first, not only to discuss the nature of the phenomena which would arise if mass of the ordinary kind should be associated with electricity in metals, but also to try experiments to detect the possible effects. He had, however, no means of predicting the presumable size of such effects if they did exist since this was before the development of the electron theory and he merely states the negative results of his experiments without any information as to the dimensions or efficiency of his apparatus. Lodge[5] also reports a negative result for such experiments.

The first attempts to detect such effects of which we have any quantitative information were made by Nichols[6] in 1906. He employed the centrifugal method, using a rotating aluminum disk and making a rubbing contact at the periphery and center with wires, which led to the electrical measuring apparatus. Such rubbing contacts, in particular the one at the rapidly moving periphery, necessarily introduce large and variable electromotive forces; nevertheless from a series of experiments Nichols was able to conclude that the mass of the carrier in metals is less than that of the hydrogen atom.

This centrifugal method of attack, which is by far the most powerful and satisfactory in the case of electrolytes, suffers necessarily in the case of metals from this necessity for some form of rubbing contact at the

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[1] Tolman, Osgerby and Stewart, J. Am. Chem. Soc., *36*, p. 466, 1914.

[2] Des Coudres, Wied. Ann., *49*, p. 284, 1893. Ibid., *57*, p. 232, 1896.

[3] Tolman, Proc. Am. Acad., *46*, p. 109, 1910; J. Am. Chem. Soc., *33*, p. 121, 1911.

[4] Maxwell, Treatise on Electricity and Magnetism, 3d edition (1892). Vol. II., pp. 211 et seq.

[5] Lodge, Modern Views of Electricity, 3d edition (1907), p. 39.

[6] Nichols, Physik. Z., *7*, p. 640, 1906.

periphery. In the case of electrolytes the difference in potential between the central and peripheral electrodes can be measured with the help of wires which are led to the center of the rotating apparatus and there make contact with connections leading to the stationary measuring apparatus. In the case of a rotating metal disk, however, if we should make connection with the periphery through a wire led to the center, the electromotive force in this wire, which is inappreciable compared with the electromotive force in electrolytes, would of course be just large enough to neutralize the electromotive force in the disk itself. For this reason the centrifugal method of attack almost necessarily involves some disastrous form of rubbing contact, and the acceleration method, which would give much smaller effects but would also involve much smaller errors, seems more suitable for metals.

Making use for this reason of the acceleration method the authors reported in 1913[1] that the effect in metals, if any, was so small that the mass of the carrier in metals was less than one two-hundredth part of that of the hydrogen atom. With the help of a much more sensitive galvanometer and eliminating one by one a number of accidental effects which appear when greater sensitiveness is reached, the authors have now apparently obtained a real effect due to the mass of the carrier in metals. The results are reported here not only because of the interest naturally attaching to the observation of an effect which has been so long looked for, but also because of the bearing which measurements of this kind have on the theory of the conduction and constitution of metals.

Elementary Theory of the Acceleration Effect.

From an elementary point of view we can develop a fairly satisfactory theory of the potentials and currents which would be produced by accelerating a metallic conductor. In the future, making use of a more intimate knowledge of the structure of matter, it may be possible to develop a more complete theory.

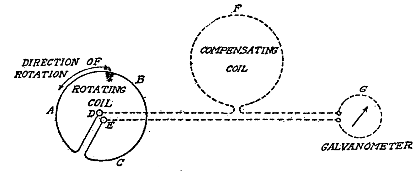

Fig. 1.

Let us consider the arrangement of conductors actually used in our experiments as shown diagrammatically in Fig. 1.

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[1] Tolman, Osgerby and Stewart, loc. cit.

ABC is a coil of wire which can be rotated in either direction about its axis at a high speed and suddenly brought to rest. The two ends of this coil of wire are brought to the center at *D* and *E* and connections made with the external circuit as indicated by the dotted lines. This external circuit contains a compensating coil *F* whose function need not detain us now, and the ballistic galvanometer *G* which measures the pulse of electric current produced on stopping the rotating coil. This pulse of electricity arises from the momentum of the electrons in the rotating coil which causes them to continue in motion after the rotation of the coil has ceased.

Let the length of the moving wire be l, its cross section s, and its resistance R_i. Let the resistance of the external part of the circuit be R_e.

Consider now one equivalent of electrons situated in the rotating coil. If m is the mass associated with an equivalent of electrons (*i. e.*, with $F = 96{,}500$ coulombs of electricity) and v is the tangential velocity of the coil just before stopping, the momentum of these electrons at the instant of stopping will be mv, and the electrons will continue in motion until the mechanical and electrical forces acting on them have destroyed this momentum mv.

Let us now try to analyze the forces acting on one equivalent of electrons in a conductor which is being accelerated.

The Electrical Force.—If E is the difference in electrical potential between the two ends of the wire the potential gradient at any point will be E/l and the *electrical force* acting on one equivalent of electrons (*i. e.*, on $F = 96{,}500$ coulombs of electricity) will be EF/l.

The Frictional Force.—If the electrons are lagging behind the rest of the metal there will be a frictional force acting on them. From the fact that Ohm's law holds in metals we may assume proportionality between such frictional forces and the relative velocity of the metal and the electrons. If U is the mobility of the electron (*i. e.*, the velocity which it attains under unit potential gradient) then U/F will be the velocity which one equivalent of electrons would attain under unit force. Hence if the electrons are lagging behind the rest of the metal with the velocity u the *frictional force* acting per equivalent will be uF/U.

The Acceleration Force.—Besides the frictional force which acts on the electrons because they are lagging behind the conductor, it is evident that there may be an additional mechanical force which would act on the electrons in a conductor which is being accelerated even if they did not lag behind the rest of the metal. From a molecular point of view this force might arise from the fact that the electrons are surrounded by atoms of the metal which are themselves being accelerated and hence in collision or interaction with these atoms the electrons receive on the average an extra component of momentum in the direction of acceleration. The magnitude of this *accelerational force*, as it might be called, is difficult to estimate. In the case of an accelerated liquid which has the density D and is being given the acceleration a it is evident that each cubic centimeter is being acted on with the force Da, and hence that a body of volume v immersed in such a liquid would be acted on by an accelerational or buoyant force of the amount vDa, this buoyant force being transmitted to the body in question by the surrounding liquid. For this reason in making calculations on accelerated liquids similar to those we are now considering in accelerated metals, the accelerational force acting on one equivalent of dissolved substance was placed equal to $\bar{v}Da$ where $\bar{v}$ was the partial equivalent volume of the solute in question. Even in the case of liquids, however, the justification for such a procedure is not entirely clear, since the buoyant force acting on a substance which after being dissolved and dispersed in a solution has the partial volume $\bar{v}$ would perhaps not necessarily be the same as the force acting on an undispersed body having the same volume v. For this reason in the even more complicated case of solids we may content ourselves with placing the buoyant or accelerational force acting on one equivalent of electrons equal to ka, where k is a constant whose value is for the present unknown. It will be seen that this is in reality equivalent to representing by ka, regardless of the exact nature of their mechanism, any other forces besides those which we have called "electrical" and "frictional," and it may be noted that this has involved the tacit assumption that they are proportional to the acceleration, which in the light of the foregoing discussion is not entirely unreasonable.

From the considerations presented in the three preceding paragraphs we may now place the total force acting on one equivalent of electrons in the interior of an accelerated metal equal to the sum of the electrical, frictional and accelerational forces, giving us

$$f = \frac{EF}{l} + \frac{uF}{U} + ka. \tag{1}$$

If now our conductor is suddenly brought to rest from the velocity v, it is evident that the integral of this force with the time must be equal to the total momentum of the electrons which has to be destroyed, giving us

$$mv = \int_0^\infty \left(\frac{EF}{l} + \frac{uF}{U} + ka \right) dt, \tag{2}$$

where m is the mass of one equivalent of electrons.

E is the potential difference at the two ends of the moving coil and hence is the electromotive force which pushes the current through the external circuit, permitting the substitution

$$E = IR_e + L_e\frac{dI}{dt}, \tag{3}$$

where R_e and L_e are the resistance and inductance of the external circuit.

Furthermore since u is the velocity with which the electrons are lagging behind the metal we may evidently write for the current

$$I = uFCs, \tag{4}$$

where C is the concentration of the electrons in equivalents per cubic centimeter and s is the cross section of the conductor. We may also note, since U is the mobility of the electrons (*i. e.*, their velocity under unit potential gradient) that the specific conductivity of the metal will be UFC, and hence the total resistance of the accelerated conductor will be

$$R_i = \frac{l}{UFCS}. \tag{5}$$

Combining (4) and (5) we obtain

$$\frac{uF}{U} = \frac{IR_iF}{l}. \tag{6}$$

Substituting (3) and (6) in (2) we obtain

$$mv = \int_0^\infty \left(\frac{IR_eF}{l} + \frac{L_eF}{l}\frac{dI}{dt} + \frac{IR_iF}{l} + ka\right)dt. \tag{7}$$

To carry out the indicated integration we may note since the current I has the value zero both at the start and finish of the process that

$$\int_0^\infty \frac{L_eF}{l}\frac{dI}{dt}\,dt = \frac{L_eF}{l}dI = 0. \tag{8}$$

It is also evident that we may write

$$\int_0^\infty Idt = Q, \tag{9}$$

where Q is the total electricity which passes through the circuit, and also write

$$\int_0^\infty adt = v, \tag{10}$$

since v is the total change in the velocity of the conductor. We obtain

$$mv = \frac{QF}{l}(R_e + R_i) + kv,$$

or solving for the total pulse of electricity and putting $(R_e + R_i) = R$ the total resistance of the circuit we obtain the desired equation

$$Q = \frac{(m - k)vl}{RF}. \tag{11}$$

This is the equation which we have tested in our experiments and have found the pulse of electricity to depend in the way indicated on the variables, velocity v, length l, and resistance R. We have also calculated from our results the value of $(m - k)$ and found this quantity to be nearly the same as the mass of an electron in free space. We shall consider the significance of this fact in the sequel.

In the next paragraphs we shall present a very complete and detailed description of our experimental apparatus, methods and measurements. We think this necessary so that the reader shall have all the material needed for a critical estimate of the significance of our results. Experiments so difficult as the ones we have undertaken where so many accidental effects have to be eliminated must necessarily be subjected to very careful scrutiny, and although we believe that our work demonstrates the existence of the predicted effect, and indeed gives its magnitude with reasonable accuracy, we nevertheless shall welcome criticisms or suggestions as to improvements.

The Experimental Apparatus.

A general view of the rotating apparatus is shown in the reproduction from a photograph, Fig. 2. The rotating wheel carrying the coil of wire to be experimented on is shown at A. It is driven from a *three-phase* 1½-horsepower motor by a belt through a gear and worm enclosed in the case at B. The small shaft C leads to a magneto for measuring speed. The large rectangular coils of wire, D, E, are used in neutralizing the earth's magnetic field and the compensating coil F is connected in series with the rotating coil. Connection with the rotating coil is made through the pair of wires G, which run over a pulley on the ceiling of the room above and are allowed to twist up. They come down again at H.

A description of the individual features of the apparatus will be necessary.

The Frame.—The frame of the apparatus was constructed of 2 × 4 in. scantling fastened together with brass bolts and erected on 4 × 6 in. sills which were bolted direct to the concrete floor. Such a construction from non-magnetic materials was found to be absolutely necessary, since an iron frame which was tried was found to change its state of magnetization when jarred by the running of the machine, and this change in magnetization affected the coil enough to produce very large deflections in our sensitive galvanometer.

The Shaft, Bearings, Gears, etc.—The exclusion of iron from any part of the apparatus near the coil made it necessary to use a Tobin bronze shaft (three fourth inch in diameter), running in brass bearings. This shaft was driven by a three-phase 440-volt 1½-horse-power motor, which was belted to a tight and loose pulley driving the gears in the gear case at *B*, Fig. 2. The shaft was provided with two thrust bearings so that it could be driven in either direction by reversing the motor. The lower thrust bearing was a steel ball under the lower end of the shaft in the gear case, and the upper bearing was an ordinary ball bearing seen in the cut just at the lower end of the bronze shaft. The steel balls in this bearing were the nearest iron to the coil and their motion did not affect the galvanometer. It was found necessary, however, to use bronze instead of steel washers in these bearings.

The gears and gear case were those belonging to a "Babcock Standard Milk Tester" and stepped up the speed in the ratio of 3 to 29. It was found necessary to arrange the gear case so that the gears were always immersed in a mixture of oil and oil-dag and even then it was necessary to replace the gears at rather frequent intervals.

The handle for applying the brake is shown at *I*.

The Magneto.—To determine the instantaneous speed of rotation at the time of stopping, the apparatus was made to drive a magneto connected to the shaft *C*. This magneto was set at a considerable distance from the rest of the apparatus so as to avoid possible magnetic disturbances. The voltage of the magneto was found to be very closely proportional to the speed according to the relation 1 centivolt = 0.728 rev. per second of the rotating coil. This voltage was read on a voltmeter conveniently placed. In cheap magnetos such as the one we used there is often considerable fluctuation of voltage owing to poor contact of the brushes. This was pretty well avoided, however, by the use of two pairs of brushes instead of two single brushes. Each pair consisted of a wire brush and a carbon brush, the wire brush of one pair being placed opposite the carbon brush of the other pair, to act as a cleaner. It was also found desirable for steady voltages to arrange a spring to prevent longitudinal motion of the armature with reference to the magnetic field.

The Rotating Wheel.—The most significant part of the apparatus is of course the rotating wheel which carries the coil of wire to be experimented on. A reproduction of this wheel is shown in the accompanying cut, Fig. 3. The coil of wire was wound in the groove *A* which has a depth of one inch and width of fifteen sixteenths inch. The ends of the

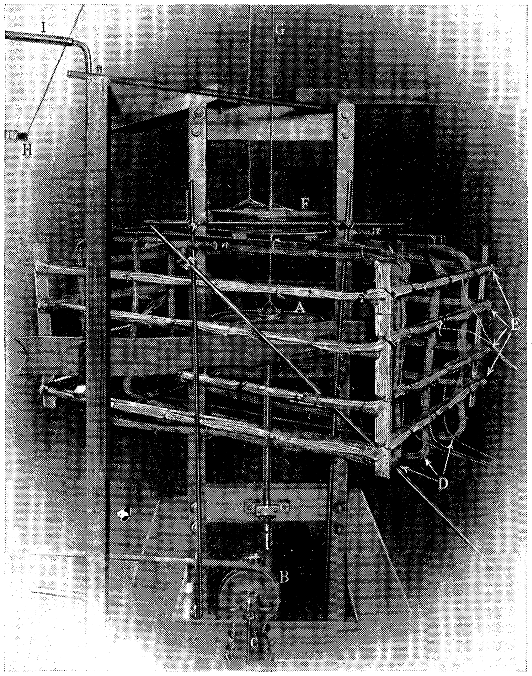

FIG. 2.

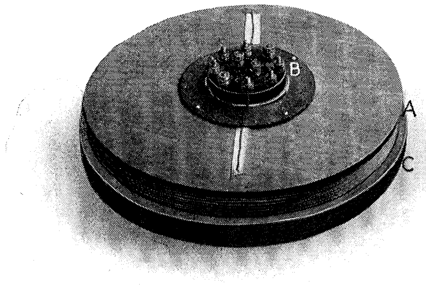

FIG. 3.

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wire were brought to binding posts set on the vulcanite slab *B*. These binding posts were of copper to help in the elimination of thermoelectric effects. The wire used was No. 20 double silk insulated copper wire carefully wound in the lathe, the successive layers being impregnated with parafin. The outside of the coil was protected by two layers of unconnected wire. The length of the wire was measured by resistance and by weight, comparing with fifty-foot samples taken from each end of the wire used, the two methods checking closely. The coil used was 46,650 cm. long and had an average diameter of 24.65 cm. Some of the wire was later removed and experiments made with a shorter coil, 30,370 cm. long with an average diameter of 24.1 cm.

The disk for the wheel was made by gluing together under extremely high pressure a number of layers of birch veneer, each one eighth inch thick, the direction of the grain being changed by 45 degrees in successive layers. Wood was chosen not only because of its high ratio of strength to density which is of course essential, in apparatus subjected to centrifugal action, but also because in experiments made with an aluminum alloy wheel, after stopping the wheel, we obtained, on releasing the brake strap, for some reason which we have not thoroughly investigated an extra pulse of electricity through the galvanometer. Using a wooden wheel we had no such trouble. In the case of metal wheels we were also afraid that currents generated in the wheel itself might make trouble for us by inducing currents in the coil.

The brake strap of heavy leather belting was arranged to pull tight around the circumference of the wheel at *C*, Fig. 3, it being of course desirable from a mechanical point of view to apply the force for stopping the wheel as near as possible to the coil of wire with its large momentum. The strap was arranged to pull as uniformly as possible on the whole circumference to prevent pulling the wheel out of line. It was found possible to stop the machine in a fraction of a second from 5,000 r.p.m.

The Twisting Wires.—One of the most important problems which had to be solved was the matter of making connection with the rotating apparatus, since these connections themselves must not introduce appreciable electromotive forces into the circuit. We finally found that small wires which were allowed to twist up were quite satisfactory. These wires went from the center of the wheel over a pulley on the ceiling of the room above and were kept taut by a small weight. This great length permitted us to make a number of runs without changing the wire. We found No. 28 to be a good size of wire to use and this was either doubly or triply silk insulated. The insulation had to be wound in such a direction as to loosen up on twisting, as otherwise the wire kinked badly. For this reason we had to use different wire in running forward and backward.

By running the machine with these twisting wires short circuited we found that their twisting introduced no appreciable electromotive forces into the circuit. We also found by testing the twisted wires after running that the insulation between the two wires remained perfectly satisfactory.

All of the electric connections were shielded from accidental leaks by White's method of mounting insulators on metal plates which are all connected together.

The Galvanometer.—The galvanometer was an extraordinarily satisfactory d'Arsonval type designed by Dr. Frank Wenner and loaned to us by the Bureau of Standards, and we wish here also to express to Dr. Wenner our deep appreciation of his courtesy in this matter. The initial high sensitiveness of the galvanometer was increased by using a very long optical path. The filaments of a 100-watt concentrated filament tungsten lamp were brought to focus by a two thirds inch objective of an ordinary microscope about 60 cm. from the concave mirror of the galvanometer, which in turn produced a new image of the filaments on a paper scale at a distance of about 10 meters.

The galvanometer was adjusted by its gravity control to have a period of about 10 seconds and was used ballistically. It was standardized with a standard solenoid and found to have with our optical path a sensitiveness of about 4.75×10^{-10} coulombs per millimeter throw, the resistance in the circuit being that actually used, 40 ohms. The sensitiveness varied slightly from day to day and was always redetermined for each set of runs, using the same resistance in the circuit as that used in the actual measurements. The standard solenoid was of such a capacity that we could conveniently use one milliampere as the current to be broken in the primary and this was measured with a Leeds and Northrup milliameter. One milliampere broken in the primary gave a throw of about 45 millimeters on the galvanometer scale. The deflections actually measured were in the neighborhood of 5 to 10 millimeters, but as nearly as we could make out the throw was proportional to the number of coulombs in the pulse of electricity. The standard solenoid was of our own construction and was standardized both by computation and by comparison with a known solenoid lent by the Physics Department of this university, the two methods leading to a satisfactory check.

The Compensating Coil.—With our sensitive galvanometer connected in series with our coil we found that there are rapid fluctuations in the earth's magnetic field which produced continual deflections of the galvanometer up to magnitudes of ten centimeters or more, which were of course enormously larger than the deflection we were going to measure.

This difficulty was overcome by connecting in series with our rotating coil a stationary "compensating coil" having the same magnetic power but wound in the opposite direction. This coil was placed directly over the rotating coil as shown in Fig. 2 at *F*. This coil had to be very carefully adjusted as to length and then almost completely neutralized the bad effects, although sometimes an automobile going rapidly past the corner of the laboratory would produce enough magnetic disturbance to give a deflection of several millimeters.

We do not know whether the continual variation in the earth's magnetic field which we observed is due to the proximity of the city of Berkeley with a probable leakage of stray and variable currents through the earth or whether the phenomenon would be observed anywhere on the surface of the earth. The fact that we got nearly as good compensation, except for such local disturbances as automobiles, when the coils were several feet apart as when they were near together indicates that the sources of the disturbance are at a considerable distance. The subject would be an extremely interesting one for further research.[1]

The Horizontal Anti-Earth Coils.—Not only does the varying part of the earth's magnetic field produce difficulties for experiments of this kind, but the steady horizontal component of the earth's magnetic field must also be neutralized, since otherwise the slight tippings of the wheel during rotation and on application of the brake, even though they are too small to be easily observable visually, are nevertheless sufficient to make the coil of wire cut enough horizontal lines of force to produce enormous deflections of the galvanometer. For this reason we arranged parallel to the horizontal component of the earth's field three rectangular coils of wire shown at *D* in Fig. 1, through which we passed a steady current of the right magnitude to neutralize the component in question. This current was arranged to run through all three coils, a shunt making the current somewhat weaker in the middle coil. The current had to be carefully controlled and also the axis of the coils adjusted with extreme care as to direction. We obtained in this way very good neutralization which was tested by loosening the wheel so that it could be tipped with a motion of about a quarter of an inch at the circumference. With the best adjustment the resulting deflections of the galvanometer were reduced almost to zero. It was found necessary to readjust the direction of the axis and the strength of the current employed at rather frequent intervals, since sometimes even within a day or two we seemed to find small but appreciable variations in the direction or magnitude of the horizontal component.

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[1] A considerable amount of work has already been done in this field. See Ebert, J., Terres. Mag. and Atm. Elec., *12*, 1 (1907).

The Vertical Anti-Earth Coils.—After neutralizing the variable part of the earth's field with the compensating coil and the steady horizontal component with the "horizontal anti-earth coils" it was possible to obtain quite reproducible results in our runs, and indeed we collected data for a number of months before we realized that the steady vertical component of the earth's field was leading to a constant error in our measurements. This error arises from the fact that on stopping the machine the centrifugal force which has been acting on the coil ceases and the coil suddenly decreases very slightly in diameter, thus diminishing the vertical flux through the coil by an amount large enough to give a throw of the same order of magnitude as that which we were after, say 10 mm. The possibility of this effect which had entirely escaped our attention was suggested to us by Professor Bridgman, of Harvard University, and we desire to express to him also in this place our grateful acknowledgment.

To do away with this error a set of four rectangular coils were arranged around the wheel on a vertical axis (see E, Fig. 1) and a steady current sent through them. The magnitude of the current was regulated well enough so that the earth's vertical field was cut down to about one per cent. of its value in the room outside the coils. This was tested by observing the deflections of a galvanometer placed in series with a small coil of wire held horizontally and arranged so that its area could be suddenly changed. Cutting the field down to one per cent. of its value was quite sufficient to make the error involved negligibly small. We had, moreover, a further test of the efficiency of our arrangement in the fact that with it there were no large or consistent differences in the experimental results obtained when the wheel was run forward or backward, while the centrifugal effect which was present when the earth's field had not been neutralized tended to increase the throw when running in one direction and decrease it when running in the other.

In constructing our apparatus we were most fortunate in being able to rely upon the skill and experience of Mr. G. F. Nelson, the department mechanician, and we wish also here to express our grateful acknowledgment to him.

EXPERIMENTAL PROCEDURE.

In carrying out the actual measurements one of us (T. D. S.) operated the machine, and the other (R. C. T.) observed the galvanometer deflections. The machine was speeded up to the desired point, the belt shifted to the loose pulley, the current thrown off the motor and the motor brought to rest by applying a special brake, thus leaving the apparatus itself the only thing in motion. The observer of the galvanometer then signalled if the galvanometer was reasonably stationary, the operator applied the brake and read the voltmeter, and the observer noted the galvanometer deflection. It was occasionally necessary to wait a short time before applying the brake because of the temporary presence in the circuit of variable accidental electromotive forces, or because of mechanical vibration of the galvanometer.

The motor operated on three-phase current so that throwing it off and on did not produce serious magnetic disturbances. It was found advisable, however, because of small disturbances produced by the moving armature, to bring the motor to complete rest before trying to measure a deflection.

The galvanometer scale was set at zero just before starting a run, but owing to the introduction of comparatively constant thermoelectric or other electromotive forces incident upon rotation it was usually somewhat off from zero at the instant of stopping. This new rest point, the point of maximum deflection, and the point to which the galvanometer returned, were always recorded.

For convenience we reversed the electrical connections whenever we changed the direction of rotation, thus getting the galvanometer throw always in the same direction.

Experimental Results.

An idea of the kind of results actually obtained is given in Table I., which gives as samples those measurements which were made with the greatest length of copper wire both for forward and backward rotation[1] at speeds in the neighborhood of 80 centivolts as measured on the voltmeter. The variation in individual measurements is fairly large but not more so than might be expected from the nature of the work. The final column gives the number of coulombs passing through the galvanometer divided by the speed of rotation as expressed in centivolts read on the voltmeter which was connected with the magneto. This should be a constant in accordance with equation (11).

The third column shows that the galvanometer might be several millimeters away from the original zero at the instant of stopping. We think that thermoelectric forces were one of the important causes for this initial deflection. The throw was *always* in the direction predicted on the basis of mobile electrons with a negative charge. The throw was always sharp and distinct except at very low speeds. The return to a final rest-point was not quite so definite as the throw, probably partly owing to the gradual change in thermoelectric forces in the circuit after the machine had been stopped. In calculating the throw we always

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[1] By forward rotation we mean anticlockwise rotation of the wheel as looked at from above.

TABLE I.

Sample Readings.

Date.	Speed in Centivolts V.	Galvanometer Readings.			Galv. Throw in Mm.	$\frac{Q\ (\text{coulombs}\times 10^{11})}{V\ (\text{centivolts})}$.
		From	To	Back to		
Forward rotation:						
Nov. 23, 1915.......	80	0	+ 6¾	− ½	6¾	3.91
" "	79	+ 5	+11	+ 4	6	3.53
" "	90	− 1	+ 8	− 1½	7	4.64
" "	88	− 1	+ 7	− 1	8	4.22
" "	80	+ 4½	+12	+ 4½	7½	4.35
Nov. 25, 1916.......	89	− ½	+ 6½	+ ½	6	3.70
" "	80	0	+ 4½	0	4½	2.64
" "	89	+ 2	+11	+ ½	9	4.66
" "	89	+ 2½	+ 8¾	+ 2½	6¼	3.24
" "	92	+ 1	+ 7½	+ 1	6½	3.25
					Average	3.81
Backward rotation:						
Dec. 6, 1915.......	77	− 3	+ 2	− 1½	5	3.15
" "	80	− 1½	+ 3½	− 1	5	3.04
" "	80	+ 2¼	+ 7¼	+ 2½	5	3.04
" "	71	− 1½	+ 4½	− ½	6	4.11
Dec. 7, 1915.......	86	− 2½	+ 4¾	+ 2½	7¼	4.09
" "	87	− ½	+ 6	− 1½	7½	3.07
" "	77	− 1¾	+ 3¼	− 1½	5	3.15
Dec. 8, 1915.......	73	+ 5½	+12¼	+ 7	6¾	4.22
" "	75	+ 3½	+ 7	+ 1½	3½	2.21
" "	70	+ 4½	+ 7	+ 2	2½	1.69
" "	88	+10¼	+14½	+ 9½	4¼	2.29
" "	72	+ 2	+ 7	− 3	5	3.29
" "	82	0	+ 6½	0	6½	3.76
" "	70	− 5	0	− 5½	5	3.39
Dec. 9, 1915.......	65	+ ½	+ 6½	+ ½	7	4.48
Dec. 10, 1915.......	87	− 6½	0	− 3½	6½	3.51
" "	82	− 2½	+ 3¼	− 2½	5¾	3.29
" "	83	− 7½	− 1½	− 7½	6	3.39
Average...........	....	....	....	....	....	3.28

took the difference between the initial point and maximum deflection, although sometimes we could note that a large displacement of the rest point corresponded to an increased or decreased throw (see for example the third throw on December 8). Owing to the width of the image of the filament and the unsteadiness of the galvanometer it was not possible to estimate the position of the galvanometer nearer than about one half or, under favorable conditions, one quarter of a millimeter.

Provided the wheel were stopped with reasonable quickness we found no effect of the speed of stopping on the magnitude of throw. The period of the galvanometer was about 10 seconds and the time necessary to stop the wheel was a fraction of a second.

It was only very rarely necessary to exclude a run from the record because of some known accident occurring during the trial, such as short circuiting, breaking of the connecting wires, or passage of an automobile, etc. A very few runs were also omitted because of a departure of very unusual magnitude arising from unknown causes.

It will be seen from the averages in the last column of Table I. that the throw under these particular circumstances was likely to be a little larger for forward than for backward running. We do not know the whole reason for this unpleasant phenomenon. We think, however, that it is very likely due to small accidental electromotive forces produced by the motion of the coil relative to the wheel when the wheel is stopped. This is borne out by some experiments which we have since made with aluminum wire where very elaborate precautions in the way of shellacking and thus binding together the successive layers of wire were necessary before we could get satisfactory results both with forward and backward running. It is significant to note, as will be seen from Table II., where forward runs are indicated by the plus and backward by the minus sign, that the accidental effect, whatever it is in the case of the shorter wire, made the backward runs give the larger result. (See Remarks under *Vertical Anti-Earth Coil.*) It should be noted that this accidental effect is only of the same order of magnitude as the errors of observation. The study and, if possible, the elimination of this effect would be very desirable. It is, however, a very difficult problem to tackle.

Variation of Q with Speed.—The most immediate purpose of the experimental work was to test the validity of equation (11),

$$Q = \frac{(m - k)lv}{RF}, \tag{11}$$

for the pulse of electricity sent through the galvanometer on stopping. For this purpose we first made a series of runs with the complete length of copper wire varying the velocity. Equation (11) being true, the number of coulombs Q should be proportional to the velocity of rotation. This is shown by Fig. 4, which is made from the data in the first eight lines of Table II., plotting Q as ordinates against the velocity v as abscissæ. The table includes runs made in both directions, the forward runs being indicated by a plus sign and the backward runs by a negative sign. It will be seen from the plot that Q is very closely proportional to the speed.

The fourth column in Table II. gives the rim velocity of the wheel in centimeters per second for the average diameter of the coil of wire. Each value of Q corresponds to a number of runs made in the neighborhood of

TABLE II.

No. of Runs.	Resistance in Ohms.	Length of Wire in Centimeters.	Velocity in Cm./Sec.	Q Coulombs $\times 10^9$.	$(m-k)\times 10^4$ $O=16$.	$\left(\frac{1}{(m-k)}\right)$ $O=16$.
23	40	46,650	+1,980	1.30	5.43	1840
15	40	46,650	−1,980	1.10	4.61	2170
10	40	46,650	+2,820	1.61	4.72	2120
9	40	46,650	+3,670	2.39	5.38	1860
18	40	46,650	−4,400	2.56	4.83	2070
10	40	46,650	+4,790	3.24	5.58	1790
11	40	46,650	+5,520	3.43	5.15	1940
11	40	46,650	−5,640	3.15	4.63	2160
12	32	30,370	+4,960	2.54	5.21	1920
12	32	30,370	−4,960	3.33	6.84	1460
Total 131					Av. 5.238	Av. 1910

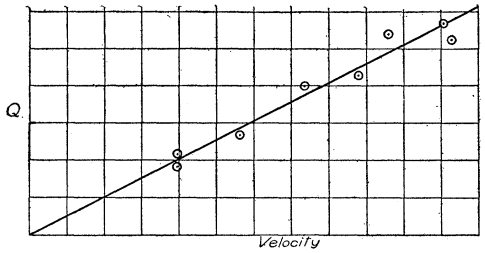

Fig. 4.

this speed, the reduction to a common speed being made by assuming that Q was proportional to the speed over the short range involved.

Variation of Q with Length.—Equation (11) also requires that Q should be proportional to the length of the wire. To test this relation, part of the wire was removed and the runs in the last two lines of Table II. were made. This change in length of wire incidentally changed the total resistance of the circuit from 40 to 32 ohms so that this must also be allowed for on the assumption that Q would be inversely proportional to the total resistance R in the circuit. We obtain from Table II. for the runs with the long wire an average value of 0.522 for the quantity QR/lv and for the runs with the short wire an average value for the same quantity of 0.624 which agree within twenty per cent., which is perhaps as close as could be expected considering the small number of runs made with the short wire.

Calculation of $(m-k)$.—We also used the data in Table II. for the purpose of calculating the value of $(m-k)$ in copper, in accordance with equation (11). The values found are given in the next to the last column and their reciprocals in the last column.[1]

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[1] The calculation of $(m-k)$ can be made directly from equation (11) in the form

In order to get an idea of the "probable error" in our determination of $(m - k)$, we given in Table III. a statement of the number of runs which gave values of $(m - k)$ lying within definite equal ranges, thus showing how the individual measurements are distributed about the mean. These results are shown by the plot in Fig. 5, where the abscissæ give the values of $(m - k)$ and the ordinates the number of determinations falling within the small range in question. The highest point of the curve is the mean value for $(m - k)$ and it will be seen that the curve closely resembles the familiar probability curve, indicating that deviations from the mean value are due to indiscriminate accidental effects. The shape of the curve also shows that further measurements with the same appa-

Table III.

Distributions of Values of $(m - k)$.

No. of Runs.	$(m-k) \times 10^4$ Lying Between
11	184 and 316
35	316 and 448
37	448 and 580
35	580 and 712
11	712 and 844
2	844 and 976

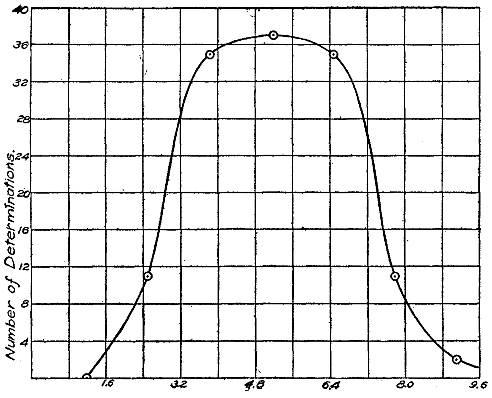

Fig. 5.

$$(m - k) = \frac{QRF}{lv}$$

provided we substitute all our values in C.G.S. units. Consider for example the calculation of the first value of $(m - k)$ in Table II. We have $Q = 1.30 \times 10^{-10}$ abcoulombs, $R = 40 \times 10^9$ abohms, $F = 9{,}650$ abcoulombs, $l = 46{,}650$ cm., $v = 1{,}980$ cm. per sec.

$$(m - k) = \frac{1.30 \times 10^{-10} \times 40 \times 10^9 \times 9{,}650}{46{,}650 \times 1{,}980} = 5.43 \times 10^{-4} \text{ gm.}$$

ratus would probably not lead to any change in the value of $(m-k)$ greater, say, than $\pm 0.8 \times 10^4$.

DISCUSSION OF RESULTS.

The Value of k.—We have calculated the value of $(m - k)$ from equation (11), which may be written

$$mv = \frac{QRF}{l} + kv. \qquad (11)$$

In this equation mv is the momentum associated with some particular equivalent of electrons situated in the wire, and since these electrons are ultimately brought to rest this momentum is equated to the time integral of the three kinds of forces—"frictional," "electrical" and "accelerational"—which are acting.

QRF/l is the sum of the amounts of momentum destroyed by the "frictional" and "electrical" forces acting on the elctrons, while kv is the momentum destroyed by what we have called the "accelerational" force. Both the "frictional" and the "accelerational" force are exerted by the main body of the wire on the electrons which it contains; the "frictional" force arises because of the motion of the electrons through the body of the wire, while the "accelerational" force arises because of the interaction of the electrons with a body of metal which is itself being accelerated and this force would exist even if the electrons were not lagging behind.

It is difficult to make any estimate as to the value of k. If, however, we assume that the electrons which are doing the conducting are largely present in interatomic spaces which form a relatively large part of the total volume of the metal then we might expect k to be small. And at first sight this might seem to be in agreement with the experimental fact that our value of $(m - k)$, 1/1910, is only slightly smaller than the accepted value for the equivalent mass of slow-moving electrons in free space, *i. e.*, 1/1845.

The Value of m.—In interpreting the results, however, there are modifying effects on the value of m which must also be considered. The mass of an electron is presumably the mass associated with the electromagnetic field which surrounds it. In accordance with the theory of the relativity of motion this mass in grams is equal to the energy of the electromagnetic field in ergs multiplied by the square of the velocity of light, so that the amount of this mass will evidently depend upon the configuration of the field. For a slow-moving electron in free space the lines of force spread out radially and run off to infinity, most of the mass, however, being located close to the electron itself. For an electron situated in the body of a metal the configuration of the field will be changed. The fact that the lines of force for electrons in a metal no longer run off to infinity will be a factor tending to decrease the electromagnetic mass, such a shortening of the lines of force being accompanied by a decrease in the potential energy of the field. On the other hand there are two important causes which might make the mass of electrons in metals larger than in free space. In the first place, the lines of force from an electron in a metal will not run out radially but will be bunched in the direction of the nearest positive charges, and such a distortion is accompanied by an increase in potential energy and hence an increase in the mass of the system. Part of the mass of such a system, however, might be thought of as belonging to the positive charge. Furthermore, an overlapping of the fields of two electrons can lead to an increase in mass, since at any point the field strengths will be directly additive while the energy density at the point in question will go up as the square of the field strengths.

Owing to these uncertainties as to the values of k and m in metals we cannot for the present draw any definite conclusions from the close correspondence of the mass of an electron in free space and our value for $(m - k)$. The evidence presumably favors the belief that k is small and m not much different from its value in free space, but of this we cannot yet be certain. One natural method of attacking the general problem is to carry out measurements with other metals than copper and see whether $(m - k)$ is always of the same order of magnitude as m in free space. Measurements are now under way on aluminum and silver, and we hope to report on these at a later time. We can already state that the results are in the same direction and of the same order of magnitude as in copper.

On the Nature of the Conducting Process in Metals.—Our purpose in carrying out the measurements that we have described has not been merely to demonstrate an effect which has long been an object of search, we have also had in mind the possibility of obtaining from our experiments information as to the nature of the conducting process in metals and indeed perhaps further information as to the nature of the electron itself.

Equation (11) which we have tested in this article was derived on the assumption that the conducting process in metals is in the nature of a drift of "free" electrons when acted on by an electric field, and the fact that the equation seems to fit the experimental facts is to some extent a verification of these assumptions. Such considerations are of particular interest at the present time in view of Sir J. J. Thomson's proposal[1] of a

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[1] Thomson, Phil. Mag., *30*, p. 192, 1915. Richardson, ibid., *30*, p. 295, 1915.

quite different theory of metallic conduction. According to his theory, a metal contains atoms which are in the nature of electrical doublets which will orient themselves parallel to any applied electrical field. These atoms are assumed to have the power of ejecting electrons in the same direction as the axis of the doublet and hence the conducting process on the basis of this theory consists in a tendency for orientation of the doublets under the action of the applied electromotive force and consequent ejection of electrons from one atom to another in the direction which the current is known to flow. It seems very doubtful to us whether such a theory can be satisfactorily brought into agreement with our experimental results, since it would seem at first sight to be merely an accidental coincidence if the *mechanical* forces which we apply should produce an orientation in the right direction and of the right amount to give the pulse of electricity whose magnitude we have calculated on the basis of the other theory and actually found experimentally.

UNIVERSITY OF CALIFORNIA,
February 25, 1916.

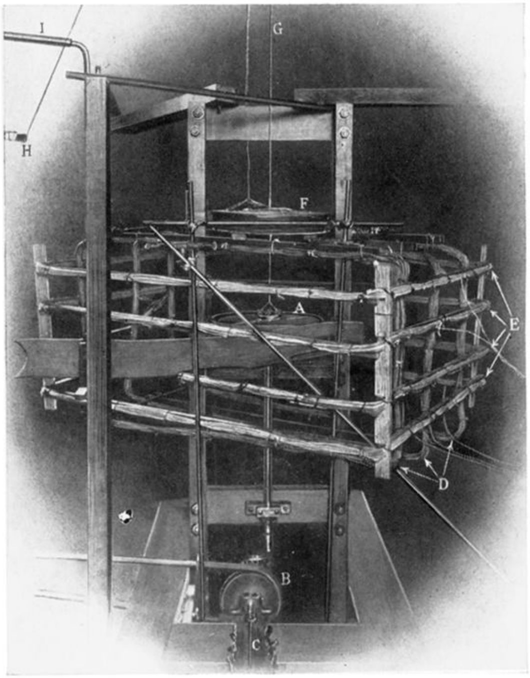

FIG. 2.

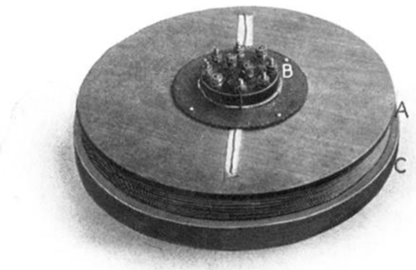

FIG. 3.