A Bayesian Probabilistic Approach to Structural Health Monitoring

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ABSTRACT

Some general issues associated with on-line structural health monitoring are discussed. In order to address the problem of determining the existence and location of damage in the presence of uncertainties, a global model-based structural health monitoring method which utilizes Bayesian probabilistic inference is developed. The results of tests using simulated data are described.

NOMENCLATURE

N_m	:	Number of modes
N_o	:	Number of observed degrees of freedom
N_d	:	Number of of degrees of freedom
N_θ	:	Number of model parameters
N_s	:	Number of measured data sets
$\hat{\omega}_r$	:	rth measured modal frequency
$\hat{\psi}_r$	:	rth measured modeshape vector
σ_{ω_r}	:	Standard deviation for $\hat{\omega}_r$
ϵ_{ψ_r}	:	Measure of variation for $\hat{\psi}_r$
$\hat{\phi}_r$	:	rth expanded modeshape vector
$\hat{\Upsilon}_n$	:	nth modal parameter set
$\mathcal{D}_{ud}$	:	Set of undamaged data sets
$\mathcal{D}_{pd}$	:	Set of potentially damaged data sets
M, C, K	:	Mass, damping and stiffness matricies
K_i	:	Substructure stiffnes matrix
$\mathcal{M}_{N_d}$	:	Model class
Γ	:	Selection matrix
θ	:	Model parameter vector
J,J_r	:	Overall and modal measures of fit
P_i^{dam}	:	ith probabilistic damage measure
P_i^{alarm}	:	ith alarm bound
t_{mon}	:	Monitoring cycle
k	:	Monitoring cycle window
R	:	Real numbers

1. INTRODUCTION

Structural health monitoring, or SHM for short, is the process of establishing some knowledge of the current condition of a structure. The ultimate goal is to determine the existence, location, and degree of damage in a structure if damage occurs. A great deal of research in the past thirty years has been aimed at establishing effective local and global methods for health monitoring in civil, mechanical, and aerospace structures. An extensive survey of global methods which use vibration characteristics to perform SHM is presented in Doebling et al. [1].

One typical global approach involves comparing structural models identified using sets of modal data (i.e. frequencies and modeshapes) from a structure before and after damage has occurred. This *model-based* SHM approach relies on structural model updating methodologies to solve the inverse problem of determining the parameters of a structural model given some modal data. The critical assumption is made that changes in the parameters of the structural model imply changes in the parts of the real structure associated with the model parameters.

There are some inherent features of structural model updating which lead to difficulties in the model-based SHM approach. The process of identifying the model parameters from the modal data is generally ill-conditioned. Thus, small changes in the modal data lead to proportionally larger changes in the model parameters. Model error and variations in the modal data due to noise, when combined with the ill–conditioning, can lead to large variations in the identified model parameters which are not due to true changes in the structure. Thus, there is uncertainty in whether changes in identified model parameters reflect damage in the structure.

Many of the available papers in the SHM literature do not consider this uncertainty. When the structure under consideration is well–characterized by the analytical model, many controlled measurements can be taken, and the measurements have very low noise levels, no significant uncertainty may be present, and ignoring it may not lead to any problems. However, in many cases, such as with civil structures, these assumptions do not apply. The analytical models rarely capture the full behavior of the structure. For instance, the model may not account for effects such as thermally-induced diurnal variations and excitation amplitude dependence of the modal parameters. Further, the available measured information is restricted by limits on the amount of instrumentation and the fact that only a few of the lower modes of a civil structure can generally be determined with confidence. Finally, measured modal data tends to show significant variation from one measurement to the next. Any SHM method applied in the case of civil structures should therefore account for the substantial uncertainty which arises in the identified structural model parameters.

In addition to neglecting the uncertainties, most methods tend to look for damage using only one set of data from the undamaged structure and another from the structure in a potentially damaged state. Such approaches may attempt an ad hoc treatment of the uncertainty by using average modal data sets over a series of modal measurements in the undamaged and potentially damaged states. In situations where the structure is only measured during infrequent periodic inspections or following a severe loading event for which structural damage is suspected, such methods are potentially useful. However, treating the problem in this fashion ignores the long–range "monitoring" goal of SHM. This goal is to continually monitor a structure so that gradual deterioration, as well as damage from severe events such as earthquakes, can be detected. Few, if any, methods explicitly consider this "monitoring" aspect of SHM.

There are a number of advantages to treating SHM as a continual process. First, the effects of noise in the data can potentially be mitigated by using multiple modal measurements. Also, by observing the structure continually, systematic changes may be separated from random fluctuations. Those systematic variations which are not due to damage, such as diurnal effects, can be included in the model to decrease model error. In this manner, effects of gradual damage, such as that due to fatigue and corrosion, are more likely to be detected.

2. BAYESIAN PROBABILISTIC SHM

This paper introduces a *continual on–line* Bayesian probabilistic SHM technique which addresses the ill–conditioning inherent in the inverse problem. The approach requires a linear structural model whose stiffness matrix is parameterized to develop a class of possible models. The parameterization involves grouping the elements of the structural model into substructures. Modal data (i.e. frequencies and incomplete modeshapes) measured from a structure are used to identify the model substructure "stiffness" parameters. In a deterministic SHM scheme, differences in the "best" parameters identified from different modal data sets would be used as indicators of damage. However, rather than consider only single "best" models for each modal data set, the probabilistic method takes uncertainties in the identified model into account by treating the problem within a framework of plausible inference. Bayes' theorem is invoked to develop a probability density function (PDF) for the model stiffness parameters conditional on measured modal data and the class of possible models. Using conditional PDFs derived from sets of modal data determined at different times, a probabilistic damage measure is developed. The probabilistic damage measure arises in answer to the question: *Based on the available modal data and acknowledging the uncertainty, what is the probability that the current model stiffness parameters are less than the corresponding undamaged stiffness parameters?* A simple graphical representation of the damage measure and the interpretation of this measure for use in SHM will be discussed. Before presenting the SHM method, a few terms are first defined.

2.1 Modal Data and Structural Model Class

The identified modal parameters consist of N_m frequencies, $\hat{\omega}_r$, and N_m generally incomplete modeshapes, $\hat{\psi}_r \in \mathrm{R}^{N_o}$, where N_o represents the number of observed degrees of freedom and $r \in 1, \ldots, N_m$. These modal parameters can be identified from measured time–domain data using any reliable modal parameter identification method. The values identified from the nth time–domain data set are referred to by $\hat{\Upsilon}_n$. A grouping of modal parameter data sets taken at different times is called *modal data*, $\mathcal{D}$.

A set of N_d degree–of–freedom (DOF) deterministic models, $\mathcal{M}_{N_d}$, which have dynamic behavior characterized by the equation of motion, $M\ddot{x} + C(\theta)\dot{x} + K(\theta)x = f(t)$, with $f, x \in \mathrm{R}^{N_d}$, and $M, C, K \in \mathrm{R}^{N_d \times N_d}$ is used as the model class. The mass matrix is assumed known with sufficient accuracy from structural drawings, and the damping matrix is assumed to be of a form so that the models possess classical normal modes. The models in $\mathcal{M}_{N_d}$ are parameterized by the structural parameters, $\theta \in \mathrm{R}^{N_\theta}$, which define $K(\theta)$ in terms of a linear combination of substructure stiffness matrices, K_i:

$$K(\theta) = K_0 + \sum_{i=1}^{N_\theta} \theta_i K_i. \quad (1)$$

The substructure stiffness matrices model the contributions of a portion of the structure to the overall stiffness matrix. The nominal model, which corresponds to simply summing the K_i, is given by $\theta_F = [1, \dots, 1]^T$. Expressing the dependence of the stiffness matrix on the structural parameters in the form given by (1) is convenient for mathematical analysis. However, the method which is developed does not preclude using a more general parameterization. For the purpose of health monitoring, the linear parameterization given is sufficient to enable determination of the existence and location of damage.

2.2 Probabilistic Framework

In the absence of noise and model error, a single model in the model class could be used to match the modal data. Differences in models identified from undamaged and potentially damaged data would then indicate changes in the structure. However, due to the aforementioned uncertainties, such a direct approach to SHM is not possible. The problem is therefore treated in a probabilistic context.

The method presented in this paper extends work done by Beck [2],[3] in probabilistic system identification. Beck used a system of plausible inference based on work by Cox [4] and Jaynes [5]. In this framework, conditional probabilities are used as measures of the plausibility of certain statements given other statements. Bayes' theorem is used to express the conditional probabilities of the model parameters given the data:

$$p(\theta|\mathcal{D}, \mathcal{M}_{N_d}) = c p(\mathcal{D}|\theta, \mathcal{M}_{N_d}) p(\theta|\mathcal{M}_{N_d}). \quad (2)$$

Here, c is a normalizing constant. The distribution $p(\theta|\mathcal{M}_{N_d})$ is the initial PDF for the model parameters based on engineering and modeling judgment. The data distribution, $p(\mathcal{D}|\theta, \mathcal{M}_{N_d})$, is the PDF for the modal parameters given the model parameters. Unless required to maintain a conditional form for a PDF, the explicit dependence of distributions on the model class, $\mathcal{M}_{N_d}$, is dropped in future expressions in order to simplify the notation.

Suppose that $\mathcal{D}$ comprises N_s data sets such that $\mathcal{D} = \mathcal{D}_{N_s} = \{\hat{\Upsilon}_1, \dots, \hat{\Upsilon}_{N_s}\}$. Using the axioms of probability, $p(\mathcal{D}|\theta)$ can be written as

$$\begin{aligned} p(\mathcal{D}_{N_s}|\theta) &= p(\hat{\Upsilon}_{N_s}|\mathcal{D}_{N_s-1}, \theta)\, p(\mathcal{D}_{N_s-1}|\theta) = \cdots \\ &= \prod_{n=1}^{N_s} p(\hat{\Upsilon}_n|\mathcal{D}_{n-1}, \theta), \end{aligned} \quad (3)$$

where $p(\hat{\Upsilon}_1|\mathcal{D}_0, \theta) = p(\hat{\Upsilon}_1|\theta)$. The assumption is made that $p(\hat{\Upsilon}_n|\mathcal{D}_{n-1}, \theta)$ is independent of $\mathcal{D}_{n-1}$. This assumption reflects that the user's uncertainty in the nth modal parameters, when a structural model is given, is not influenced by the previous modal data. Thus, (2) becomes

$$p(\theta|\mathcal{D}_{N_s}) = c p(\theta|\mathcal{M}_{N_d}) \prod_{n=1}^{N_s} p(\hat{\Upsilon}_n|\theta). \quad (4)$$

This result shows that within the Bayesian framework, new data can be incorporated into the PDF for the model parameters in a systematic and consistent fashion by simply extending the product by one term. The details of forming $p(\hat{\Upsilon}_n|\theta, \mathcal{M}_{N_d})$ and $p(\theta|\mathcal{M}_{N_d})$ appear in Vanik [6]. Once the necessary choices are made, the final form of the PDF for θ conditional on $\mathcal{D}$ is given as:

$$p(\theta|\mathcal{D}) = k \exp\left[-\frac{1}{2} J(\theta)\right] \quad (5)$$

where the *overall measure of fit* (MOF), is

$$J(\theta) = (\theta - \theta_F)^T S^{-1} (\theta - \theta_F) + \sum_{r=1}^{N_m} J_r(\theta), \quad (6)$$

and the *modal measure of fit* (MMOF) for mode r is

$$J_r(\theta) = \sum_{n=1}^{N_s} \left[\frac{\left\| (K - \hat{\omega}_r^2(n) M) \hat{\phi}_r(\theta) \right\|_{M^{-1}}^2}{\sigma_{\omega_r^2}^2 \|\hat{\phi}_r(\theta)\|_M^2} + \frac{\hat{\phi}_r(\theta)^T \Gamma^T \left(I - \psi_r(n) \psi_r^T(n)\right) \Gamma \hat{\phi}_r(\theta)}{\epsilon_{\psi_r}^2 \|\Gamma \hat{\phi}_r(\theta)\|^2} \right]. \quad (7)$$

The matrix S is a diagonal matrix of variances which reflect the initial level of uncertainty in the analytical model based on engineering and modeling judgment. The parameter $\sigma_{\omega_r^2}$ is the experimentally determined standard deviation for the rth measured frequency. The parameter $\epsilon_{\psi_r}^2$ is an experimentally determined measure of the variation in the rth modeshape. The vectors $\hat{\phi}_r(\theta) \in \mathrm{R}^{N_d}$ are *optimally expanded modeshapes*, which minimize the MOF with respect to ϕ_r for a fixed θ. The matrix Γ picks the observed degrees of freedom from $\hat{\phi}_r$. Although the PDF in (5) gives a non-zero probability for non-physical negative stiffness values, the amount of probability volume less than zero is generally negligible, so truncation of the PDF for negative values followed by a re-normalization is not necessary.

The PDF (5) on θ is integrated over all N_θ parameters but one to get the marginal distribution for a single parameter. For each parameter, θ_i, $i \in 1, \dots, N_\theta$, this gives

$$p(\theta_i|\mathcal{D}) = \int_{\Theta^i} p(\theta_i, \theta^i|\mathcal{D}) d\theta^i \quad (8)$$

where $\theta^i = [\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_{N_\theta}]^T$ and Θ^i is the set where θ^i varies across its full possible range. This integration cannot be performed analytically. Therefore, the integral is evaluated using an asymptotic expansion technique based on Laplace's method [7]. The final result gives

$$p(\theta_i|\mathcal{D}) = \varphi\left[\frac{\theta_i - \hat{\theta}_i}{\sqrt{e_i^T L(\hat{\theta})^{-1} e_i}}\right], \tag{9}$$

where φ is the Gaussian PDF with zero mean and unit variance, $\hat{\theta}$ maximizes $p(\theta|\mathcal{D})$, $L(\hat{\theta})$ is the Hessian matrix of $J(\theta)$ evaluated at $\hat{\theta}$, and e_i is a vector with the i^{th} component equal to one, and all other components zero. Using the marginal distributions, the question which was posed at the beginning of this section can now be answered.

2.3 Probabilistic Damage Measure

The data from a structure in a known undamaged state is *undamaged data*, $\mathcal{D}_{ud}$, while data from the same structure in an unknown state is called *potentially damaged data*, $\mathcal{D}_{pd}$. The marginal PDF for the i^{th} model parameters derived using only the undamaged data, $\mathcal{D}_{ud}$, and potentially damaged data, $\mathcal{D}_{pd}$, are $p^{ud}(\theta_i^{ud}|\mathcal{D}_{ud})$ and $p^{pd}(\theta_i^{pd}|\mathcal{D}_{pd})$ respectively. The probability that the i^{th} potentially damaged parameter is less than the i^{th} undamaged parameter is

$$P_i^{dam} = P\{\theta_i^{pd} < \theta_i^{ud}|\mathcal{D}_{ud}, \mathcal{D}_{pd}\}. \tag{10}$$

The quantity P_i^{dam} is called the *probabilistic damage measure*, or simply the *damage measure*, since it is a probabilistic measure of the variation of θ_i^{pd} below θ_i^{ud}. Using the marginal distributions, P_i^{dam} can be shown to be

$$P_i^{dam} = 1 - \int_{-\infty}^{\infty} C^{ud}(\theta_i)p^{pd}(\theta_i)d\theta_i, \tag{11}$$

where $C^{ud}(\cdot)$ is the cumulative distribution for θ_i^{ud}. The damage measure is calculated and monitored for each model parameter in order to perform SHM. Changes in P_i^{dam}, rather than changes in the θ_i, are studied to detect structural damage. C^{ud} and p^{pd} are evaluated using (9) with $\mathcal{D} = \mathcal{D}_{ud}$ and $\mathcal{D}\mathcal{D}_{pd}$ respectively.

2.4 Using The P_i^{dam} For SHM

In a continual on–line monitoring scenario, many modal data measurements will be available. As noted, these modal data are grouped into $\mathcal{D}_{ud}$ and $\mathcal{D}_{pd}$. A question arises as to which data from $\mathcal{D}_{pd}$ should be used to form the potentially damaged marginal PDFs and calculate P_i^{dam}. If only the most recently measured modal parameter set is used, then damage could be detected as soon as it occurs. However, the damage measure would be sensitive to noise in the modal parameters and could therefore give rise to misleading conclusions. If many sets of data are used, the effects of noise will be mitigated. Thus, smaller levels of damage will be detectable. Unfortunately, if damage occurs, the marginal PDF, and thus P_i^{dam}, will be strongly biased by the undamaged data already in $\mathcal{D}_{pd}$, so many additional measurements must be made before the damaged data can overcome the bias to indicate the existence of damage. In between these extremes are choices which trade off between sensitivity, noise mitigation, and bias.

Rather than consider only one such choice, P_i^{dam} is calculated for a range of modal parameter sets from $\mathcal{D}_{pd}$. Thus, P_i^{dam} is calculated for the most recently measured data set, the two most recently measured data sets, and so forth until a limiting number of previous data sets, N_{win}. Each time a modal measurement is performed, P_i^{dam} is recalculated for all of the different subgroupings of modal parameter sets. The number of modal measurements performed is the current *monitoring cycle*, t_{mon}. The *monitoring cycle window* or simply *window*, k, indicates how many previous data sets from $\mathcal{D}_{pd}$, starting at t_{mon}, are used to calculate P_i^{dam}. Thus, P_i^{dam} is a function of t_{mon} and k. Calculating P_i^{dam} in this fashion, as both t_{mon} and k vary, leads to the novel concept of monitoring the fluctuation of the damage measure as a function of time and the amount of data used. In testing with simulated data, this approach has shown promise as a means of detecting and locating a wide range of levels of damage.

The measure of the likely variation in P_i^{dam} for a given k due to noise in the modal parameters when no damage is present is characterized using $P_i^{alarm}(k)$, where

$$P_i^{alarm}(k) = 1 - \int_{-\infty}^{\infty} C^{ud}(\theta_i)\varphi\left[\frac{\theta_i - (\hat{\theta}_i^{ud} - \gamma\sigma_i^{mod}(k))}{\frac{N_s^{ud}}{k}\sqrt{e_i^T L(\hat{\theta}^{ud})^{-1} e_i}}\right] d\theta_i. \tag{12}$$

The model $\hat{\theta}_i^{ud}$ maximizes $p^{ud}(\theta|\mathcal{D}_{ud})$. The term $\sigma_i^{mod}(k)$ is the sample standard deviation for $\hat{\theta}_i^{ud}$ determined using a bootstrap technique. In the bootstrap procedure multiple $\hat{\theta}^{ud}(k)$, $k \in 1, \ldots, N_s^{ud}$, are determined from modal values simulated assuming Gaussian PDFs. The mean and standard deviation for the Gaussian PDFs on the modal parameters are the sample means and standard deviations based on the observed data. N_s^{ud} is the number of undamaged data sets. The term $\frac{N_s^{ud}}{k}$ is the proper scaling to reflect the increased uncertainty in the identified $\hat{\theta}^{ud}$ as fewer than N_s^{ud} data sets are used to calculate P_i^{dam}. The parameter γ determines the relative frequency of indicating damage when there is none (i.e. a *false alarm*) and missing damage when it exists (i.e. a *missed alarm*). For each window, k, the resulting Gaussian PDF, $\varphi[\cdot]$, reflects the expected variation in $p^{pd}(\cdot)$ as new undamaged data is taken.

For a given monitoring cycle, if $P_i^{dam}(t_{mon}, k)$ exceeds $P_i^{alarm}(k)$ for any k, an alarm is sounded that the substructure for which the excessive variation occurred may be "damaged":

$$
\begin{aligned}
P_i^{dam}(t_{mon}, k) &\geq P_i^{alarm}(k) \text{ for any } k \Rightarrow \text{“Alarm”} \\
&< P_i^{alarm}(k) \text{ for all } k \Rightarrow \text{“No Alarm.”}
\end{aligned} \tag{13}
$$

Thus, smaller values of γ increase the chance of false alarms while larger values increase missed alarms.

Since P_i^{alarm} is a soft level rather that a hard level, additional decisions should be made when the damage alarm is set in order to determine whether the level was exceeded due to damage, or an extreme data set. In addition to the alarm criterion in (13), therefore, other criteria should be considered in order to determine the difference between false and true alarms. In practice with simulated data, $P_i^{dam}(t_{mon}, k)$ is found to have characteristic behaviors depending on whether the structure is undamaged or damaged. Therefore, the behavior of $P_i^{dam}(t_{mon}, k)$ can be studied by a human or an expert system as a function of t_{mon} and k in order to determine the state of damage. During the description of the simulation testing in the next section, the behaviors of $P_i^{dam}(t_{mon}, k)$ for a simple structure are discussed, and some guidelines for use in determining a true state of damage from a false one are given.

2.5 Applying the SHM Approach

In practice, the proposed method could be applied as an on–line automated structural health monitoring system through the following procedure. This procedure assumes that the structure under consideration has already been instrumented, and the model class and form of the PDFs for the application have already been determined.

- During an *initialization phase*, take many measurements from the structure in the undamaged state and establish a stable reference undamaged PDF
- Start the *monitoring phase* wherein the structure is measured periodically and $P_i^{dam}(t_{mon}, k)$ is calculated after each measurement.
- If $P_i^{dam}(t_{mon}, k) < P_i^{alarm}(k)\ \forall i, k$, wait for the next set of data.
- If $P_i^{dam}(k) \geq P_i^{alarm}(k)$ for any i, k, consult an expert system (human or computer encoded logic) about how to proceed. The expert system decides whether the alarm appears false, true, or uncertain based on criteria establish through previous investigations.

3. ILLUSTRATIVE EXAMPLES

This section reports on the results of testing the method. During the testing, 2–DOF and 10–DOF shear structure models are used to explore the various features, strengths, and limitations of the probabilistic SHM approach. The lumped masses are $m_1 = 2 \times 10^4$ kg and $m_2 = 1 \times 10^4$ kg. The substructure stiffness matrices for the 2-DOF case are

$$
K_1 = \begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad K_2 = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \tag{14}
$$

where the "undamaged" model has $k_1 = 1200$ kN/m and $k_2 = 1000$ kN/m. The model parameters, θ_1 and θ_2, therefore scale the "undamaged" interstory stiffnesses, so that $K(\theta) = \theta_1 K_1 + \theta_2 K_2$. The 10–DOF shear structure model is of the same form with $m_1, m_2, m_3 = 3 \times 10^4$ kg, $m_4, \ldots, m_9 = 2 \times 10^4$ kg, $m_{10} = 1 \times 10^4$ kg, $k_1, k_2, k_3 = 4.8 \times 10^4$ kN/m, $k_4, \ldots, k_8 = 4.4 \times 10^4$ kN/m, and $k_9, k_{10} = 4.0 \times 10^4$ kN/m. The K_i in (1) have zero elements except for 2×2 submatricies of the form of (14).

Noisy "measured" modal parameters are generated by adding random values chosen from zero–mean Gaussian distributions to the modal parameters of a model from the model class. A coefficient of variation of 0.02 is used for the frequencies and the modeshapes. In practice, the noise levels generally vary from mode to mode, and worsen for higher modes. Also, for a given application, the relationship between the frequency and modeshape noise levels may not be the same used in this testing. Adding these levels of complexity is not considered necessary in this exploratory study.

The monitoring procedure suggested at the end of the previous section is implemented using the synthetically generated modal data. Figures 1(a) to 1(d) show results from the 2–DOF example where the modal data comprises two modes with full modeshapes. The figures show $P_1^{dam}(t_{mon}, k)$ as the monitoring cycle and window vary for different levels of damage in the first substructure. Recall, an increase in the monitoring cycle, t_{mon}, indicates that another modal parameter set has been measured, and an increase in the window parameter, k, indicates that more of the previous potentially damaged data is being used to calculate P_1^{dam}. Figure 1(a) shows the results with no damage. Figures 1(b) to 1(d) show the results with 2%, 5%, and 10% reduction in stiffness k_1 while holding k_2 fixed. Damage is present during the first depicted monitoring cycle in each of Figures 1(b) to 1(d). When damage is present, there is a marked increase in P_1^{dam} over successive monitoring cycles. The damage measure for the undamaged substructure, P_2^{dam}, behaves similarly to the undamaged measure in Figure 1(a). For these examples, therefore, there is no "smearing" of the damage into adjacent substructures.

Figures 2(a) and 2(b) show P_4^{dam} and P_5^{dam} for the 10–DOF example with a 15% reduction in stiffness in the fifth story. The modal data consists of only two modes with full modeshapes in each mode. The time sequence of plots for the fifth substructure damage measure clearly indicate change in that substructure, while those for the fourth substructure damage measure indicate no change in that substructure. Other cases were run with only partial modeshape information and more modes, and damage could still be detected in most cases. In a few cases, however, potential damage was indicated in undamaged substructures. This "smearing" effect in SHM has also been observed by other researchers [8]. Investigation is continuing into how to handle the smearing. Research is also underway in determining how many modal parameters per measurement are needed in order to successfully apply the proposed SHM approach.

Many more analyses were performed than can be presented in this paper. These few are depicted in order to illustrate the behavior and the use of the probabilistic SHM method. The manner in which the alarm function is exceeded provides some way to distinguish types of damage. For large levels of damage in the i^{th} substructure, the P_i^{dam} will quickly be driven to 1 for all k, so such damage is quickly detected. For moderate levels of damage, P_i^{dam} will not shift to 1 immediately, as in the large damage case, but will still tend toward 1 for most of the k. Low levels of damage will not cause P_i^{dam} to exceed the alarm level for small values of k with few monitoring cycles. However, as more damaged data is acquired, the probability of variation should begin to rise above the alarm level for large values of k since the effects of random noise are being reduced. Therefore, small levels of damage may be eventually detected by monitoring the structure over longer times and tracking the behavior of the P_i^{dam}. Alarms when there is no damage do not show the behaviors described for the damaged cases. Thus, recognition of these features can be programmed into an expert system to assist in the verification of an alarm when one is set.

Note that considering the data over long periods of time will not mitigate regularly persistent variations such as those due to diurnal changes. Suppose, however, that such effects can be observed while the structure is in its undamaged state, and different sets of "undamaged" data can be associated with different environmental conditions. Then, different undamaged PDFs formed from these data sets can be used as the reference PDF depending on the measured conditions. This is one way in which this type of model error can be accounted for in the proposed SHM framework.

4. CONCLUSIONS AND FUTURE WORK

This work has discussed the issues associated with uncertainty in applying SHM to real structures, and presented a method for continual on-line implementation which takes those factors into consideration. A novel approach to monitoring which involves studying the variation in time of a probabilistic damage measure was introduced. This approach may enable small levels of damage to be detected through monitoring of the structure over long times. Some preliminary results of testing on simulated data were shown, but additional work is required in a number of areas. The method must be tested on more complex simulated structures, and with data from real structures. Also, the variation of P_i^{dam} with and without damage must be further characterized so that a set of rules for use by an expert system or end-user can be established. Finally, implementation in an automated fashion must be developed to provide real-time monitoring.

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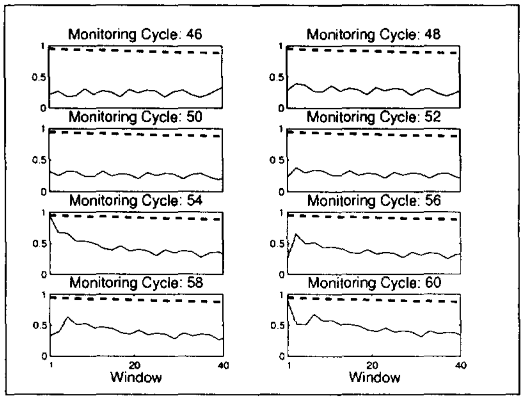
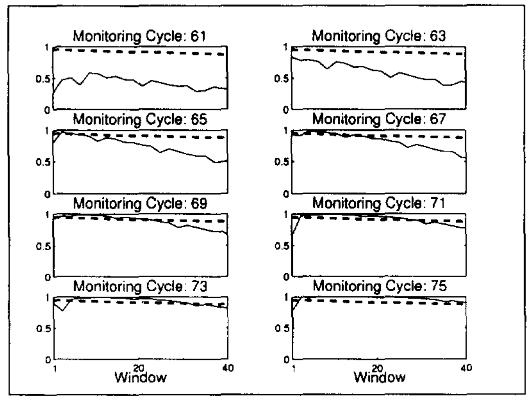
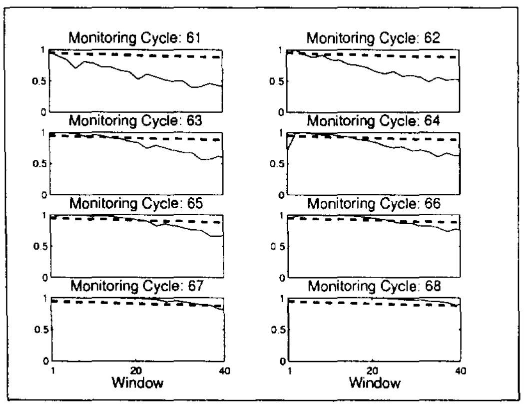


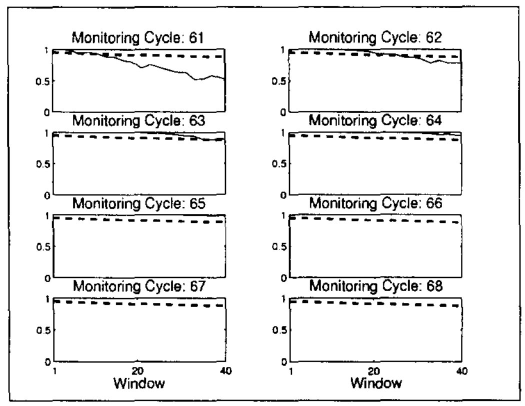
(a) No Damage



(b) 2% Damage

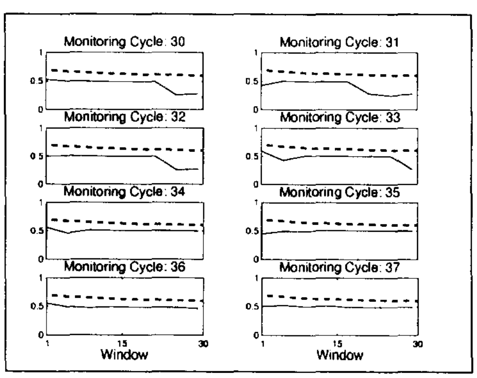


(c) 5% Damage

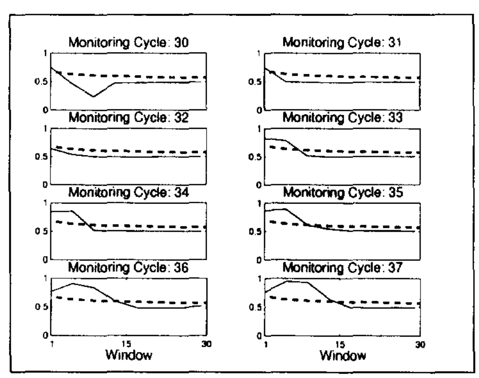


(d) 10% Damage

Figure 1 2–DOF Case, 2 modes, full modeshapes, P_1^{dam} over fifteen ((a) and (b)) or eight ((c) and (d)) monitoring cycles, (a) undamaged, (b) 2%, (c) 5%, and (d) 10% damage in first story, dashed line is the alarm function for $\gamma = 1.7$.



(a) P_4^{dam}



(b) P_5^{dam}

Figure 2 10–DOF Case, 2 modes, full modeshapes, P_4^{dam} and P_5^{dam} over eight monitoring cycles, 15% damage in fifth story, $\gamma = 3$.