Quantum Correlated D Decays at Super$B$

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We present the prospects for studying quantum correlated charm decays at the $\psi(3770)$ using 0.5–1.0 ab^{-1} of data at Super$B$. The impact of studying such double tagged decays upon measurements in other charm environments will be discussed.

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1 Introduction

Super$B$ is a next-generation high-luminosity asymmetric-energy e^+e^- collider that aims to collect 50 to 100 times more data than the B-Factories of the last decade, BABAR and Belle. The center-of-mass energy for the majority of the program will be at or near the $\Upsilon(4S)$ resonance with the designed peak luminosity of 10^{36} s^{-1}cm^{-2}. The goal is to collect $75\,\mathrm{ab}^{-1}$ over five years. Additional runs are also planned at $D\overline{D}$ threshold $\psi(3770)$ to collect 0.5–1.0 ab^{-1} over a few months. Super$B$'s physics programs include, but not limited to, heavy-flavor $B_{u,d,s}$, D, and τ physics. Super$B$ will be able to search for new physics at energy scale up to 10–100 TeV through rare/forbidden decay searches, CP violation, and precision CKM matrix measurements [1].

With $75\,\mathrm{ab}^{-1}$ of data near $\Upsilon(4S)$, $\mathcal{O}(10^{11})$ charm mesons will be created. They come from continuum production $e^-e^- \to c\bar{c}$, as well as B decays. Many charm analyses identify a D meson through $D^{*+} \to D^0\pi^+$ process. Consequently the reconstruction efficiencies are relatively low.

With $0.5\,\mathrm{ab}^{-1}$ of data at $\psi(3770)$, one can expect approximately 1.8×10^9 $D^0\overline{D}^0$ and 1.5×10^9 D^+D^- events. This amount is more than an order of magnitude larger than the current charm factory BESIII [4] will collect. As a $J^{PC} = 1^{--}$ state, $\psi(3770) \to D\overline{D}$ is in a quantum entangled, anti-symmetric state. If one D decays to state α at time t_1 and the other to β at t_2, the decay amplitude $\mathcal{M}$ is

$$\mathcal{M} = \frac{1}{\sqrt{2}}\left[\langle\alpha|H|D^0(t_1)\rangle\langle\beta|H|\overline{D}^0(t_2)\rangle - \langle\beta|H|D^0(t_2)\rangle\langle\alpha|H|\overline{D}^0(t_1)\rangle\right]. \tag{1}$$

A neutral meson mixing system can be described by a 2×2 effective Hamiltonian with non-vanishing off-diagonal terms

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}. \tag{2}$$

The eigenstates $|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$ satisfy

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}, \quad |p|^2 + |q|^2 = 1. \tag{3}$$

The eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \frac{i}{2}\Gamma_{1,2} = \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right). \tag{4}$$

Here we have assumed CPT conservation. The time evolution of $\psi(3770) \to D\overline{D} \to \alpha(t_1)\beta(t_2)$ system can then be expressed as

$$\begin{aligned} d\Gamma/dt \propto\ & (|a_+|^2 + |a_-|^2)\cosh(y\Gamma\Delta t) + (|a_+|^2 - |a_-|^2)\cos(x\Gamma\Delta t) \\ & -2\mathcal{R}e(a_+^* a_-)\sinh(y\Gamma\Delta t) + 2\mathcal{I}m(a_+^* a_-)\sinh(x\Gamma\Delta t), \end{aligned} \tag{5}$$

where $\Delta t = t_2 - t_1$, $a_+ \equiv \bar{A}_\alpha A_\beta - A_\alpha \bar{A}_\beta$, $a_- \equiv -\frac{q}{p}\bar{A}_\alpha \bar{A}_\beta + \frac{p}{q} A_\alpha A_\beta$, $M = (M_{11} + M_{22})/2$ $\Gamma = (\Gamma_{11} + \Gamma_{22})/2$, $x = (m_1 - m_2)/\Gamma$, and $y = (\Gamma_1 - \Gamma_2)/(2\Gamma)$; $A_x(\bar{A}_x)$ is the decay amplitude of D ($\overline{D}$) to X.

2 Charm mixing measurements

The mixing in neutral D system is expected to be very small. The short-distance $|\Delta F| = 2$ comes from box diagrams. The diagrams with b quark in the loop is CKM-suppressed (with V_{ub} in the vertex), and the ones with s and d quarks are GIM-suppressed. The long-distance contributions come from diagrams that connect D^0 and $\overline{D}^0$ through on-shell states (e.g., $K\overline{K}$). These long-distance contributions are expected to be $\mathcal{O}(10^{-3})$ but the theoretical calculation is difficult [3]. CP violation induced by mixing is therefore expected very small too. Observations of large mixing and/or CP violation are considered clear signs of new physics beyond the standard model.

Charm mixing has been firmly established at B-factories [2] using continuum events in the data taken near the $\Upsilon(4S)$ resonance. Both x and y terms are approximately 0.5%. These analyses use the charge of the soft pion from $D^{*+} \to D^0\pi^+$ (or its charge conjugate process) to identify the initial flavor of the D meson, and reconstruct a final state that is accessible by both D^0 and $\overline{D}^0$. The decay time distribution for the D meson tagged by π^+ ($N(t)$) and π^- ($\bar{N}(t)$) are

$$N(t) \propto \left[1 + \frac{x^2+y^2}{4}|\lambda_f|^2(\Gamma t)^2 + |\lambda_f|(y\cos(\overline{\delta_f + \phi_f}) - x\sin(\overline{\delta_f + \phi_f})(\Gamma t)\right] \tag{6}$$

$$\bar{N}(t) \propto \left[1 + \frac{x^2+y^2}{4}|\lambda_f|^{-2}(\Gamma t)^2 + |\lambda_f|^{-1}(y\cos(\overline{\delta_f - \phi_f}) - x\sin(\overline{\delta_f - \phi_f})(\Gamma t)\right], \tag{7}$$

where $x\Gamma t,\ y\Gamma t \ll 1$, $\lambda_f = (q\bar{A}_f)/(pA_f)$, and $\phi_f = \psi_f + \phi_m$, where δ_f (ψ_f) is the relative strong (weak) phase in decay, and ϕ_m is the mixing phase $\arg(q/p)$.

The cleanest mode used in this method is $D \to K^+\pi^-$, which is Cabibbo-favored in $\overline{D}^0$ decays but doubly-Cabibbo suppressed in D^0 decays. One does not measure x and y directly. Rather, the observables are rotated by the strong phase difference $\delta_{K\pi}$:

$$x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi} \tag{8}$$

$$y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}. \tag{9}$$

Independent measurements of strong phase difference are needed.

Strong phase differences can be measured in $\psi(3770) \to D\overline{D}$ decays with a "double-tag" technique. Due to the quantum-entangled nature of the system, when one D decays to a CP final state, the other D is projected to the orthogonal state,

which is a linear combination of D^0 and $\overline{D}^0$, and its decay branching fraction is sensitive to the relative strong phase of $D^0 \to f$ and $\overline{D}^0 \to f$. For example, for $f = K^-\pi^+$, the effective branching fraction of the double-tag event is [5],

$$\mathcal{F}_{S_\pm,K^-\pi^+} \simeq \mathcal{B}_{S_\pm}\mathcal{B}_{K^-\pi^+}(1 \pm 2r\cos\delta_{K\pi} + R_{\mathrm{WS}} + y), \tag{10}$$

where $\mathcal{B}_{S_\pm}$ and $\mathcal{B}_{K^-\pi^+}$ are the branching fractions of D^0 decaying to $CP\pm$ and $K^-\pi^+$ final states, respectively, $\langle K^+\pi^-|D^0\rangle/\langle K^+\pi^-|\overline{D}^0\rangle = re^{-i\delta_{K\pi}}$, and R_M is the wrong-sign total decay rate ratio, $R_M \equiv \Gamma(\overline{D}^0 \to K^-\pi^+)/\Gamma(D^0 \to K^-\pi^+) = r^2 + ry' + (x^2+y^2)/2$. CLEO-c [5] has demonstrated this technique with $281\,\mathrm{pb}^{-1}$ of data and obtained $\delta_{K\pi} = (22^{+11+\ 9}_{-12-11})^\circ$ or $[-7^\circ, +61^\circ]$ interval at 95% confidence level.

Another powerful method of measuring D^0-$\overline{D}^0$ mixing is using a time-dependent Dalitz-plot analysis with three-body decays. With this method, one can avoid strong phase ambiguity and resolve x and y by exploiting strong phase variation and interferences of resonances on the Dalitz plot. The most power mode of this kind is $D^0 \to K^0_S\pi^+\pi^-$. The time-dependent decay amplitude of a state created as D^0 or $\overline{D}^0$ at $t = 0$ can be expressed as [6],

$$\begin{aligned}
\mathcal{M}(s_{12}, s_{13}, t) &= A_D(s_{12}, s_{13})\frac{e_1(t)+e_2(t)}{2} + \frac{q}{p}\bar{A}_D(s_{12}, s_{13})\frac{e_1(t)-e_2(t)}{2}, && (11)\\
\bar{\mathcal{M}}(s_{12}, s_{13}, t) &= \bar{A}_D(s_{12}, s_{13})\frac{e_1(t)+e_2(t)}{2} + \frac{p}{q}A_D(s_{12}, s_{13})\frac{e_1(t)-e_2(t)}{2}, && (12)
\end{aligned}$$

where A_D ($\bar{A}_D$) is the decay amplitude of D^0 ($\overline{D}^0$) as a function of invariant mass squared $s_{12} \equiv m_-^2 = (p_{K^0_S} + p_{\pi^-})^2$, $s_{13} \equiv m_+^2 = (p_{K^0_S} + p_{\pi^+})^2$, and $e_{1,2}(t) = \exp[-i(m_{1,2} - i\Gamma_{1,2}/2)t]$. Using this method, Belle [7] and *BABAR* [8] measured $x = (0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14})\%$, $y = (0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08})\%$, and $x = (0.16 \pm 0.23 \pm 0.12 \pm 0.8)\%$, $y = (0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$, respectively, where the first uncertainties are statistical, the second are systematic, and the third are Dalitz plot model uncertainty.

With $75\,\mathrm{ab}^{-1}$ of at $\Upsilon(4S)$ at Super$B$, the statistical uncertainty can be reduced by a factor of 10. Since major systematic uncertainties are in fact statistical in nature, estimated from data control samples and simulated events, they will also be improved with more data. However, the Dalitz plot model uncertainty may not improve much without other input; it will become the dominant uncertainty at Super$B$ [1].

To avoid Dalitz plot model dependence, Giri *et al* [9] proposed a method, originally for measuring the CKM angle γ in $B^+ \to D[K^0_S\pi^+\pi^-]K^+$ decays using time-dependent Dalitz plot analysis. In this method, the Dalitz plot phase space is divided into N pairs of bins; two bins in each pair is mirror-symmetric about the line $s_{12} = s_{13}$

the Dalitz plane. One then can define

$$c_i \equiv \int_i dp A_{12,13} A_{13,12} \cos(\delta_{12,13} - \delta_{13,12}), \tag{13}$$

$$s_i \equiv \int_i dp A_{12,13} A_{13,12} \sin(\delta_{12,13} - \delta_{13,12}), \tag{14}$$

$$T_i \equiv \int_i dp A_{12,13}^2, \tag{15}$$

where $\delta_{1j,1k} \equiv \delta(s_{1j}, s_{1k})$, and $A_{1j,1k}$ is the magnitude of the D decay amplitude $A_D(s_{1j}, s_{1k}) = A_{1j,1k} \exp(i\delta_{1j,1k})$. The integral is over the phase space of the bin i. Here we have used the fact that $A_D(s_{12}, s_{13}) = \bar{A}_D(s_{13}, s_{12})$. The c_i and s_i contain unknown strong phase difference $\delta_{12,13} - \delta_{13,12}$, and thus unknown, but T_i can be measured with flavor tagged D^0 decays. For mirror bins, i and $\bar{i}$, $c_i = c_{\bar{i}}$ and $s_i = -s_{\bar{i}}$. With charm mixing, the number of events in bin i at time t is [10]

$$T_i'(t) \propto e^{-\Gamma t}[T_i + \sqrt{T_i T_{\bar{i}}}(c_i y + s_i x)\Gamma t + \mathcal{O}((x^2 + y^2)(\Gamma t)^2)]. \tag{16}$$

One can fit all bins simultaneously to extract mixing parameters (x, y) if (s_i, c_i) are known.

Again, using entangled $\psi(3770) \to D\overline{D}$, one can measure s_i and c_i. If one D decays into a *CP* eigenstate, the other D is in an orthogonal state. We denote these two states as $D^0_\pm \equiv (D^0 \pm \overline{D}^0)/\sqrt(2)$. The amplitude and partial decay width of the second D can be written as [9]

$$A(D^0_\pm \to K^0_S(p_1)\pi^-(p_2)\pi^+(p_3)) = \frac{1}{\sqrt{2}}(A_D(s_{12}, s_{13}) \pm A_D(s_{13}, s_{12})),$$

$$d\Gamma(D^0_\pm \to K^0_S(p_1)\pi^-(p_2)\pi^+(p_3)) = \frac{1}{2}(A_{12,13}^2 + A_{13,12}^2) \pm A_{12,13} A_{13,12} \cos(\delta_{12,13} - \delta_{13,12}) dp, \tag{17}$$

where p_i in parentheses are the momentum of the corresponding particle. We can then measure c_i using

$$c_i = \frac{1}{2}\left[\int_i d\Gamma(D^0_+ \to K^0_S(p_1)\pi^-(p_2)\pi^+(p_3)) - \int_i d\Gamma(D^0_- \to K^0_S(p_1)\pi^-(p_2)\pi^+(p_3))\right]. \tag{18}$$

If we can bin the Dalitz plot so that c_i and s_i are nearly constant in each bin, (c_i, s_i) can be determined with high precision

$$c_i = \sum_j c_j = \sum_j A_j A_{\bar{j}} \cos(\delta_j - \delta_{\bar{j}}) \Delta p_j = \sum_j \sqrt{T_j T_{\bar{j}}} \cos(\delta_j - \delta_{\bar{j}}), \tag{19}$$

$$s_i = \sum_j \sqrt{T_j T_{\bar{j}}} \sin(\delta_j - \delta_{\bar{j}}) = \sum_j \pm\sqrt{T_j T_{\bar{j}} - c_{\bar{j}}^2}. \tag{20}$$

Fit	$x \times 10^3$	$y \times 10^3$	$\delta^\circ_{K^+\pi^-}$	$\delta^\circ_{K^+\pi^-\pi^0}$
(a)	$3.01^{+3.12}_{-3.39}$	$10.10^{+1.69}_{-1.72}$	$41.3^{+22.0}_{-24.0}$	43.8 ± 26.4
Stat.	(2.76)	(1.36)	(18.8)	(22.4)
(b)	$xxx^{+0.72}_{-0.75}$	$xxx \pm 0.19$	$xxx^{+3.7}_{-3.4}$	$xxx^{+4.6}_{-4.5}$
Stat.	(0.18)	(0.11)	(1.3)	(2.9)
(c)	$xxx \pm 0.42$	$xxx \pm 0.17$	$xxx \pm 2.2$	$xxx^{+3.3}_{-3.4}$
Stat.	(0.18)	(0.11)	(1.3)	(2.7)
(d)	$xxx \pm 0.20$	$xxx \pm 0.12$	$xxx \pm 1.0$	$xxx \pm 1.1$
Stat.	(0.17)	(0.10)	(0.9)	(1.1)

Table 1: D^0-$\overline{D}^0$ mixing parameters (x, y) and strong phases obtained from χ^2 fits to observables obtained either from *BABAR* or from their projections to Super*B*. Fit a) is for $482\,\mathrm{fb}^{-1}$ from *BABAR* alone and this is scaled up in b) to $75\,\mathrm{ab}^{-1}$ at $\Upsilon(4S)$ for Super*B*. Fit c) includes strong phase information projected to come from a BES III run at $D\bar{D}$ threshold, and d) is what would be possible from a $500\,\mathrm{fb}^{-1}$ $D\bar{D}$ threshold run at Super*B*. The uncertainties due to statistical limitation alone are shown below each fit result.

CLEO-c [11, 12] measured s_i and c_i for $D \to K^0_S\pi^+\pi^-$ and $D \to K^0_S K^+K^-$ with $818\,\mathrm{pb}^{-1}$ of data on $\psi(3770)$ resonance. They also estimated the impact on the measurement of the CKM angle γ. They found their s_i and c_i are consistent with that calculated from the Dalitz plot model used in *BABAR* analysis, and the reduction of Dalitz plot model dependence is substantial.

3 Projected precisions in Super*B* era

In Super*B*'s physics reach studies [1], the expected precisions in D^0-$\overline{D}^0$ mixing parameters in various scenarios on the Super*B* time scale are estimated. First the results from *BABAR*'s $482\,\mathrm{fb}^{-1}$ are extrapolated to Super*B*'s target of $75\,\mathrm{ab}^{-1}$ near $\Upsilon(4S)$, without any independent inputs of strong phase measurements. Then the improvement due to better precision in strong phase measurements using $D\overline{D}$ threshold data are estimated, first from the forthcoming BESIII runs and from Super*B* plan ($0.5\,\mathrm{ab}^{-1}$ integrated luminosity).

The results, including current average values from *BABAR*, are summarize in Table 1, and the corresponding confidence regions are shown in Fig. 1.

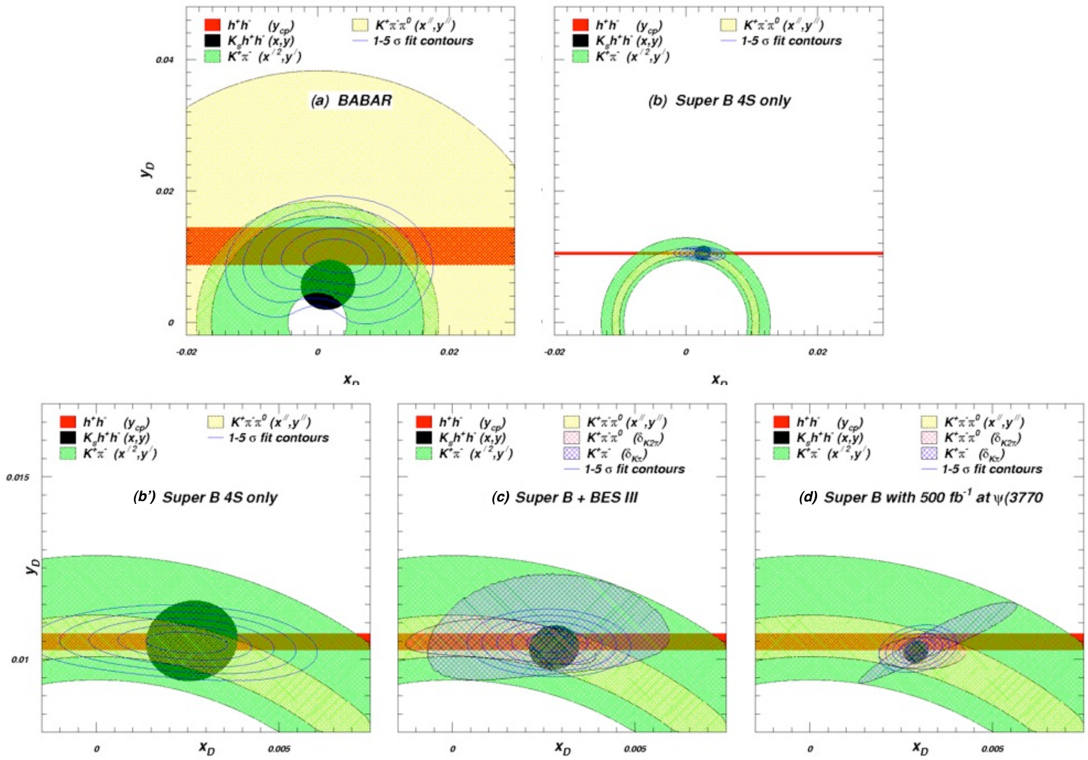

Figure 1: The confidence regions of D^0-$\overline{D}^0$ mixing parameters (x, y) in various scenarios described in the text and in Table 1. Shaded areas indicate the coverage of measured observables lying within their 68.3% confidence region. Contours enclosing 68.3% (1σ), 95.45% (2σ), 99.73% (3σ), 99.994% (4σ) and $1 - 5.7 \times 10^{-7}$ two-dimensional confidence regions from the χ^2 fit to these results are drawn as solid lines.

4 Time-dependent CP asymmetry

Using coherent $\psi(3770) \to D^0\overline{D}^0$ decays, one can perform time-dependent CP asymmetry studies analogous to $\Upsilon(4S) \to B^0\overline{B}^0$ in B-factories. If a neutral D meson decays to a final state at t_1 that can identify the sign of its c-quark, e.g., lepton charge in semileptonic decays, the other D meson must be in an orthogonal state, i.e., the opposite flavor to the first D. The time-dependent decay rate of the second D meson into a CP eigenstate can be derived from Eq. 5:

$$A(\Delta t) = \frac{\overline{\Gamma}(\Delta t) - \Gamma(\Delta t)}{\overline{\Gamma}(\Delta t) + \Gamma(\Delta t)} = 2e^{y\Gamma\Delta t}\frac{(|\lambda_f|^2 - 1)\cos(x\Gamma\Delta t) + 2\mathcal{I}m\lambda_f \sin(x\Gamma\Delta t)}{(1 + |\lambda_f|^2)(1 + e^{2y\Gamma\Delta t}) + 2(1 - e^{2y\Gamma\Delta t})\mathcal{R}e\lambda_f}, \quad (21)$$

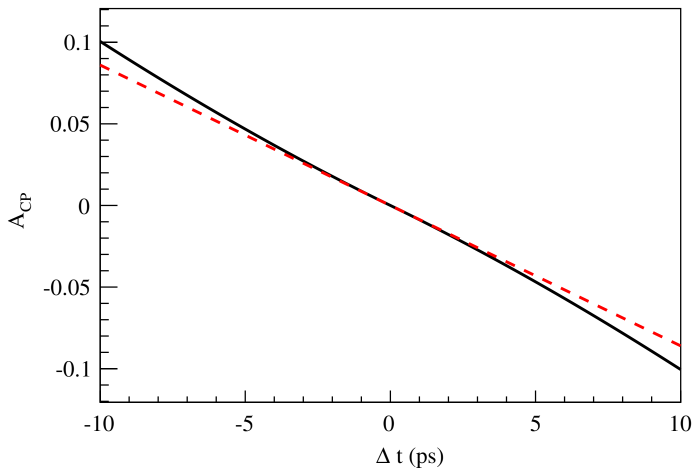

Figure 2: Distribution of time-dependent CP asymmetry for D^0 mesons with $\mathcal{R}e\lambda_f = \mathcal{I}m\lambda_f = 1/\sqrt{2}$ and $CP = -1$. These distributions assume $q/p = 1$ and the solid (dashed) line corresponds to $y = 0.01(0.00)$.

where $\Delta t = t_2 - t_1$, and $\lambda_f = (q\bar{A}_f)/(pA_f)$.

Measuring time-dependent CP asymmetry in D^0-$\overline{D}^0$ system is much more difficult than in B^0-$\overline{B}^0$ system. The reason is that charm mixing rate is very small; both x and y are $\mathcal{O}(1\%)$ for D^0-$\overline{D}^0$, whereas $x \sim \mathcal{O}(1)$ for B^0-$\overline{B}^0$. This effect is illustrated in Fig. 2 [13], in which one can see that even with a large CP-violating phase ($\arg(\lambda_f) = \pi/4$) the CP asymmetry is only a few percent within $|\Delta t| < 10$ ps (more than 20 times the D^0 lifetime). In contrast, within the same Δt range, the CP asymmetry for B^0 meson exhibits 1.5 full sinusoidal oscillations already.

At Super$B$, the design beam spot is much smaller ($\sigma_x \sim 8\ \mu$m, $\sigma_y \sim 40$ nm, $\sigma_z \sim 200\ \mu$m) than that in BABAR. This makes fitting for the primary vertex possible (and meaningful) even if no charged tracks originating from the primary vertex. As illustrated in Fig. 3, the charm mesons from $\psi(3770)$ decay fly away from the primary vertex for $\mathcal{O}(100\ \mu\text{m})$, depending on the center-of-mass frame boost. One can perform a beam-spot constraint fit on the $\psi(3770) \to D\overline{D}$ system to simultaneously fit for both flight lengths (L_1 and L_2) and convert them to decay times. At near $\Upsilon(4S)$, one studies charm physics using continuum data; the soft pion from D^* decays are used to identify the initial flavor of the charm meson.

Super$B$ has conducted studies to evaluate the sensitivities to mixing parameters (x, y) and CP-violating parameters q/p using $0.5\,\text{ab}^{-1}$ of $D\overline{D}$ threshold data alone and compared that with using $75\,\text{ab}^{-1}$ of data near $\Upsilon(4S)$. The preliminary results that used several two-body charm decays with various combination of CP/flavor-tags shows that the uncertainties with $\psi(3770)$ data are about six times larger than those with $\Upsilon(4S)$ data. On this topic along, $0.5\,\text{ab}^{-1}$ of $D\overline{D}$ threshold data is clearly not as

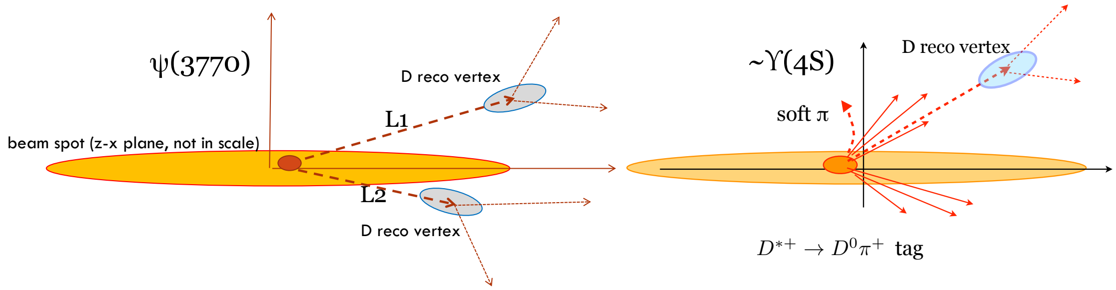

Figure 3: Illustrations of charm meson reconstructions with beam spot at Super$B$ for (left) $\psi(3770) \to D\overline{D}$ events and for (right) continuum events near $\Upsilon(4S)$.

competitive as $\Upsilon(4S)$ data. However, one should be reminded that the former only requires a few months of data taking, while the latter will take five years according to the plan.

5 Summary

The precision of charm mixing measurements will be limited by the uncertainties of strong phases and Dalitz plot model by the time Super$B$ collected its targeted data near $\Upsilon(4S)$. One can mitigate this situation by utilizing the quantum correlation of charm decays in $\psi(3770) \to D\overline{D}$ with BESIII data. With a months-long run at $D\overline{D}$ threshold at Super$B$, it is possible to improve the precision by another factor of two. With a boost of the center-of-mass frame, time-dependent CP asymmetry measurements can also be performed in $\psi(3770) \to D\overline{D}$ data, but the precision is not as competitive as the much larger $\Upsilon(4S)$ data.

Finally, not discussed in this paper but worth noting here, charm threshold data have advantages to $\Upsilon(4S)$ data in several areas due to the low background and the fact that the whole event can be fully reconstructed (double tag), in addition to the quantum correlation. These areas include rare decays ($D^0 \to \gamma\gamma$, $\mu\mu(X)$, etc.), leptonic/semileptonic charm decays, form factor measurements, CPT violation, CP violation in $D \to V\gamma$ that probes chromomagnetic dipole operator [14], and others. It certainly adds to the breadth of Super$B$ physics programs.

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