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Erratum: Group symmetry and covariance regularization

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The paper contains a notational error in the statement of Schur’s lemma and the paragraph immediately succeeding it. The corrected version is below. We would like to thank Ilya Soloveychik for pointing out the error to us.

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Lemma 2.1 (Schur’s lemma [37]). For a finite group $G$ there are only finitely many inequivalent irreducible representations (indexed by $I$) $\vartheta_1, \ldots, \vartheta_{|I|}$ of dimensions $m_1, \ldots, m_{|I|}$. Every linear representation of $G$ has a canonical decomposition

$$\rho = s_1 \vartheta_1 \oplus \cdots \oplus s_{|I|} \vartheta_{|I|},$$

and $s_i$ is the multiplicity of the $i^{th}$ irreducible representation. Correspondingly, there is an isotypic decomposition of $\mathbb{C}^N$ into invariant subspaces $W_i$:

$$\mathbb{C}^N = W_1 \oplus \cdots \oplus W_{|I|}$$

where each $W_i$ is again a direct sum of isomorphic copies $W_i = W_{i1} \oplus \cdots \oplus W_{is_i}$. ¹

A basis of this decomposition (that depends only on the group) transforming with respect to the matrices $\vartheta(g)$ is called symmetry adapted, and can be explicitly computed algorithmically [20, 37]. This basis defines a change of coordinates by a unitary matrix $T \in \mathbb{C}^{N \times N}$. One of the main consequences of Schur’s lemma is that the symmetry adapted basis block diagonalizes the fixed point subspace, i.e. every matrix in the fixed point subspace is commonly diagonalized by $T$. If $M \in \mathcal{W}_G$ is a matrix in the fixed point subspace, then changing coordinates with respect to $T$ decomposes $M$ into a block diagonal form as follows:

$$T^* M T = \begin{bmatrix} M_1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & M_{|I|} \end{bmatrix}, \quad M_i = \begin{bmatrix} B_i & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & B_i \end{bmatrix}. \quad (3)$$

¹Schur’s lemma, as stated classically, provides a decomposition over the complex field $\mathbb{C}^N$. However, a real version can be adapted in a straightforward manner [37, pp. 106–109], [23] and the irreducible real representations are called absolutely irreducible.

3057
In the above decomposition the diagonal blocks $M_i \in \mathbb{R}^{m_i \times m_i}$ can be further decomposed into $m_i$ repeated diagonal copies $B_i \in \mathbb{R}^{s_i \times s_i}$ (recall that $m_i$ are the dimensions of the irreducible representations and $s_i$ are the multiplicities). Thus, the symmetry restriction $M \in \mathcal{W}_G$ reduces the degrees of freedom in the problem of interest. This observation plays a central role in our paper.

\footnote{Note that in general the diagonal blocks in (3) are complex. In this paper, for the sake of simplicity, we will assume that the $M_i$ are real; indeed this is the case in all the examples that we consider. The complex case can be handled in a straightforward manner by working with concentration bounds for complex Gaussian covariance matrices. Doing so leaves the overall behaviour of the sample complexity unchanged (up to constant factors).}