Supplementary Online Material

Materials and Methods

Experiments

Uniaxial compression tests were performed in a G200 Nanoindenter (Agilent Technologies) using the dynamic contact module (DCM) fitted with a 7 micron diameter diamond flat punch. Each compression test was conducted under nominal constant displacement rate ranging from 0.1 nm/s to 1000 nm/s, controlled through a feedback loop method as the nanoindenter is inherently a load-controlled instrument. Compression tests were performed on single-crystalline, cylindrical nano-pillars with diameters ranging from 75 nm to 1000 nm and aspect ratios (height/diameter) between 3:1 and 6:1. Nano-pillars of five different materials were used: Au, Cu, Mo, Nb, and Ta. With the exception of Cu, all nano-pillars were prepared by a subtractive technique using a focused ion beam (FIB) on well-annealed electropolished (100) crystals, which involves milling out the matrix material and leaving the sample in the center [1]. Cu nano-pillars were prepared by electroplating Cu into cylindrical holes patterned by electron beam lithography into PMMA template, as described in more detail in [2]. Examples of a Nb pillar before and after compression along with examples of resulting stress-strain curves are shown in Figure 1 A-C in the main paper, with the schematic depiction of the experiment presented in Figure 1D. Details of these experimental tests and results for Au, Cu, Mo, Nb, and Ta can be found in [2, 3, 1, 4, 5]. A discussion of the displacement rates and the resolution of the measurements is given in [6]. An in-situ uniaxial compression movie combined with instantaneous stress-strain plot is included in the supplementary material.

Data Analysis

In order to identify the slips, the displacement time series \(d(t)\) (that is the height of the nano-pillar as a function of time) was numerically differentiated to obtain \(V(t) = d'(d(t))/dt\). Linear interpolation between discrete measurement times rendered \(V(t)\) for arbitrary times \(t\). Slip-sizes were extracted from the fluctuations of \(V(t)\) around the mean displacement rate \(V_{\text{thr}}\). A slip beginning at time \(t_1\) and ending at time \(t_2\) is defined by

\[
V(t_1) = V(t_2) = V_{\text{thr}} \quad \text{and} \quad V(t) > V_{\text{thr}} \quad \text{for all} \ t \ \text{with} \ t_1 < t < t_2.
\]

The size of the slip is \(s = d(t_2) - d(t_1)\). This method is consistent with related analyses in [6, 7]. Alternative definitions of the avalanche sizes used in the literature, and the associated avalanche size distribution exponents are discussed below.

Review of Measures of Avalanches

In the experiments described in this paper, avalanches are characterized by a starting time \(t_{\text{start}}\), an ending time \(t_{\text{end}} > t_{\text{start}}\), and a slip velocity \(V(t)\), which is a function of \(t\) where \(t_{\text{start}} \leq t \leq t_{\text{end}}\). As explained in the main paper \(V(t)\) is obtained by differentiating the displacement time series \(d(t)\) of the nano-crystal during compression \(V(t) = d'(d(t))/dt\). In general \(V(t)\) is a measure of the collective speed of the dislocations during a slip event. In theory

\[
V(t_{\text{start}}) = V(t_{\text{end}}) = 0 \quad \text{and} \quad V(t) > 0 \quad \text{for} \quad t_{\text{start}} < t < t_{\text{end}}.
\]
In practice our definitions of avalanche beginnings and endings are guided by threshold velocities,

\[ V(t_{\text{start}}) = V(t_{\text{end}}) = V_{\text{thr}} \text{ and } V(t) > V_{\text{thr}} \text{ for } t_{\text{start}} < t < t_{\text{end}} \]  

(2)

with \( V_{\text{thr}} \) the mean displacement rate in each experiment. There are several ways to characterize the size of such an avalanche. In our work, we discuss avalanche sizes in terms of the total slip displacement \( S \),

\[ S \equiv \int_{t_{\text{start}}}^{t_{\text{end}}} dt V(t) = d(t_{\text{end}}) - d(t_{\text{start}}). \]  

(3)

In a related theoretical study (see reference [25] in the main paper) we have also used the energy that is released during an avalanche:

\[ E \equiv \int_{t_{\text{start}}}^{t_{\text{end}}} dt V^2(t). \]  

(4)

For each of these quantities, we can use simulations and renormalization group theory to determine the scaling behavior of the probability density functions of the avalanche sizes. The avalanche sizes are distributed according to

\[ D(S, \tau) \sim S^{-\kappa} f_S \left( S(1 - \tau/\tau_c)^{\frac{1}{\nu}} \right), \]  

(5)

where \( D(S, \tau) \) is the probability density function of \( S \), \( \tau \) is the stress, \( \tau_c \) is the critical stress, \( \kappa \) and \( \sigma \) are universal critical exponents, and \( f_S(x) \) is a universal scaling function that drops off exponentially for large values of \( x \). Given this relation, we can derive the distribution of the energies as shown in (Karin A. Dahmen, “Hysteresis, Avalanches, and Disorder Induced Critical Scaling: A Renormalization Group Approach”, PhD Thesis, Cornell University 1995) to be

\[ D_E(E, \tau) \sim E^{-1 - \frac{\kappa - 1}{1 - \frac{\sigma}{\nu}}} f_E \left( E(1 - \tau/\tau_c)^{2 - \frac{\sigma}{\nu}} \right), \]  

(6)

where \( D_E \) is the density function of the energies, the exponent \( \nu \) defines the correlation length \( \xi \sim (1 - \tau/\tau_c)^{-\nu} \), the dynamic exponent \( z \) is defined through the scaling of avalanche duration \( T \sim \xi^z \), and \( f_E \) is a universal scaling function. Another definition of avalanche size used in some reports (e.g. references [3-7] in the main paper) is the peak amplitude \( A \), defined as

\[ A \equiv \max_{t_{\text{start}} \leq t \leq t_{\text{end}}} \{ V(t_i) \}, \]  

(7)

where the \( t_i \) are the times at which the slip velocity \( V(t) \) is measured, or the peak amplitude squared

\[ E' \equiv \left( \max_{t_{\text{start}} \leq t \leq t_{\text{end}}} \{ V(t_i) \} \right)^2, \]  

(8)

which essentially measures the peak acoustic emission energy in an avalanche. There are some advantages to using \( S \) and \( E \) over \( A \) and \( E' \), in terms of theory, analysis and experiment.

Distributions of \( S \) and \( E \) have easily derived scaling forms. \( A \) is more complicated, it involves taking the maximum on the set of measured velocities \( V(t_i) \) within a given avalanche. Viewed as random variables, the \( V(t_i) \) in an avalanche are not independent. This can be seen by looking at the power spectrum (the square modulus of the Fourier transform of \( V(t_i) \)), which also exhibits a power law (see reference [25] in the main paper). Since the power spectrum is not white noise, the autocovariance function is not a delta function, which implies that the \( V(t_i) \) are not independent. This, combined with the complexity of conditioning on the definition of an avalanche (\( V \) is zero at the beginning and end of an avalanche, but not in the middle), ensures that the scaling of \( A \) is difficult to derive.

A statistical analysis using extremal values like \( A \) and \( E' \) as opposed to averaged quantities like \( S \) and \( E \) is disadvantageous. Extremal values have more fluctuations than averaged values, meaning more data will be necessary to see clear convergence of the distributions. Also, \( S \) and \( E \) provide different information about an avalanche, which can be seen by the fact that \( D \) and \( D_E \) involve different exponents. This is not
the case for $A$ and $E'$, they are simply related algebraically and provide the exact same information about avalanches.

Finally from an experimental perspective, $S$ and $E$ are associated with more easily observable physical phenomena: $S$ is the total displacement and $E$ is the total released energy of an avalanche. Both can be observed without the need to record the entire time series $V(t_i)$ of each avalanche. $A$ on the other hand requires the observation of the entire time trace $V(t_i)$, for example through acoustic emission experiments (see reference [3] from the main paper). Peak values have less significant interpretations as they do not represent the entire avalanche but just its momentary behavior.

Based on the above arguments we chose $S$ as the best suited definition for the size of an avalanche in the main paper.

Details of Histograms in Main Paper

Here we provide details on the figures provided in the main paper. Recall that Figure 1c in the main paper shows stress-strain curves for various materials, while Figures 2-5 in the main paper show cumulative histograms. Each table below corresponds to one figure, and provides the following details for each plot:

1. the number of pillars used to produce each plot
2. the number of data points in the plot (which is equal to the number of measured avalanches used to create the cumulative distributions)
3. whether the crystal structure of the respective material is face-centered cubic (fcc) or body-centered cubic (bcc)

<table>
<thead>
<tr>
<th>Figure 1c: stress-strain curves</th>
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<tbody>
<tr>
<td>Plot Name</td>
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<tr>
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<tr>
<td>Nb 868 nm, 2 nm/s</td>
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<tr>
<td>Mo 800 nm, 10 nm/s</td>
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<td>Au 250 nm, 0.1 nm/s</td>
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<td>Ta 400 nm, 2 nm/s</td>
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<th>Figure 2: various materials/sizes/rates</th>
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<tr>
<td>Plot Name</td>
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<tr>
<td>Au 900 nm, 0.1 nm/s</td>
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<tr>
<td>Mo 800 nm, 0.1 nm/s</td>
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<td>Cu 500 nm, 0.2 nm/s</td>
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<td>Nb 900 nm, 2 nm/s</td>
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<th>Figure 3: Au &amp; Mo, 800 nm, various rates</th>
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<td>Plot Name</td>
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<tr>
<td>Au 0.1 nm/s, 0.1 nm/s</td>
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<tr>
<td>Au 1 nm/s, 0.1 nm/s</td>
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<tr>
<td>Au 10 nm/s, 0.2 nm/s</td>
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<td>Mo 0.1 nm/s, 2 nm/s</td>
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<td>Mo 1 nm/s, 2 nm/s</td>
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<td>Mo 10 nm/s, 2 nm/s</td>
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<th>Figure 4: Cu, 2 nm/s, various sizes</th>
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<td>Plot Name</td>
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<td>250 nm</td>
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<td>500 nm</td>
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Use of Cumulative Histograms

For a collection of displacement-time series taken at the same experimental parameters (material, size, and nominal displacement rate) we find a collection of $N$ slips of size $S_i, i = 1 \ldots N$. We characterize the statistical properties of this collection of slips by computing a histogram. To make an $M$ bin histogram, we choose a set of bin edges $\{e_j\}_{j=1}^M$. We then plot $\{x_j, y_j\}_{j=1}^M$ where

$$x_j = m(e_j, e_{j+1}),$$

$$y_j = \frac{n(e_j, e_{j+1}, \{S_i\})}{N(e_{j+1} - e_j)}.$$

Here, $m$ is a function that is generally chosen to be the mean of its arguments, either arithmetic or geometric. The function $n$ is just the number of slips that fall between $e_j$ and $e_{j+1}$. Note that this normalization guarantees that integrating the histogram numerically across its domain yields 1, imitating the probability density function $D(S)$. Hence when we plot the normalized histograms, we label the $x$-axis $S$ and the $y$-axis $D(S)$.

In general, choosing smaller bins allows one to see the distribution with greater resolution. However this reduces the number of counts in each bin and increases noise. Hence a balance is required. The most common choice of edges corresponds to bins of uniform width. However, this is inconvenient for power law distributions. Uniform bins are too large for the small events and too small for the large events because of the rapid decay of the probability density function $D(S)$, leading to a highly uneven distribution of statistical noise.

Instead, for histograms of power-law distributed events, one generally chooses logarithmic binning. This is achieved by choosing $e_j = a r^{j-1}$, for some constants $a$ and $r$. Generally, the function $m$ is taken to be the geometric mean. We can observe in Figure 5a below that the logarithmic binning is an improvement over the linear binning. However, the tail of the distribution $D(S)$ at large $S$ is still relatively noisy. This is because $(e_{j+1} - e_j) \sim x_j$, but $D(S) \sim S^{-\alpha}$ where in our case $\alpha = \kappa + \sigma = 2$ is the exponent of the integrated avalanche size distribution in equation (2) in the main paper. Thus, in our case the number of counts in the $j$th bin $n_j \equiv n(e_j, e_{j+1}, \{S_i\})$ will scale as $n_j \sim x_j^{-1} \sim x_j^{-\alpha}$. Hence we still have a dearth of events at the largest bins, and we still have difficulty balancing resolution at the small events with noise at the large ones.

An alternative approach is to avoid the use of bins entirely. Rather than calculate a histogram which attempts to mimic the probability density function (pdf), we calculate a cumulative histogram which mimics the cumulative distribution function (cdf). The cdf is defined by

$$C(S) = \int_{-\infty}^{S} D(S) dS.$$

Hence, it is the fraction of events below a certain value. In our case, this is particularly convenient: if our density $D(S) \sim S^{-\alpha}$ is a power law, then $C(S) \sim S^{-\alpha+1}$ is a power law as well. Note that $C(-\infty) = 0$ and $C(+\infty) = 1$. In practice, we actually use the complement of this function $1 - C(S)$, the fraction of events larger than $S$.

To create a cumulative histogram for $\{S_i\}_{i=1}^N$, we begin by sorting the $\{S_j\}$ so that $S_1$ is the size of the smallest slip and $S_N$ is the size of the largest slip. We then plot $\{x_j, y_j\}_{j=1}^{N-1}$, where

$$x_j = m(s_j, s_{j+1}),$$

$$y_j = \frac{N - j}{N}.$$
Figure S1: a) The same data plotted in two histograms with different binning. One plot shows linear binning, the other shows logarithmic binning. Notice how the linear binning has poor resolution on small events, and is noisy for large events. The logarithmic binning is much improved, as it is less noisy for the large events. Both binning methods use 30 bins; the number of avalanches collected for the histogram is over 3000. b) The same data as in a), now plotted on a cumulative histogram. Excellent resolution and minimal noise clearly show the scaling behavior of the cumulative distribution $C(S)$ from small to large events. Also clearly seen is the cut-off at large events.

Again, $m$ can be either the arithmetic or geometric mean of its arguments. In practice, the $\{S_i\}$ are so closely spaced that this choice has barely any effect on the plot. The $y$-value ranges from just under 1 to just over 0, as expected. Notice that no binning is necessary here; we avoid the trade-off between noise and resolution. Every single individual event directly impacts the plot, so resolution is maximal. On the other hand, because the distribution is integrated over $x$ the statistical errorbars are reduced compared to the probability density distribution. The reason is that the integration in the cumulative distribution avoids distributing the data among bins with potentially low counts and high statistical error bars. It also avoids the need to justify the choice of number of bins, a parameter that could conceivably impact the perceived power law exponent. Figure S1b shows the cumulative approach applied to the same data as in Figure S1a.

**Integrated vs Non-Integrated Exponents**

In the section of our paper that summarizes the results of our mean field theory model, we distinguish between distributions that are integrated in stress versus those that are binned in stress. The stress-binned distribution of slip sizes is a function of stress, while the stress-integrated distribution is obtained by integrating the stress-binned distribution over all stresses. The scaling form of the stress-binned distribution is given by

$$D(S, \tau) \sim S^{-\kappa} f_S \left( S \cdot (\tau_c - \tau)^{1/\sigma} \right),$$

where $D$ gives the relative likelihood of observing a slip of size $S$ at stress $\tau$, $\tau_c$ is the critical (flow) stress and $f_S$ is the universal scaling function. $\kappa$ and $1/\sigma$ are universal scaling exponents, with values in mean field theory of 1.5 and 2.0 respectively. However, when we deform a crystal from stress 0 to some stress $\tau_{max} \leq \tau_c$ and consider the distribution of all the slips that occur along the way, we need to integrate $D(S, \tau)$ over stress to find the integrated form:

$$D_{int}(S, \tau_{max}) \sim \int_0^{\tau_{max}} d\tau S^{-\kappa} f_S \left( S \cdot (\tau_c - \tau)^{1/\sigma} \right).$$
Substituting \( u = S(τ_c − τ)^{1/σ} \), we have

\[
D_{\text{int}}(S, τ_{\text{max}}) \sim \int_{Sτ_c^{1/σ}}^{S(τ_c − τ_{\text{max}})^{1/σ}} \frac{−σ}{S(u/S)^{σ(1/σ−1)}} du S^{−κ}f_S(u) \\
\sim \int_{Sτ_c^{1/σ}}^{S(τ_c − τ_{\text{max}})^{1/σ}} \frac{−σ \cdot S^{−σ}}{u^{1−σ}} du S^{−κ}f_S(u) \\
\sim S^{−(σ+κ)} \int_{Sτ_c^{1/σ}}^{S(τ_c − τ_{\text{max}})^{1/σ}} \frac{f_S(u)}{u^{1−σ}} du .
\]

Notice that at the upper bound, the numerator of the integrand is \( f_S(Sτ_c^{1/σ}) \). This is just the scaling function modifying the power law if we evaluate \( D(S, τ)|_{τ=0} \). At \( τ = 0 \) the system is far from criticality and the cutoff size \( S_{\text{max}} \sim 1/(τ_c − τ)^{1/σ} \) of the power law region of \( D_{\text{int}}(S, τ) \) is small. Equivalently for almost all \( S \) the scaling function \( f_S(S(τ_c − τ)^{1/σ}) = f_S(u) \) is also small at \( τ = 0 \). Since \( f_S(x) \) decays exponentially for large \( x \) we can replace the upper bound of integration with infinity and obtain

\[
D_{\text{int}}(S, τ_{\text{max}}) \sim S^{−(σ+κ)} g \left( S \cdot (τ_c − τ_{\text{max}})^{1/σ} \right) .
\]

(9)

Note that the universal scaling function \( g(x) \) has the same argument as \( f_S(x), x = S(τ_c − τ_{\text{max}})^{1/σ} \), while the distribution exponent has changed from \( κ = 1.5 \) to \( κ + σ = 1.5 + 0.5 = 2.0 \). Evaluating this at the critical stress, \( τ_{\text{max}} = τ_c \), we obtain the distribution of avalanche sizes integrated from zero stress to the critical stress:

\[
D_{\text{int}}(S, τ_{\text{max}} = τ_c) \sim S^{−(σ+κ)} g(0) \sim S^{−(σ+κ)} ,
\]

(10)

recovering equation (2) in the paper.

In the main paper we use experimental data to verify this prediction of the mean field theory, and test for the exponents \( κ = 1.5 \) and \( κ + σ = 2.0 \) for the stress-binned and stress-integrated avalanche size distributions, respectively. Testing the result for the stress-binned distribution \( D(S) \) is challenging: the theory describes the distribution of slips that occur at an exact value of stress \( τ \). Since with finite data essentially no events occur at one exact value of stress, it is necessary to bin in stress. If the bins are too large, we are back in the integrated regime. As the bins get small however, obtaining sufficient statistics to generate a meaningful histogram with small statistical error bars becomes more and more difficult. It is therefore necessary to collect large amounts of data to precisely test the predictions for the stress-binned distributions. The result of such an analysis is shown in Figure 5 of the main paper.

**Finite-Size Effects**

In the model described in [8], the parameter describing the distance of the system from criticality is the stress. Above the critical stress \( τ_c \), the system deforms continuously. For any stress below \( τ_c \), the system will eventually reach a pinned state where it stops deforming. As stress is increased slowly from 0 towards \( τ_c \), the system corresponds by deforming suddenly, transitioning from one pinned state to another. The maximum size of these sudden deformations, or slips, is controlled by the closeness to criticality,

\[
S_{\text{max}} \sim \frac{1}{(τ_c − τ)^{1/σ}}.
\]

However, this is all based on a model where the system size is infinite. In a real, finite system, there is always a hard limit to slip sizes based on some parameter of system size:

\[
S_{\text{max}} \sim L^{d_f},
\]

where \( L \) is the linear dimension of the system and \( d_f \) is an appropriate fractal dimension.

If we want to see how well the model captures the behavior of the system, we need to take finite-size effects into account. In particular, we need to find a suitable range for the stress where finite size effects do
greater than $s$. Consider a cumulative histogram that has $T$ samples; suppose we are trying to compute the value of the complementary cumulative distribution $C(s)$ at some value of $s$ for which $k$ of the $N$ samples have a value greater than $s$. Let us refer to $C(s)$ at the $s$ value of interest as $p_s$ then $p$ is the probability of a sample being greater than $s$. Since $k$ of the $N$ samples were greater than $s$, our best estimate of $p$ is clearly $p = \frac{k}{N}$. We can go farther than this however. The probability of $p$ taking some value given our observation of $k$ (and holding $N$ fixed) is given by

$$P(p | k) = P(k | p) \propto P(p) / P(k),$$

(11)

where $P$ refers to the probability of the contained variable taking on a value, and $P(k)$ indicates a conditional probability. $P(k)$ can be ignored as we are only dealing with one value of $k$, and thus it is a constant. For $P(p)$, the priors, we assume a uniform distribution on $[0, 1]$. So we have

$$P(p | k) \propto P(k | p) = \binom{N}{k} p^k (1 - p)^{N-k}.$$  

(12)

Normalizing and integrating, we find the cumulative distribution function to be

$$P(p < x | k) = I_x (k + 1, N - k + 1)$$

(13)

where $I_x (a, b)$ is the regularized incomplete beta function. To find error bars, we set $p_{\text{low}}$ and $p_{\text{high}}$, the upper and lower error bounds on $p$, such that

$$I_{p_{\text{low}}} (k + 1, N - k + 1) = 0.025$$

$$I_{p_{\text{high}}} (k + 1, N - k + 1) = 0.975,$$

(14)

(15)

Fitting and Error Analysis

To put error bars on our cumulative histograms, we used a Bayesian technique with a 95% confidence interval [9]. Consider a cumulative histogram that has $N$ samples; suppose we are trying to compute the value of the complementary cumulative distribution $C(s)$ at some value of $s$ for which $k$ of the $N$ samples have a value greater than $s$. Let us refer to $C(s)$ at the $s$ value of interest as $p_s$ then $p$ is the probability of a sample being greater than $s$. Since $k$ of the $N$ samples were greater than $s$, our best estimate of $p$ is clearly $p = \frac{k}{N}$. We can go farther than this however. The probability of $p$ taking some value given our observation of $k$ (and holding $N$ fixed) is given by

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$$I_{p_{\text{low}}} (k + 1, N - k + 1) = 0.025$$

$$I_{p_{\text{high}}} (k + 1, N - k + 1) = 0.975,$$
Figure S2: A plot showing how the scale of slips increases as one approaches criticality. $\tau_c - \tau$ is plotted in normalized units where $\tau_c$ is equal to 1. For farther distances from criticality towards the right of the graph, there is a consistent, roughly power law increase of event scale with approach to criticality. However, extremely close to criticality this breaks down as the finite system size limits what would otherwise be larger events. Each plotted point comes from averaging over events occurring in consecutive ranges of 0.1 stress in normalized units. So the point closest to criticality is from averaging over all slips that occurred at stresses between $\tau_c$ and 0.9$\tau_c$, the next point from events between 0.9$\tau_c$ and 0.8$\tau_c$, and so on.
That is, the lower and upper 2.5% of probability, leaving us with a 95% interval in between.

We used a similar approach to estimate the power law exponent $\alpha$ of real data, one that is slightly modified from the approach used by Newman et al [10]. If we assume that data \( \{x_i\} \) is distributed as a power law between $x_{\text{min}}$ and $x_{\text{max}}$, and once again assume uniform priors, we have

$$P(\alpha|\{x_i\}) \propto \prod_{i=1}^{N} \frac{\alpha - 1}{x_{\text{min}}} \frac{1}{1 - S^{1-\alpha}} \left( \frac{x}{x_{\text{min}}} \right)^{-\alpha},$$

where $S \equiv \frac{x_{\text{max}}}{x_{\text{min}}}$. We now have a probability density on $\alpha$; we simply pick as our estimate the value of $\alpha$ that has maximum probability. The statistical error in this case from fitting $\alpha$ is relatively small, however $\alpha$ is quite sensitive to the choice of $x_{\text{min}}$. We thus determine the error in $\alpha$ by considering a reasonable range of values for $x_{\text{min}}$ and then use the amount by which $\alpha$ varied in that range as the error.

References


